



TRƯỜNG ĐẠI HỌC  
**SƯ PHẠM KỸ THUẬT TP. HỒ CHÍ MINH**

HCMC University of Technology and Education

**Faculty of Information Technology**

# **Search for Solutions**

## **(ARTIFICIAL INTELLIGENCE)**

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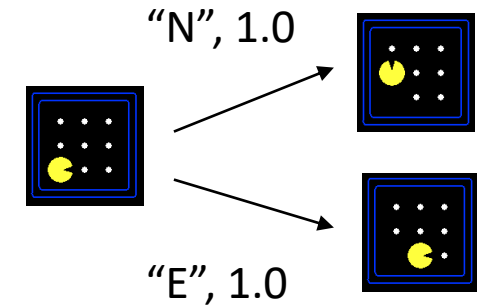
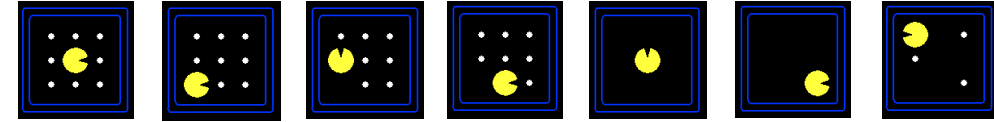
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(slides adapted from Dan Klein, Pieter Abbeel, Anca Dragan, et al)

# Search Problems

- In AI, search techniques are universal problem-solving methods

- A **search problem** consists of:
  - A state space
  - A successor function (with actions, costs)
  - A start state and a goal test



- A **solution** is a sequence of actions (a plan) which transforms the start state to a goal state

# Search Problems

- Terminologies:
  - Search: Searching is a step by step procedure to solve problem.
  - 3 main factors: Search Space; Start State; Goal test
  - Search tree: represent a search problem.
  - Actions.
  - Transition model.
  - Path Cost.
  - Solution.
  - Optimal Solution.

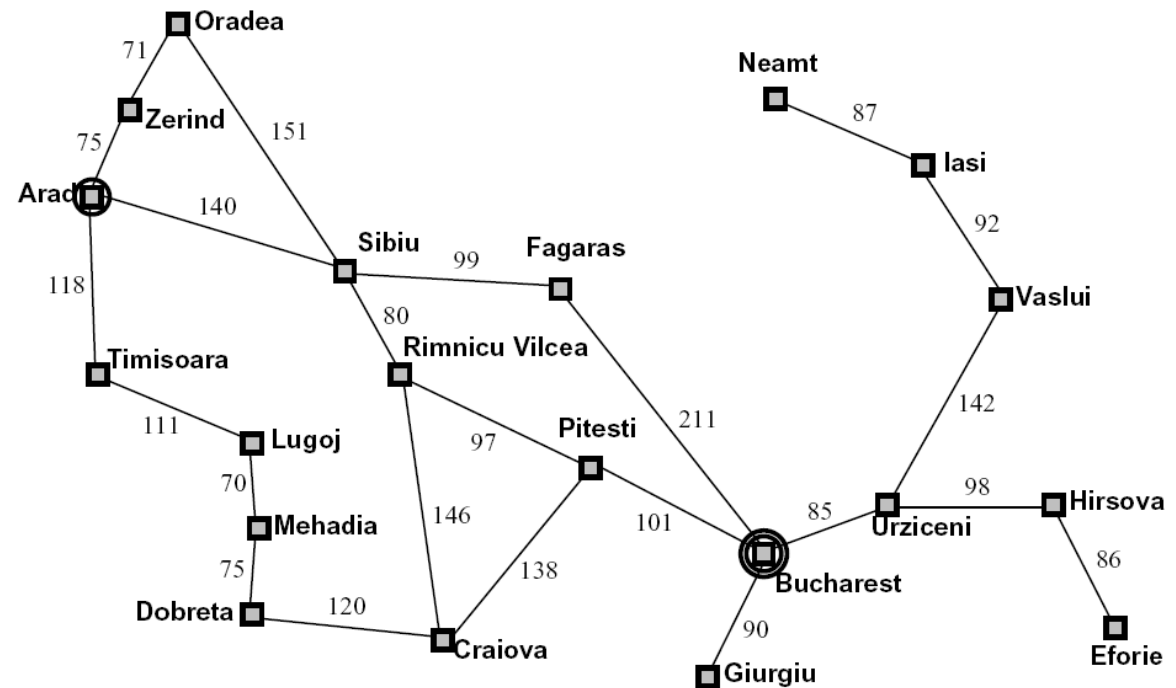
# Search Problems

## Components of search problem

- State space:
  - Cities
- Successor function:
  - Action: Roads
  - Cost

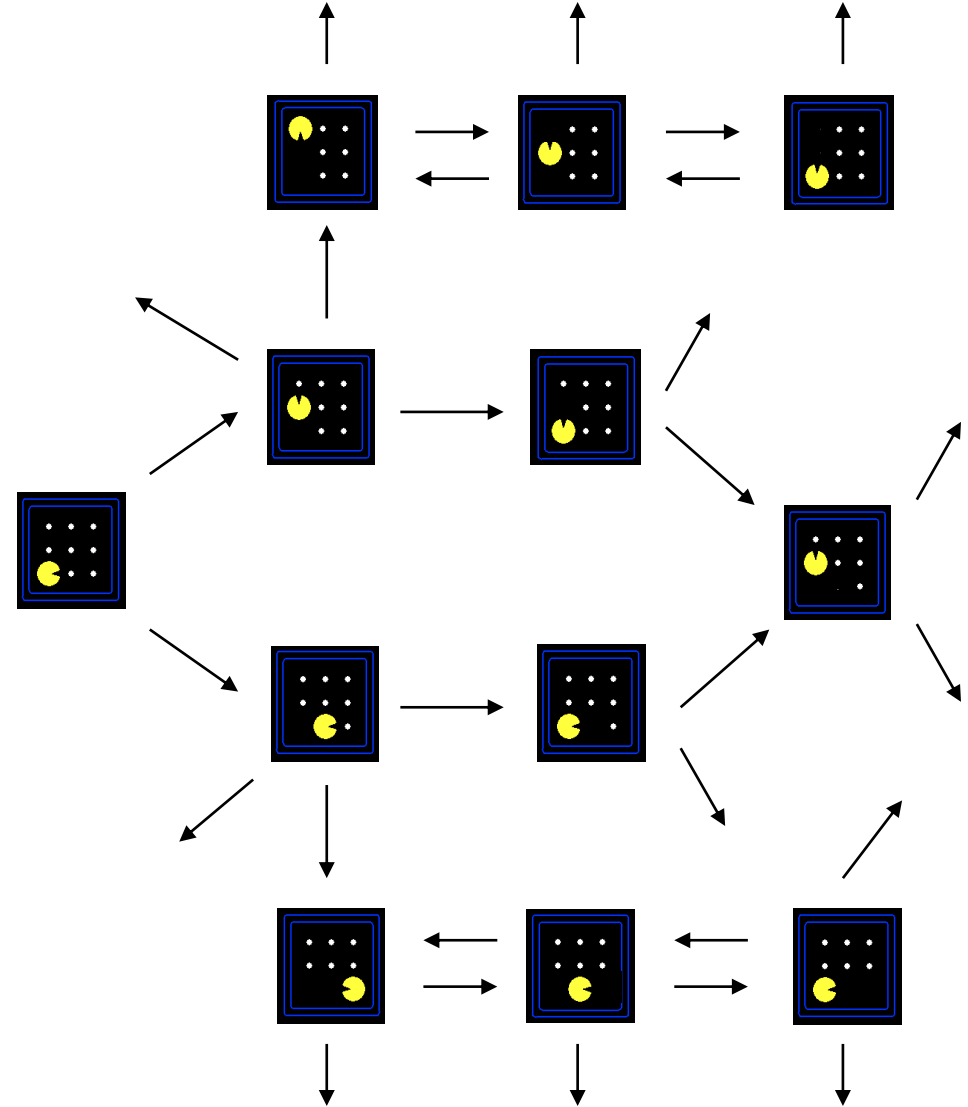
Go to adjacent city  
with cost = distance
- Start state:
  - Arad
- Goal test:
  - Is state == Bucharest?
- Solution?

### Example: Traveling in Romania



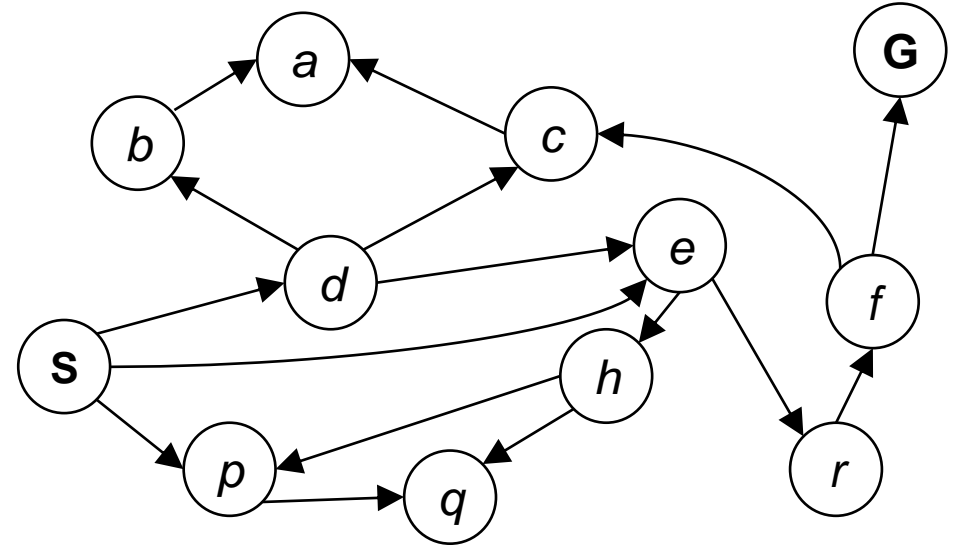
# State Space Graphs

- State space graph: A mathematical representation of a search problem
  - Nodes are (abstracted) world configurations
  - Arcs represent successors (action results)
  - The goal test is a set of goal nodes (maybe only one)
- In a state space graph, each state occurs only once!
- We can rarely build this full graph in memory (it's too big), but it's a useful idea



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*Tiny state space graph for a tiny search problem*

# Search Trees

- **Example:**

Searching for a solution to the 8-puzzle.

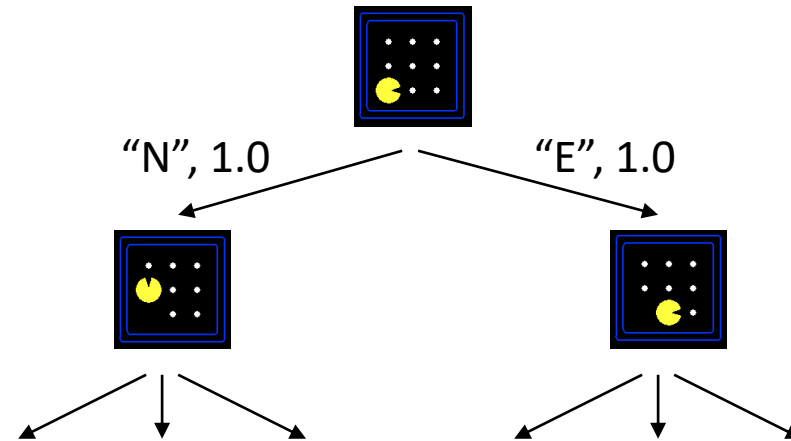
2	8	3
1	6	4
7		5

Start state



1	2	3
8		4
7	6	5

Goal



This is now  
/ start



Possible  
futures

- A search tree:
  - A “what if” tree of plans and their outcomes
  - The start state is the root node
  - Children correspond to successors
  - Nodes show states, but correspond to PLANS that achieve those states
  - For most problems, we can never actually build the whole tree

# Search Trees

**Example:**  
Searching for a solution to the 8-puzzle.

Aside: in this tree, immediate duplicates are removed.

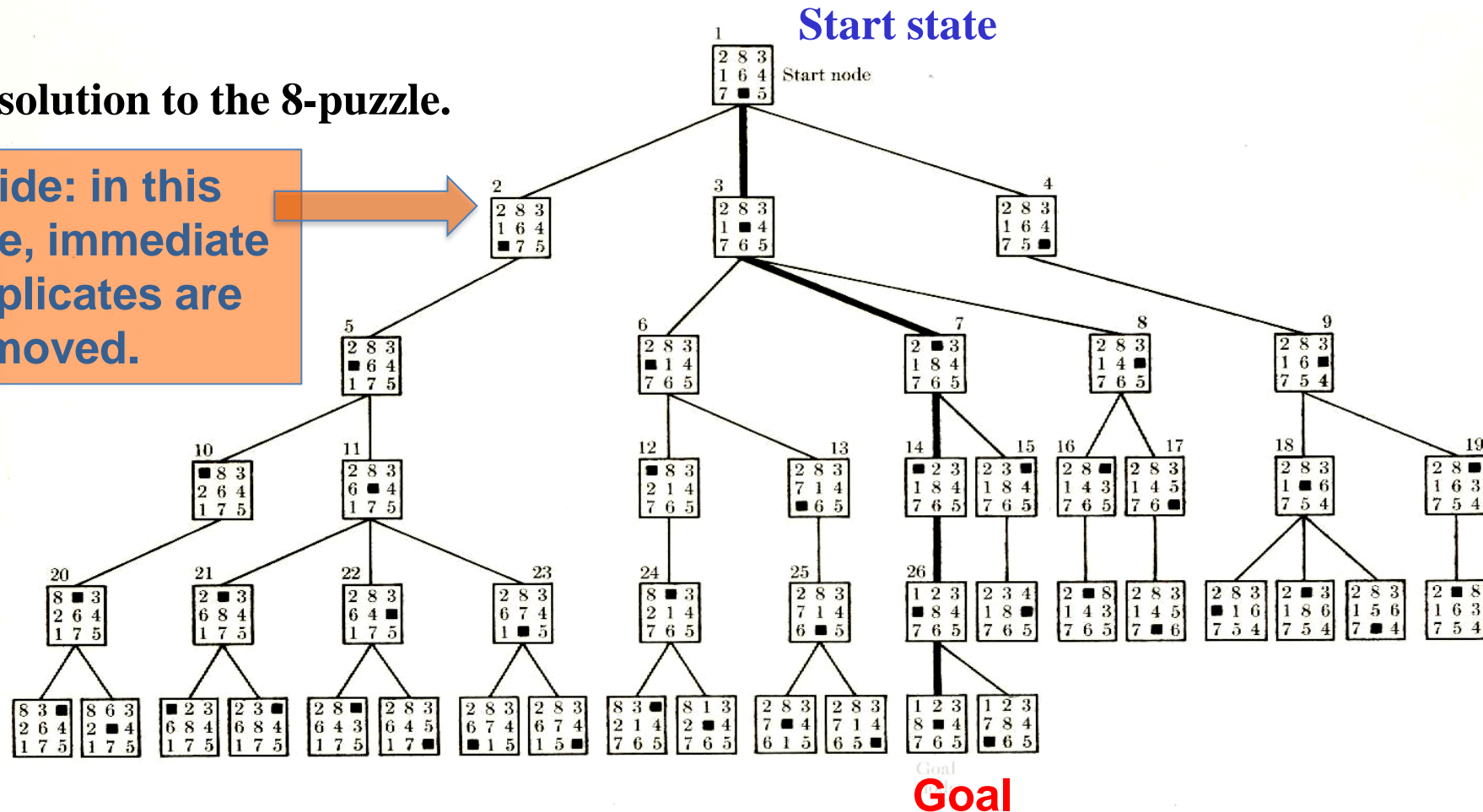


FIG. 3-2 The tree produced by a breadth-first search. (More detail soon.)

Branching factor 1, 2, or 3 (max). So, approx. 2 --- # nodes roughly doubles at each level. Number states of explored nodes grows exponentially with depth.



# Tree Search

- General Tree Search

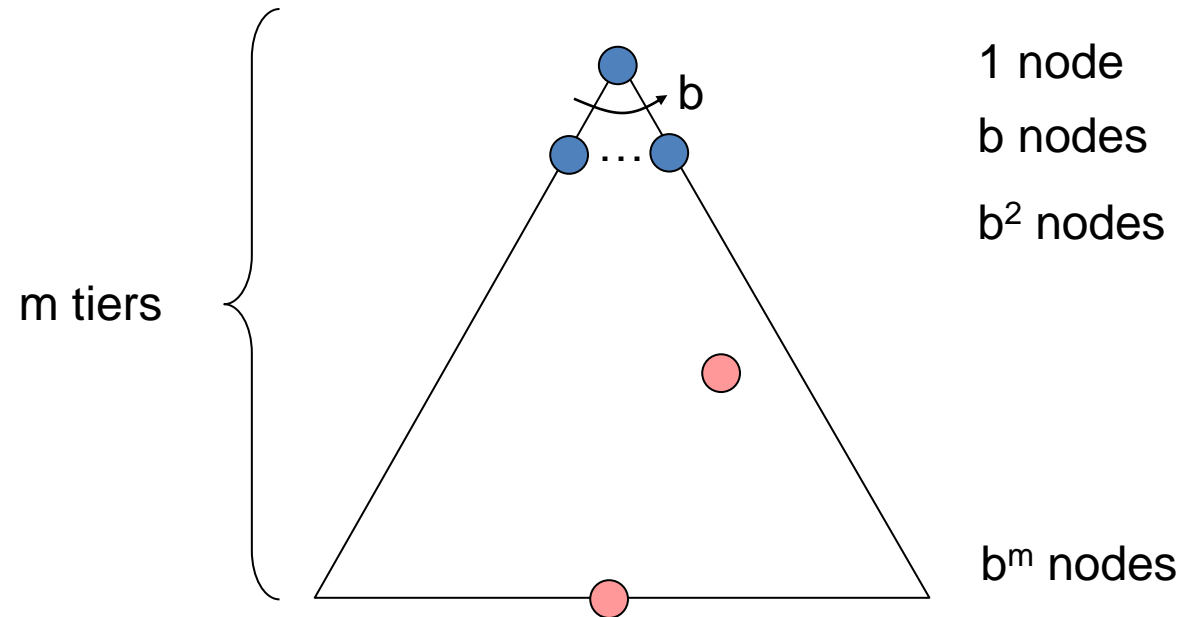
```
function TREE-SEARCH(problem, strategy) returns a solution, or failure
  initialize the search tree using the initial state of problem
  loop do
    if there are no candidates for expansion then return failure
    choose a leaf node for expansion according to strategy
    if the node contains a goal state then return the corresponding solution
    else expand the node and add the resulting nodes to the search tree
  end
```

- Important ideas:
  - Fringe
  - Expansion
  - Exploration strategy
- Main question: which fringe nodes to explore?

# Search Algorithm Properties

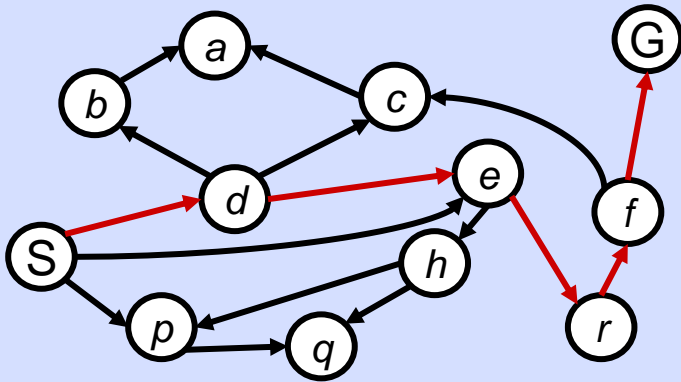
## Search Algorithm Properties

- Complete: Guaranteed to find a solution if one exists?
- Optimal: Guaranteed to find the least cost path?
- Time complexity?
- Space complexity?
- Cartoon of search tree:
  - $b$  is the branching factor
  - $m$  is the maximum depth
  - solutions at various depths
- Number of nodes in entire tree?
  - $1 + b + b^2 + \dots + b^m = O(b^m)$



# State Space Graphs vs. Search Trees

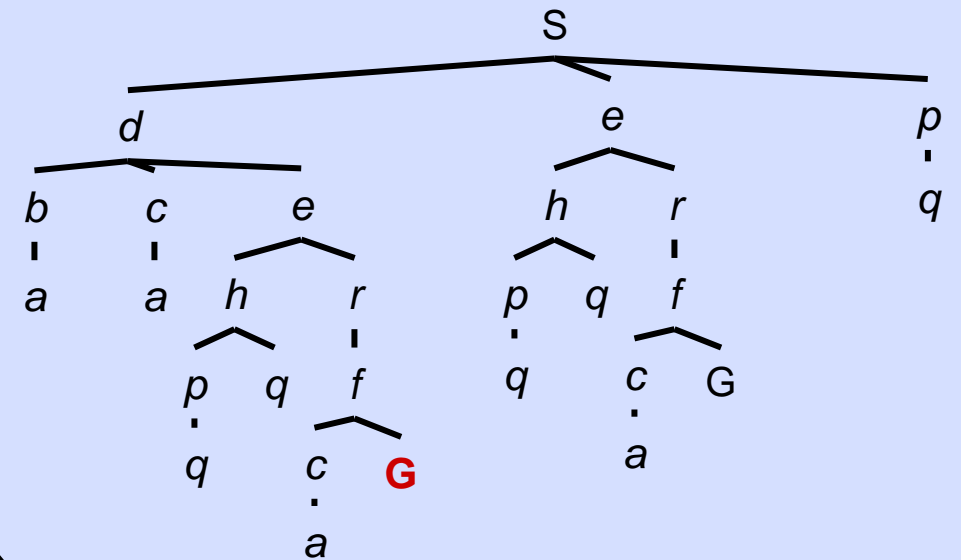
## State Space Graph



*Each NODE in in the search tree is an entire PATH in the state space graph.*

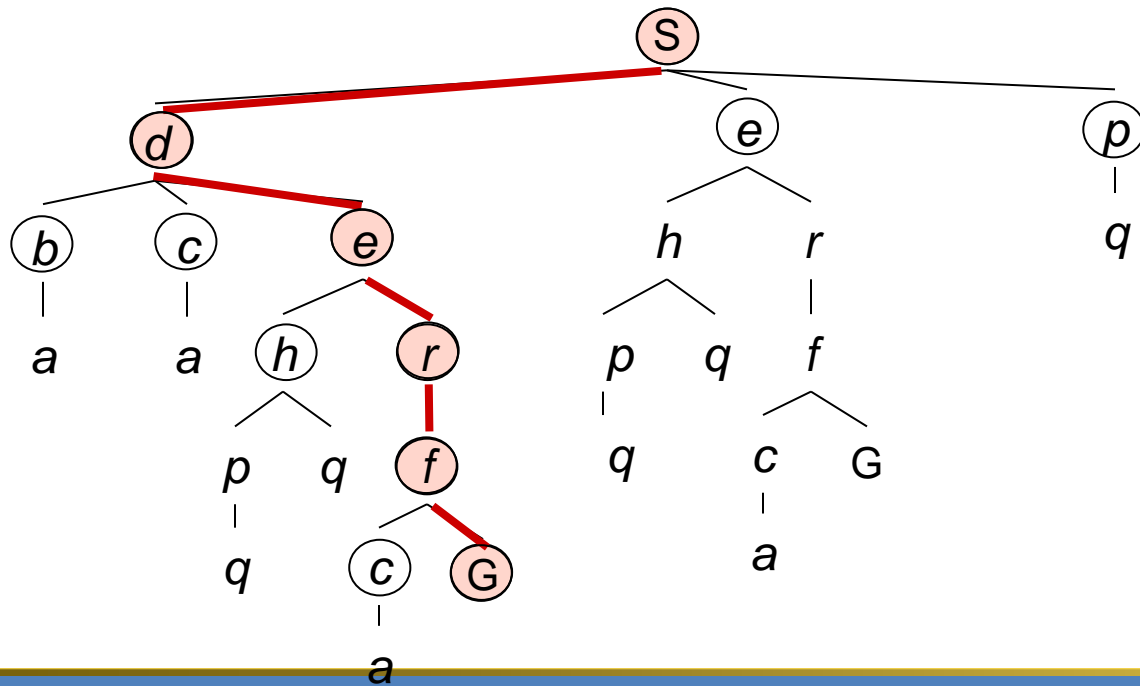
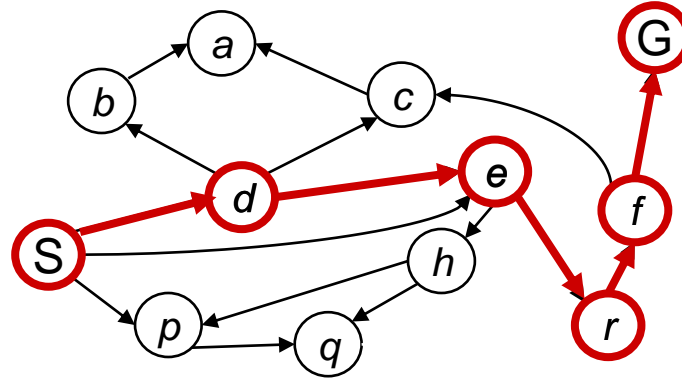
*We construct both on demand – and we construct as little as possible.*

## Search Tree



# State Space Graphs vs. Search Trees

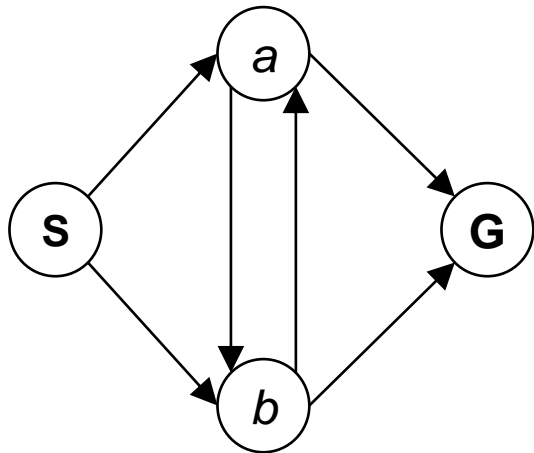
- Example: Tree Search



~~s~~  
~~s → d~~  
s → e  
s → p  
s → d → b  
s → d → c  
~~s → d → e~~  
s → d → e → h  
~~s → d → e → r~~  
~~s → d → e → r → f~~  
s → d → e → r → f → c  
~~s → d → e → r → f → G~~

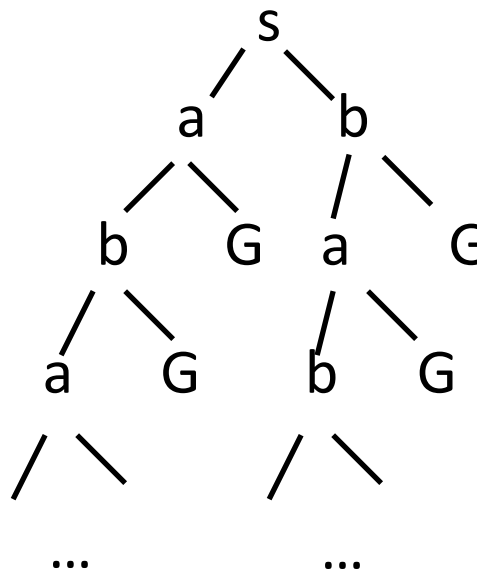
# Quiz: State Space Graphs vs. Search Trees

Consider this 4-state graph:



Có bao nhiêu node trên cây tìm kiếm tương ứng với đồ thị trạng thái sau?

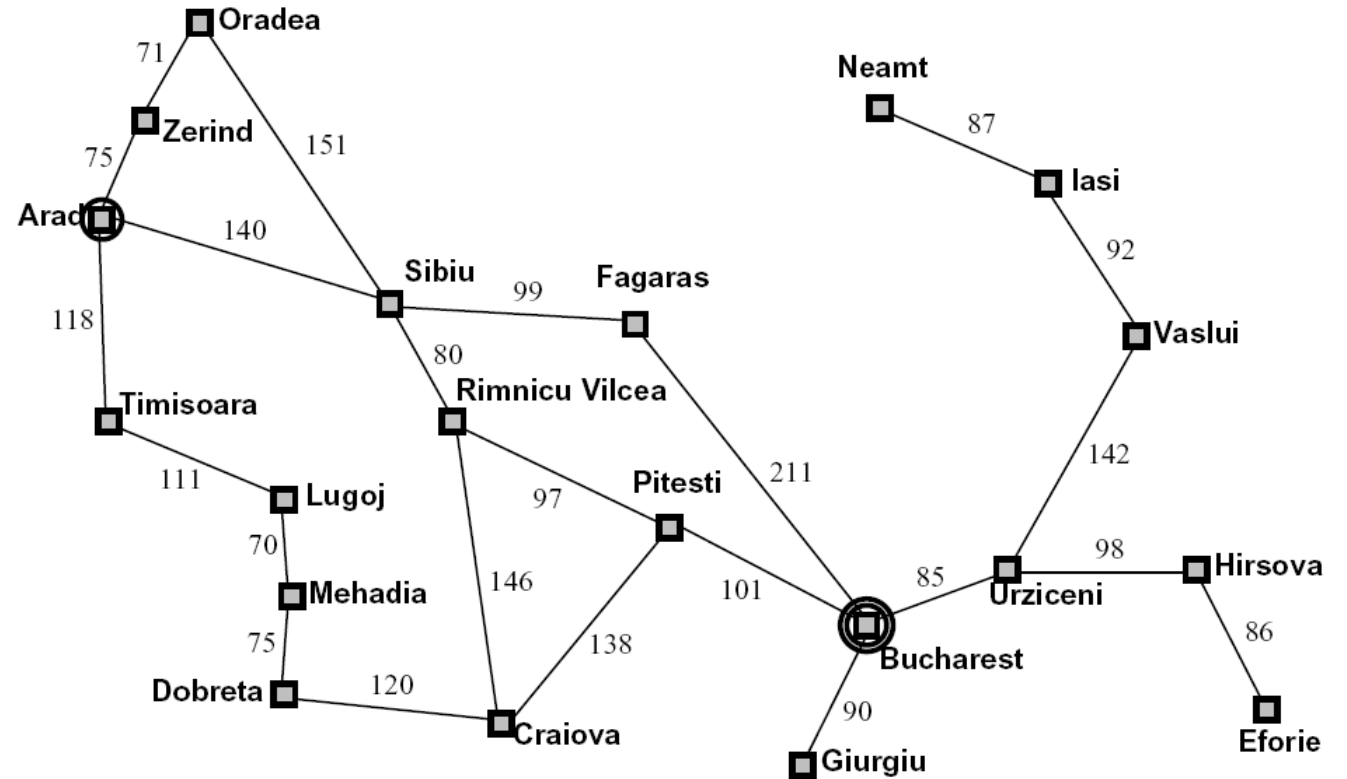
How big is its search tree (from S)?



Important: Lots of repeated structure in the search tree!

# Assignment

- Path planning using Roma city map
  - Starting state: **Arad**
  - Goal state: **Bucharest**
- Building a searching tree (to search a solution)



trạng thái xuất phát

2	6	5
	8	7
4	3	1

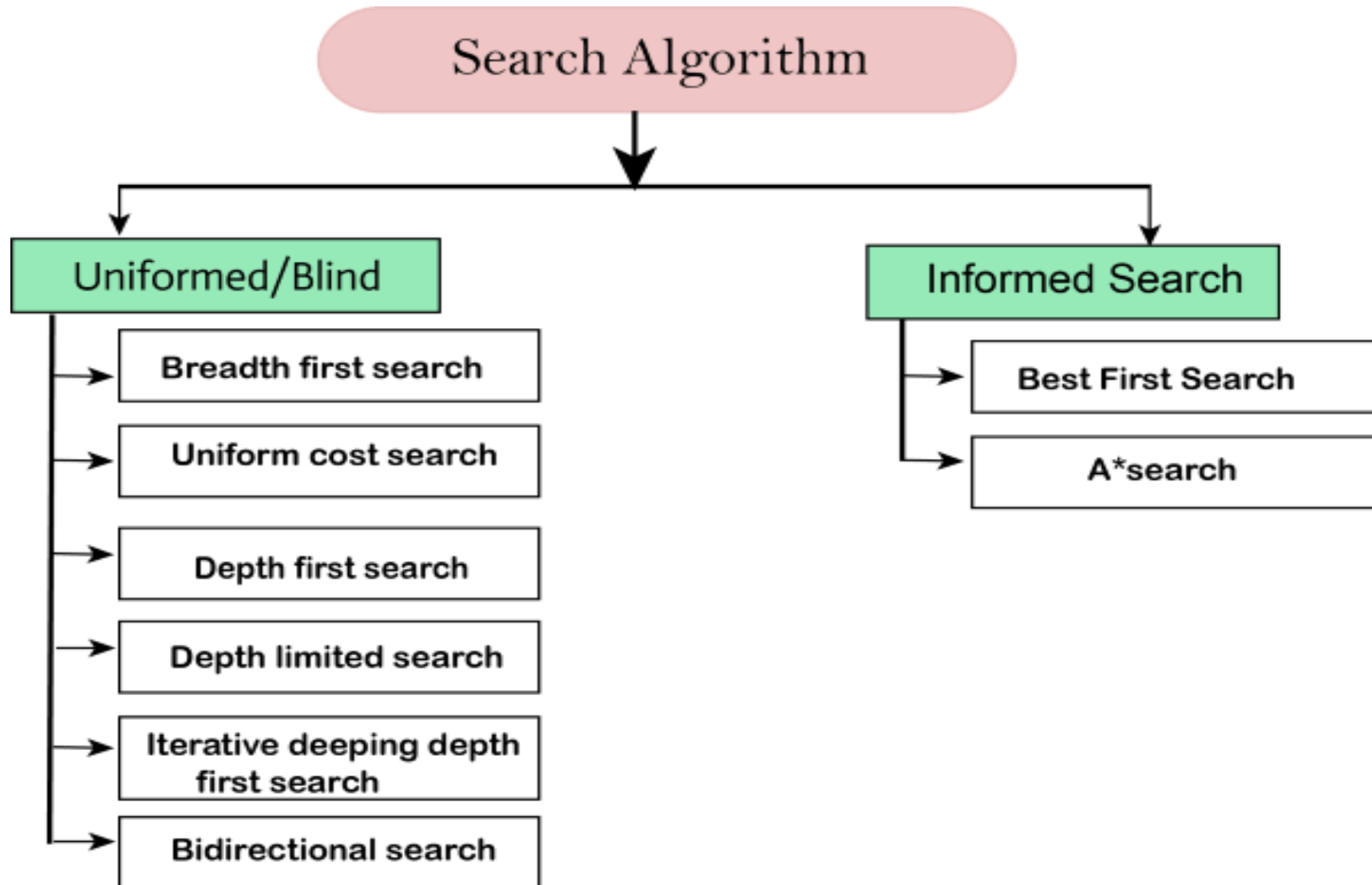
trạng thái đích

1	2	3
4	5	6
7	8	

□

# Search Algorithm

- Search approaches



# Uninformed search algorithms

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Breadth-first search

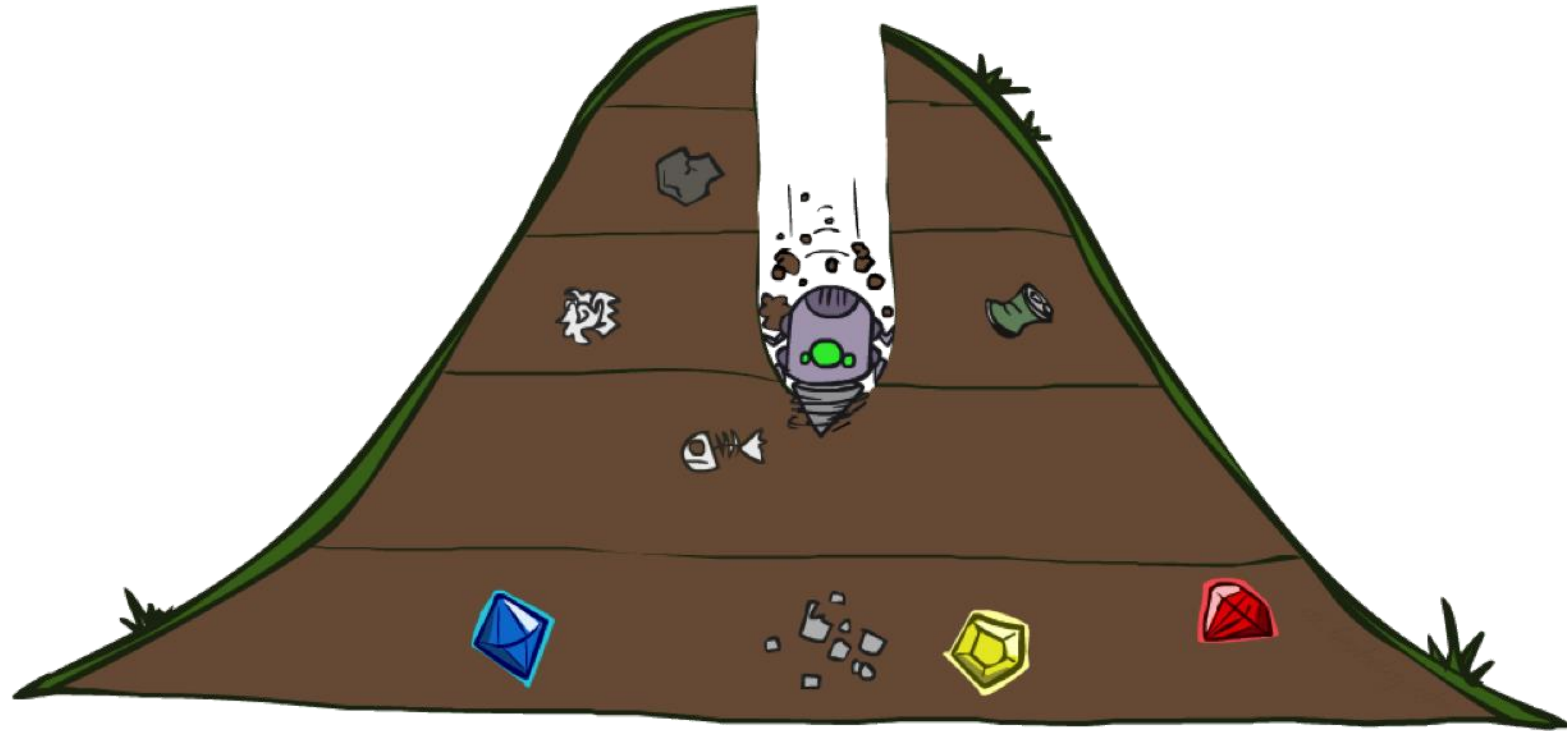
Depth-first search

Iterative Deepening

Uniform Cost Search



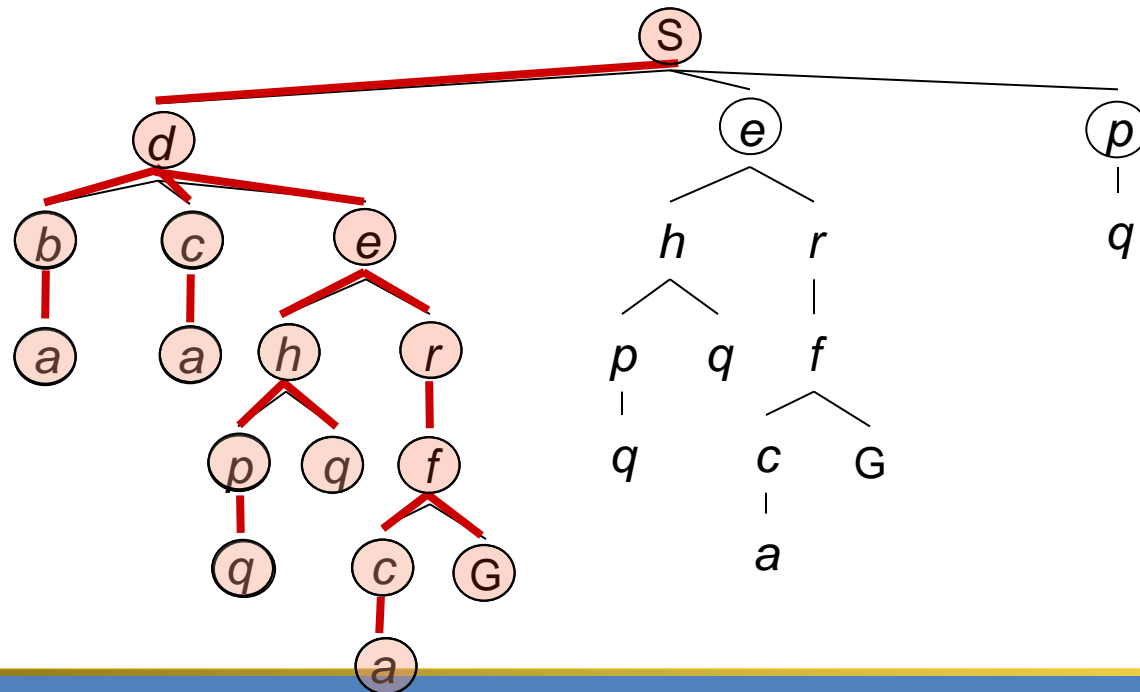
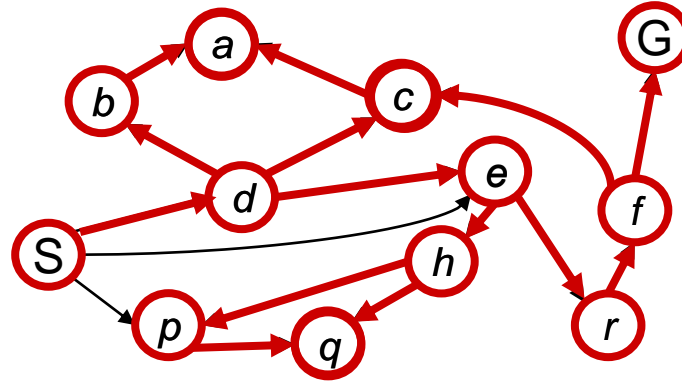
# Depth-First Search



# Depth-First Search

*Strategy: expand a  
deepest node first*

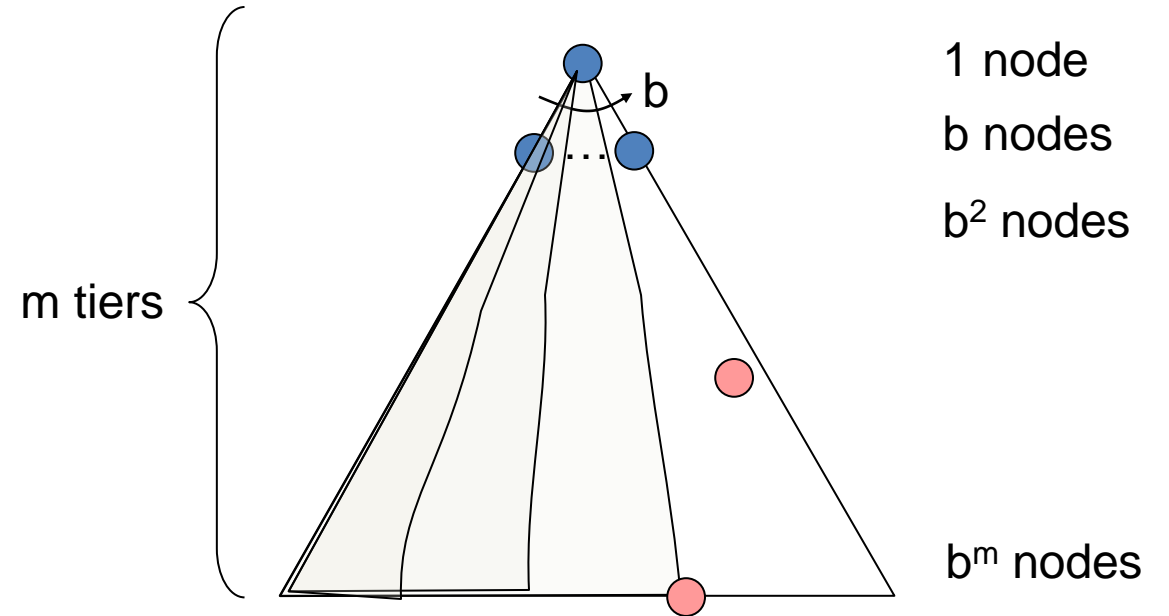
*Implementation:  
Fringe is a LIFO stack*



# Depth-First Search

## DFS Properties

- What nodes DFS expand?
  - Some left prefix of the tree.
  - Could process the whole tree!
  - If  $m$  is finite, takes time  $O(b^m)$
- How much space does the fringe take?
  - Only has siblings on path to root, so  $O(b^m)$
- Is it complete?
  - $m$  could be infinite, so only if we prevent cycles (more later)
- Is it optimal?
  - No, it finds the “leftmost” solution, regardless of depth or cost



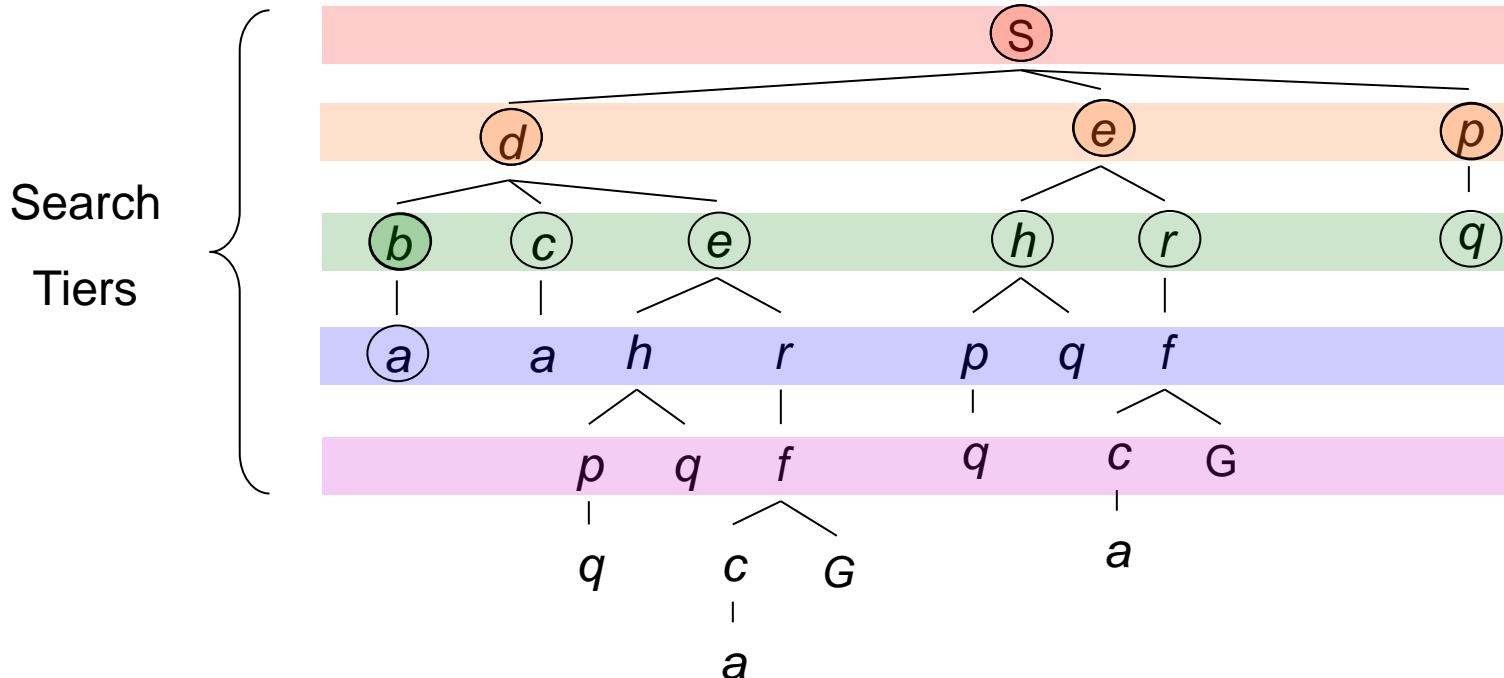
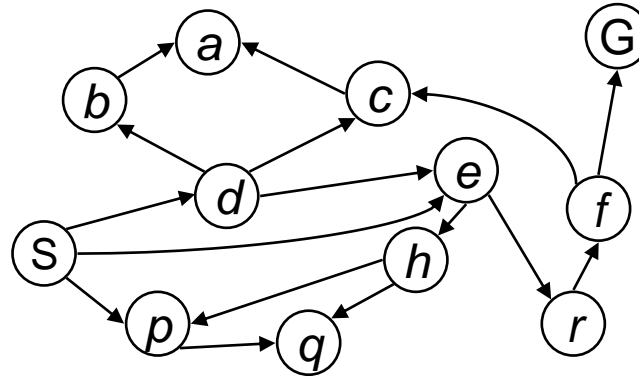
# Breadth-First Search



# Breadth-First Search

*Strategy: expand a shallowest node first*

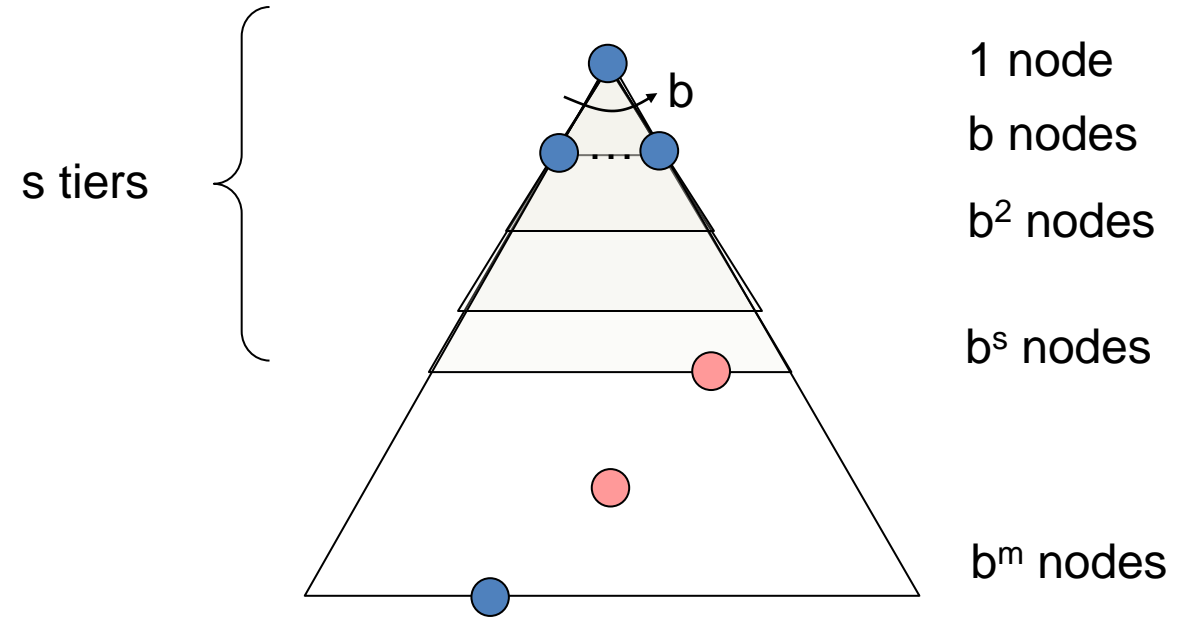
*Implementation: Fringe is a FIFO queue*



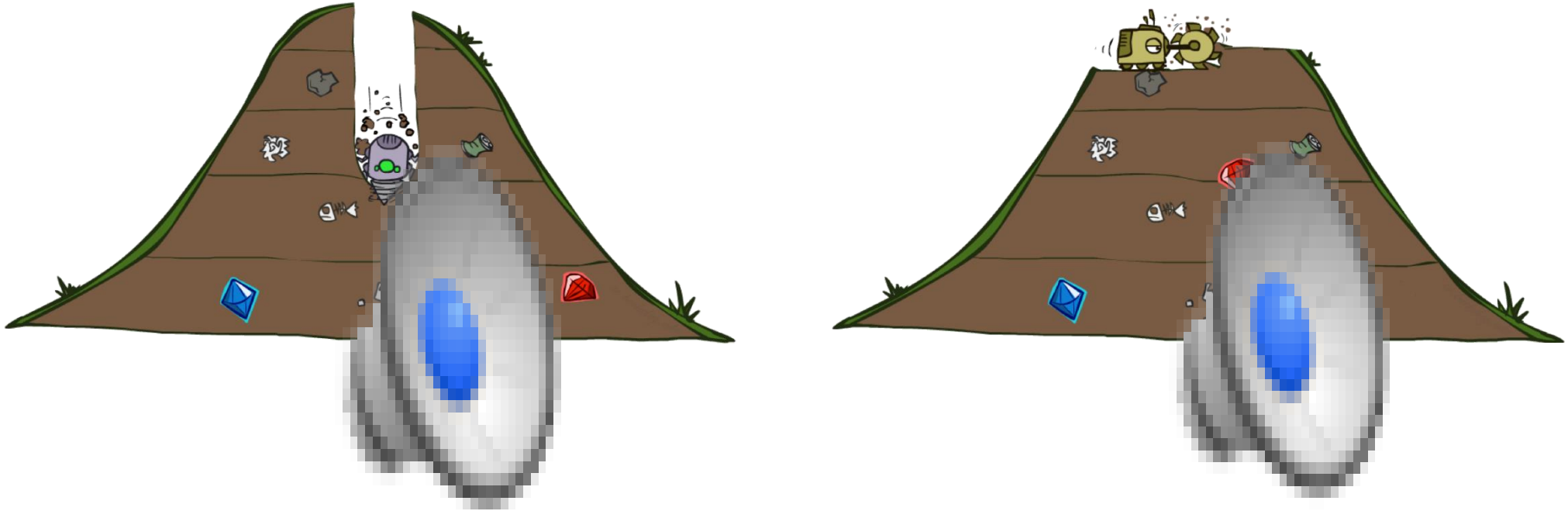
# Breadth-First Search

## BFS Properties

- What nodes does BFS expand?
  - Processes all nodes above shallowest solution
  - Let depth of shallowest solution be  $s$
  - Search takes time  $O(b^s)$
- How much space does the fringe take?
  - Has roughly the last tier, so  $O(b^s)$
- Is it complete?
  - $s$  must be finite if a solution exists, so yes!
- Is it optimal?
  - Only if costs are all 1 (more on costs later)



# DFS vs BFS

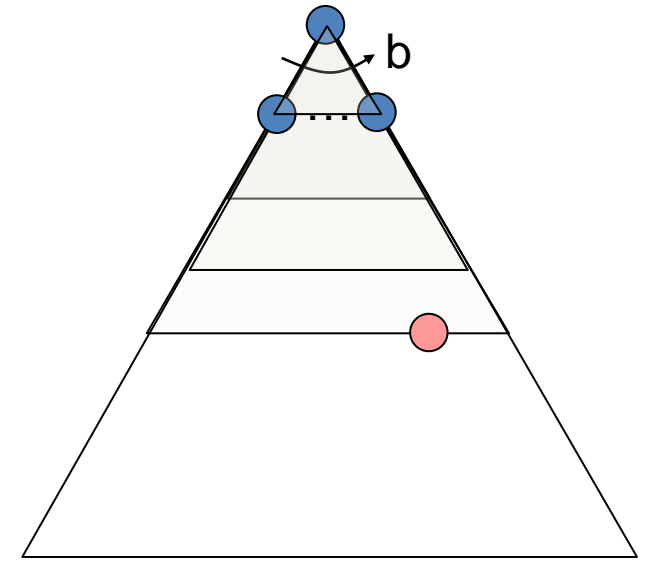


Video of  
Demo Maze  
Water  
DFS/BFS

- When will BFS outperform DFS?
- When will DFS outperform BFS?

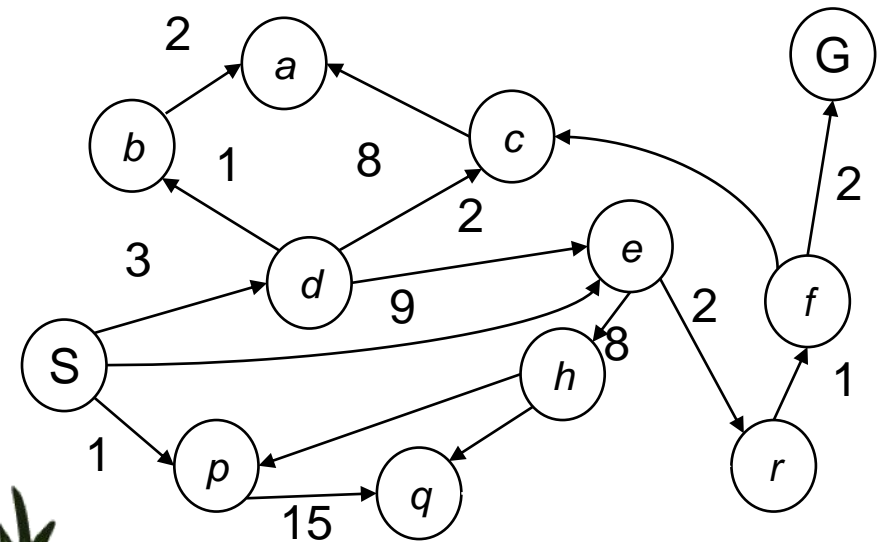
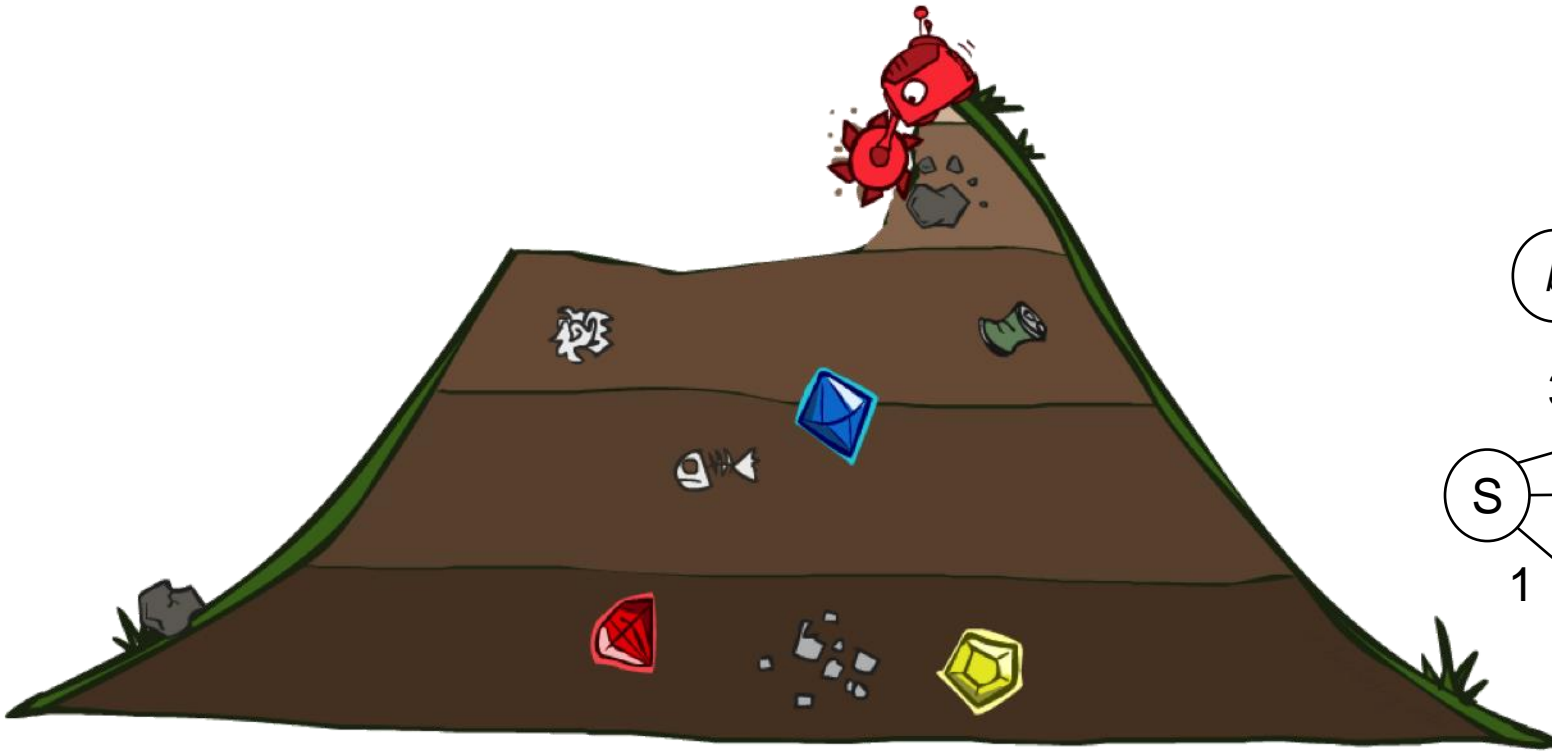
# Iterative Deepening

- Idea: get DFS's space advantage with BFS's time / shallow-solution advantages
  - Run a DFS with depth limit 1. If no solution...
  - Run a DFS with depth limit 2. If no solution...
  - Run a DFS with depth limit 3. ....
- Isn't that wastefully redundant?
  - Generally most work happens in the lowest level searched, so not so bad!





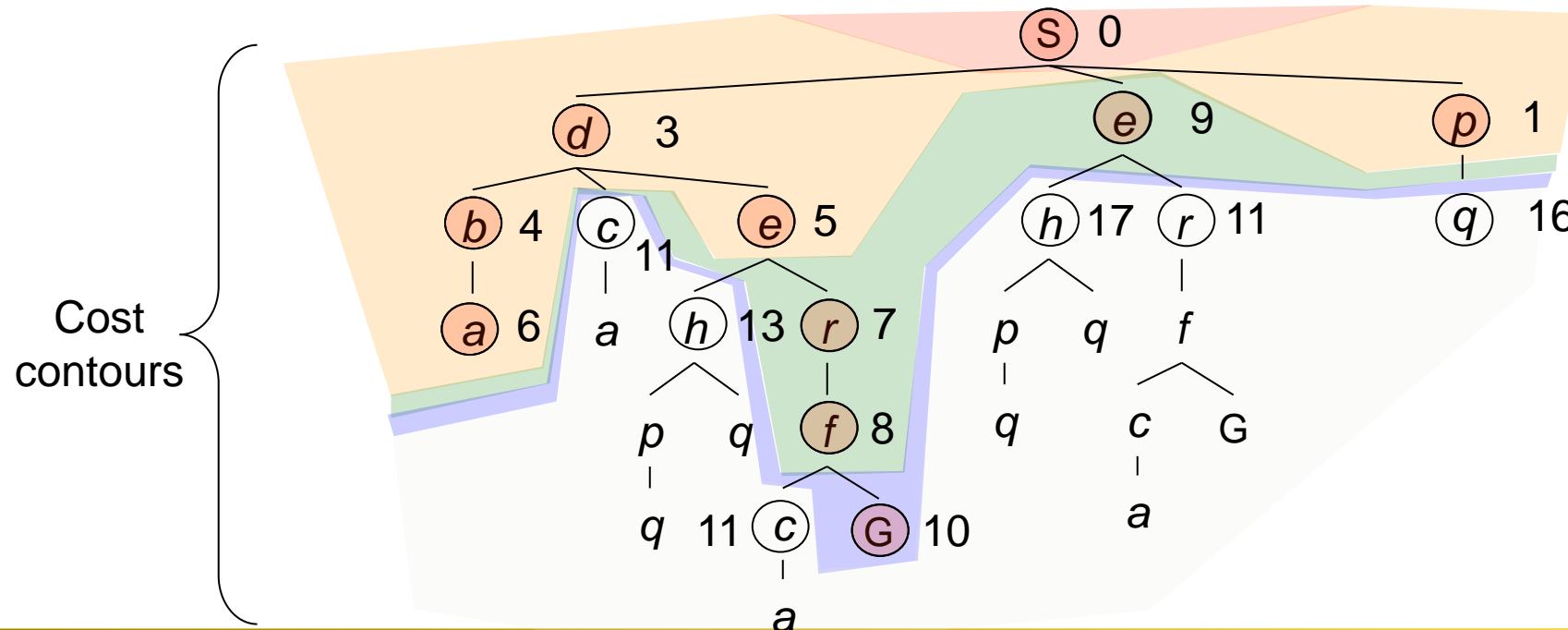
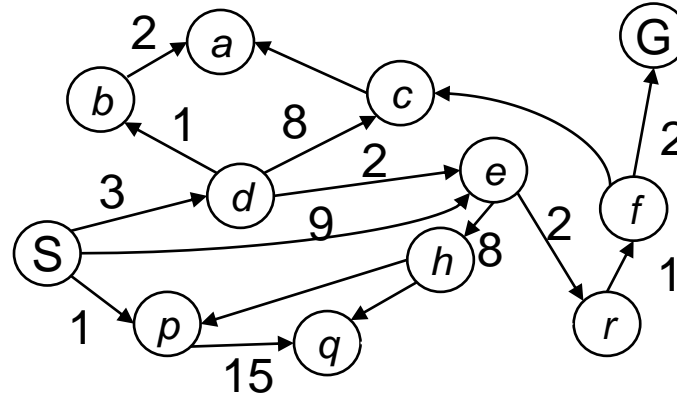
# Uniform Cost Search



# Uniform Cost Search

Strategy: expand a  
cheapest node first:

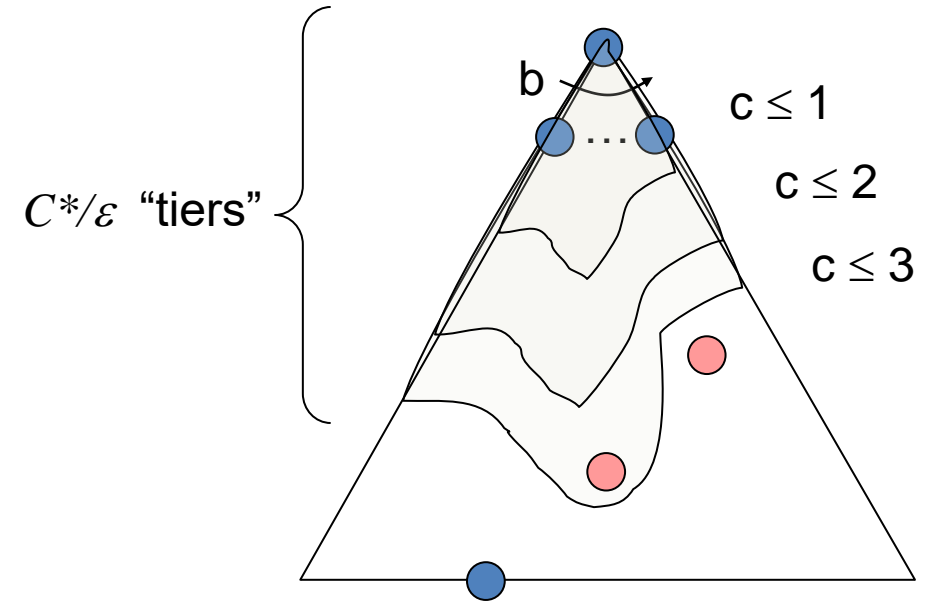
Fringe is a priority queue  
(priority: cumulative cost)



# Uniform Cost Search

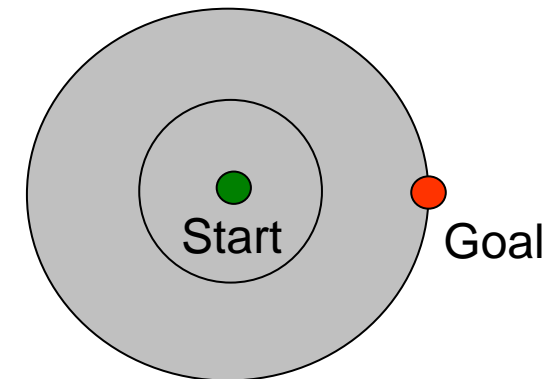
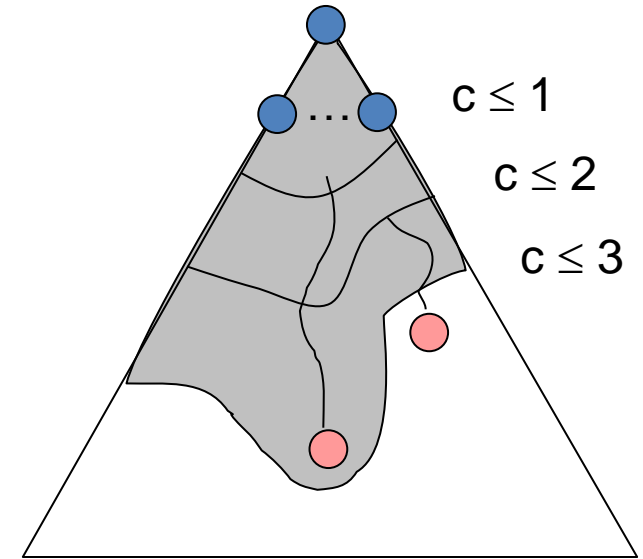
## UCS Properties

- What nodes does UCS expand?
  - Processes all nodes with cost less than cheapest solution!
  - If that solution costs  $C^*$  and arcs cost at least  $\epsilon$ , then the “effective depth” is roughly  $C^*/\epsilon$
  - Takes time  $O(b^{C^*/\epsilon})$  (exponential in effective depth)
- How much space does the fringe take?
  - Has roughly the last tier, so  $O(b^{C^*/\epsilon})$
- Is it complete?
  - Assuming best solution has a finite cost and minimum arc cost is positive, yes!
- Is it optimal?
  - Yes! (Proof next lecture via  $A^*$ )



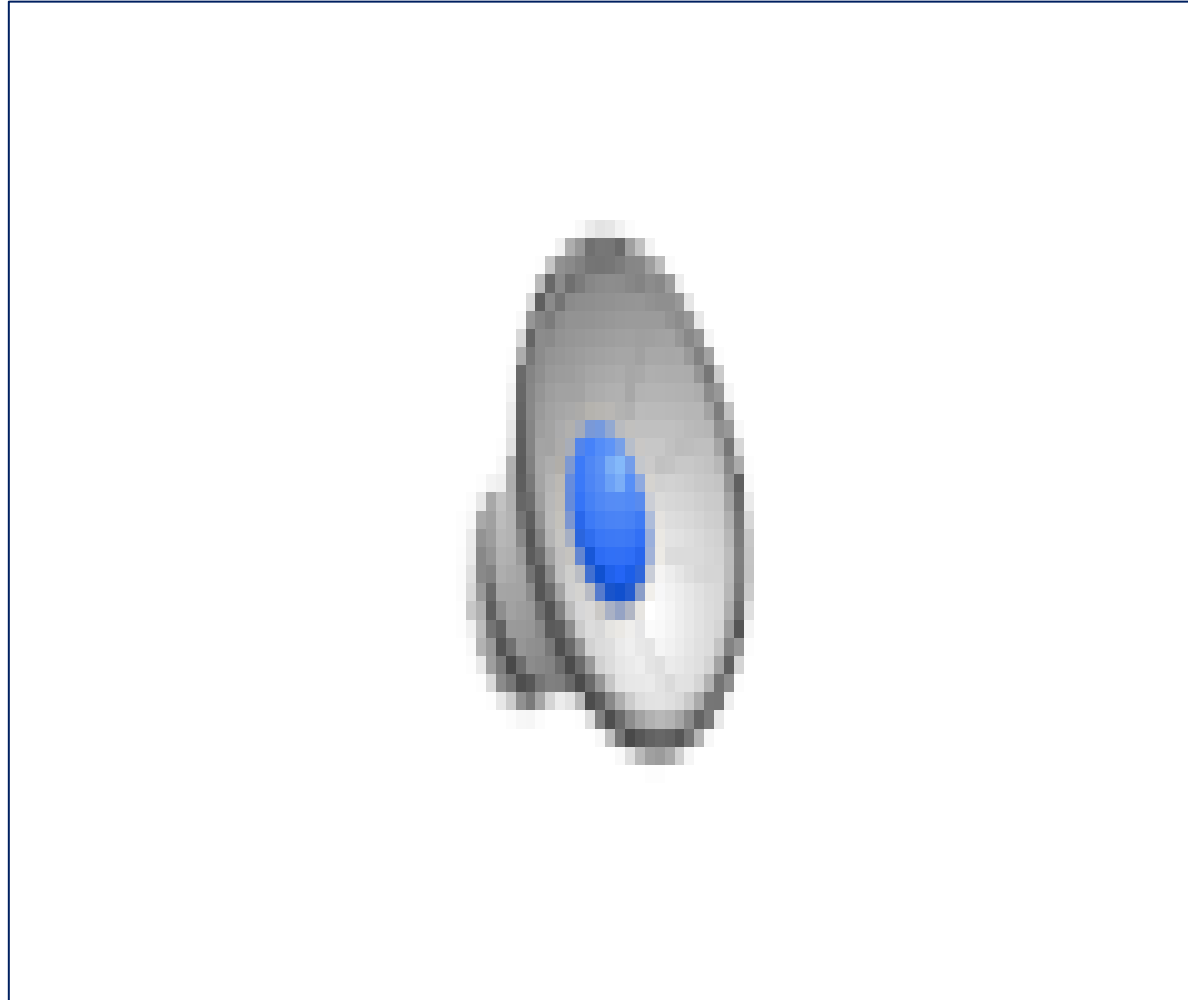
# Uniform Cost Search

- Strategy: expand lowest path cost
- good: UCS is complete and optimal!
- The bad:
  - Explores options in every “direction”
  - No information about goal location



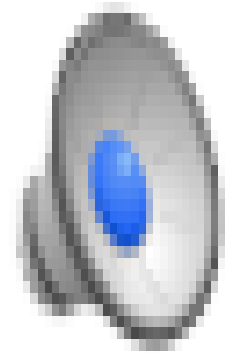
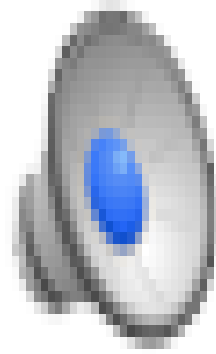
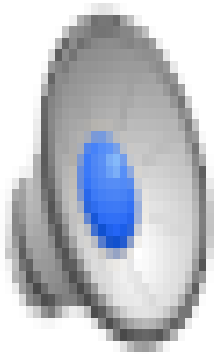
# Uniform Cost Search

- Video of Demo Empty UCS



# Video of Demo Empty UCS

Demo Maze with Deep/  
Shallow Water:  
DFS, BFS or UCS?

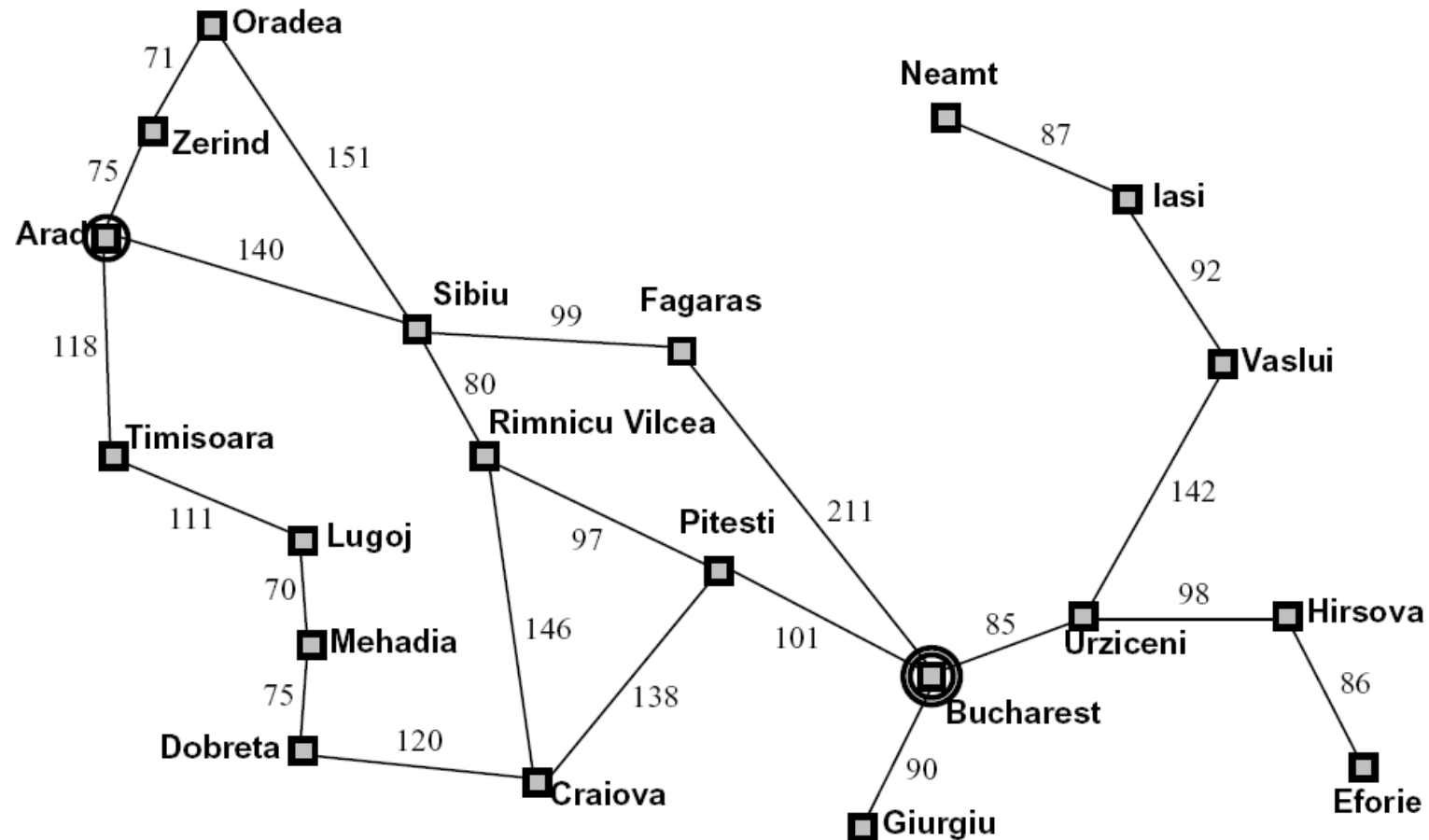


# Assignment

- Thể hiện quá trình tìm đường đi bằng phương pháp DFS, BFS, ID sâu dần và UCS, với

➤ Trạng thái đầu: **Arad**

➤ Trạng thái đích: **Bucharest**



(\*) Viết chương trình python

**Thanks for your attention!**

**Q&A**

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