Given

Slew Attitude $\left([\phi, \theta, \psi]^T\right)$

Property	Value
Initial Slew Attitude	$\begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^\circ$
Desired Slew Attitude	$\begin{bmatrix} 30 & 50 & 70 \end{bmatrix}^\circ$

Rate $([\dot{\phi}, \dot{\theta}, \dot{\psi}]^T)$

Property	Value
Initial Rate	$\begin{bmatrix} 15 & -15 & 15 \end{bmatrix}^\circ /s$
Desired Rate	$\begin{bmatrix} 0.2 & 0.2 & 0.2 \end{bmatrix}^\circ /s$

Moment of Inertia (I)

Property	Value
MOI of the Reaction Wheel	$0.00042\, kgm^2$
MOI of the satellite	$\begin{bmatrix} 2.10 & 0.00 & 0.01 \\ 0.00 & 2.30 & -0.03 \\ 0.01 & -0.03 & 1.72 \end{bmatrix} kgm^2$

Torque (τ) and Angular Momentum (H)

Property	Value
Disturbance Torque acting on the satellite (sinusoidal in ECI Frame)	$\begin{bmatrix} 10^{-6} & 10^{-6} & 10^{-6} \end{bmatrix} Nm$
Maximum Torque generated by the Reaction Wheel	$0.015\, Nm$
Angular Momentum generated by the Reaction wheel	$0.035\, Nms$

Approach

Problem Approach

- Using a PD control
- Propagating attitude using quaternions
- Computing attitude error through quaternion error matrix
- Computing rate error through angular velocity error vector

Assumptions

- Satellite behaves as a rigid body.
- Moments of inertia are constant over time.
- Reaction wheels are ideal, with no friction losses, misalignments, significant delays, or backlashes.
- Reaction wheel dynamics are fast enough to be considered instantaneous, with no motor dynamics modeled.
- Reaction wheels' spin axes are aligned with the satellite's body frame axes.

Assumptions (Contd.)

- No external disturbances other than the specified sinusoidal torque.
- Rotation sequence is 3-2-1 (Yaw-Pitch-Roll or Z-Y-X).
- Angles and rates given are related to roll, pitch, and yaw in that order.
- Satellite's orbit and orbital perturbations are not considered, as the focus is on attitude control.
- No sensor modeling, since assuming all parameters are perfectly measured in an ideal system.

References

- Optimization of Pyramidal Reaction Wheel Configuration for Minimizing Angular Momentum by Ali Kasiri, Farhad Fani Saberi, and Mehdi Kashkul
- Spacecraft Dynamics and Control: A Practical Engineering Approach by Marcel J. Sidi

Equations

Quaternion Error

$$\boldsymbol{q}_e = \frac{1}{2}\begin{bmatrix} q_{d_4} & q_{d_3} & -q_{d_2} & q_{d_1} \\ -q_{d_3} & q_{d_4} & q_{d_1} & q_{d_2} \\ q_{d_2} & -q_{d_1} & q_{d_4} & q_{d_3} \\ -q_{d_1} & -q_{d_2} & -q_{d_3} & q_{d_4} \end{bmatrix}\begin{bmatrix} -q_1 \\ -q_2 \\ -q_3 \\ q_4 \end{bmatrix}$$

where,

- $\boldsymbol{q}_e$ is the error quaternion
- $\boldsymbol{q}_d = [q_{d_1}, q_{d_2}, q_{d_3}, q_{d_4}]^T$ is the desired quaternion
- $\boldsymbol{q} = [q_1, q_2, q_3, q_4]^T$ is the current quaternion

Angular Velocity Error

$$\omega_e = \omega_d - \omega$$

where,

- ω_e is the angular velocity error
- ω_d is the desired angular velocity
- ω is the current angular velocity

PD Control

$$
\begin{aligned}
T_{pd_x} &= 2k_{p_x} q_{e_1} q_{e_4} + k_{d_x} \omega_{x_e} \\
T_{pd_y} &= 2k_{p_y} q_{e_2} q_{e_4} + k_{d_y} \omega_{y_e} \\
T_{pd_z} &= 2k_{p_z} q_{e_3} q_{e_4} + k_{d_z} \omega_{z_e}
\end{aligned}
$$

where,

- $\boldsymbol{T}_{pd} = [T_{pd_x}, T_{pd_y}, T_{pd_z}]^T$ are the control torques
- $k_p = [k_{p_x}, k_{p_y}, k_{p_z}]^T$ are the proportional gains
- $k_d = [k_{d_x}, k_{d_y}, k_{d_z}]^T$ are the derivative gains

Disturbance Torque

$$\boldsymbol{\tau}_{d_{SBF}} = R_{ECItoSBF}\boldsymbol{\tau}_{d_{ECI}}$$
$$= \begin{bmatrix} 1-2(q_3^2+q_4^2) & 2(q_2q_3-q_1q_4) & 2(q_2q_4+q_1q_3) \\ 2(q_2q_3+q_1q_4) & 1-2(q_2^2+q_4^2) & 2(q_3q_4-q_1q_2) \\ 2(q_2q_4-q_1q_3) & 2(q_3q_4+q_1q_2) & 1-2(q_2^2+q_3^2) \end{bmatrix} \begin{bmatrix} 10^{-6}\sin(t) \\ 10^{-6}\sin(t) \\ 10^{-6}\sin(t) \end{bmatrix}$$

where,

- $\boldsymbol{\tau}_{d_{ECI}}$ is the external disturbance vector in Earth-Centered Inertial frame
- $\boldsymbol{\tau}_{d_{SBF}}$ is the external disturbance vector in Satellite Body-Fixed frame
- $R_{ECItoSBF}$ is the rotation matrix from ECI to SBF frame

Reaction Wheel Configuration Matrix

$$
\begin{aligned}
\boldsymbol{\tau}_{rw} &= A^{\dagger}\boldsymbol{\tau}_{pd} \\
\boldsymbol{\tau}_{c} &= A\boldsymbol{\tau}_{rw} \\
&= \begin{bmatrix} \cos(\beta) & 0 & -\cos(\beta) & 0 \\ 0 & \cos(\beta) & 0 & -\cos(\beta) \\ \sin(\beta) & \sin(\beta) & \sin(\beta) & \sin(\beta) \end{bmatrix} \boldsymbol{\tau}_{rw}
\end{aligned}
$$

where,

- $\boldsymbol{\tau}_{rw}$ is a vector of torques generated by the 4 reaction wheels
- $A, A^{\dagger}$ is the reaction wheel distrbution matrix and its pseduoinverse
- $\boldsymbol{\tau}_{c}$ is a vector of the final three axes torque output

Dynamics Equation of Motion

$$\boldsymbol{\tau}_c + \boldsymbol{\tau}_{d_{SBF}} = I\dot{\boldsymbol{\omega}} + AI_{rw}\dot{\boldsymbol{\omega}}_{rw} + \boldsymbol{\omega} \times (I\boldsymbol{\omega} + AI_{rw}\boldsymbol{\omega}_{rw})$$
$$\dot{\boldsymbol{\omega}}_{rw} = \frac{\boldsymbol{\tau}_{rw}}{I_{rw}}$$

where,

- I is the satellite moment of inertia matrix
- I_{rw} is the reaction wheel moment of inertia
- $\boldsymbol{\omega}$ is the angular velocity of the satellite
- $\boldsymbol{\omega}_{rw}$ is the angular velocity of the reaction wheel

Quaternion Kinematics

$$\begin{aligned}\dot{\boldsymbol{q}} &= \frac{1}{2}\Omega\boldsymbol{q} \\ &= \frac{1}{2}\begin{bmatrix} 0 & \omega_z & -\omega_y & \omega_x \\ -\omega_z & 0 & \omega_x & \omega_y \\ \omega_y & -\omega_x & 0 & \omega_z \\ -\omega_x & -\omega_y & -\omega_z & 0 \end{bmatrix}\boldsymbol{q}\end{aligned}$$

where,

- $\dot{\boldsymbol{q}}$ is the quaternion rate
- Ω is an angular velocity skew symmetric matrix

Euler Angles to Quaternion for ZYX Rotation Sequence

$$
\begin{aligned}
q_1 &= \sin\left(\frac{\psi}{2}\right)\sin\left(\frac{\theta}{2}\right)\sin\left(\frac{\phi}{2}\right) + \cos\left(\frac{\psi}{2}\right)\cos\left(\frac{\theta}{2}\right)\cos\left(\frac{\phi}{2}\right) \\
q_2 &= -\sin\left(\frac{\psi}{2}\right)\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\phi}{2}\right) + \cos\left(\frac{\psi}{2}\right)\cos\left(\frac{\theta}{2}\right)\sin\left(\frac{\phi}{2}\right) \\
q_3 &= \sin\left(\frac{\psi}{2}\right)\cos\left(\frac{\theta}{2}\right)\sin\left(\frac{\phi}{2}\right) + \cos\left(\frac{\psi}{2}\right)\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\phi}{2}\right) \\
q_4 &= \sin\left(\frac{\psi}{2}\right)\cos\left(\frac{\theta}{2}\right)\cos\left(\frac{\phi}{2}\right) - \cos\left(\frac{\psi}{2}\right)\sin\left(\frac{\theta}{2}\right)\sin\left(\frac{\phi}{2}\right)
\end{aligned}
$$

where,

- ϕ is the satellite's longitudinal axis (Roll) rotation
- θ is the satellite's transverse axis (Pitch) rotation
- ψ is the satellite's normal axis (Yaw) rotation

Quaternion to Euler Angles for ZYX Rotation Sequence

$$\phi = \tan^{-1}\left(\frac{2(q_1q_2 + q_3q_4)}{q_1^2 - q_2^2 - q_3^2 + q_4^2}\right)$$
$$\theta = -\sin^{-1}\left(2(q_2q_4 - q_1q_3)\right)$$
$$\psi = \tan^{-1}\left(\frac{2(q_1q_4 + q_2q_3)}{q_1^2 + q_2^2 - q_3^2 - q_4^2}\right)$$