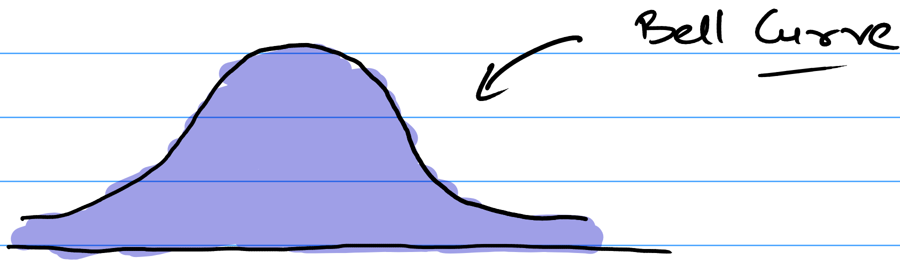
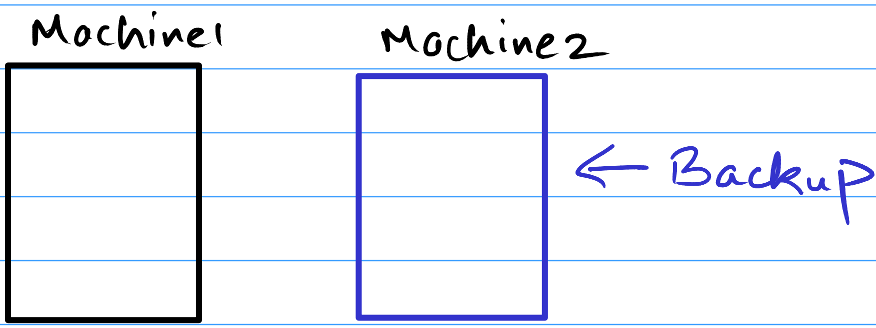
The Normal Distribution





{Vanilla, Chocolate, Strawberry}

Scenario:

Complaint: Sashi got a complaint that the quantity of ice-cream served was less compared to last time!

Sol^n: Sashi should measure the weight of each ice-cream OR she can set an acceptable limit & check if the prob. of weight of ice-cream is falling within or outside the acceptable range.

Manufacturer: " Mean Weight = 95 gm
S.D. = 11 gm

if acceptable range $\in [84, 117]$ gm

Then, Prob. of weight of ice-cream falling within & outside this range?

Sol^n: Acceptable range ∈ [84, 117] gm

⇒ it is a cont. distribution

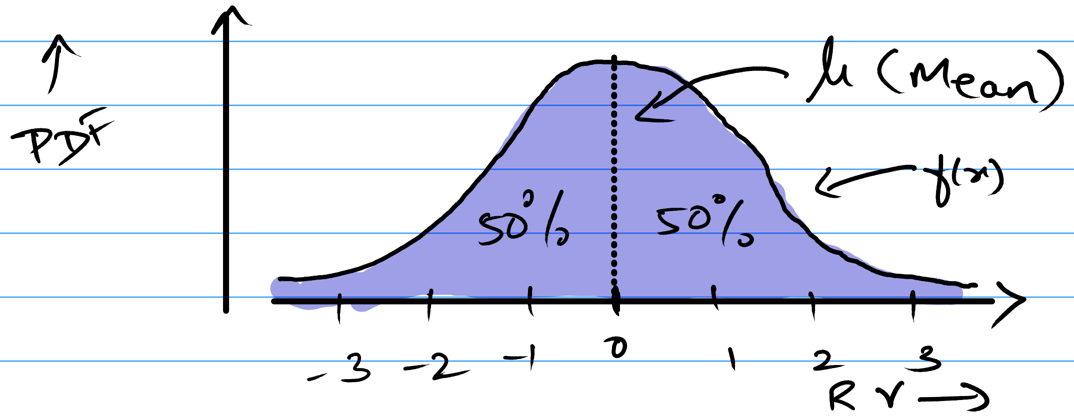
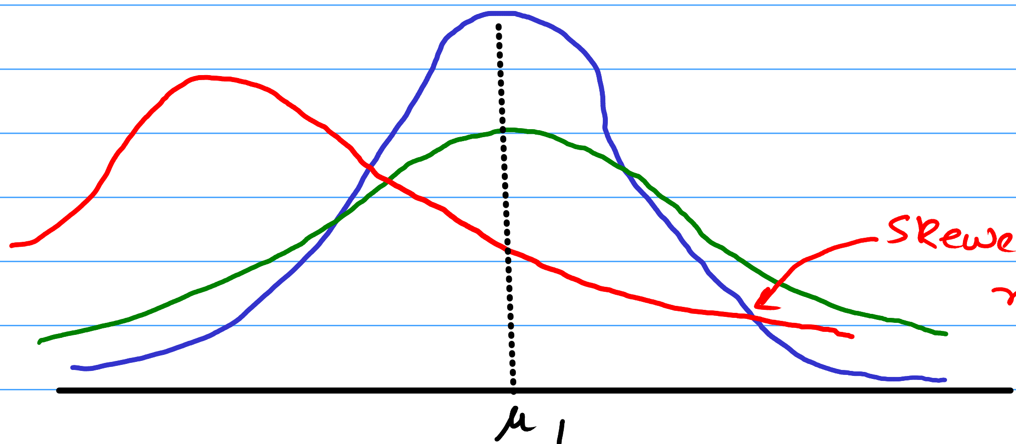
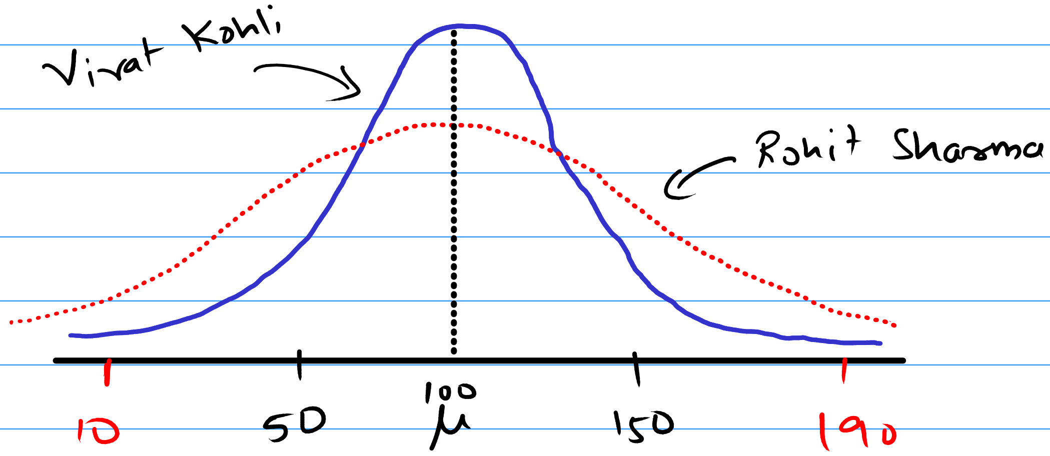


Fig. Normal Dist.

$$\text{Area} = \int_{-\infty}^{\infty} f(x)\,dx = 1$$



$\mu_{Blue} = \mu_{Green}$ | $\text{Variance}_{green} > \text{Variance}_{blue}$



Assuming $\mu_V = \mu_R = 100$ Runs

$\text{Variance}_R > \text{Variance}_V$

The PDF (Normally distributed RV):

$$f(x, \mu, \sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$$

where $x \in \mathbb{R} \Rightarrow (-\infty, \infty)$

x is a Normal RV
μ = Mean
σ = standard Deviation

$\pi = 3.14$
$e = 2.718$

Standard Normal Distribution – PDF

* $\mu = 0$
* $\sigma = 1$

PDF : $f(x) = \dfrac{1}{\sqrt{2\pi}} \cdot e^{-x^2/2}$

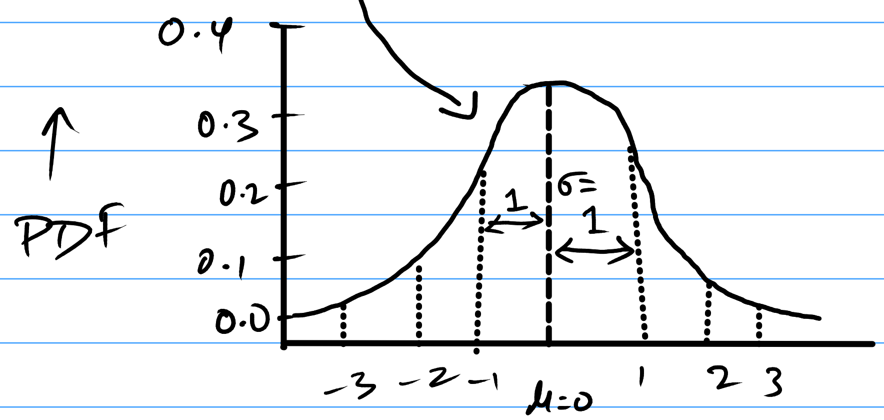
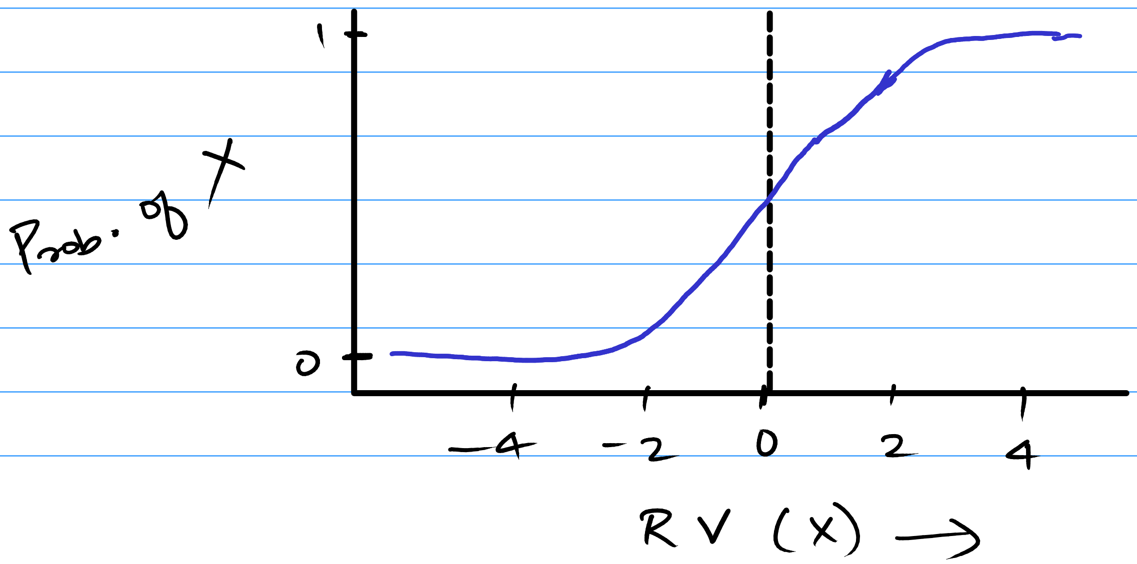


Fig. Standard Normal Dist.

CDF of a Normal Dist:

$$F(x) = \int_{-\infty}^{x} f(x)\,dx = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} \cdot e^{-x^2/2}\,dt$$



- Weight (x) : $0 < x < 84$
- w (x) : $117 < x < \infty$
- weight(x) : $84 \leq x \leq 117$

Z-score (Standard Score) = total # of S.D.'s from the mean by a RV.

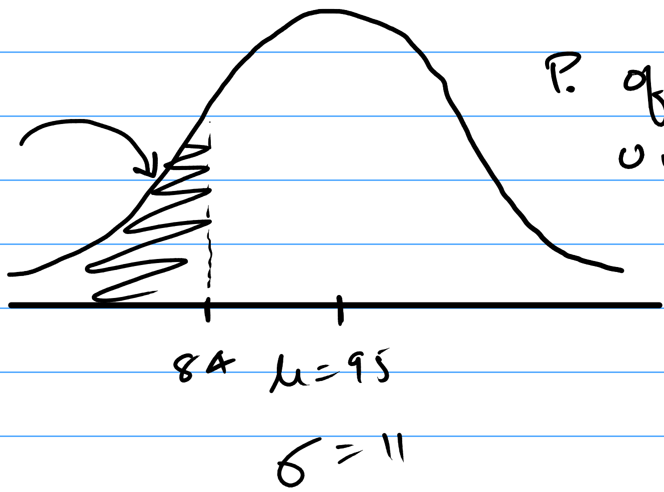
$$Z = \frac{x - \mu}{\sigma}$$

$\mu = 95$
$\sigma = 11$

- P (weight < 84gm) :

$$Z = \frac{x-\mu}{\sigma} = \frac{84-95}{11} = -1$$

P = 15.87%



P. of weight under 84gm

P of weight over 117 gms

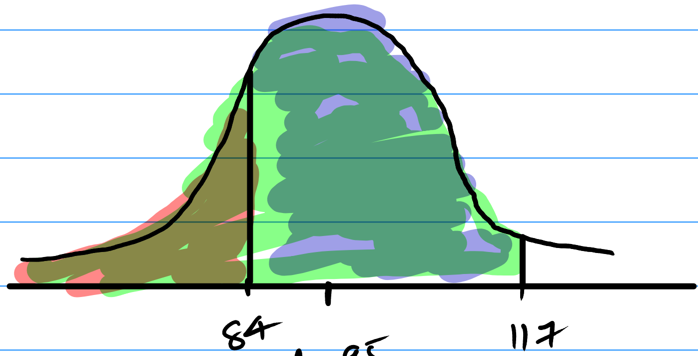
$\mu = 95$

$\sigma = 11$

$= 1 - \text{CDF}(117)$

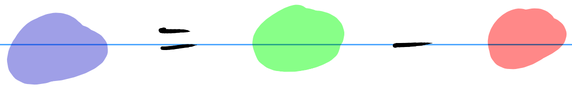
$= 0.0227$

$\approx 2.27\%$



$\mu = 95$

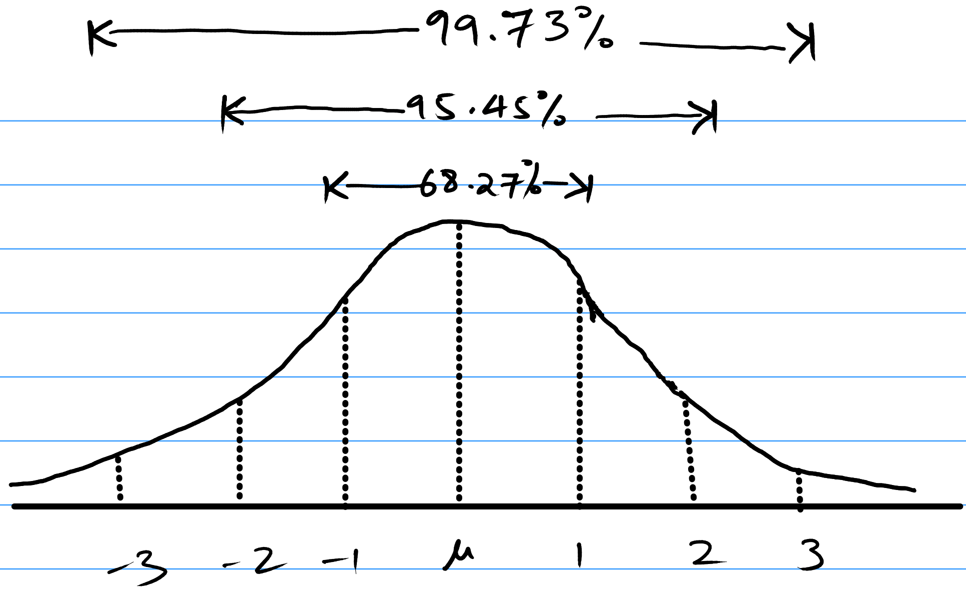
$\sigma = 1$



$= 81.85\%$

Empirical Rule:

* 68.27% $\in$ 1 SD of Mean
* 95.45% $\in$ 2 SD of Mean
* 99.73% $\in$ 3 SD of Mean


99.73%
95.45%
68.27%
-3
-2
-1
μ
1
2
3