진법 강의노트

Numeric Representation

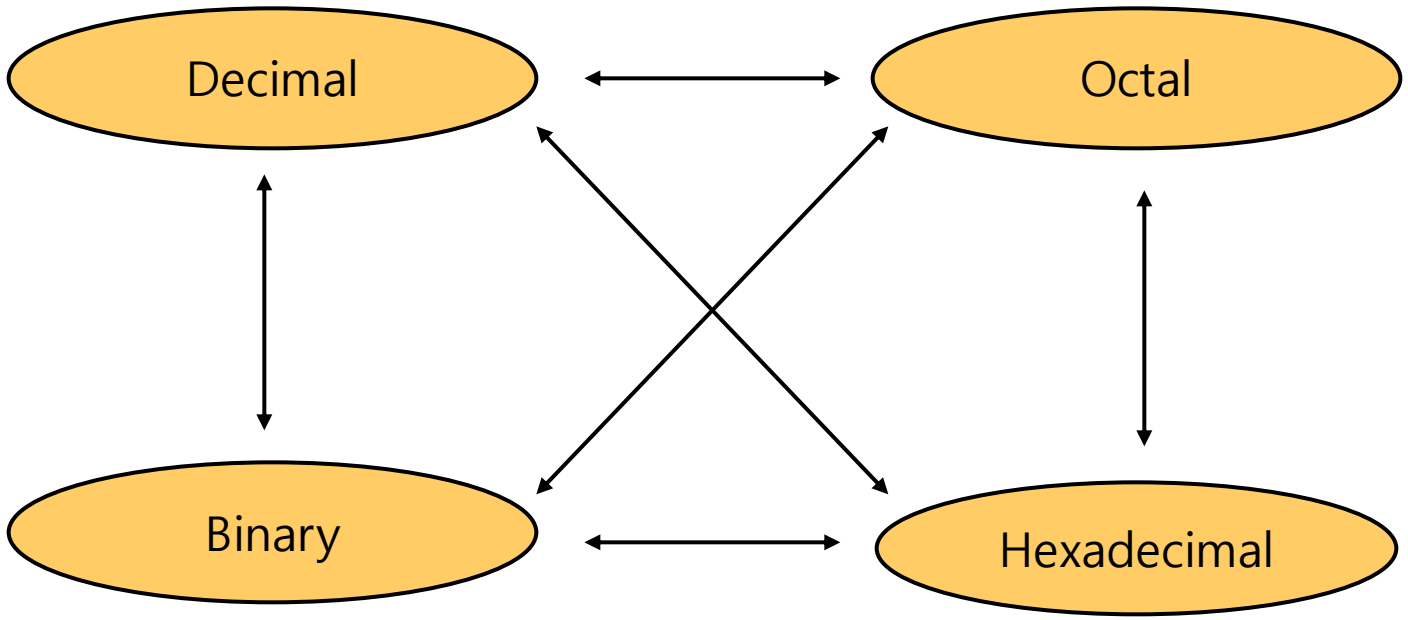
System	Base	Symbols	Used by humans?	Used in computers?
Decimal	10	0, 1, … 9	Yes	No
Binary	2	0, 1	No	Yes
Octal	8	0, 1, … 7	No	No
Hexa-decimal	16	0, 1, … 9, A, B, … F	No	No

Number Counting

Base-10	Base-2	Base-8	Base-16
0	0	0	0
1	1	1	1
2	10	2	2
3	11	3	3
4	100	4	4
5	101	5	5
6	110	6	6
7	111	7	7

Base-10	Base-2	Base-8	Base-16
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F
16	10000	20	10
17	10001	21	11
18	10010	22	12

Transitions Between Representations



How Decimal Number is Represented

- Each digit in a decimal number is multiplied by power of 10 with the digit index starting from 0
  - 125_{10}

  $1 * 10^3 = 100$

  $2 * 10^2 = 20$

  $5 * 10^0 = 5$

How Binary Number is Represented

- Similar to Decimal number, each digit in the binary representation is multiplied by power of 2 with the digit index starting from 0
  - $1111101_2 = 125_{10}$

  $1 * 2^6 = 64$

  $1 * 2^5 = 32$

  $1 * 2^4 = 16$

  $1 * 2^3 = 8$

  $1 * 2^2 = 4$

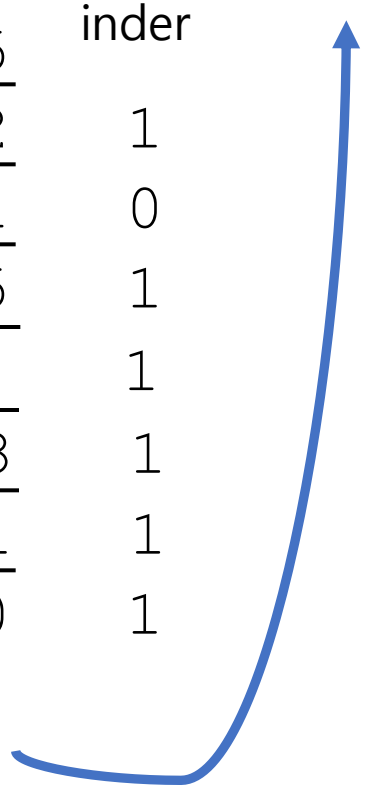
  $0 * 2^1 = 0$

  $1 * 2^0 = 1$
- Similar steps apply to Octal and Hexa-Decimal

How to Convert Decimal to Binary

$125_{10} = ?_2$

		Remainder
2	125	
2	62	1
2	31	0
2	15	1
2	7	1
2	3	1
2	1	1
	0	1



Output sequence

Converting to Octal and Hexa-decimal follows the same sequence

$125_{10} = 1111101_2$

Converting From Binary

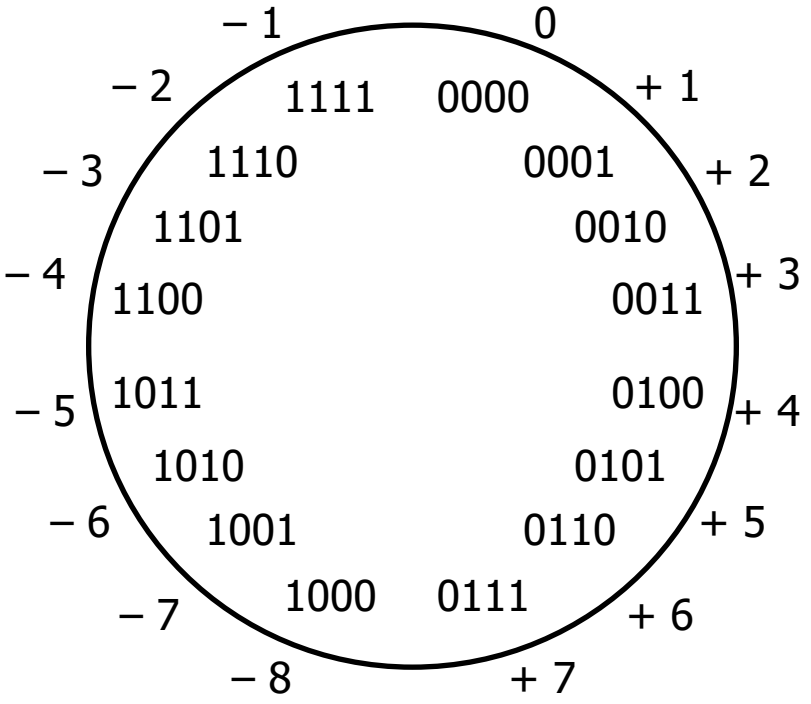
- Note that Binary number is represented with 0 and 1
  - 1 Bit is necessary
- Octal is represented with 0, 1, 2, 3, 4, 5, 6, 7
  - 3 Bit is necessary
    - Note that each bit can be 0 or 1
  - 000_2 → 0, 001_2 → 1, 010_2 → 2, 011_2 → 3
  - 100_2 → 4, 101_2 → 5, 110_2 → 6, 111_2 → 7
- In order to convert Binary value to Octal, convert every three digits to Octal number
  - $1111101_2 = 175_8$

Converting From Binary

- How about transforming binary to hexa-decimal?
- Note that Hexa-decimal can be represented with 4 bit
  - 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E
  - 0000_2 ~ 1111_2
  - Similar to Octal, convert every four digits into hexa-decimal number
  - 1111101_2 Hexa-decimal value?

Express Negative Value In Binary Format

- Two's complement
  - A way for a computer to understand (express) negative value in a binary format
- MSB (the left-most bit) decides the sign of a value
  - 0 : Positive
  - 1 : Negative
  - MSB stands for Most Significant Bit
  - LSB stands for Least Significant Bit



How Two's Complement Works

- If the MSB is 1, it will take -2^{i-1}, where i is the bit length
- Converting an integer to negative
  - 0001 0011 → 19
- First, flip every digit (0 → 1, 1 → 0)
  - 1110 1100
- Next, add 1 and add other digits number
  - 1110 1101 → -128 + 64 + 32 + 8 + 4 + 1 = -128 + 109 = -19