1. Introduction

An air conditioning company manufactures a type of air conditioner, this air conditioner is manufactured at plants belonging to the company and needs to be shipped to customers. In order to ship the goods to the customers the company needs to decide which warehouse to operate in order to store the goods while waiting for the delivery.
The company can produce air conditioners at six plants across the country and stock these units in any of four different warehouses before sending them to five different customers. Below a picture to summarize the cost of manufacturing and shipping of units between each plant and warehouses.

	Warehouse 1	Warehouse 2	Warehouse 3	Warehouse 4	Fixed Cost	Capacity
Plant 1	$700	$1,000	$900	$1,200	$55,000	300
Plant 2	$800	$ 500	$600	$ 700	$40,000	200
Plant 3	$850	$ 600	$700	$ 500	$45,000	300
Plant 4	$600	$ 800	$500	$ 600	$50,000	250
Plant 5	$500	$ 600	$450	$ 700	$42,000	350
Plant 6	$700	$ 600	$750	$ 500	$40,000	400

In a similar way each warehouse has a fixed cost in order to be considered “active” and a cost associated with the delivery of goods from each warehouse to each customer. Below is a picture to summarize the situation.

	Customer 1	Customer 2	Customer 3	Customer 4	Customer 5	Fixed Cost
Warehouse 1	$40	$80	$60	$90	$50	$40,000
Warehouse 2	$60	$50	$75	$40	$35	$50,000
Warehouse 3	$55	$40	$65	$60	$80	$35,000
Warehouse 4	$80	$30	$80	$50	$60	$60,000

Each customer has a monthly demand which needs to be satisfied as per picture below.

	Customer 1	Customer 2	Customer 3	Customer 4	Customer 5
Demand	200	300	200	150	250

2. Mathematical representation

Sets

- Plants be the set of plant indices
- Warehouses the set of warehouse indices.
- Customer the set of customer indices

Parameters

- c_{pw} cost of manufacturing and shipping a unit from plant p to warehouse w
- cap_p represents the production capacity of plant p
- f_p fixed cost of operating plant p
- c_{wc} cost of shipping from warehouse w to customer c
- f_w fixed cost of operating warehouse w
- $demand_c$ demand of customer c

Variables

- x_{pw} number of units to be shipped from plant p to warehouse w
- y_p binary variable which indicates whether plant p is operational or not
- y_w binary variable which indicates whether warehouse w is operational or not
- x_{wc} number of units to be shipped from warehouse w to customer c

Objective function

Minimize the total cost which includes the manufacturing and shipping costs, operational costs of plants and warehouses and shipping costs from warehouses to customers:

$$\text{Minimize } \sum_{p\in Plants, w\in Warehouses} c_{pw} * x_{pw} + \sum_{p\in Plants} f_p * y_p + \sum_{w\in Warehouses} f_w * y_w + \sum_{w\in Warehouse, c\in Customers} c_{wc} * x_{wc}$$

Constraints

- Capacity constraint for plants

$$\sum_{w\in Warehouses} x_{pw} <= cap_p * y_p \quad \text{for all } p\in Plants$$

- Production quantity to meet customer demand

$$\sum_{p\in Plants, w\in Warehouses} x_{pw} >= demand_c \quad \text{for all } c\in Customers$$

- Demand must be satisfied from goods shipped from Warehouses

$$\sum_{w\in Warehouses} x_{wc} >= demand_c \quad \text{for all } c\in Customers$$

- Goods shipped from only active warehouses

$$x_{wc} <= y_w * demand_c \quad \text{for all } w \in Warehouses, c \in Customers$$

- Flow conservation at warehouses

$$\sum_{p \in Plants} x_{pw} = \sum_{c \in Customers} demand_c * y_w \quad \text{for all } w \in Warehouses$$

3. Conclusions

The solution returned by CPLEX gives the following results:

- Optimal integer solution found with objective equal to 889500
- Warehouse operated: Warehouse3
- Plants operated: Plant2, Plant3, Plant4, Plant5
- total number of iterations: 64

Below a picture of the results returned by the solver.

```
CPLEX 12.10.0.0: optimal integer solution; objective 889500
64 MIP simplex iterations
0 branch-and-bound nodes
y_w [*] :=
Warehouse1  0
Warehouse2  0
Warehouse3  1
Warehouse4  0
;

y_p [*] :=
Plant1  0
Plant2  1
Plant3  1
Plant4  1
Plant5  1
Plant6  0
;

x_pw [*,*]
:      Warehouse1 Warehouse2 Warehouse3 Warehouse4    :=
Plant1      0          0           0          0
Plant2      0          0         200          0
Plant3      0          0         300          0
Plant4      0          0         250          0
Plant5      0          0         350          0
Plant6      0          0           0          0
;
```

```
x_wc :=
Warehouse1 Customer1     0
Warehouse1 Customer2     0
Warehouse1 Customer3     0
Warehouse1 Customer4     0
Warehouse1 Customer5     0
Warehouse2 Customer1     0
Warehouse2 Customer2     0
Warehouse2 Customer3     0
Warehouse2 Customer4     0
Warehouse2 Customer5     0
Warehouse3 Customer1   200
Warehouse3 Customer2   300
Warehouse3 Customer3   200
Warehouse3 Customer4   150
Warehouse3 Customer5   250
Warehouse4 Customer1     0
Warehouse4 Customer2     0
Warehouse4 Customer3     0
Warehouse4 Customer4     0
Warehouse4 Customer5     0
;
```