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Optimization of Pyramidal Reaction Wheel Configuration for Minimizing Angular Momentum

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***Abstract*— many spacecraft using more than 3 reaction wheels for both redundancy and extra maneuverability capabilities. Actuators configuration plays an effective role in the performance of the attitude control system. This paper deals with the management of 4 reaction wheels used simultaneously in the pyramidal configuration. Due to the saturation phenomenon, it is very important to minimizing the angular momentum in the case of using momentum exchange devices. Therefore this paper investigates the best tilt-angle of wheels in pyramidal configuration to perform the maneuver with minimum angular momentum.**

***Index Terms*— Attitude control system, Momentum management, Over actuated spacecraft, Reaction wheel configuration,**

I. INTRODUCTION

Reaction wheels are cost-effective actuators commonly used in attitude control systems of spacecraft to perform high-precision, high-accuracy, and agility. At least three reaction wheels (RWs) are needed for 3-axis attitude control of the spacecraft [1]. Nevertheless, despite all the well-known advantages of RWs they suffer from momentum saturation and bearing failure [2]. If one RW becomes damaged, then the satellite's attitude can no longer be adequately controlled. Obviously, replacing any hardware in space is very risky and expensive, if not impossible [3]. Thus, the principles of product assurance and reliability are of utmost importance in the space industry. The most practical solution to deal with reaction wheel failure is using an extra wheel as the hardware redundancy. Accordingly, Many spacecraft employ more than three wheels, both for (i) redundancy and for the (ii) additional maneuvering capability of the extra wheels [4]. Among all the configuration of 4 reaction wheels, pyramidal configuration is more practical case, which also attracted huge attention. Figure 1 shows 4 RWs in the pyramidal configuration.

Shirazi investigated the optimal RWs tilt-angle in the pyramidal configuration in a minimum power consumption point of view [5] Jin focused on fault-tolerant control method based on dynamics inversion and time-delay control approach for satellites with four reaction wheels in pyramidal configuration [6]. Reynolds stated that Spacecraft reaction wheel maneuvers are limited by the maximum torque and/or angular momentum which the wheels can provide in each direction. He analyzed 4 RWs momentum envelope (workspace) specifications in different configurations [7].

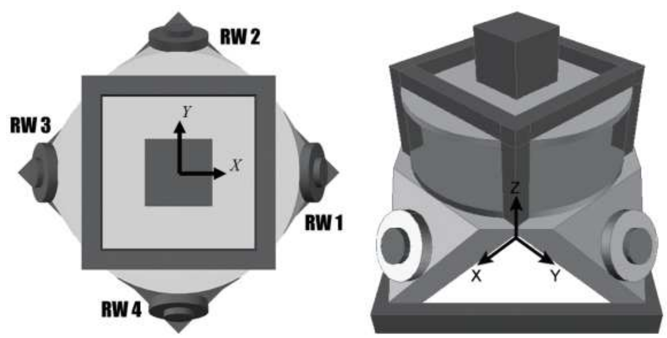

Figure 1. Pyramidal configuration of 4 RWs

Lebedev and Hogan designed a control strategy to obtain perfect tracking attitude control with continuous momentum dumping based on 4-RWs [8, 9]. Kron studied several ways for optimal and simultaneous use of 4-RWs to achieve high pointing-accuracy in the presence of torque and angular velocity constraints [10]. Zhang designed a controller based on sliding mode observer to eliminate RWs misalignment and installation deviation in pyramidal configuration [11]. Several configurations based on four reaction wheels are investigated in order to identify the most suitable configuration that provides maximum pointing accuracy by Ismail [12]. Schaub developed a power-optimal RW torque distribution strategy that minimizes the instantaneous electrical power requirements [13]. He claimed that the proposed control strategy is able to reduce the amount of mechanical power and energy required by about 10–20%, while only marginally increasing the average required torque. Blenden developed an analytical instantaneous power-optimal attitude control for a spacecraft using an integrated RW array system allowing for energy storage and return [14]. He showed that in the case of over actuated attitude control system it is possible to regenerate electrical power in RWs slowing down phase.

In the case of using reaction wheel array, minimization of the wheels' angular momentum is a matter of great concern in order to prevent frequent saturation and control performance degradation. However, no discussion is made about the 4-RWs angular-momentum-minimization strategy in pyramidal configuration so far, despite being of great theoretical and practical interest. Therefore, the question of which tilt-angle of RWs performs maneuver with minimum angular momentum has remained unanswered. The main goal of this paper is to find the optimum RWs tilt-angle in pyramidal configuration to perform the maneuver with minimum angular momentum.

This paper is organized as follows: the rigid spacecraft attitude dynamics and kinematic equations are presented in section 2. The conventional PD-based "quaternion-error" controller is proposed in the 3rd section. simulation-based optimization of the RWs tilt-angle and angular momentum minimization strategy are presented in chapter 4. Finally, the conclusions are summarized in Section 5.

II. EQUATION OF MOTION

Rigid spacecraft dynamic and kinematic will be modeled in this section

A. Dynamic Modeling

The dynamic equation of a rigid spacecraft equipped with reaction wheels can be found in many references. We used the dynamic equation presented in [4] by Sidi because of familiar notations.

$$\begin{aligned} T_x &= \dot{h}_x + \dot{h}_{wx} + (\omega_y h_z + \omega_z h_y) + (\omega_y h_{wz} + \omega_z h_{wy}) \\ T_y &= \dot{h}_y + \dot{h}_{wy} + (\omega_z h_x + \omega_x h_z) + (\omega_z h_{wx} + \omega_x h_{wz}) \\ T_z &= \dot{h}_z + \dot{h}_{wz} + (\omega_x h_y + \omega_y h_x) + (\omega_x h_{wy} + \omega_y h_{wx}) \end{aligned} \tag{1}$$

where $T = [T_x, T_y, T_z]^T \epsilon\, \boldsymbol{\mathcal{R}}^3$ represents total torque (control and disturbance torque) applied to the body frame. $h_w = [h_{wx}, h_{wy}, h_{wz}]^T \epsilon\, \boldsymbol{\mathcal{R}}^3$ is RWs angular momentum. $h = [h_x, h_y, h_z]^T \epsilon\, \boldsymbol{\mathcal{R}}^3$ is spacecraft body angular momentum. $\omega = [\omega_x, \omega_y, \omega_z]^T \epsilon\, \boldsymbol{\mathcal{R}}^3$ denotes body angular velocity with respect to the inertia frame.

B. Kinematic modeling

The quaternion parameters $\boldsymbol{Q} = [q_4, \boldsymbol{q}^T]^T \epsilon\, \boldsymbol{\mathcal{R}} \times \boldsymbol{\mathcal{R}}^3$ are used to define the spacecraft's attitude orientation between the body-fixed frame and the inertial frame due to their non-singularity. The kinematic equations of a spacecraft are given in Eq. (2) [4].

$$\begin{aligned} \dot{q}_1 &= \frac{1}{2}(q_4\omega_x - q_3\omega_y + q_2\omega_z) \\ \dot{q}_2 &= \frac{1}{2}(q_3\omega_x - q_4\omega_y - q_1\omega_z) \\ \dot{q}_3 &= \frac{1}{2}(-q_2\omega_x + q_1\omega_y + q_4\omega_z) \\ \dot{q}_4 &= \frac{1}{2}(-q_1\omega_x - q_2\omega_y - q_3\omega_z) \end{aligned} \tag{2}$$

Quaternion parameters are less intuitive, so the simulation results will be transferred to the Euler angles according to the Eq. (3), at the end.

$$\begin{aligned} \emptyset &= arctan\left(\frac{2(q_1q_2 + q_3q_4)}{q_1^2 - q_2^2 - q_3^2 + q_4^2}\right) \\ \theta &= -\arcsin(2(q_2q_4 - q_1q_3)) \\ \psi &= \arctan(\frac{2(q_1q_4 + q_2q_3)}{q_1^2 + q_2^2 - q_3^2 - q_4^2}) \end{aligned} \tag{3}$$

C. Reaction Wheels Control Strategy

Reaction wheels configuration plays an important role in the performance and character of the attitude control system. In comparison with other configurations of 4 RWs, the Pyramidal configuration has some advantages in the implementation phase. RWs tilt-angle (β) can be optimized to achieve a special goal such as minimum power consumption, minimum tracking error, or heist agility. In this paper, we will optimize β to perform the maneuver with minimum angular momentum. Wheels tilt-angle in the pyramidal configuration is shown in figure 2.

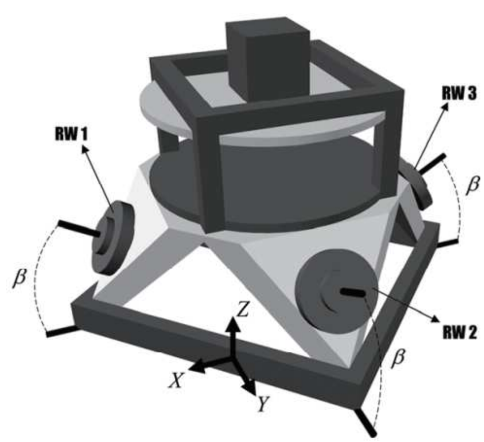

Figure 2. Tilt-angle of RWs in a pyramidal configuration

The torques delivered by the wheels are called $T_i = [T_1, T_2, T_3, T_4]^T \epsilon\, \boldsymbol{\mathcal{R}}^4$ and should be converted to the control torques along the three body axes. Thus we have the following relations:

$$\begin{bmatrix} T_{C_x} \\ T_{C_y} \\ T_{C_z} \end{bmatrix} = \begin{bmatrix} c\beta & 0 & -c\beta & 0 \\ 0 & c\beta & 0 & -c\beta \\ s\beta & s\beta & s\beta & s\beta \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix} \tag{4}$$

Where $s\beta = \sin(\beta)$ and $c\beta = \cos(\beta)$. The control torque vector is computed by the quaternion error approach described in Section III. We can calculate the components T_i, which are the control torques to be applied by each one of the four wheels as follows:

$$T_i = A^* T_C \tag{5}$$

Where A^* is the pseudo inverse of the coefficient matrix.

III. CONTROLLER DESIGN

Here we used the standard PD-type controller called "quaternion error" introduced in [4]. In this control approach, the control torques are formulated as follows:

$$\begin{aligned} T_{C_x} &= -2K_{p_x}\boldsymbol{q_{E1}}\boldsymbol{q_{E4}} - K_{d_x}\boldsymbol{\omega_{ex}} \\ T_{C_y} &= -2K_{p_y}\boldsymbol{q_{E2}}\boldsymbol{q_{E4}} - K_{d_y}\boldsymbol{\omega_{ey}} \\ T_{C_z} &= -2K_{p_z}\boldsymbol{q_{E3}}\boldsymbol{q_{E4}} - K_{d_z}\boldsymbol{\omega_{ez}} \end{aligned} \tag{6}$$

where $\left[T_{c_x}, T_{c_y}, T_{c_z}\right]^T \epsilon\, \boldsymbol{\mathcal{R}^3}$ denotes the control torque, $\boldsymbol{q_E} \epsilon\, \boldsymbol{\mathcal{R}} \times \boldsymbol{\mathcal{R}^3}$ is the attitude error quaternion between (i) current and (ii) desired attitude quaternions, as presented in Eq. (7), and $\boldsymbol{\omega_e} = \left[\omega_{e_x}, \omega_{e_y}, \omega_{e_z}\right]^T \epsilon\, \boldsymbol{\mathcal{R}^3}$ is the angular velocity error vector, as presented in Eq. (8). $\left[K_{p_x}, K_{p_y}, K_{p_z}\right]^T$ and $\left[K_{d_x}, K_{d_y}, K_{d_z}\right]^T$ are proportional and derivative gains, respectively.

$$\boldsymbol{q_E} = \frac{1}{2}\begin{bmatrix} q_{T1} & q_{T2} & q_{T3} & q_{T4} \\ -q_{T2} & q_{T1} & q_{T4} & -q_{T3} \\ -q_{T3} & -q_{T4} & q_{T1} & q_{T2} \\ -q_{T4} & q_{T3} & -q_{T2} & q_{T1} \end{bmatrix}\begin{bmatrix} q_{S1} \\ q_{S2} \\ q_{S3} \\ q_{S4} \end{bmatrix} \tag{7}$$

where $\boldsymbol{q_E}$, q_{Ti} and q_{Si}, (i=1,2,3,4) are the attitude error quaternions, desired attitude quaternions, and the satellite's current attitude quaternions, respectively.

$$\boldsymbol{\omega_e} = \boldsymbol{\omega} - \boldsymbol{\omega_d} \tag{8}$$

Where $\boldsymbol{\omega}$ denotes the satellite's current body angular velocity vector, and $\boldsymbol{\omega_d} = \left[\omega_{d_x}, \omega_{d_y}, \omega_{d_z}\right]^T \epsilon\, \boldsymbol{\mathcal{R}^3}$ denotes desired angular velocity vector.

Appropriate proportional and derivative gains to generate sufficient control torques can be calculated as following[4].

$$\begin{array}{lll} \text{X channel} & k_{p_x} = \omega_n^2 I_x & k_{d_x} = 2\xi\omega_n I_x \\ \text{Y channel} & k_{p_y} = \omega_n^2 I_y & k_{d_y} = 2\xi\omega_n I_y \\ \text{Z channel} & k_{p_z} = \omega_n^2 I_z & k_{d_z} = 2\xi\omega_n I_z \end{array} \tag{9}$$

where I refers to the moment of inertia of each channel. ω_n denotes closed-loop system natural frequency and ξ is the closed-loop damping ratio. In fact, ω_n and ξ determine the behavior of satellite controlled motion. We selected $\omega_n = 0.5$ and $\xi = 1$ to achieve the fastest and smooth (Non-oscillating) response[4].

IV. RWs INCIDENCE ANGLE OPTIMIZATION

Optimization will be done based on simulation results in this section. Because the RWs power consumption, angular momentum and saturation time are deeply dependent to the maneuver type (high-angle maneuver, agile maneuver, slow maneuver and etc.), we considered the following maneuver types to ensure our achievements based on simulation results. Shirazi considered only one maneuver ($[\phi_{com}, \theta_{com}, \psi_{com}] = [30, 20, -40]deg$) to find the optimal tilt-angle which minimizes the power consumption[5]. As shown in table I, We are considered a maneuver envelope, including all the possible rest-to-rest maneuvers similar to what was selected in reference [5].

TABLE I. Maneuver Envelope

Number	Maneuver $[\psi_{com}, \theta_{com}, \phi_{com}]$	Number	Maneuver $[\psi_{com}, \theta_{com}, \phi_{com}]$
1	$[30, 20, -40]deg$	4	$[20, -40, 30]deg$
2	$[20, 30, -40]deg$	5	$[-40, 30, 20]deg$
3	$[30, -40, 20]deg$	6	$[-40, 20, 30]deg$

The dynamic, kinematic, and controller have been simulated in Simulink. Specifications of the spacecraft and orbital elements are presented in table II.

TABLE II. Simulation Data

Parameter	Data	
Spacecraft Specification		
Spacecraft Category	Mini Satellite	
Mission	Earth Observation	
Moments of inertia matrix	$I = \begin{bmatrix} 4.2 & 0 & 0 \\ 0 & 4.4 & 0 \\ 0 & 0 & 4.2 \end{bmatrix} [\text{kg}m^2]$	
Mass	100 kg	
Initial attitude	$[\phi_0, \theta_0, \psi_0] = [0,0,0]\ deg$	
Initial angular body rate	$[p_0, q_0, r_0] = [0,0,0]$ *rad/sec*	
Orbital Elements		
Orbit Type	Circular	
Altitude	540 km	
Orbital frequency	0.0011 rad/sec	
Period	7.6317e+03 s	
Inclination	85 deg	
Actuator Specification		
Actuator Type	4 Reaction Wheels in Pyramidal Config.	
Maximum Torque	0.05 Nm	
Maximum Rate	5000 RPM	
Moment of Inertia	4.2e-4	
Controller Parameters		
$K_{p_x} = 1.05$	$K_{p_y} = 1.1$	$K_{p_z} = 1.05$
$K_{d_x} = 4.2$	$K_{d_y} = 4.4$	$K_{d_z} = 4.2$

Figure 3 shows the block diagram of the attitude control system simulation.

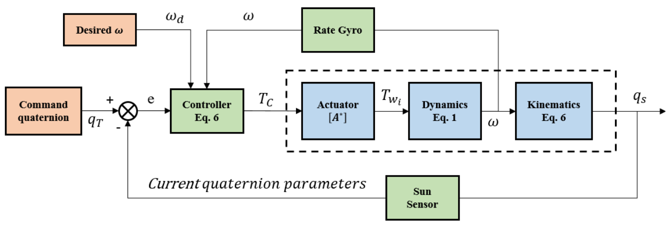

Figure 3. Block diagram of the attitude control system

Variation of RWs array angular velocity with respect to the tilt-angle is depicted in Figure 4.

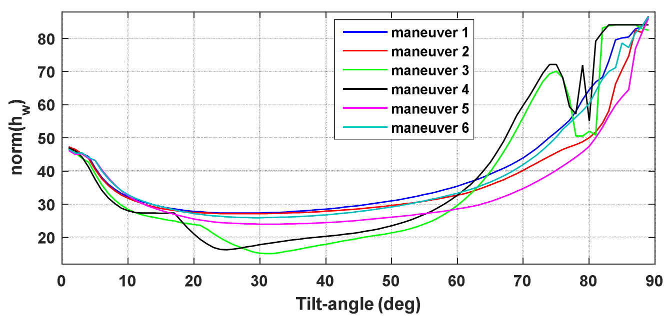

Figure 4. Block diagram of the attitude control system

According to the figure 4 it can be claimed that minimum RWs array angular momentum achieves in almost $\beta = 25^{\circ}\ to\ 33^{\circ}$ for most of the maneuvers.

The following figure shows a 3D plot (3D view of figure 4), which is useful to find out the behavior of tilt-angle and wheels array angular momentum with respect to the maneuver type.

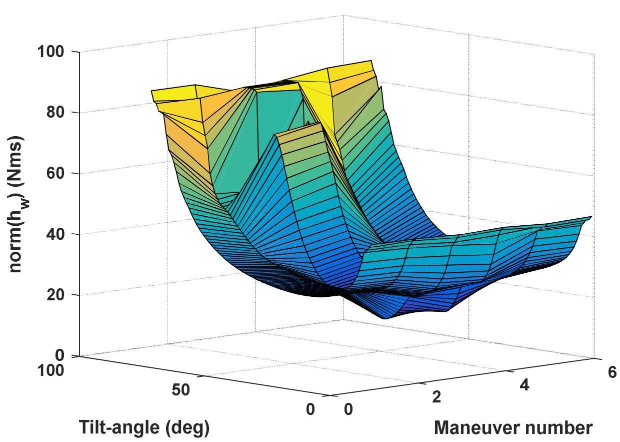

Figure 5. RWs array angular momentum as a function of wheels tilt-angle based on maneuver type.

We tried to surmise the useful data of Figures 4 and 5 by presenting figure 6. Figure 6 shows the minimum angular momentum norm of the wheels array and their corresponding tilt-angle for each maneuver.

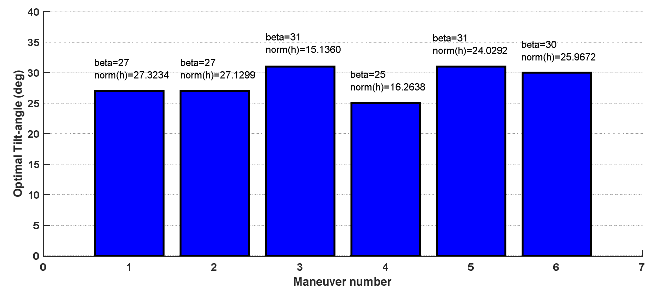

Figure 6. The exact amount of optimal tilt-angle and angular momentum of wheels array.

Attitude pointing accuracy is one of the most important problems for almost all space missions. We cannot ignore the attitude control system's accuracy in wheels array configuration design. Table III presents the norm of steady-state error attitude response, which was obtained in the case of using optimum tilt-angles were introduced in figure 6.

TABLE III. Comprehensive Data of attitude control system's performance

No.	$\|h_w\|_{min}$ [Nms]	Tilt-angle [deg]	e_{ss} [rad]
1	27.3234	27	1.0281e-11
2	27.1299	27	9.04e-11
3	15.1360	31	1.0064e-13
4	16.2638	25	2.5265e-14
5	24.0292	31	7.9838e-11
6	25.9672	30	1.0067e-10

According to table III it can be claimed that $\beta = 28.5^{\circ}$ is the average of optimal tilt-angles and can be the best option to perform maneuver envelope presented in table I with minimum angular momentum and acceptable pointing error.

Therefore we defined $\beta = 28.5^{\circ}$ as the optimal tilt-angle for performing maneuver envelope with minimum wheels angular momentum. Thus, we now need to evaluate the attitude control system's performance in $\beta = 28.5^{\circ}$. The following figure shows the norm of wheels angular momentum in different maneuvers based on $\beta = 28.5^{\circ}$.

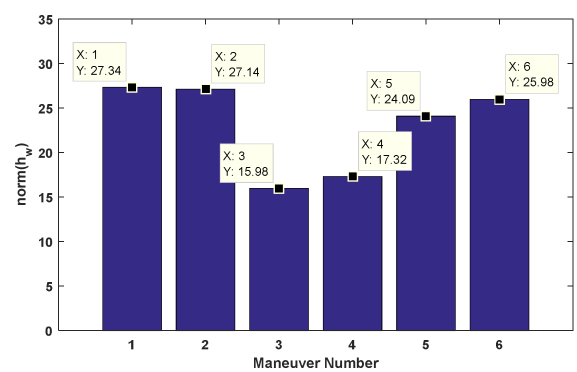

Figure 7. RWs array angular momentum norm based on $\beta = 28.5^{\circ}$

Steady-state pointing error norm in different maneuvers based on $\beta = 28.5^{\circ}$ is depicted in figure 8.

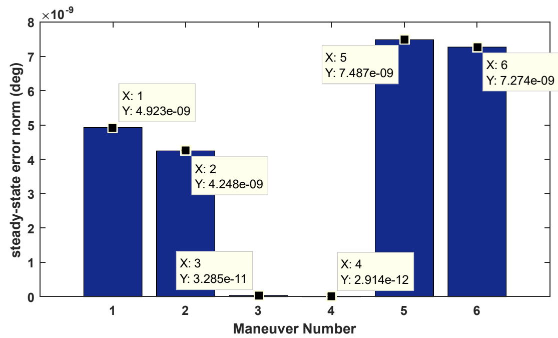

Figure 8. Attitude pointing error norm based on $\beta = 28.5^{\circ}$

According to figure 7 and 8 we are allowed to choose $\beta = 28.5^{\circ}$ the optimal tilt-angle for both (i)minimizing the wheels array angular momentum and (ii)achieving suitable pointing-accuracy as a tradeoff.

Following figures show the time-history of output in 50 seconds. It should be noted that the simulation condition is considered as table IV.

IV. Simulation data

Statement	Emount
Initial condition	$[\phi_0, \theta_0, \psi_0] = [0,0,0]\ deg$
RWs incidence angle	$\beta = 28.5^{\circ}$
Desired attitude	$[\phi_d, \theta_d, \psi_d] = [-30,20,15]\ deg$
Gravity gradient distrubance	$T_{d_x} = \frac{3\mu}{2r_0^3}(I_z - I_y)\sin(2\phi)\,cos^2(\theta)$ $T_{d_y} = \frac{3\mu}{2r_0^3}(I_z - I_x)\sin(2\theta)\cos(\phi)$ $T_{d_z} = \frac{3\mu}{2r_0^3}(I_x - I_y)\sin(2\theta)\sin(\phi)$

Euler angles of the satellite during maneuver is presented in figure 9.

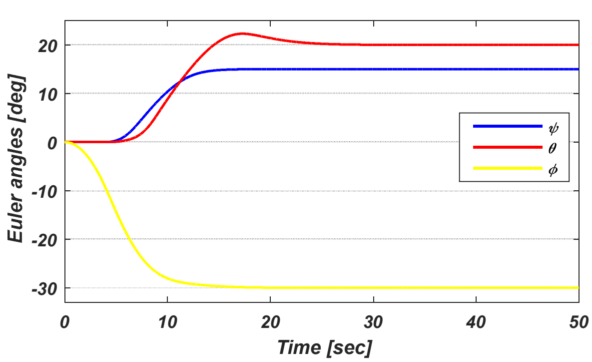

Figure 9. Euler angles time response

Figure 10 shows the 3D plot of the ouler angles time history.

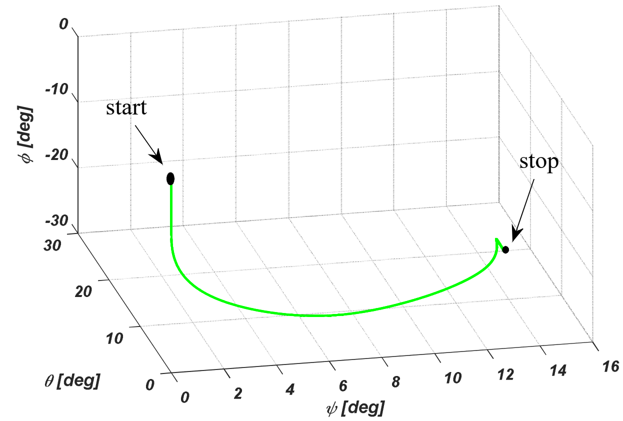

Figure 10. Euler angles 3D time response

Figure 11 presents the norm of RWs array angular momentum.

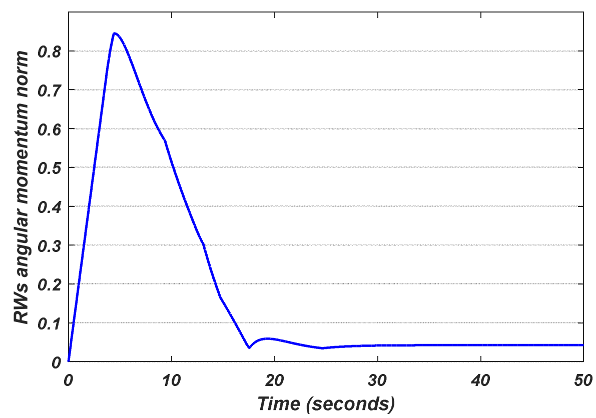

Figure 11. Norm of the RWs angular momentum

V. Conclusion

Reaction wheels configuration plays an important role in the character and performance of the attitude control system. The location and tilt-angle of the wheels can be optimized to achieve any specific purpose (minimum pointing error, minimum power consumption, minimum control effort or maximum agility). In this paper, the authors investigated the optimum tilt-angle of 4 reaction wheels located in pyramidal configuration to perform a preselected maneuver envelope with minimum angular momentum. Based on simulation results, it can be claimed that $\beta = 28.5^{\circ}$ is the optimal tilt-angle from the wheels array angular momentum point of view.

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