ACM-ICPC Cheat Sheet

School Name

Team Name

2017 年 11 月 5 日

目 录

1 头文件模板

```cpp
#include <bits/stdc++.h> // c++0x only
#include <iostream>
#include <cstdio>
#include <cstring>
#include <algorithm>
#include <string>
#include <vector>
#include <queue>
#include <stack>
#include <set>
#include <map>
#include <cmath>
#include <iomanip>
#include <functional>
#include <cstdlib>
#include <climits>
#include <cctype>
using namespace std;
#define REP(i,x) for(int i = 0; i < (x); i++)
#define DEP(i,x) for(int i = (x) - 1; i >= 0; i--)
#define FOR(i,x) for(__typeof(x.begin())i=x.begin(); i!=x.end();
↪ i++)
#define CLR(a,x) memset(a, x, sizeof(a))
#define MO(a,b) (((a)%(b)+(b))%(b))
#define ALL(x) (x).begin(), (x).end()
#define SZ(v) ((int)v.size())
#define UNIQUE(v) sort(ALL(v)); v.erase(unique(ALL(v)), v.end())
#define out(x) cout << #x << ": " << x << endl;
#define fastcin ios_base::sync_with_stdio(0);cin.tie(0);
typedef long long ll;
typedef unsigned long long ull;
typedef pair<int, int> PII;
typedef vector<int> VI;
#define INF 0x3f3f3f3f
#define MOD 1000000007
#define EPS 1e-8
#define MP(x,y) make_pair(x,y)
#define MT(x,y...) make_tuple(x,y) // c++0x only
#define PB(x) push_back(x)
#define IT iterator
#define X first
#define Y second
```

2 数学

2.1 素数

2.1.1 埃氏筛

```
// O(n log log n) 筛出 MAXN 内所有素数
// notprime[i] = 0/1 0 为素数 1 为非素数
const int MAXN = 1000100;
bool notprime[MAXN] = {1, 1};    // 0/1 为非素数
void GetPrime() {
    for (int i = 2; i < MAXN; i++)
        if (!notprime[i] && i <= MAXN / i)    // 筛到 √n 为止
            for (int j = i * i; j < MAXN; j += i)
                notprime[j] = 1;
}
```

2.1.2 欧拉筛

```
// O(n) 得到欧拉函数 phi[]、素数表 prime[]、素数个数 tot
// 传入的 n 为函数定义域上界
const int MAXN = 100010;
bool vis[MAXN];
int tot, phi[MAXN], prime[MAXN];
void CalPhi(int n) {
    set(vis, 0); phi[1] = 1; tot = 0;
    for (int i = 2; i < n; i++) {
        if (!vis[i]) {
            prime[tot++] = i;
            phi[i] = i - 1;
        }
        for (int j = 0; j < tot; j++) {
            if (i * prime[j] > n) break;
            vis[i * prime[j]] = 1;
            if (i % prime[j] == 0) {
                phi[i * prime[j]] = phi[i] * prime[j];
                break;
            }
            else phi[i * prime[j]] = phi[i] * (prime[j] - 1);
        }
    }
}
```

2.1.3 随机素数判定

```
// O(s log n) 内判定 2^63 内的数是不是素数, s 为测定次数
bool Miller_Rabin(ll n, int s) {
    if (n == 2) return 1;
```

```
    if (n < 2 || !(n & 1)) return 0;
    int t = 0; ll  x, y, u = n - 1;
    while ((u & 1) == 0) t++, u >>= 1;
    for (int i = 0; i < s; i++) {
        ll a = rand() % (n - 1) + 1;
        ll x = Pow(a, u, n);
        for (int j = 0; j < t; j++) {
            ll y = Mul(x, x, n);
            if (y == 1 && x != 1 && x != n - 1) return 0;
            x = y;
        }
        if (x != 1) return 0;
    }
    return 1;
}
```

2.1.4 分解质因数

```
// 函数返回素因数个数
// 数组以 fact[i][0]^{fact[i][1]} 的形式保存第 i 个素因数
ll fact[100][2];
int getFactors(ll x) {
    int cnt = 0;
    for (int i = 0; prime[i] <= x / prime[i]; i++) {
        fact[cnt][1] = 0;
        if (x % prime[i] == 0 ) {
            fact[cnt][0] = prime[i];
            while (x % prime[i] == 0) {
                fact[cnt][1]++;
                x /= prime[i];
            }
            cnt++;
        }
    }
    if (x != 1) {
        fact[cnt][0] = x;
        fact[cnt++][1] = 1;
    }
    return cnt;
}
```

2.2 欧拉函数

2.2.1 求一个数的欧拉函数

```
long long Euler(long long n) {
    long long rt = n;
    for (int i = 2; i * i <= n; i++)
```

```
        if (n % i == 0) {
            rt -= rt / i;
            while (n % i == 0) n /= i;
        }
    if (n > 1) rt -= rt / n;
    return rt;
}
```

2.2.2 筛法求欧拉函数

```
const int N = 10001;
int phi[N] = {0, 1};
void CalEuler() {
    for (int i = 2; i < N; i++)
        if (!phi[i]) for (int j = i; j < N; j += i) {
                if (!phi[j]) phi[j] = j;
                phi[j] = phi[j] / i * (i - 1);
            }
}
```

2.3 扩展欧几里得-乘法逆元

2.3.1 扩展欧几里得

```
void exgcd(ll a, ll b, ll &d, ll &x, ll &y) {
    if (!b) {d = a; x = 1; y = 0;}
    else {exgcd(b, a % b, d, y, x); y -= x * (a / b);}
}
```

2.3.2 求 ax+by=c 的解

```
// 引用返回通解: X = x + k * dx, Y = y–k * dy
// 引用返回的 x 是最小非负整数解, 方程无解函数返回 0
#define Mod(a,b) (((a)%(b)+(b))%(b))
bool solve(ll a, ll b, ll c, ll &x, ll &y, ll &dx, ll &dy) {
    if (a == 0 && b == 0) return 0;
    ll d, x0, y0; exgcd(a, b, d, x0, y0);
    if (c % d != 0) return 0;
    dx = b / d; dy = a / d;
    x = Mod(x0 * c / d, dx); y = (c - a * x) / b;
// y = Mod(y0 * c / d, dy); x = (c - b * y) / a;
    return 1;
}
```

2.3.3 乘法逆元

```
// 利用 exgcd 求 a 在模 m 下的逆元，需要保证 gcd(a, m) == 1.
ll inv(ll a, ll m) {
    ll x, y, d; exgcd(a, m, d, x, y);
    return d == 1 ? (x + m) % m : -1;
}
// a < m 且 m 为素数时，有以下两种求法
ll inv(ll a, ll m) {
    return a == 1 ? 1 : inv(m % a, m) * (m - m / a) % m;
}
ll inv(ll a, ll m) {
    return Pow(a, m - 2, m);
}
```

2.4 模线性方程组

2.4.1 中国剩余定理

```
// X = r[i]%m[i], 要求 m[i] 两两互质
// 引用返回通解 X = re + k * mo
void crt(ll r[], ll m[], ll n, ll &re, ll &mo) {
    mo = 1, re = 0;
    for (int i = 0; i < n; i++) mo *= m[i];
    for (int i = 0; i < n; i++) {
        ll x, y, d, tm = mo / m[i];
        exgcd(tm, m[i], d, x, y);
        re = (re + tm * x * r[i]) % mo;
    } re = (re + mo) % mo;
}
```

2.4.2 一般模线性方程组

```
// X = r[i]%m[i], m[i] 可以不两两互质
// 引用返回通解 X = re + k * mo, 函数返回是否有解
bool excrt(ll r[], ll m[], ll n, ll &re, ll &mo) {
    ll x, y, d; mo = m[0], re = r[0];
    for (int i = 1; i < n; i++) {
        exgcd(mo, m[i], d, x, y);
        if ((r[i] - re) % d != 0) return 0;
        x = (r[i] - re) / d * x % (m[i] / d);
        re += x * mo;
        mo = mo / d * m[i];
        re %= mo;
    } re = (re + mo) % mo;
    return 1;
}
```

2.5 组合数学

2.5.1 一般组合数

```
// 0 ≤ m ≤ n ≤ 1000
const int maxn = 1010;
ll C[maxn][maxn];
void CalComb() {
    C[0][0] = 1;
    for (int i = 1; i < maxn; i++) {
        C[i][0] = 1;
        for (int j = 1; j <= i; j++)
            C[i][j] = (C[i - 1][j - 1] + C[i - 1][j]) % mod;
    }
}

// 0 ≤ m ≤ n ≤ 10^5, 模 p 为素数
const int maxn = 100010;
ll f[maxn];
void CalFact() {
    f[0] = 1;
    for (int i = 1; i < maxn; i++)
        f[i] = (f[i - 1] * i) % mod;
}
ll C(int n, int m) {
    return f[n] * inv(f[m] * f[n - m] % mod, mod) % mod;
}
```

2.5.2 Lucas 定理

```
// 1 ≤ n,m ≤ 10^9, 1 < p < 10^5, p 是素数
const int maxp = 100010;
ll f[maxp];
void CalFact(ll p) {
    f[0] = 1;
    for (int i = 1; i <= p; i++)
        f[i] = (f[i - 1] * i) % p;
}
ll Lucas(ll n, ll m, ll p) {
    ll ret = 1;
    while (n && m) {
        ll a = n % p, b = m % p;
        if (a < b) return 0;
        ret = (ret * f[a] * Pow(f[b] * f[a - b] % p, p - 2, p)) % p;
        n /= p; m /= p;
    }
    return ret;
}
```

2.5.3 大组合数

```
// 0 ≤ n ≤ 10^9, 0 ≤ m ≤ 10^4, 1 ≤ k ≤ 10^9 + 7
vector<int> v;
int dp[110];
ll Cal(int l, int r, int k, int dis) {
    ll res = 1;
    for (int i = l; i <= r; i++) {
        int t = i;
        for (int j = 0; j < v.size(); j++) {
            int y = v[j];
            while (t % y == 0) {
                dp[j] += dis;
                t /= y;
            }
        }
        res = res * (ll)t % k;
    }
    return res;
}
ll Comb(int n, int m, int k) {
    set(dp, 0); v.clear(); int tmp = k;
    for (int i = 2; i * i <= tmp; i++) {
        if (tmp % i == 0) {
            int num = 0;
            while (tmp % i == 0) {
                tmp /= i;
                num++;
            }
            v.pb(i);
        }
    } if (tmp != 1) v.pb(tmp);
    ll ans = Cal(n - m + 1, n, k, 1);
    for (int j = 0; j < v.size(); j++) {
        ans = ans * Pow(v[j], dp[j], k) % k;
    }
    ans = ans * inv(Cal(2, m, k, -1), k) % k;
    return ans;
}
```

2.5.4 Polya 定理

```
// 推论：一共 n 个置换，第 i 个置换的循环节个数为 gcd(i,n)
// N*N 的正方形格子，c^(n^2) + 2c^((n^2+3)/4) + c^((n^2+1)/2) + 2c^(n(n+1)/2) + 2c^(n(n+1)/2)
// 正六面体，(m^8 + 17m^4 + 6m^2)/24
// 正四面体，(m^4 + 11m^2)/12
// 长度为 n 的项链串用 c 种颜色染
ll solve(int c, int n) {
```

```
    if (n == 0) return 0;
    ll ans = 0;
    for (int i = 1; i <= n; i++)
        ans += Pow(c, __gcd(i, n));
    if (n & 1)
        ans += n * Pow(c, n + 1 >> 1);
    else
        ans += n / 2 * (1 + c) * Pow(c, n >> 1);
    return ans / n / 2;
}
```

2.6 快速乘 + 快速幂

```
ll Mul(ll a, ll b, ll mod) {
    ll t = 0;
    for (; b; b >>= 1, a = (a << 1) % mod)
        if (b & 1) t = (t + a) % mod;
    return t;
}

ll Pow(ll a, ll n, ll mod) {
    ll t = 1;
    for (; n; n >>= 1, a = (a * a % mod))
        if (n & 1) t = (t * a % mod);
    return t;
}
```

2.7 莫比乌斯反演

2.7.1 莫比乌斯

```
// F(n) = \sum_{d|n} f(d) \Rightarrow f(n) = \sum_{d|n} \mu(d)F(\frac{n}{d})
// F(n) = \sum_{n|d} f(d) \Rightarrow f(n) = \sum_{n|d} \mu(\frac{d}{n})F(d)
long long ans;
const int MAXN = 1e5 + 1;
int n, x, prime[MAXN], tot, mu[MAXN];
bool check[MAXN];
void calmu() {
    mu[1] = 1;
    for (int i = 2; i < MAXN; i++) {
        if (!check[i]) {
            prime[tot++] = i;
            mu[i] = -1;
        }
        for (int j = 0; j < tot; j++) {
            if (i * prime[j] >= MAXN) break;
            check[i * prime[j]] = true;
```

```
            if (i % prime[j] == 0) {
                mu[i * prime[j]] = 0;
                break;
            } else {
                mu[i * prime[j]] = -mu[i];
            }
        }
    }
}
```

2.7.2 n 个数中互质数对数

```
// 有 n 个数（n ≤ 10^5），问这 n 个数中互质的数的对数
#include <cstdio>
#include <cstring>
#include <cstdlib>
using namespace std;
long long ans;
const int MAXN = 1e5 + 1;
int n, x, prime[MAXN], _max, b[MAXN], tot, mu[MAXN];
bool check[MAXN];
void calmu() {
    mu[1] = 1;
    for (int i = 2; i < MAXN; i++) {
        if (!check[i]) {
            prime[tot++] = i;
            mu[i] = -1;
        }
        for (int j = 0; j < tot; j++) {
            if (i * prime[j] >= MAXN) break;
            check[i * prime[j]] = true;
            if (i % prime[j] == 0) {
                mu[i * prime[j]] = 0;
                break;
            } else {
                mu[i * prime[j]] = -mu[i];
            }
        }
    }
}
int main() {
    calmu();
    while (scanf("%d", &n) == 1) {
        memset(b, 0, sizeof(b));
        _max = 0; ans = 0;
        for (int i = 0; i < n; i++) {
            scanf("%d", &x);
            if (x > _max) _max = x;
```

```
            b[x]++;
        }
        int cnt;
        for (int i = 1; i <= _max; i++) {
            cnt = 0;
            for (long long j = i; j <= _max; j += i)
                cnt += b[j];
            ans += 1LL * mu[i] * cnt * cnt;
        }
        printf("%lld\n", (ans - b[1]) / 2);
    }
    return 0;
}
```

2.7.3 VisibleTrees

```
// gcd(x,y)==1 的对数 x ≤ n,y ≤ m
int main() {
    calmu();
    int n, m;
    scanf("%d %d", &n, &m);
    if (n < m) swap(n, m);
    ll ans = 0;
    for (int i = 1; i <= m; ++i) {
        ans += (ll)mu[i] * (n / i) * (m / i);
    }
    printf("%lld\n", ans);
    return 0;
}
```

2.8 其他

2.8.1 Josephus 问题

```
#include <iostream>
using namespace std;
int main() {
    int num, m, r
    while (cin >> num >> m) {
        r = 0;
        for (int k = 1; k <= num; ++k)
            r = (r + m) % k;
        cout << r + 1 << endl;
    }
    return 0;
}
```

2.8.2 数位问题

```
// n^n 最左边一位数
int leftmost(int n) {
    double m = n * log10((double)n);
    double g = m - (long long)m;
    g = pow(10.0, g);
    return (int)g;
}

// n! 位数
int count(ll n) {
    return n == 1 ? 1 : (int)ceil(0.5 * log10(2 * M_PI * n) + n *
    ↪ log10(n) - n * log10(M_E));
}
```

2.9 相关公式

约数定理：若 $n = \prod_{i=1}^{k} p_i^{a_i}$，则

1. 约数个数 $f(n) = \prod_{i=1}^{k}(a_i + 1)$
2. 约数和 $g(n) = \prod_{i=1}^{k}(\sum_{j=0}^{a_i} p_i^j)$

小于 n 且互素的数之和为 $n\varphi(n)/2$
若 $gcd(n, i) = 1$，则 $gcd(n, n - i) = 1 (1 \leq i \leq n)$
错排公式：$D(n) = (n-1)(D(n-2) + D(n-1)) = \sum_{i=2}^{n} \frac{(-1)^k n!}{k!} = [\frac{n!}{e} + 0.5]$
威尔逊定理：$p\ is\ prime\ \Rightarrow (p-1)! \equiv -1 (mod\ p)$
欧拉定理：$gcd(a, n) = 1 \Rightarrow a^{\varphi(n)} \equiv 1 (mod\ n)$
欧拉定理推广：$gcd(n, p) = 1 \Rightarrow a^n \equiv a^{n \% \varphi(p)} (mod\ p)$
素数定理：对于不大于 n 的素数个数 $\pi(n)$，$\lim_{n \to \infty} \pi(n) = \frac{n}{\ln n}$
位数公式：正整数 x 的位数 $N = log10(n) + 1$
斯特灵公式 $n! \approx \sqrt{2\pi n}(\frac{n}{e})^n$
设 $a > 1, m, n > 0$, 则 $gcd(a^m - 1, a^n - 1) = a^{gcd(m,n)} - 1$
设 $a > b, gcd(a, b) = 1$, 则 $gcd(a^m - b^m, a^n - b^n) = a^{gcd(m,n)} - b^{gcd(m,n)}$

$$G = gcd(C_n^1, C_n^2, ..., C_n^{n-1}) = \begin{cases} n, & n \text{ is prime} \\ 1, & n \text{ has multy prime factors} \\ p, & n \text{ has single prime factor } p \end{cases}$$

$gcd(Fib(m), Fib(n)) = Fib(gcd(m, n))$
若 $gcd(m, n) = 1$, 则:
1. 最大不能组合的数为 $m * n - m - n$
2. 不能组合数个数 $N = \frac{(m-1)(n-1)}{2}$

$(n+1)lcm(C_n^0, C_n^1, ..., C_n^{n-1}, C_n^n) = lcm(1, 2, ..., n+1)$
若 p 为素数，则 $(x + y + ... + w)^p \equiv x^p + y^p + ... + w^p (mod\ p)$
卡特兰数：1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, 208012
$h(0) = h(1) = 1, h(n) = \frac{(4n-2)h(n-1)}{n+1} = \frac{C_{2n}^n}{n+1} = C_{2n}^n - C_{2n}^{n-1}$

$$a_{n+m}=\sum_{i=0}^{m-1}b_i a_{n+i}\Rightarrow\begin{pmatrix}a_{n+m}\\a_{n+m-1}\\\vdots\\a_{n+1}\end{pmatrix}=\begin{pmatrix}b_{m-1}&\cdots&b_1&b_0\\1&\cdots&0&0\\\vdots&\ddots&\vdots&\vdots\\0&\cdots&1&0\end{pmatrix}\begin{pmatrix}a_{n+m-1}\\a_{n+m-2}\\\vdots\\a_n\end{pmatrix}$$

$$a_{n+m}=\sum_{i=0}^{m-1}b_i a_{n+i}+c\Rightarrow\begin{pmatrix}a_{n+m}\\a_{n+m-1}\\\vdots\\a_{n+1}\\1\end{pmatrix}=\begin{pmatrix}b_{m-1}&\cdots&b_1&b_0&c\\1&\cdots&0&0&0\\\vdots&\ddots&\vdots&\vdots&\vdots\\0&\cdots&1&0&0\\0&\cdots&0&0&1\end{pmatrix}\begin{pmatrix}a_{n+m-1}\\a_{n+m-2}\\\vdots\\a_n\\1\end{pmatrix}$$

3 字符串

3.1 KMP

```
// 返回 y 中 x 的个数
int ne[N];
void initkmp(char x[], int m) {
    int i, j; j = ne[0] = -1; i = 0;
    while (i < m) {
        while (j != -1 && x[i] != x[j])
            j = ne[j];
        ne[++i] = ++j;
    }
}
int kmp(char x[], int m, char y[], int n) {
    int i, j, ans; i = j = ans = 0;
    initkmp (x, m);
    while (i < n) {
        while (j != -1 && y[i] != x[j]) j = ne[j];
        i++; j++;
        if (j >= m) {
            ans++; j = ne[j];
        }
    }
```

```
    return ans;
}
```

3.2 Manacher 最长回文子串

```
// O(n) 求解最长回文子串
const int N = 1000100;
char s[N], str[N << 1];
int p[N << 1];
void Manacher(char s[], int &n) {
    str[0] = '$';
    str[1] = '#';
    for (int i = 0; i < n; i++) {
        str[(i << 1) + 2] = s[i];
        str[(i << 1) + 3] = '#';
    }
    n = 2 * n + 2;
    str[n] = 0;
    int mx = 0, id;
    for (int i = 1; i < n; i++) {
        p[i] = mx > i ? min(p[2 * id - i], mx - i) : 1;
        while(str[i - p[i]] == str[i + p[i]]) p[i]++;
        if (p[i] + i > mx) {
            mx = p[i] + i;
            id = i;
        }
    }
}
int solve(char s[]) {
    int n = strlen(s);
    Manacher(s, n);
    int res = 0;
    for (int i = 0; i < n; i++)
        res = max(res, p[i]);
    return res - 1;
}
```

3.3 AC 自动机

```
#include <cstdio>
#include <cstring>
using namespace std;
#define rep(i,a,n) for (int i=a;i<n;i++)
const int AC_SIGMA = 26, AC_V = 29, AC_N = 500100;
struct AC_automaton {
    struct node {
        node *go[AC_V], *fail, *fa;
```

```
        int fg, id;
    } pool[AC_N], *cur, *root, *q[AC_N];
    node* newnode() {
        node *p = cur++;
        memset(p->go, 0, sizeof(p->go));
        p->fail = p->fa = NULL; p->fg = 0;
        return p;
    }
    void init() { cur = pool; root = newnode();}
    node* append(node *p, int w) {
        if (!p->go[w]) p->go[w] = newnode(), p->go[w]->fa = p;
        return p = p->go[w];
    }
    void build() {
        int t = 1;
        q[0] = root;
        rep(i, 0, t) rep(j, 0, AC_SIGMA) if (q[i]->go[j]) {
            node *v = q[i]->go[j], *p = v->fa->fail;
            while (p && !p->go[j]) p = p->fail;
            if (p) v->fail = p->go[j]; else v->fail = root;
            q[t++] = q[i]->go[j];
        } else {
            node *p = q[i]->fail;
            while (p && !p->go[j]) p = p->fail;
           if (p) q[i]->go[j] = p->go[j]; else q[i]->go[j] = root;
        }
    }
    int query(char s[]) {
        node *p = root;
        int res = 0;
        for (int i = 0; s[i]; i++) {
            p = p->go[s[i] - 'a'];
            node *v = p;
            while (v != root) {
                res += v->fg;
                v->fg = 0;
                v = v->fail;
            }
        }
        return res;
    }
} T;
typedef AC_automaton::node ACnode;


const int MAXN = 1000000 + 1000;
char txt[MAXN];
```

```
int main() {
#ifdef MANGOGAO
    freopen("data.in", "r", stdin);
#endif

    int t;
    scanf("%d", &t);
    while (t--) {
        int n;
        scanf("%d", &n);
        T.init();
        char s[55];
        rep(i, 0, n) {
            ACnode *p = T.root;
            scanf("%s", s);
            for (int j = 0; s[j]; j++)
                p = T.append(p, s[j] - 'a');
            p->fg++;
        }
        T.build();
        scanf("%s", txt);
        printf("%d\n", T.query(txt));
    }
    return 0;
}
```

4 数据结构

4.1 树状数组

```
// O(log n) 查询和修改数组的前缀和
// 注意下标应从 1 开始 n 是全局变量
int bit[N], n;
int sum(int i){
    int s = 0;
    while(i){
        s += bit[i];
        i -= i&-i;
    }
    return s;
}
void add(int i, int x){
    while(i<=n){
        bit[i] += x;
        i += i&-i;
    }
}
```

4.2 线段树

4.2.1 声明

```
#define lson rt<<1              // 左儿子
#define rson rt<<1|1            // 右儿子
#define Lson l,m,lson          // 左子树
#define Rson m+1,r,rson        // 右子树
void PushUp(int rt);            // 用 lson 和 rson 更新 rt
void PushDown(int rt[, int m]);    // rt 的标记下移, m 为区间长度
↪ (若与标记有关)
void build(int l, int r, int rt);    // 以 rt 为根节点, 对区间 [l,
↪ r] 建立线段树
void update([...,] int l, int r, int rt)    // rt[l, r] 内寻找目标
↪ 并更新
int query(int L, int R, int l, int r, int rt) // rt-[l, r] 内查询
↪ [L, R]
```

4.2.2 单点更新-区间查询

```
const int maxn = 50010;
int sum[maxn << 2];
void PushUp(int rt) {
    sum[rt] = sum[lson] + sum[rson];
}
void build(int l, int r, int rt) {
    if (l == r) {scanf("%d", &sum[rt]); return;}    // 建立的时候
    ↪ 直接输入叶节点
    int m = (l + r) >> 1;
    build(Lson); build(Rson);
    PushUp(rt);
}
void update(int p, int add, int l, int r, int rt) {
    if (l == r) {sum[rt] += add; return;}
    int m = (l + r) >> 1;
    if (p <= m) update(p, add, Lson);
    else update(p, add, Rson);
    PushUp(rt);
}
int query(int L, int R, int l, int r, int rt) {
    if (L <= l && r <= R) {return sum[rt];}
    int m = (l + r) >> 1, s = 0;
    if (L <= m) s += query(L, R, Lson);
    if (m < R) s += query(L, R, Rson);
    return s;
}
```

4.2.3 区间更新-区间查询

```
// seg[rt] 用于存放懒惰标记，注意 PushDown 时标记的传递
const int maxn = 101010;
int seg[maxn << 2], sum[maxn << 2];
void PushUp(int rt) {
    sum[rt] = sum[lson] + sum[rson];
}
void PushDown(int rt, int m) {
    if (seg[rt] == 0) return;
    seg[lson] += seg[rt];
    seg[rson] += seg[rt];
    sum[lson] += seg[rt] * (m - (m >> 1));
    sum[rson] += seg[rt] * (m >> 1);
    seg[rt] = 0;
}
void build(int l, int r, int rt) {
    seg[rt] = 0;
    if (l == r) {scanf("%lld", &sum[rt]); return;}
    int m = (l + r) >> 1;
    build(Lson); build(Rson);
    PushUp(rt);
}
void update(int L, int R, int add, int l, int r, int rt) {
    if (L <= l && r <= R) {
        seg[rt] += add;
        sum[rt] += add * (r - l + 1);
        return;
    }
    PushDown(rt, r - l + 1);
    int m = (l + r) >> 1;
    if (L <= m) update(L, R, add, Lson);
    if (m < R) update(L, R, add, Rson);
    PushUp(rt);
}
int query(int L, int R, int l, int r, int rt) {
    if (L <= l && r <= R) return sum[rt];
    PushDown(rt, r - l + 1);
    int m = (l + r) >> 1, ret = 0;
    if (L <= m) ret += query(L, R, Lson);
    if (m < R) ret += query(L, R, Rson);
    return ret;
}
```

4.3 字典树

```
struct Node {
    char c;
```

```
    Node* next[26];
    Node(char cc) {
        c = cc;
        REP(i, 26)next[i] = NULL;
    }
    ~Node() {
        REP(i, 26) if (next[i] != NULL) {
            next[i]->~Node();
            delete next[i];
            next[i] = NULL;
        }
    }
    bool empty() {
        REP(i, 26)if (next[i])return 0;
        return 1;
    }
};

class Trie {
public:
    Node *rt;
    Trie() {
        rt = new Node('*');
    }
    ~Trie() {
        rt->~Node();
    }
    void insert(char s[]) {
        Node *p = rt;
        for (int i = 0; s[i]; i++) {
            int d = s[i] - 'A';
            if (!p->next[d])
                p->next[d] = new Node(s[i]);
            p = p->next[d];
        }
    }
    int find(char s[]) {
        Node *p = rt;
        for (int i = 0; s[i]; i++) {
            int d = s[i] - 'A';
            if (!p->next[d]) return 0;
            p = p->next[d];
        }
        return 1;
    }
    void remove(char s[]) {
        stack<Node*> st;
        Node *pp = rt;
```

```
        for (int i = 0; s[i]; i++) {
            int d = s[i] - 'A';
            if (!pp->next[d]) return;
            st.push(pp);
            pp = pp->next[d];
        }
        pp->~Node();
        while (!st.empty()) {
            Node *p = st.top(); st.pop();
            p->next[pp->c - 'A'] = NULL;
            pp = p;
            bool f = 1;
            REP(i, 26) if (p->next[i]) f = 0;
            if (f) {
                p->~Node();
                if (!st.empty())st.top()->next[p->c - 'A'] = NULL;
            }
            else break;
        }
        if (rt == NULL) rt = new Node('*');
    }
};
```

4.4 RMQ

```
const int MAXN = 200000 + 100;
int mmax[MAXN][30], mmin[MAXN][30];
int a[MAXN], n, k;

void init() {
    for (int i = 1; i <= n; i++) {
        mmax[i][0] = mmin[i][0] = a[i];
    }
    for (int j = 1; (1 << j) <= n; j++)
        for (int i = 1; i + (1 << j) - 1 <= n; i++) {
            mmax[i][j] = max(mmax[i][j - 1], mmax[i + (1 << (j -
            ↪ 1))][j - 1]);
            mmin[i][j] = min(mmin[i][j - 1], mmin[i + (1 << (j -
            ↪ 1))][j - 1]);
        }
}

// op=0/1 返回 [l,r] 最大/小值
int rmq(int l, int r, int op) {
    int k = 0;
    while ((1 << (k + 1)) <= r - l + 1) k++;
    if (op == 0) return max(mmax[l][k], mmax[r - (1 << k) + 1][k]);
```

```
    return min(mmin[l][k], mmin[r - (1 << k) + 1][k]);
}
```

5 图论

5.1 并查集

```
const int MAXN = 128;
int n, fa[MAXN], ra[MAXN];
void init() {
    for (int i = 0; i <= n; i++) {
        fa[i] = i; ra[i] = 0;
    }
}
int find(int x) {
    if (fa[x] != x) fa[x] = find(fa[x]);
    return fa[x];
}
void unite(int x, int y) {
    x = find(x); y = find(y); if (x == y) return;
    if (ra[x] < ra[y]) fa[x] = y;
    else {
        fa[y] = x; if (ra[x] == ra[y]) ra[x]++;
    }
}
bool same(int x, int y) {
    return find(x) == find(y);
}
```

5.2 最小生成树

5.2.1 Kruskal

```
vector<pair<int, PII> > G;
void add_edge(int u, int v, int d) {
    G.pb(mp(d, mp(u, v)));
}
int Kruskal(int n) {
    init(n);
    sort(G.begin(), G.end());
    int m = G.size();
    int num = 0, ret = 0;
    for (int i = 0; i < m; i++) {
        pair<int, PII> p = G[i];
        int x = p.Y.X;
        int y = p.Y.Y;
        int d = p.X;
```

```
        if (!same(x, y)) {
            unite(x, y);
            num++;
            ret += d;
        }
        if (num == n - 1) break;
    }
    return ret;
}
```

5.2.2 Prim

```
// 耗费矩阵 cost[][], 标号从 0 开始,0~n-1
// 返回最小生成树的权值, 返回-1 表示原图不连通
const int INF = 0x3f3f3f3f;
const int MAXN = 110;
bool vis[MAXN];
int lowc[MAXN];
int Prim(int cost[][MAXN], int n) {
    int ans = 0;
    set(vis, 0);
    vis[0] = 1;
    for (int i = 1; i < n; i++)
        lowc[i] = cost[0][i];
    for (int i = 1; i < n; i++) {
        int minc = INF;
        int p = -1;
        for (int j = 0; j < n; j++)
            if (!vis[j] && minc > lowc[j]) {
                minc = lowc[j];
                p = j;
            }
        if (minc == INF) return -1;
        vis[p] = 1;
        for (int j = 0; j < n; j++)
            if (!vis[j] && lowc[j] > cost[p][j]) lowc[j] =
            ↪ cost[p][j];
    }
    return ans;
}
```

5.3 最短路

5.3.1 Dijkstra-邻接矩阵

```
// N 为点数最大值 求 s 到所有点的最短路
// 要求边权值为非负数 模板为有向边
// dis[x] 为起点到点 x 的最短路 inf 表示无法走到
```

```cpp
// 记得初始化
const int N = 100;    // 点数最大值
const int INF = 0x3f3f3f3f;
int G[N][N], dis[N];
bool vis[N];
void init(int n) {
    set(G, 0x3f);
}
void add_edge(int u, int v, int w) {
    G[u][v] = min(G[u][v], w);
}
void Dijkstra(int s, int n) {
    set(vis, 0);
    set(dis, 0x3f);
    dis[s] = 0;
    for (int i = 0; i < n; i++) {
        int x, minDis = INF;
        for (int j = 0; j < n; j++) {
            if (!vis[j] && dis[j] <= minDis) {
                x = j;
                minDis = dis[j];
            }
        }
        vis[x] = 1;
        for (int j = 0; j < n; j++)
            dis[j] = min(dis[j], dis[x] + G[x][j]);
    }
}
```

5.3.2 Dijkstra-邻接表数组

```cpp
// 点最大值: MAX_N  边最大值: MAX_E
// 求起点 s 到每个点 x 的最短路 dis[x]
const int MAX_N = "Edit";    // 点数最大值
const int MAX_E = "Edit";
const int INF = 0x3F3F3F3F;
int tot;
int Head[MAX_N], vis[MAX_N], dis[MAX_N];
int Next[MAX_E], To[MAX_E], W[MAX_E];
void init() {
    tot = 0;
    memset(Head, -1, sizeof(Head));
}
void add_edge(int u, int v, int d) {
    W[tot] = d;
    To[tot] = v;
    Next[tot] = Head[u];
    Head[u] = tot++;
```

```
}
void Dijkstra(int s, int n) {
    memset(vis, 0, sizeof(vis));
    memset(dis, 0x3F, sizeof(dis));
    dis[s] = 0;
    for (int i = 0; i < n; i++) {
        int x, min_dis = INF;
        for (int j = 0; j < n; j++) {
            if (!vis[j] && dis[j] <= min_dis) {
                x = j;
                min_dis = dis[j];
            }
        }
        vis[x] = 1;
        for (int j = Head[x]; j != -1; j = Next[j]) {
            int y = To[j];
            dis[y] = min(dis[y], dis[x] + W[j]);
        }
    }
}
```

5.3.3 Dijkstra-邻接表向量

```
// MAXN: 点数最大值
// 求起点 s 到所有点 x 的最短路 dis[x]
// 记得初始化
const int MAXN = "Edit";
const int INF = 0x3F3F3F3F;
vector<int> G[MAXN];
vector<int> GW[MAXN];
bool vis[MAXN];
int dis[MAXN];
void init(int n) {
    for (int i = 0; i < n; i++) {
        G[i].clear();
        GW[i].clear();
    }
}
void add_edge(int u, int v, int w) {
    G[u].push_back(v);
    GW[u].push_back(w);
}
void Dijkstra(int s, int n) {
    memset(vis, false, sizeof(vis));
    memset(dis, 0x3F, sizeof(dis));
    dis[s] = 0;
    for (int i = 0; i < n; i++) {
        int x;
```

```
        int min_dis = INF;
        for (int j = 0; j < n; j++) {
            if (!vis[j] && dis[j] <= min_dis) {
                x = j;
                min_dis = dis[j];
            }
        }
        vis[x] = true;
        for (int j = 0; j < (int)G[x].size(); j++) {
            int y = G[x][j];
            int w = GW[x][j];
            dis[y] = min(dis[y], dis[x] + w);
        }
    }
}
```

5.3.4 Dijkstra-优先队列

```
// pair< 权值, 点 >
// 记得初始化
const int MAXN = "Edit";
const int INF = 0x3F3F3F3F;
typedef pair<int, int> PII;
typedef vector<PII> VII;
VII G[MAXN];
int vis[MAXN], dis[MAXN];
void init(int n) {
    for (int i = 0; i < n; i++)
        G[i].clear();
}
void add_edge(int u, int v, int w) {
    G[u].push_back(make_pair(w, v));
}
void Dijkstra(int s, int n) {
    memset(vis, 0, sizeof(vis));
    memset(dis, 0x3F, sizeof(dis));
    dis[s] = 0;
    priority_queue<PII, VII, greater<PII> > q;
    q.push(make_pair(dis[s], s));
    while (!q.empty()) {
        PII t = q.top();
        int x = t.second;
        q.pop();
        if (vis[x]) continue;
        vis[x] = 1;
        for (int i = 0; i < (int)G[x].size(); i++) {
            int y = G[x][i].second;
            int w = G[x][i].first;
```

```
            if (!vis[y] && dis[y] > dis[x] + w) {
                dis[y] = dis[x] + w;
                q.push(make_pair(dis[y], y));
            }
        }
    }
}
```

5.3.5 Bellman-Ford(可判负环)

```
// 求出起点 s 到每个点 x 的最短路 dis[x]
// 存在负环返回 1 否则返回 0
// 记得初始化
const int MAX_N = "Edit";    // 点数最大值
const int MAX_E = "Edit";    // 边数最大值
const int INF = 0x3F3F3F3F;
int From[MAX_E], To[MAX_E], W[MAX_E];
int dis[MAX_N], tot;
void init() {tot = 0;}
void add_edge(int u, int v, int d) {
    From[tot] = u;
    To[tot] = v;
    W[tot++] = d;
}
bool Bellman_Ford(int s, int n) {
    memset(dis, 0x3F, sizeof(dis));
    dis[s] = 0;
    for (int k = 0; k < n - 1; k++) {
        bool relaxed = 0;
        for (int i = 0; i < tot; i++) {
            int x = From[i], y = To[i];
            if (dis[y] > dis[x] + W[i]) {
                dis[y] = dis[x] + W[i];
                relaxed = 1;
            }
        }
        if (!relaxed) break;
    }
    for (int i = 0; i < tot; i++)
        if (dis[To[i]] > dis[From[i]] + W[i])
            return 1;
    return 0;
}
```

5.3.6 SPFA

```
// G[u] = mp(v, w)
// SPFA() 返回 0 表示存在负环
const int MAXN = "Edit";
const int INF = 0x3F3F3F3F;
vector<pair<int, int> > G[MAXN];
bool vis[MAXN];
int dis[MAXN];
int inqueue[MAXN];
void init(int n) {
    for (int i = 0; i < n; i++)
        G[i].clear();
}
void add_edge(int u, int v, int w) {
    G[u].push_back(make_pair(v, w));
}
bool SPFA(int s, int n) {
    memset(vis, 0, sizeof(vis));
    memset(dis, 0x3F, sizeof(dis));
    memset(inqueue, 0, sizeof(inqueue));
    dis[s] = 0;
    queue<int> q;    // 待优化的节点入队
    q.push(s);
    while (!q.empty()) {
        int x = q.front();
        q.pop();
        vis[x] = false;
        for (int i = 0; i < G[x].size(); i++) {
            int y = G[x][i].first;
            int w = G[x][i].second;
            if (dis[y] > dis[x] + w) {
                dis[y] = dis[x] + w;
                if (!vis[y]) {
                    q.push(y);
                    vis[y] = true;
                    if (++inqueue[y] >= n) return 0;
                }
            }
        }
    }
    return 1;
}
```

5.3.7 Floyd 算法

```
// O(n^3) 求出任意两点间最短路
const int MAXN = "Edit";
```

```
const int INF = 0x3F3F3F3F;
int G[MAXN][MAXN];
void init(int n) {
    memset(G, 0x3F, sizeof(G));
    for (int i = 0; i < n; i++)
        G[i][i] = 0;
}
void add_edge(int u, int v, int w) {
    G[u][v] = min(G[u][v], w);
}
void Floyd(int n) {
    for (int k = 0; k < n; k++)
        for (int i = 0; i < n; i++)
            for (int j = 0; j < n; j++)
                G[i][j] = min(G[i][j], G[i][k] + G[k][j]);
}
```

5.4 拓扑排序

5.4.1 邻接矩阵

```
// 存图前记得初始化
// Ans 存放拓排结果，G 为邻接矩阵，deg 为入度信息
// 排序成功返回 1，存在环返回 0
const int MAXN = "Edit";
int Ans[MAXN];          // 存放拓扑排序结果
int G[MAXN][MAXN];      // 存放图信息
int deg[MAXN];          // 存放点入度信息
void init() {
    memset(G, 0, sizeof(G));
    memset(deg, 0, sizeof(deg));
    memset(Ans, 0, sizeof(Ans));
}
void add_edge(int u, int v) {
    if (G[u][v]) return;
    G[u][v] = 1;
    deg[v]++;
}
bool Toposort(int n) {
    int tot = 0;
    queue<int> que;
    for (int i = 0; i < n; ++i)
        if (deg[i] == 0) que.push(i);
    while (!que.empty()) {
        int v = que.front(); que.pop();
        Ans[tot++] = v;
        for (int i = 0; i < n; ++i)
            if (G[v][i] == 1)
```

```
                if (--deg[t] == 0) que.push(t);
    }
    if (tot < n) return false;
    return true;
}
```

5.4.2 邻接表

```
// 存图前记得初始化
// Ans 排序结果, G 邻接表, deg 入度, map 用于判断重边
// 排序成功返回 1, 存在环返回 0
const int MAXN = "Edit";
typedef pair<int, int> PII;
int Ans[MAXN];
vector<int> G[MAXN];
int deg[MAXN];
map<PII, bool> S;
void init(int n) {
    S.clear();
    for (int i = 0; i < n; i++)G[i].clear();
    memset(deg, 0, sizeof(deg));
    memset(Ans, 0, sizeof(Ans));
}
void add_edge(int u, int v) {
    if (S[make_pair(u, v)]) return;
    G[u].push_back(v);
    S[make_pair(u, v)] = 1;
    deg[v]++;
}
bool Toposort(int n) {
    int tot = 0; queue<int> que;
    for (int i = 0; i < n; ++i)
        if (deg[i] == 0) que.push(i);
    while (!que.empty()) {
        int v = que.front(); que.pop();
        Ans[tot++] = v;
        for (int i = 0; i < G[v].size(); ++i) {
            int t = G[v][i];
            if (--deg[t] == 0) que.push(t);
        }
    }
    if (tot < n) return false;
    return true;
}
```

5.5 欧拉回路

5.5.1 判定

定理 5.1. 无向图 *G* 存在欧拉通路的充要条件是：*G* 为连通图，并且 *G* 仅有两个奇度结点或无奇度结点。

推论 5.1. *(1)* 当 *G* 是仅有两个奇度结点的连通图时，*G* 的欧拉通路必以此两个结点为端点。*(2)* 当 *G* 时无奇度结点的连通图时，*G* 必有欧拉回路。*(3) G* 为欧拉图（存在欧拉回路）的充要条件是 *G* 为无奇度结点的连通图。

定理 5.2. 有向图 *D* 存在欧拉通路的充要条件是：*D* 为有向图，*D* 的基图连通，并且所有顶点的出度与入度都相等；或者除两个顶点外，其余顶点的出度与入度都相等，而这两个顶点中一个顶点的出度与入度只差为 *1*，另一个顶点的出度与入度之差为 *-1*。

推论 5.2. *(1)* 当 *D* 除出、入度之差为 *1*，*-1* 的两个顶点之外，其余顶点的出度与入度都相等时，*D* 的有向欧拉通路必以出、入度之差为 *1* 的顶点作为始点，以出、入度之差为 *-1* 的顶点作为终点。*(2)* 当 *D* 的所有顶点的出、入度都相等时，*D* 中存在有向欧拉回路。*(3)* 有向图 *D* 为有向欧拉图的充要条件是 *D* 的基图为连通图，并且所有顶点的出、入度都相等。

5.5.2 求解

```
#define MAXN 200
struct stack {
    int top, node[MAXN];
} s;

int G[MAXN][MAXN];    // 邻接矩阵
int n;    // 顶点个数

void dfs(int x) {
    int i;
    s.node[++s.top] = x;
    for (int i = 0; i < n; i++)
        if (G[i][x] > 0) {
            G[i][x] = G[x][i] = 0;
            dfs(i);
            break;
        }
}
void Fleury(int x) {
    int i, b;
    s.node[s.top = 0] = x;
    while (s.top >= 0) {
        b = 0;
        for (int i = 0; i < n; i++)
            if (G[s.node[s.top]][i] > 0) {
                b = 1;
                break;
```

```
            }
        if (b == 0) {
            printf("%d ", s.node[s.top] + 1);
            s.top--;
        }
        else {
            s.top--;
            dfs(s.node[s.top + 1]);
        }
    }
    printf("\n");
}

int main() {
    int i, j;
    int m, s, t; // 边数, 读入的边的起点和终点
    int degree, num, start; // 每个顶点的度、奇度顶点个数、欧拉回路
    ↪ 的起点
    scanf("%d%d", &n, &m);
    set(G, 0);
    for (i = 0; i < m; i++) {
        scanf("%d%d", &s, &t)
        G[s - 1][t - 1] = G[t - 1][s - 1] = 1;
    }
    num = 0; start = 0;
    for (i = 0; i < n; i++) {
        degree = 0;
        for (j = 0; j < n; j++)
            degree += G[i][j];
        if (degree & 1) {
            start = i;
            num++;
        }
    }
    if (num == 0 || num == 2) Fleury(start);
    else puts("No Euler path");
    return 0;
}
```

6 计算几何

6.1 定义

```
#define eps 1e-8
#define pi M_PI
#define zero(x) ((fabs(x)<eps?1:0))
#define sgn(x) (fabs(x)<eps?0:((x)<0?-1:1))
```

```
#define mp make_pair
#define X first
#define Y second

struct point {
    double x, y;
    point(double a = 0, double b = 0) {x = a; y = b;}
    point operator - (const point& b) const {
        return point(x - b.x, y - b.y);
    }
    point operator + (const point &b) const {
        return point(x + b.x, y + b.y);
    }
    // 两点是否重合
    bool operator == (point& b) {
        return zero(x - b.x) && zero(y - b.y);
    }
    // 点积 (以原点为基准)
    double operator * (const point &b) const {
        return x * b.x + y * b.y;
    }
    // 叉积 (以原点为基准)
    double operator ^ (const point &b) const {
        return x * b.y - y * b.x;
    }
    // 绕 P 点逆时针旋转 a 弧度后的点
    point rotate(point b, double a) {
        double dx, dy; (*this - b).split(dx, dy);
        double tx = dx * cos(a) - dy * sin(a);
        double ty = dx * sin(a) + dy * cos(a);
        return point(tx, ty) + b;
    }
    // 点坐标分别赋值到 a 和 b
    void split(double &a, double &b) {
        a = x; b = y;
    }
};

struct line {
    point s, e;
    line() {}
    line(point ss, point ee) {s = ss; e = ee;}
};
```

6.2 位置关系

6.2.1 两点间距离

```
double dist(point a, point b) {
    return sqrt((a - b) * (a - b));
}
```

6.2.2 直线与直线的交点

```
// <0, *> 表示重合; <1, *> 表示平行; <2, P> 表示交点是 P;
pair<int, point> spoint(line l1, line l2) {
    point res = l1.s;
    if (sgn((l1.s - l1.e) ^ (l2.s - l2.e)) == 0)
        return mp(sgn((l1.s - l2.e) ^ (l2.s - l2.e)) != 0, res);
    double t = ((l1.s - l2.s) ^ (l2.s - l2.e)) / ((l1.s - l1.e) ^
    ↪ (l2.s - l2.e));
    res.x += (l1.e.x - l1.s.x) * t;
    res.y += (l1.e.y - l1.s.y) * t;
    return mp(2, res);
}
```

6.2.3 判断线段与线段相交

```
bool segxseg(line l1, line l2) {
    return
        max(l1.s.x, l1.e.x) >= min(l2.s.x, l2.e.x) &&
        max(l2.s.x, l2.e.x) >= min(l1.s.x, l1.e.x) &&
        max(l1.s.y, l1.e.y) >= min(l2.s.y, l2.e.y) &&
        max(l2.s.y, l2.e.y) >= min(l1.s.y, l1.e.y) &&
        sgn((l2.s - l1.e) ^ (l1.s - l1.e)) * sgn((l2.e-l1.e) ^
        ↪ (l1.s - l1.e)) <= 0 &&
        sgn((l1.s - l2.e) ^ (l2.s - l2.e)) * sgn((l1.e-l2.e) ^
        ↪ (l2.s - l2.e)) <= 0;
}
```

6.2.4 判断线段与直线相交

```
bool segxline(line l1, line l2) {
    return sgn((l2.s - l1.e) ^ (l1.s - l1.e)) * sgn((l2.e-l1.e) ^
    ↪ (l1.s - l1.e)) <= 0;
}
```

6.2.5 点到直线距离

```
point pointtoline(point P, line L) {
    point res;
    double t = ((P - L.s) * (L.e-L.s)) / ((L.e-L.s) * (L.e-L.s));
```

```
    res.x = L.s.x + (L.e.x - L.s.x) * t;
    res.y = L.s.y + (L.e.y - L.s.y) * t;
    return dist(P, res);
}
```

6.2.6 点到线段距离

```
point pointtosegment(point p, line l) {
    point res;
    double t = ((p - l.s) * (l.e-l.s)) / ((l.e-l.s) * (l.e-l.s));
    if (t >= 0 && t <= 1) {
        res.x = l.s.x + (l.e.x - l.s.x) * t;
        res.y = l.s.y + (l.e.y - l.s.y) * t;
    }
    else res = dist(p, l.s) < dist(p, l.e) ? l.s : l.e;
    return res;
}
```

6.2.7 点在线段上

```
bool PointOnSeg(point p, line l) {
    return
        sgn((l.s - p) ^ (l.e-p)) == 0 &&
        sgn((p.x - l.s.x) * (p.x - l.e.x)) <= 0 &&
        sgn((p.y - l.s.y) * (p.y - l.e.y)) <= 0;
}
```

6.3 多边形

6.3.1 多边形面积

```
double area(point p[], int n) {
    double res = 0;
    for (int i = 0; i < n; i++)
        res += (p[i] ^ p[(i + 1) % n]) / 2;
    return fabs(res);
}
```

6.3.2 点在凸多边形内

```
// 点形成一个凸包，而且按逆时针排序（如果是顺时针把里面的 <0 改为
↪ >0)
// 点的编号 : [0,n)
// -1 : 点在凸多边形外
// 0  : 点在凸多边形边界上
// 1  : 点在凸多边形内
int PointInConvex(point a, point p[], int n) {
```

```
    for (int i = 0; i < n; i++) {
        if (sgn((p[i] - a) ^ (p[(i + 1) % n] - a)) < 0)
            return -1;
        else if (PointOnSeg(a, line(p[i], p[(i + 1) % n])))
            return 0;
    }
    return 1;
}
```

6.3.3 点在任意多边形内

```
// 射线法,poly[] 的顶点数要大于等于 3, 点的编号 0~n-1
// -1 : 点在凸多边形外
// 0  : 点在凸多边形边界上
// 1  : 点在凸多边形内
int PointInPoly(point p, point poly[], int n) {
    int cnt;
    line ray, side;
    cnt = 0;
    ray.s = p;
    ray.e.y = p.y;
    ray.e.x = -100000000000.0; // -INF, 注意取值防止越界
    for (int i = 0; i < n; i++) {
        side.s = poly[i];
        side.e = poly[(i + 1) % n];
        if (PointOnSeg(p, side))return 0;
        //如果平行轴则不考虑
        if (sgn(side.s.y - side.e.y) == 0)
            continue;
        if (PointOnSeg(sid e.s, r ay)) {
            if (sgn(side.s.y - side.e.y) > 0) cnt++;
        }
        else if (PointOnSeg(side.e, ray)) {
            if (sgn(side.e.y - side.s.y) > 0) cnt++;
        }
        else if (segxseg(ray, side)) cnt++;
    }
    return cnt % 2 == 1 ? 1 : -1;
}
```

6.3.4 判断凸多边形

```
// 点可以是顺时针给出也可以是逆时针给出
// 点的编号 1~n-1
bool isconvex(point poly[], int n) {
    bool s[3];
    set(s, 0);
```

```
    for (int i = 0; i < n; i++) {
        s[sgn((poly[(i + 1) % n] - poly[i]) ^ (poly[(i + 2) % n] -
        ↪  poly[i]) ) + 1] = 1;
        if (s[0] && s[2]) return 0;
    }
    return 1;
}
```

6.3.5 小结

```
#include <stdlib.h>
#include <math.h>
#define MAXN 1000
#define offset 10000
#define eps 1e-8
#define zero(x) (((x)>0?(x):-(x))<eps)
#define _sign(x) ((x)>eps?1:((x)<-eps?2:0))
struct point{double x,y;};
struct line{point a,b;};

double xmult(point p1,point p2,point p0){
    return (p1.x-p0.x)*(p2.y-p0.y)-(p2.x-p0.x)*(p1.y-p0.y);
}

// 判定凸多边形，顶点按顺时针或逆时针给出，允许相邻边共线
int is_convex(int n,point* p){
    int i,s[3]={1,1,1};
    for (i=0;i<n&&s[1]|s[2];i++)
        s[_sign(xmult(p[(i+1)%n],p[(i+2)%n],p[i]))]=0;
    return s[1]|s[2];
}

// 判定凸多边形，顶点按顺时针或逆时针给出，不允许相邻边共线
int is_convex_v2(int n,point* p){
    int i,s[3]={1,1,1};
    for (i=0;i<n&&s[0]&&s[1]|s[2];i++)
        s[_sign(xmult(p[(i+1)%n],p[(i+2)%n],p[i]))]=0;
    return s[0]&&s[1]|s[2];
}

// 判点在凸多边形内或多边形边上，顶点按顺时针或逆时针给出
int inside_convex(point q,int n,point* p){
    int i,s[3]={1,1,1};
    for (i=0;i<n&&s[1]|s[2];i++)
        s[_sign(xmult(p[(i+1)%n],q,p[i]))]=0;
    return s[1]|s[2];
}
```

```
// 判点在凸多边形内，顶点按顺时针或逆时针给出，在多边形边上返回 0
int inside_convex_v2(point q,int n,point* p){
    int i,s[3]={1,1,1};
    for (i=0;i<n&&s[0]&&s[1]|s[2];i++)
        s[_sign(xmult(p[(i+1)%n],q,p[i]))]=0;
    return s[0]&&s[1]|s[2];
}

// 判点在任意多边形内，顶点按顺时针或逆时针给出
// on_edge 表示点在多边形边上时的返回值,offset 为多边形坐标上限
int inside_polygon(point q,int n,point* p,int on_edge=1){
    point q2;
    int i=0,count;
    while (i<n)
      for (count=i=0,q2.x=rand()+offset,q2.y=rand()+offset;i<n;i++)
            if
            ↪  (zero(xmult(q,p[i],p[(i+1)%n]))&&(p[i].x-q.x)*(p[(i+1)%n].x-q.x)<e
                return on_edge;
            else if (zero(xmult(q,q2,p[i])))
                break;
            else if
            ↪  (xmult(q,p[i],q2)*xmult(q,p[(i+1)%n],q2)<-eps&&xmult(p[i],q,p[(i+1
                count++;
    return count&1;
}

inline int opposite_side(point p1,point p2,point l1,point l2){
    return xmult(l1,p1,l2)*xmult(l1,p2,l2)<-eps;
}

inline int dot_online_in(point p,point l1,point l2){
    return
    ↪  zero(xmult(p,l1,l2))&&(l1.x-p.x)*(l2.x-p.x)<eps&&(l1.y-p.y)*(l2.y-p.y)<eps
}

// 判线段在任意多边形内，顶点按顺时针或逆时针给出，与边界相交返回 1
int inside_polygon(point l1,point l2,int n,point* p){
    point t[MAXN],tt;
    int i,j,k=0;
    if (!inside_polygon(l1,n,p)||!inside_polygon(l2,n,p))
        return 0;
    for (i=0;i<n;i++)
        if
        ↪  (opposite_side(l1,l2,p[i],p[(i+1)%n])&&opposite_side(p[i],p[(i+1)%n],l
            return 0;
        else if (dot_online_in(l1,p[i],p[(i+1)%n]))
            t[k++]=l1;
        else if (dot_online_in(l2,p[i],p[(i+1)%n]))
```

```
            t[k++]=l2;
        else if (dot_online_in(p[i],l1,l2))
            t[k++]=p[i];
    for (i=0;i<k;i++)
        for (j=i+1;j<k;j++){
            tt.x=(t[i].x+t[j].x)/2;
            tt.y=(t[i].y+t[j].y)/2;
            if (!inside_polygon(tt,n,p))
                return 0;
        }
    return 1;
}

point intersection(line u,line v){
    point ret=u.a;
    double
    ↪ t=((u.a.x-v.a.x)*(v.a.y-v.b.y)-(u.a.y-v.a.y)*(v.a.x-v.b.x))

            ↪ /((u.a.x-u.b.x)*(v.a.y-v.b.y)-(u.a.y-u.b.y)*(v.a.x-v.b.x));
    ret.x+=(u.b.x-u.a.x)*t;
    ret.y+=(u.b.y-u.a.y)*t;
    return ret;
}

point barycenter(point a,point b,point c){
    line u,v;
    u.a.x=(a.x+b.x)/2;
    u.a.y=(a.y+b.y)/2;
    u.b=c;
    v.a.x=(a.x+c.x)/2;
    v.a.y=(a.y+c.y)/2;
    v.b=b;
    return intersection(u,v);
}

// 多边形重心
point barycenter(int n,point* p){
    point ret,t;
    double t1=0,t2;
    int i;
    ret.x=ret.y=0;
    for (i=1;i<n-1;i++)
        if (fabs(t2=xmult(p[0],p[i],p[i+1]))>eps){
            t=barycenter(p[0],p[i],p[i+1]);
            ret.x+=t.x*t2;
            ret.y+=t.y*t2;
            t1+=t2;
        }
```

```
    if (fabs(t1)>eps)
        ret.x/=t1,ret.y/=t1;
    return ret;
}
```

6.4 整数点问题

6.4.1 线段上整点个数

```
int OnSegment(line l) {
    return __gcd(fabs(l.s.x - l.e.x), fabs(l.s.y - l.e.y)) + 1;
}
```

6.4.2 多边形边上整点个数

```
int OnEdge(point p[], int n) {
    int i, ret = 0;
    for (i = 0; i < n; i++)
        ret += __gcd(fabs(p[i].x - p[(i + 1) % n].x), fabs(p[i].y
        ↪ - p[(i + 1) % n].y));
    return ret;
}
```

6.4.3 多边形内整点个数

```
int InSide(point p[], int n) {
    int i, area = 0;
    for (i = 0; i < n; i++)
        area += p[(i + 1) % n].y * (p[i].x - p[(i + 2) % n].x);
    return (fabs(area) - OnEdge(n, p)) / 2 + 1;
}
```

6.5 圆

```
point waixin(point a, point b, point c) {
    double a1 = b.x - a.x, b1 = b.y - a.y, c1 = (a1 * a1 + b1 * b1)
    ↪ / 2;
    double a2 = c.x - a.x, b2 = c.y - a.y, c2 = (a2 * a2 + b2 * b2)
    ↪ / 2;
    double d = a1 * b2 - a2 * b1;
    return point(a.x + (c1 * b2 - c2 * b1) / d, a.y + (a1 * c2 - a2
    ↪ * c1) / d);
}
```

6.6 经典题

```cpp
#include <cstdio>
#include <cmath>
#include <algorithm>
using namespace std;
const int N = 100100;
struct Point {
    double x, y;
};
int n;
Point p[N], tmp[N];

bool cmp(Point a, Point b) {return a.x == b.x ? a.y < b.y : a.x <
↪ b.x;}
bool cmpy(Point a, Point b) {return a.y < b.y;}
double dis(Point a, Point b) {
    double dx = a.x - b.x;
    double dy = a.y - b.y;
    return sqrt(dx * dx + dy * dy);
}
double solve(int l, int r) {
    double d = 1e20;
    if (l == r) return d;
    if (l + 1 == r) return dis(p[l], p[r]);
    int mid = l + r >> 1;
    double d1 = solve(l, mid);
    double d2 = solve(mid + 1, r);
    d = min(d1, d2);
    int k = 0;
    for (int i = l; i <= r; i++)
        if (fabs(p[i].x - p[mid].x) <= d)
            tmp[k++] = p[i];
    sort(tmp, tmp + k, cmpy);
    for (int i = 0; i < k; i++)
        for (int j = i + 1; j < k; j++) {
            if (tmp[j].y - tmp[i].y > d) break;
            d = min(d, dis(tmp[i], tmp[j]));
        }
    return d;
}
int main() {
    while (scanf("%d", &n) && n != 0) {
        for (int i = 0; i < n; i++)
            scanf("%lf %lf", &p[i].x, &p[i].y);
        sort(p, p + n, cmp);
        printf("%.2lf\n", solve(0, n - 1) / 2);
    }
```

```
    return 0;
}
```

7 动态规划

7.1 最大子序列和

```
// 传入序列 a 和长度 n, 返回最大子序列和
// 限制最短长度: 用 cnt 记录长度, rt 更新时判断
int MaxSeqSum(int a[], int n) {
    int rt = 0, cur = 0;
    for (int i = 0; i < n; i++) {
        cur += a[i];
        rt = rt < cur ? cur : rt;
        cur = cur < 0 ? 0 : cur;
    }
    return rt;
}
```

7.2 最长上升子序列 LIS

```
// 序列下标从 1 开始, LIS() 返回长度, 序列存在 lis[] 中
#define N 100100
int n, len, a[N], b[N], f[N];
int Find(int p, int l, int r) {
    int mid;
    while (l <= r) {
        mid = (l + r) >> 1;
        if (a[p] > b[mid]) l = mid + 1;
        else r = mid - 1;
    }
    return f[p] = l;
}
int LIS(int lis[]) {
int len = 1;
f[1] = 1;
b[1] = a[1];
    for (int i = 2; i <= n; i++) {
        if (a[i] > b[len]) b[++len] = a[i], f[i] = len;
        else b[Find(i, 1, len)] = a[i];
    }
    for (int i = n, t = len; i >= 1 && t >= 1; i--)
        if (f[i] == t)
            lis[--t] = a[i];
    return len;
}
```

7.3 最长公共上升子序列 LCIS

```
// 序列下标从 1 开始
int LCIS(int a[], int b[], int n, int m) {
    set(dp, 0);
    for (int i = 1; i <= n; i++) {
        int ma = 0;
        for (int j = 1; j <= m; j++) {
            dp[i][j] = dp[i - 1][j];
            if (a[i] > b[j]) ma = max(ma, dp[i - 1][j]);
            if (a[i] == b[j]) dp[i][j] = ma + 1;
        }
    }
    return *max_element(dp[n] + 1, dp[n] + 1 + m);
}
```