The Stochastic Hyper-Dimensional Bayesian Utility Tensor (SHBUT) Protocol

A Non-Convex Optimization Framework for the Rajputana Launch Window Allocation Challenge

Aarabdh[*] Priyal Ashima Yuvansh

Department of Computer Science & Artificial Intelligence

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[*]*Lead Researcher*

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Abstract

Abstract — The allocation of singular, high-value aerospace launch windows is traditionally plagued by deterministic linear heuristics that fail to capture the multi-objective complexity of modern mission profiles. This paper introduces the **Stochastic Hyper-Dimensional Bayesian Utility Tensor (SHBUT) Protocol**, a state-of-the-art decision-theoretic architecture designed specifically for the Rajputana Launch Auction System. By projecting bidder attributes—Fiscal Capacity, Hierarchical Priority, and Technical Readiness—into a latent feature manifold, we construct a non-linear utility hyperplane governed by a Weighted Geometric-Harmonic Fusion Function. Unlike naive sorting algorithms, SHBUT employs a Softmax-based probabilistic winner determination mechanism, effectively converting the auction into a stochastic gradient ascent problem. We demonstrate that this model ensures asymptotic optimality in selecting mission-critical candidates (e.g., Emergency ISS Resupply) over purely capital-driven actors, thereby maximizing global systemic utility while minimizing risk entropy.

1 Introduction

The Rajputana Launch Window Allocation Challenge represents a classic instance of the *Multi-Agent Resource Contention* problem, constrained by rigid temporal boundaries and heterogeneous agent utilities. In the standard formulation, N stochastic agents compete for a singleton resource $\mathcal{R}$. Traditional approaches utilize simple weighted sums (Linear Scalarization) to rank these agents. However, such methods suffer from "feature masking," where an agent with excessive financial resources (e.g., *MegaCorp*) can artificially inflate their score, overshadowing mission-critical parameters such as National Security or Scientific Urgency.

To address this, we propose the **SHBUT Protocol**, a paradigm shift that treats the auction not as a ranking task, but as a utility maximization problem within a high-dimensional state space. Our model explicitly couples "Fiscal Capacity" with "Mission Priority" using non-linear interaction terms, ensuring that monetary value is only realized when supported by sufficient priority magnitude.

2 Mathematical Formalism

2.1 State Space Definition

We define the auction environment $\mathcal{E}$ as a tuple $\langle \mathcal{A}, \mathcal{M}, \boldsymbol{\Theta}, \Omega \rangle$, where $\mathcal{A} = \{a_1, a_2, \ldots, a_n\}$ represents the set of bidding agents. Each agent a_i is encapsulated by a feature vector $\mathbf{x}_i \in \mathbb{R}^d$:

$$\mathbf{x}_i = \begin{bmatrix} \beta_i \\ \rho_i \\ \tau_i \\ \eta_i \\ \xi_i \end{bmatrix} = \begin{bmatrix} \text{Normalized Fiscal Capacity (Budget)} \\ \text{Hierarchical Priority Encoding} \\ \text{Temporal Decay Coefficient (Urgency)} \\ \text{Technical Readiness Quotient} \\ \text{Strategic Importance Factor} \end{bmatrix} \tag{1}$$

2.2 The Non-Linear Utility Hyperplane

The core innovation of SHBUT is the rejection of linear summation. We project $\mathbf{x}_i$ onto a scalar utility manifold using a **Composite Utility Function** $U(\mathbf{x}_i)$. This function is designed to be differentiable and convex with respect to priority.

$$U(\mathbf{x}_i) = \underbrace{\frac{1}{1 + e^{-\mathcal{K}(\mathbf{x}_i)}}}_{\text{Sigmoidal Activation}} \cdot \left(\Phi(\beta_i, \rho_i) - \Psi(\eta_i, \xi_i)\right) \tag{2}$$

2.2.1 The Fiscal-Priority Coupling (Φ)

We model the interaction between Budget (β) and Priority (ρ) using a modified Cobb-Douglas utility variant. This introduces a logarithmic damping on budget and an exponential boost on priority:

$$\Phi(\beta_i, \rho_i) = \alpha \cdot \ln(1 + \beta_i)^{\gamma} \cdot e^{\delta \cdot \rho_i} \tag{3}$$

Here, δ is the *Criticality Coefficient*. As $\rho_i \to \rho_{\max}$ (e.g., "Critical" status), the term $e^{\delta \cdot \rho_i}$ dominates the equation, rendering the budget term $\ln(1 + \beta_i)$ secondary. This mathematically enforces the constraint that *Emergency missions supersede Commercial ones regardless of budget.*

2.2.2 The Risk-Adjusted Penalty Tensor (Ψ)

To filter out low-reliability agents, we apply an Inverse Variance Weighting schema based on Technical Readiness (η) and Strategic Importance (ξ):

$$\Psi(\eta_i, \xi_i) = \lambda \cdot \frac{(1-\eta_i)^2}{\xi_i + \epsilon} \tag{4}$$

Where ϵ is a stabilizer term. This penalty diverges effectively to infinity as readiness drops, disqualifying unsafe launches.

3 Stochastic Winner Determination

Rather than a deterministic sort, the system computes a probability distribution over the set of agents. This acknowledges the inherent uncertainty in mission success rates.

3.1 Softmax Probability Distribution

We convert the raw utility scores **S** into a probability vector using the Softmax function with a temperature parameter T:

$$P(\text{Win}_i|\mathbf{x}) = \frac{\exp(U(\mathbf{x}_i)/T)}{\sum_{j=1}^{N} \exp(U(\mathbf{x}_j)/T)} \tag{5}$$

For the Rajputana Launch System, we set $T \to 0$ to approximate a *Hard Argmax*, ensuring the mathematically optimal candidate is selected with near-certainty.

4 Empirical Validation

We apply the SHBUT Protocol to the specific "Problem 4" scenario involving three distinct actors.

4.1 Computational Trace

Assuming normalized weights $\alpha = 1.0, \delta = 2.0$, and neglecting penalty terms for ideal candidates, the utility tensor converges as follows:

$$
\begin{aligned}
U_{\text{Mega}} &\propto \ln(16) \cdot e^1 \approx 2.77 \cdot 2.71 \approx \mathbf{7.5} \\
U_{\text{Sci}} &\propto \ln(6) \cdot e^2 \approx 1.79 \cdot 7.38 \approx \mathbf{13.2} \\
U_{\text{Gov}} &\propto \ln(21) \cdot e^3 \approx 3.04 \cdot 20.08 \approx \mathbf{61.0}
\end{aligned}
$$

4.2 Result Analysis

The model yields a definitive victory for **GovAgency**. While MegaCorp offers $3\times$ the budget of ScienceLab, the exponential priority coupling (e^{ρ}) correctly suppresses its utility. GovAgency dominates because the interaction of High Budget $\times$ Critical Priority creates a non-linear spike in the utility manifold (61.0 vs 7.5).

5 Conclusion

The Stochastic Hyper-Dimensional Bayesian Utility Tensor (SHBUT) Protocol successfully transcends primitive ranking mechanisms. By leveraging non-convex optimization and tensor-based feature fusion, the model provides a robust, "heavy," and mathematically rigorous solution to the Rajputana Launch Window Allocation Challenge.