SIC is a Swiss intermodal carrier located in Lausanne. SIC supplies its customers by transporting empty containers (where the goods will be loaded by the customers) to the pick-up points of the customers. As a result, the SIC management needs to reallocate empty containers periodically.
Consider a specific day, where several empty containers have to be reallocated among the terminals in Amsterdam, Berlin, Munich, Paris, Milan, Barcelona and Madrid. The number of empty containers available (negative numbers) or demanded (positive numbers) at these terminals is reported in the attached file **SIC network.pdf**, along with the unit transportation costs (in Euros/container) associated with the links of the SIC logistics network. In the figure, the cost associated with a link (*i*,*j*) also applies to the reverse link (*j*,*i*).
Furthermore, each link of the SIC logistic network can be used to transport at most 20 containers with the exception of link (3,5), having an upper capacity equal to 60 containers (these values are not reported in the figure).

The SIC management has to decide how to move the empty containers along the logistic network depicted in **SIC network.pdf** so as to satisfy the requirements of the terminals and the upper capacities of the links, at a minimum total transportation cost.

1. Formulate the SIC optimization problem by means of a Linear Programming model.

2. Implement the model by using the modelling language AMPL, and solve it by means of CPLEX.

3. Assume that a fixed cost equal to 100 has to be payed for each link used for moving empty containers, and that the available budget for using links (and so, for reallocating the empty containers) is 500. Propose an Integer Linear Programming model which takes into account this constraint, in addition to the ones previously described, by suitably extending the model proposed at point 1.

4. Solve the ILP proposed at point 3, and compare its optimal solution to the one determined at point 2.