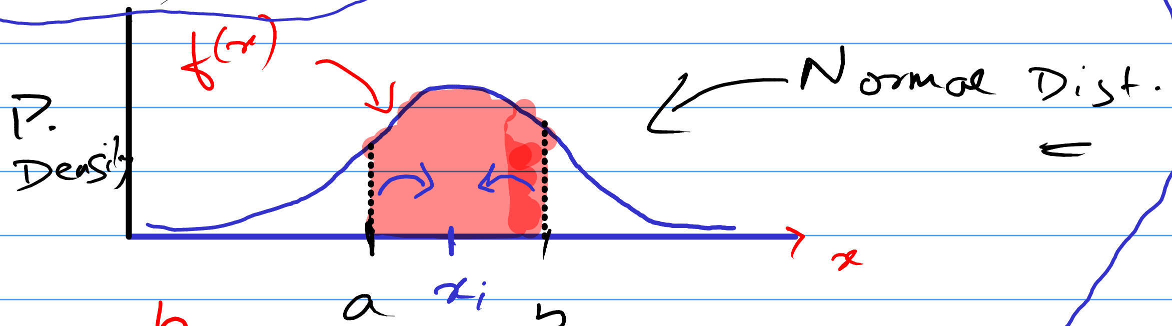
Continuous Distributions

↳ Continuous Data

Probability Density Function (PDF)

$$P(X = x_i) = 0 \quad \forall \; x \in \mathbb{R}$$

CTS data



$$P = \int_a^b f(x)\,dx$$

$$P(X = x) = \int_{x_i}^{x_i} f(x)\,dx = 0$$

For a cont. RV (X) :

$$f(x) = \lim_{\Delta \to 0^+} \frac{P(x < X \leq x + \Delta)}{\Delta} \quad \text{— (1)}$$

Here, the function f(x) gives us the Probability Density value at a certain point x. This function is the limit of the interval probability divided by the interval length as this interval length (delta) approached 0.

$$P(x < X \leq x+\Delta) = F(x+\Delta) - F(x) \quad -(2)$$

From eq^n (1) & (2), we can get,

$$f(x) = \lim_{\Delta \to 0^+} \frac{F(x+\Delta) - F(x)}{\Delta}$$

$$f(x) = \frac{dF(x)}{dx} \quad \text{or} \quad F'(x)$$

where $F(x)$ is differentiable @ $\underline{x}$.

The PDF of a given cts RV explains the relative likelihood or probability for our cts RV to take on a given value in an interval.

PDF (f) of X (cts RV) with $F(x)$ (CDF):

$$f(x) = \frac{dF(x)}{dx} \quad \text{or} \quad F'(x)$$

* PDF has 2 properties:

1. $f(x) \geq 0 \quad \forall \; x \in \mathbb{R}$
2. $\int_{-\infty}^{\infty} f(x)\,dx = 1$

Cumulative Distribution Function (CDF):

$$\text{Prob} = \text{CDF} = F(x) = \int_{-\infty}^{x} f(u)\,du$$

$$P(a \le x \le b) = \int_{a}^{b} f(x)\,dx$$

Cont. Distributions

cts Uniform　　Normal　　Log-Normal ...

1. Continuous Uniform Distribution

X (Cont. RV) b/w a & b

$$f(x) = \frac{1}{b-a},$$

where $x \in [a,b]$

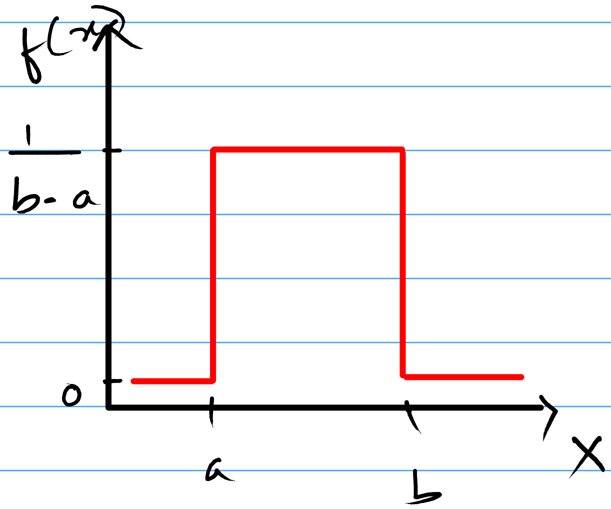
So, I could say, In the interval $[a,b]$,

X's CDF (Prob):

$$F(x) = \frac{x-a}{b-a} \quad \text{where } x \in [a,b]$$

PDF of a Cts Uniform Dist:

$$f(x) = \begin{cases} \frac{1}{b-a}, & \forall \; a \leq x \leq b \\ 0, & \forall \; x < a \text{ and } x > b \end{cases}$$

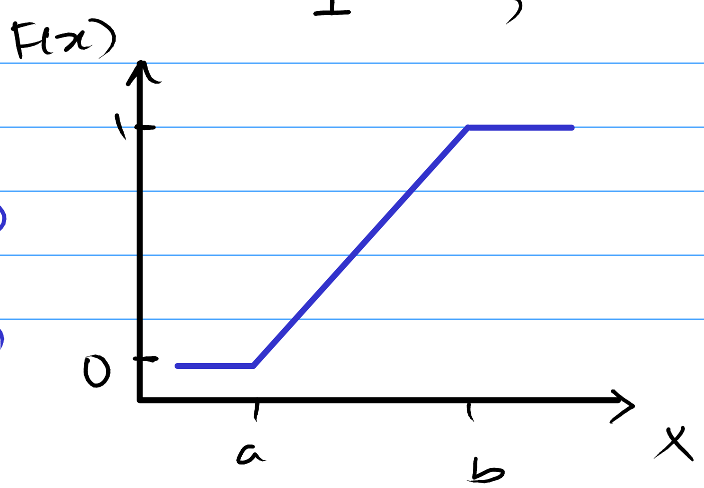


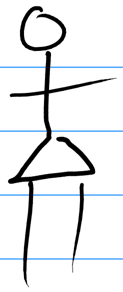
PDF

CDF of a Cts Uniform Dist:

$$F(x) = \begin{cases} 0, & \forall \; x < a \\ \frac{x-a}{b-a}, & \forall \; a \leq x \leq b \\ 1, & \forall \; x > b \end{cases}$$

CDF of Cts Uniform Dist.





4oz disher spoon $\rightarrow$ 113 gm/scoop

Suppose, permissible error margin = 10%

Q) (i) $P(\omega \in [101.7\text{ gm}, 124.3\text{ gm}])$ for a single scoop of ice-cream.

Sol^n: (i) With the help of CUD:

Experiment: $\omega \in [101.7, 124.3]$

RV (X):

Let X = weight of a single ice-cream scoop.

$\Rightarrow$ X can be any value b/w 101.7 gm & 124.3 gm

PDF & CDF:

The single ice-cream scoop weight $\in [101.7, 124.3]$

PDF, $f(x) = \frac{1}{b-a} = \frac{1}{124.3 - 101.7}$

$= 0.04423$

Thus, X is a CUD with prob 4.423% $\forall$ values b/w 101.7gm & 124.3 gm.

(ii) P(Weight being 103 gm for the ice-cream scoop)?

$x = 103$

CDF, $f(x=103) = \frac{x-a}{b-a} = \frac{103-101.7}{124.3-101.7}$

$= 0.0575$.

Thus, the prob of weight being 103 gm for the ice-cream scoop $= 0.575$ or 5.75%