Digital Logic Design

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Today’s Topic

Tabular Method

Tabular Method: Quine-McCluskey

- Developed in the mid-50's
- Finds the minimized representation of a Boolean function
- Provides systematic way of generating all prime implicants then extracting a minimum set of primes covering the on-set
- Accomplishes this by repeatedly applying the Uniting theorem
  - Uniting theorem: ab + ab' = a(b+b') = a*1 = a

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Quine-McCluskey Algorithm

- Algorithm steps
  - Find all the prime implicants
  - Find all the essential prime implicants
  - Select a minimal set of remaining prime implicants that covers remaining 1s

Quine-McCluskey Example (1)

- Minimize F = a'b'c' + a'b'c + ab'c + abc' + abc
  - Step 1: Find all the prime implicants
    - List all elements of on-set and don't care set, represented as a binary number
    - Group minterms according to the number of 1's in the minterm

a'b'c'	→	(0) 000	G0	(0) 000	group G0 contains all minterms containing zero 1's
a'b'c	→	(1) 001	G1	(1) 001	group G1 contains all minterms containing one 1
ab'c	→	(5) 101	G2	(5) 101	group G2 contains all minterms containing two 1's
abc'	→	(6) 110		(6) 110	
abc	→	(7) 111	G3	(7) 111	group G3 contains all minterms containing three 1's

this grouping strategy will help us compare the minterms systematically

Quine-McCluskey Example (1)

– Step 1: Find all the prime implicants (cont')
  - Compare each entry in G_i to each entry in G_{i+1}
    – If they differ by 1 bit, we can apply the uniting theorem and eliminate a literal
    – Add check to minterm/implicant to remind us that it is not a prime implicant (combined with another element to form a larger implicant)

G0	✓	(0)	000
G1	✓	(1)	001
G2	✓	(5)	101
	✓	(6)	110
G3	✓	(7)	111

G0	(0,1)	00-
G1	(1,5)	-01
G2	(5,7)	1-1
	(6,7)	11-

no new implicants are generated – end of step 1

we have found all prime implicants (ones without check marks)

Quine-McCluskey Example (1)

- Step 2: Find all essential prime implicants
  - Create prime implicant chart
  - Columns are minterm indicies, rows are the prime implicants we determined

F = a'b'c' + a'b'c + ab'c + abc' + abc

(000) (001) (010) (110) (111)

	0	1	5	6	7
(0,1) 00-					
(1,5) -01					
(5,7) 1-1					
(6,7) 11-					

derived in Step1

Quine-McCluskey Example (1)

- Step 2: Find all essential prime implicants (cont')
  - Place "X" in a row if the prime implicant covers the minterm
  - Essential prime implicants are found by looking for rows with a single "X"
    - If minterm is covered by one and only one prime implicant – it's an essential prime implicant
  - Add essential prime implicants to the cover

essential prime implicant

	0	1	5	6	7
(0,1) 00-	X	X			
(1,5) -01		X	X		
(5,7) 1-1			X		X
(6,7) 11-				X	X

Cover:

F = a'b' + ac

Quine-McCluskey Example (1)

- Step 3: Select a minimal set of remaining prime implicants that covers remaining 1s
  - Step 2 determined essential prime implicants, and added to cover
    - Essential prime implicants may cover other minterms, cross out all minterms covered by the prime implicants
    - Minterm only needs to be covered once

	0	1	5	6	7
(0,1) 00-	X	X			
(1,5) -01		X	X		
(5,7) 1-1			X		X
(6,7) 11-				X	X

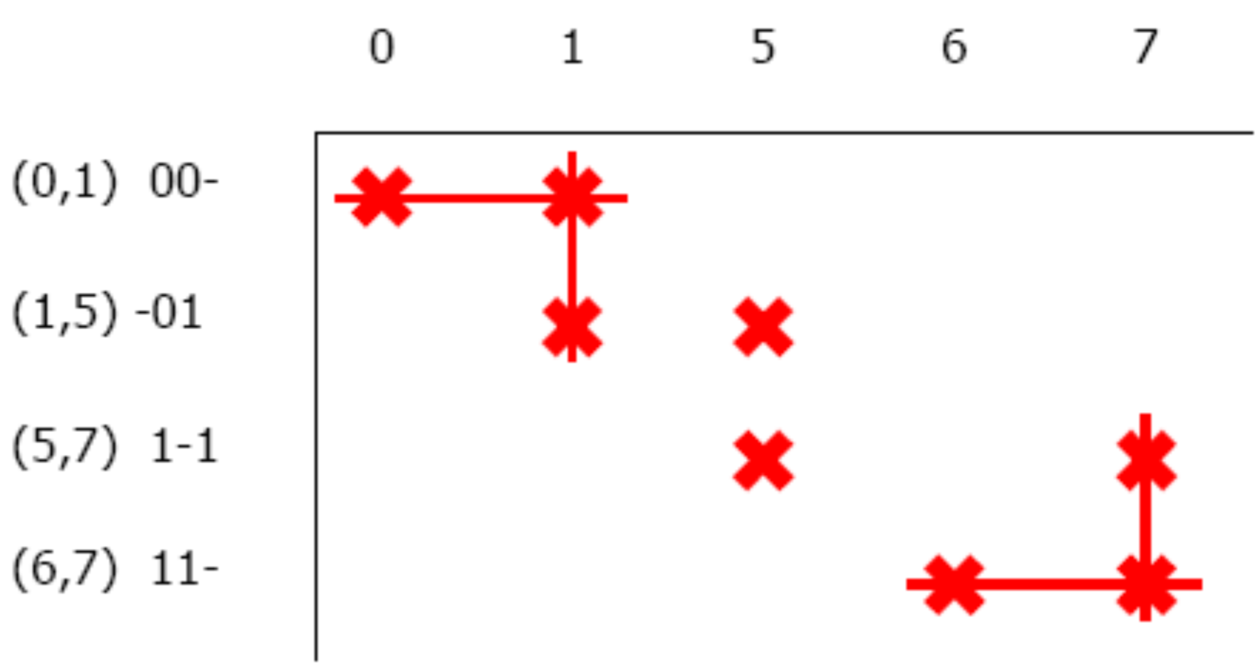
Cover:

F = a'b' + ac

Quine-McCluskey Example (1)

– Step 3: Select a minimal set of remaining prime implicants that covers the on set of the function (cont’)
  - Based on which minterms are left, add minimal set of prime implicants to cover

Only minterm 5 remains – either prime implicant (1,5) or (5,7) will work



Cover:

F = a’b’ + ac + b’c

Quine-McCluskey Example (2)

- Minimize F = w'x'y'z' + w'x'yz + w'x'yz' + w'xy'z' + w'xyz + w'xyz' + wxy'z + wxyz + wx'y'z + wx'yz

w'x'y'z'	→	(0) 0000
w'x'yz	→	(3) 0011
w'x'yz'	→	(2) 0010
w'xy'z'	→	(4) 0100
w'xyz	→	(7) 0111
w'xyz'	→	(6) 0110
wxy'z	→	(13) 1101
wxyz	→	(15) 1111
wx'y'z	→	(9) 1001
wx'yz	→	(11) 1011

G0	(0) 0000
G1	(2) 0010
	(4) 0100
G2	(3) 0011
	(6) 0110
	(9) 1001
G3	(7) 0111
	(11) 1011
	(13) 1101
G4	(15) 1111

Quine-McCluskey Example (2)

G0	✓	(0)	0000
G1	✓	(2)	0010
	✓	(4)	0100
G2	✓	(3)	0011
	✓	(6)	0110
	✓	(9)	1001
G3	✓	(7)	0111
	✓	(11)	1011
	✓	(13)	1101
G4	✓	(15)	1111

(0,2) ?
(0,4) ?
(2,3) ?
(2,6) ?
(2,9) ? N
(4,3) ? N
(4,6) ?
(4,9) ? N
(3,7) ?
(3,11) ?
(3,13) ? N
(6,7) ?
(6,11) ? N
(6,13) ? N
(9,7) ? N
(9,11) ?
(9,13) ?
(7,15) ?
(11,15) ?
(13,15) ?

G0	✓	(0,2)	00-0
	✓	(0,4)	0-00
G1	✓	(2,3)	001-
	✓	(2,6)	0-10
	✓	(4,6)	01-0
G2	✓	(3,7)	0-11
	✓	(3,11)	-011
	✓	(6,7)	011-
	✓	(9,11)	10-1
	✓	(9,13)	1-01
G3	✓	(7,15)	-111
	✓	(11,15)	1-11
	✓	(13,15)	11-1

Quine-McCluskey Method: The Other Way

- Minimize the following function
  - $f(x_1,....,x_4) = \sum m(0,4,8,10,11,12,13,15)$

List 1

0	0 0 0 0	✓
4 8	0 1 0 0 1 0 0 0	✓ ✓
10 12	1 0 1 0 1 1 0 0	✓ ✓
11 13	1 0 1 1 1 1 0 1	✓ ✓
15	1 1 1 1	✓

List 2

0,4 0,8	0 x 0 0 x 0 0 0	✓ ✓
8,10 4,12 8,12	1 0 x 0 x 1 0 0 1 x 0 0	 ✓ ✓
10,11 12,13	1 0 1 x 1 1 0 x	
11,15 13,15	1 x 1 1 1 1 x 1	

List 3

0,4,8,12	x x 0 0

  - What is the set of prime implicants?

$P = \{10x0, 101x, 110x, 1x11, 11x1, xx00\}$
$= \{p1, p2, p3, p4, p5, p6\}$

Quine-McCluskey Method: The Other Way

	Prime implicant	*0*	*4*	*8*	*10*	*11*	*12*	*13*	*15*
	p_1 = 1 0 x 0			✓	✓				
	p_2 = 1 0 1 x				✓	✓			
	p_3 = 1 1 0 x						✓	✓	
	p_4 = 1 x 1 1					✓			✓
	p_5 = 1 1 x 1							✓	✓
e.p.i.	p_6 = x x 0 0	✓	✓	✓			✓		

Prime implicant	*10*	*11*	*13*	*15*
p_1	✓			
p_2	✓	✓		
p_3			✓	
p_4		✓		✓
p_5			✓	✓

Quine-McCluskey Method: The Other Way

row dominance

Prime implicant	Minterm 10	11	13	15
p_1	√			
p_2	√	√		
p_3			√	
p_4		√		√
p_5			√	√

Prime implicant	Minterm 10	11	13	15
p_2	√	√		
p_4		√		√
p_5			√	√

p2 dominates p1 and p5 dominates p3

The final cover is :

$C = \{p2, p5, p6\}$

$= \{101x, 11x1, xx00\}$

Quine-McCluskey Method: The Other Way

- Minimize the following function
  - $f(x_1,\ldots,x_4) = \sum m(0,2,5,6,7,8,9,13)+D(1,12,15)$

List 1

0	0 0 0 0	✓
1	0 0 0 1	✓
2	0 0 1 0	✓
8	1 0 0 0	✓
5	0 1 0 1	✓
6	0 1 1 0	✓
9	1 0 0 1	✓
12	1 1 0 0	✓
7	0 1 1 1	✓
13	1 1 0 1	✓
15	1 1 1 1	✓

List 2

0,1	0 0 0 x	✓
0,2	0 0 x 0	
0,8	x 0 0 0	✓
1,5	0 x 0 1	✓
2,6	0 x 1 0	
1,9	x 0 0 1	✓
8,9	1 0 0 x	✓
8,12	1 x 0 0	✓
5,7	0 1 x 1	✓
6,7	0 1 1 x	
5,13	x 1 0 1	✓
9,13	1 x 0 1	✓
12,13	1 1 0 x	✓
7,15	x 1 1 1	✓
13,15	1 1 x 1	✓

List 3

0,1,8,9	x 0 0 x
1,5,9,13	x x 0 1
8,9,12,13	1 x 0 x
5,7,13,15	x 1 x 1

$P = \{00x0, 0x10, 011x, x00x, xx01, 1x0x, x1x1\}$
$= \{p1, p2, p3, p4, p5, p6, p7\}$