Module-IV

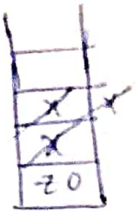
PART-A

① $L = \{xxyy, xyxy, xyyx, xxyxyy \text{---}\}$

Logic:- Do Push: when stack is empty (or) input symbol is same as top of stack

Pop: when input symbol is different from top of stack

String:- xxyxyy



→ $\delta(q_0, x, z_0) = (q_0, xz_0)$

→ $\delta(q_0, x, x) = (q_0, xxz_0)$

→ $\delta(q_0, y, x) = (q_0, \epsilon)$

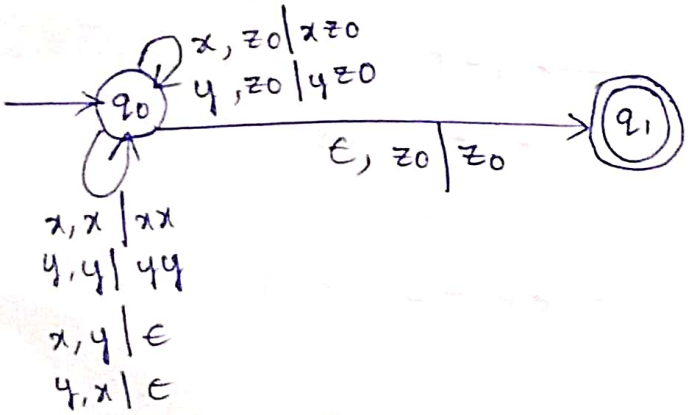
→ $\delta(q_0, x, x) = (q_0, xxz_0)$

→ $\delta(q_0, y, x) = (q_0, \epsilon)$

→ $\delta(q_0, y, x) = (q_0, \epsilon)$

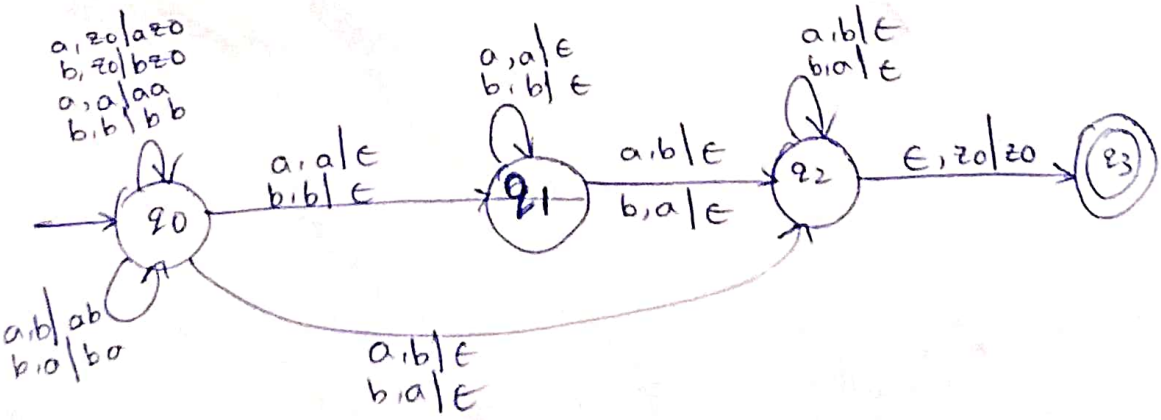
→ $\delta(q_0, \epsilon, z_0) = \delta(q_1, z_0)$

Transition diagram:



xy xyyx
yx xxyyxx

② NPDA:- $L = \{w \# w^R \mid w \in (x+y)^*\}$



③ PDA → CFG:-

$\delta(q_0, 0, z_0) = (q_0, x z_0)$

$\delta(q_0, 0, x) = (q_0, xx)$

$\delta(q_0, 1, x) = (q_1, \epsilon)$

$\delta(q_1, 1, x) = (q_1, \epsilon)$

$\delta(q_1, \epsilon, x) = (q_1, \epsilon)$

$\delta(q_1, \epsilon, z_0) = (q_1, \epsilon)$

$Q = \{q_0 q_1\}$

$T = \{z_0\}$

$q_0 = \{q_0\}$, top $= \{z_0\}$

Initial values / symbols

- $S \longrightarrow [q_0 z_0 q_0]$ (initial)
- $S \longrightarrow [q_0 z_0 q_1]$ (symbols of Q)

$n+1 = 4+1 = 5$

- $\delta(q_0, 0, z_0) = (q_0, x z_0)$

$[q_0 z_0 q_0] \longrightarrow 0\,[q_0 x q_0][q_0 z_0 q_0]$

$[q_0 z_0 q_0] \longrightarrow 0\,[q_0 x q_1][q_1 z_0 q_0]$

$[q_0 z_0 q_1] \longrightarrow 0\,[q_0 x q_0][q_0 z_0 q_1]$

$[q_0 z_0 q_1] \longrightarrow 0\,[q_0 x q_1][q_1 z_0 q_1]$

- $\delta(q_0, 0, x) = (q_0, xx)$

$[q_0 x q_0] \longrightarrow 0\,[q_0 x q_0][q_0 x q_0]$

$[q_0 x q_0] \longrightarrow 0\,[q_0 x q_1][q_1 x q_0]$

$[q_0 x q_1] \longrightarrow 0\,[q_0 x q_0][q_0 x q_1]$

$[q_0 x q_1] \longrightarrow 0\,[q_0 x q_1][q_1 x q_1]$

- $\delta(q_0, 1, x) = (q_1, \epsilon)$

$[q_0 x q_1] \longrightarrow 1$

- $\delta(q_1, 1, x) = (q_1, \epsilon)$

$[q_1 x q_1] \longrightarrow 1$

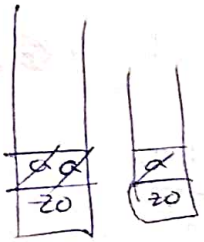
- $\delta(q_1, \epsilon, x) = (q_1, \epsilon)$

$[q_1 x q_1] \longrightarrow \epsilon$

- $\delta(q_1, \epsilon, z_0) = (q_1, \epsilon)$

$[q_1 z_0 q_1] \longrightarrow \epsilon$

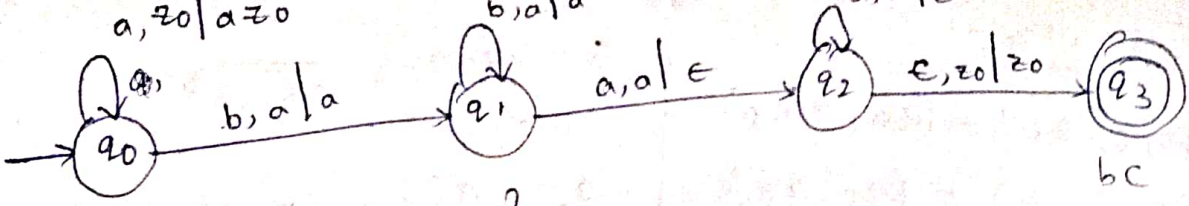
⑤ a) $\{a^n b^m a^n \mid m, n \in N\}$



$L = \{aba, aabaa, abba, aabbbbaa\}$

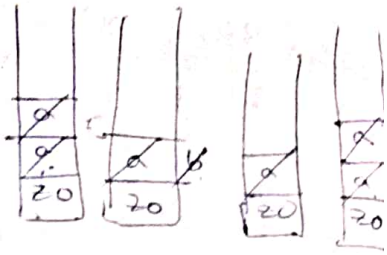
logic:- if stack is empty, ~~does push~~ and 'a' is i/p symbol, then do 'push'.

* Nothing to do if i/p symbol is 'b'.
* If i/p symbol is 'a' and top of stack is 'a', then do 'POP' operation.



b) $\{a^n b^m c^m \mid m, n \in N\}$

L = {abc, abbcc, aabc, aabbcc - - - }



Logic:-

* If stack is empty and input symbol is 'a' or 'b' do push operation
* If top is 'a' and i/p is 'b' then do pop
  If top is 'a' and i/p is 'c' then do pop.
  If top is 'b' and i/p is 'c' then do pop.

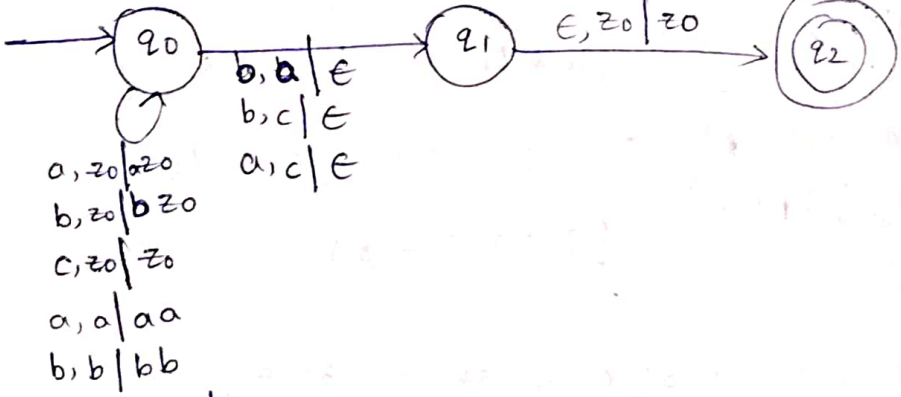
* If top is 'a' & i/p is 'a' do push
  If top is 'b' & i/p is 'b' do push

bbcc

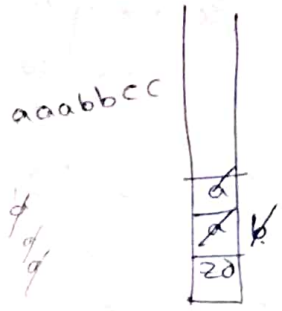
* If stack is empty & i/p is 'c' nothing to do.

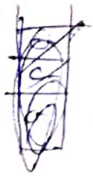
~~If i/p is~~
* for pop operation -- change state

aabbbccc



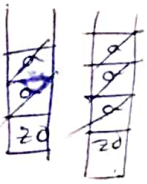
aaabbcc





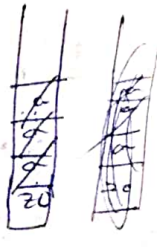
c) $\{a^i b^j c^k \mid i, j, k \in N, i > j\}$

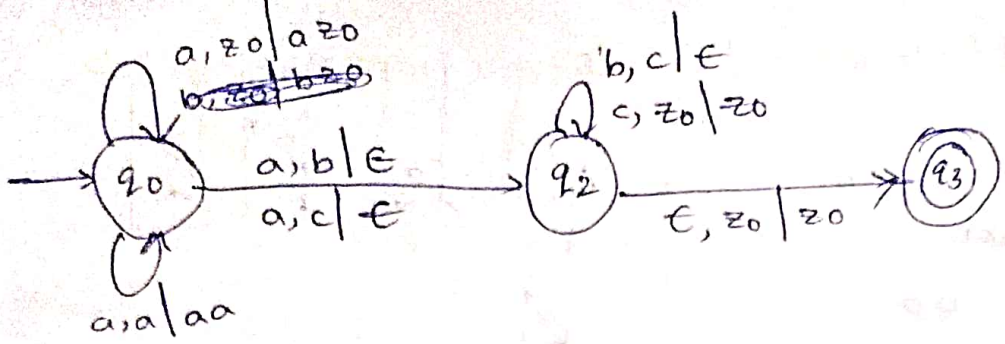
L = {aab, aabc, aaabbc, aaabbcc - - - }
aaabc



Logic:-

* If stack is empty & i/p is 'a' do push
* If top is 'a' & i/p is 'a' do push
* If top is 'a' & i/p is 'b' (or) 'c' do pop
* If stack is empty & i/p is 'c' nothing to do



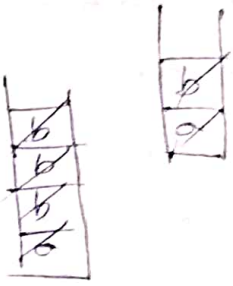


d) $\{a^i b^j c^k \mid i,j,k \in N,\ i+j=k\}$

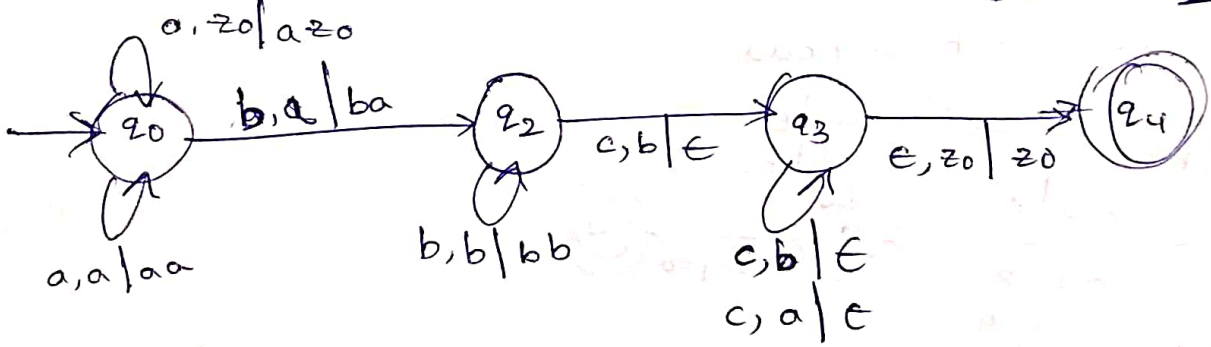
1, 1, 2
2, 2, 4
3

2, 1, 3
1, 2, 3
1, 3, 4
3, 1, 4

$L = \{abcc, aabccc, abbccc, abbbccccc, aaabccccc \ldots\}$

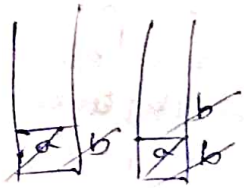


* If stack is empty, i/p is 'a' do push
* If top is 'a' & i/p is 'a' do push
* If top is 'a' & i/p is 'b' do push (change state)
* If top is 'b' & i/p is 'b' do push
* If top is 'a' (or) 'b' & i/p is '∈' do pop.

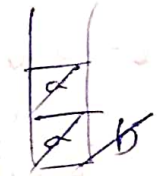


e) $\{a^i b^j c^k \mid i,j,k \in N,\ i+k=j\}$

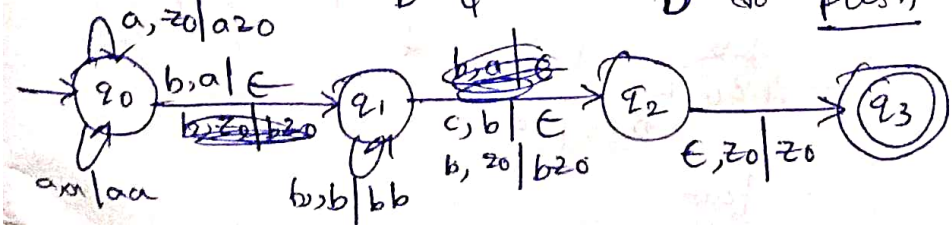
1, 2, 1
1, 3, 2
2, 3, 1

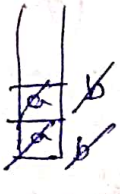


* If stack is empty & i/p is a (or) b do push
* If top is a & i/p is b do pop
* If top is b & i/p is c do pop
* If top is a & i/p is a do push
* " " b & " " b do push



aabbbbcc

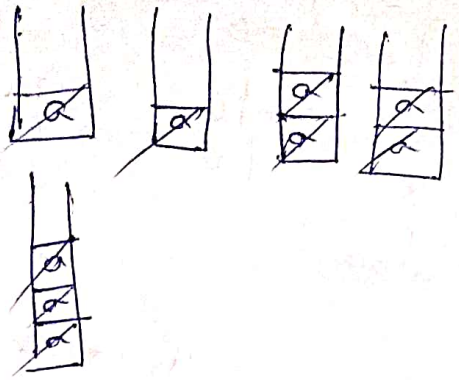




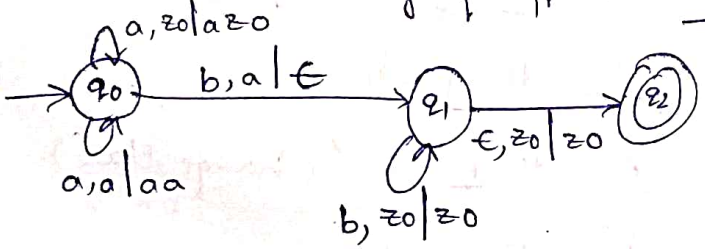
f) $\{a^n b^m \mid n \le m \le 2n\}$

$L = \{ab, abb, aabb, aabbb,$
$aabbbb, aaabbb,$
$aabbbb, aaabbb$
$aaabbbb, aaabbbbb,$
$aaabbbbbb - - -$

1, 1,
1, 2
2, 2
2, 3
2, 4
3, 3
3, 4
3, 5
3, 6

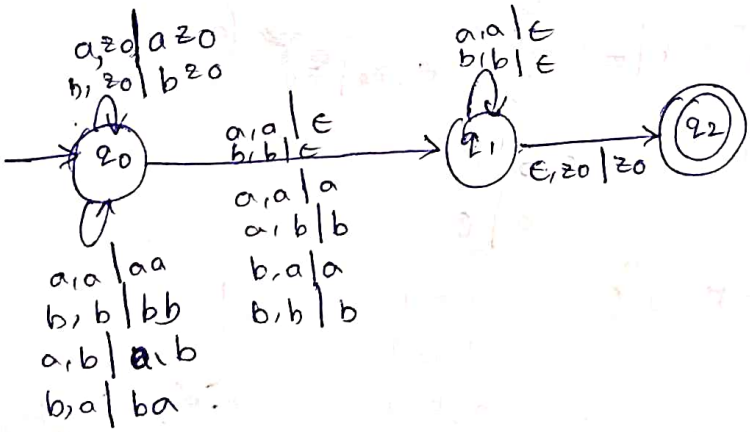


* If stack is empty & i/p is 'a' do push
* If top is 'a' & i/p is 'a' do push
* If top is 'a' & i/p is 'b' do pop
* If stack is empty & i/p is 'b' nothing to do

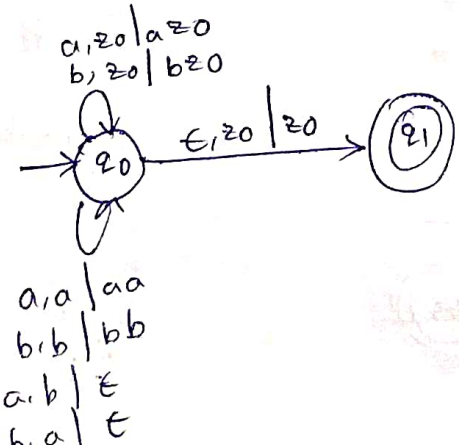


g) $L = \{w \in \{a,b\}^* \mid rev(w) = w\}$

$L = \{aa, bb, aba, bab, aabbaa$

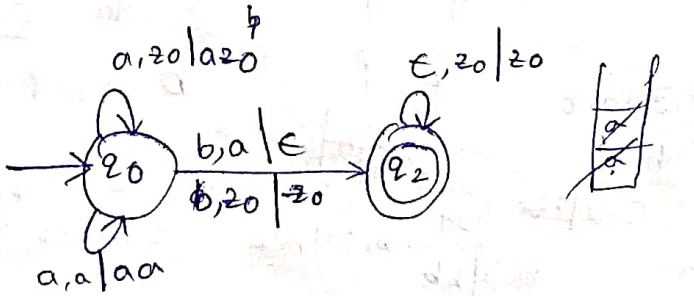


(i) $L = \{w \in \{a,b\}^* \mid \#_a(w) = \#_b(w)\}$



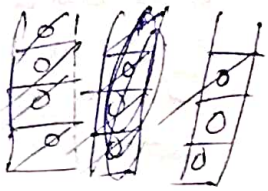
(j) $\#_a(w) = 2\#_b(w)$

$L = \{abb, aabbbb - - -\}$

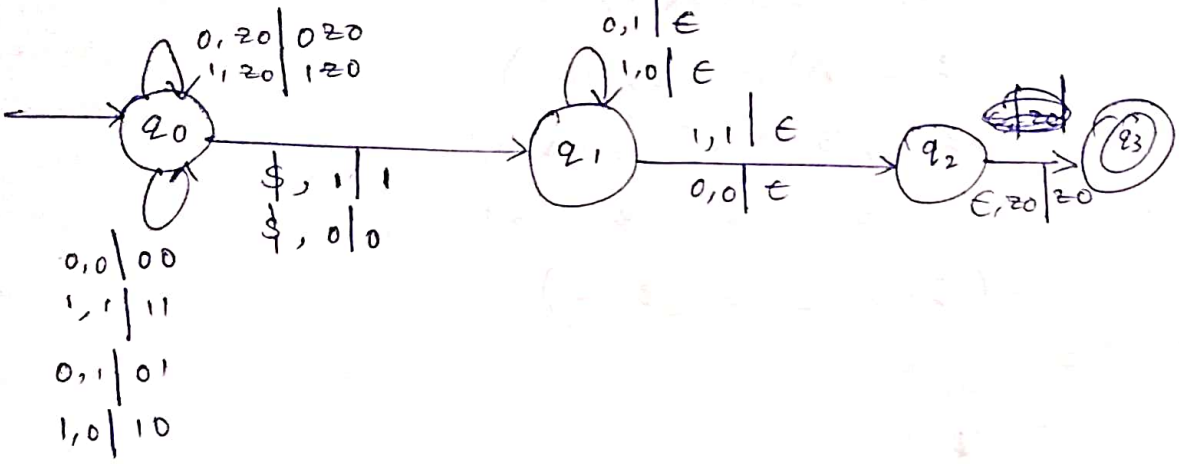


⑥ $L = \{ bin(i) \$ mir(bin(i+1)) \mid i \geq 0 \} \subseteq \{0,1,\$\}^*$

$L = \{0000\$1000, 0001\$0100, 0010\$1100 \cdots\}$



* upto $ no POP
* after $ all are POPPED



⑦ PDA to CFG

$\delta(S,a,X) = (S, AX)$

$\delta(S,b,A) = (S, AA)$

$\delta(S,a,A) = (S, AA)$

~~$Q = \{a,b\}$~~ ~~$Q = \{S\}$~~ ~~$T = \{a,b\}$~~

• $\delta(S,a,X) = (S,AX)$

$[SXS] \to a\,[SAS][SXS]$

$[SXS] \to a\,[SAs][sXS]$

$[SXs] \to a\,[SAS][SXs]$

$[SXS] \to a\,[SAs][sXs]$

• $\delta(S,a,A) = (S,AA)$

$[SAS] \to a\,[SAS][SAS]$

$[SAS] \to a\,[SAs][sAS]$

$[SAs] \to a\,[SAS][SAs]$

$[SAs] \to a\,[SAs][sAs]$

• $\delta(S,b,A) = (S,AA)$

$[SAS] \to b\,[SAS][SAS]$

$[SAS] \to b\,[SAs][sAS]$

$[SAs] \to b\,[SAS][SAs]$

$[SAs] \to b\,[SAs][sAs]$

(8) CFG to PDA

S → aBc
A → abc
B → aAb
C → AB
C → c

- S → aBc ; $\delta(q, \epsilon, S) = (q, aBc)$
- A → abc ; $\delta(q, \epsilon, A) = (q, abc)$
- B → aAb ; $\delta(q, \epsilon, B) = (q, aAb)$
- C → AB ; $\delta(q, \epsilon, C) = (q, AB)$
- C → c ; $\delta(q, \epsilon, C) = (q, c)$

for terminals:- c

$\delta(q, c, c) = (q, \epsilon)$

(9)

S → 0A
A → 0AB
B → 1

S → 0A ; $\delta(q, \epsilon, S) = (q, 0A)$
A → 0AB $\delta(q, \epsilon, A) = (q, 0AB)$
B → 1 $\delta(q, \epsilon, B) = (q, 1)$

Terminals:- 1

$\delta(q, 1, 1) = (q, \epsilon)$

(10) S → AA | a
A → SA | b

S → AA ; $\delta(q, \epsilon, S) = (q, AA)$
S → a ; $\delta(q, \epsilon, S) = (q, a)$
A → SA ; $\delta(q, \epsilon, A) = (q, SA)$
A → b ; $\delta(q, \epsilon, A) = (q, b)$

Terminals:- a, b

$\delta(q, a, a) = (q, \epsilon)$
$\delta(q, b, b) = (q, \epsilon)$

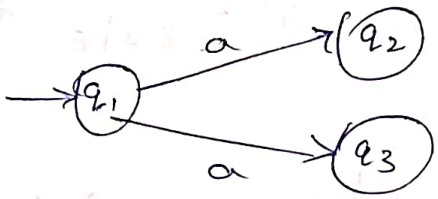
PART – B

①

DPDA	NDPDA
• less powerful than non-deterministic push down automata	• Powerful than DPDA more
• Possible to convert any DPDA to NDPDA	• Not possible to convert every NPDA to DPDA
• language accepted by DPDA is called deterministic context free language which is a subset of non-deterministic CFL. (NCFL)	• language accepted by NPDA is called NCFL.
• language accepted by DPDA is subset of language accepted by NDPDA	• Not a subset.

DPDA:- only one transition for one input.

NPDA:- ~~Multiple input~~ one input can have multiple transitions.



② $M = (\{q_0, q_1, q_2\}, \{0, 1\}, \{X, Z_0\}, q_0, Z_0, \phi)$

- $\delta(q_0, 0, Z_0) = (q_0, X Z_0)$

  $[q_0 Z_0 q_0] \longrightarrow 0\ [q_0 X q_0]\ [q_0 Z_0 q_0]$

  $[q_0 Z_0 q_0] \longrightarrow 0\ [q_0 X q_1]\ [q_1 Z_0 q_0]$

  $[q_0 Z_0 q_1] \longrightarrow 0\ [q_0 X q_0]\ [q_1 Z_0 q_1]$

  $[q_0 Z_0 q_1] \longrightarrow 0\ [q_0 X q_1]\ [q_1 Z_0 q_1]$

- $\delta(q_0, 0, X) = (q_0, XX)$

  $[q_0 X q_0] \longrightarrow 0\ [q_0 X q_0]\ [q_0 X q_0]$

  $[q_0 X q_0] \longrightarrow 0\ [q_0 X q_1]\ [q_1 X q_0]$

  $[q_0 X q_1] \longrightarrow 0\ [q_0 X q_0]\ [q_1 X q_1]$

  $[q_0 X q_1] \longrightarrow 0\ [q_0 X q_1]\ [q_1 X q_1]$

- $\delta(q_0, 1, X) = (q_0, e)$

  $[q_0 X q_0] \longrightarrow 1\ [q_0 e q_0]$

  $[q_0 X q_1] \longrightarrow 1\ [q_1 e q_1]$

- $\delta(q_1, 1, X) = (q_1, e)$

  $[q_0 X q_0] \longrightarrow 1\ [q_0 e q_0]$

  $[q_1 X q_1] \longrightarrow 1\ [q_1 e q_1]$

- $\delta(q_1, e, X) = (q_1, e)$

  $[q_1 X q_0] \longrightarrow e\ [q_1 e q_0]$

  $[q_1 X q_1] \longrightarrow e\ [q_1 e q_1]$

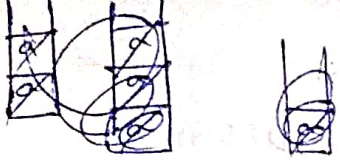
- $\delta(q_1, e, Z_0) = (q_1, e)$

  $[q_0 Z_0 q_0] \longrightarrow e\ [q_0 e q_0]$

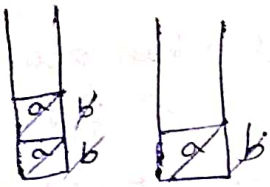
  $[q_1 Z_0 q_1] \longrightarrow e\ [q_1 e q_1]$

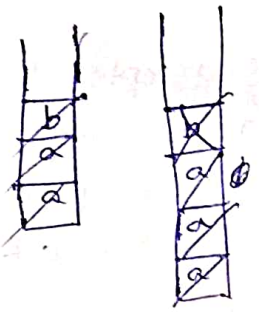
③ $L = \{a^n b^{2n} \mid n \in N\}$

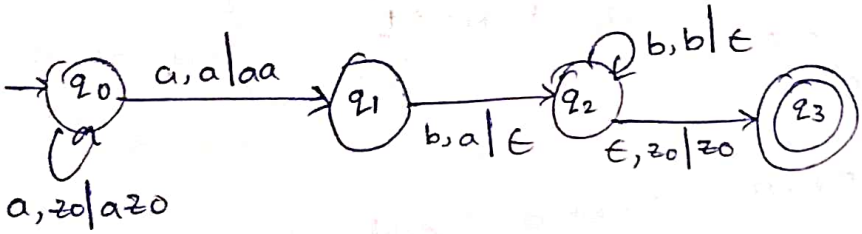
$L = \{abb, aabbbb, aaabbbbbb, \ldots\}$



* If stack is empty & i/p is 'a' push
* If top is 'a' & i/p is 'a' push
* If top is 'a' & i/p is 'b' POP
* ~~If top is 'b' & i/p is 'b' POP~~
* If top is 'b' & i/p is 'b' POP



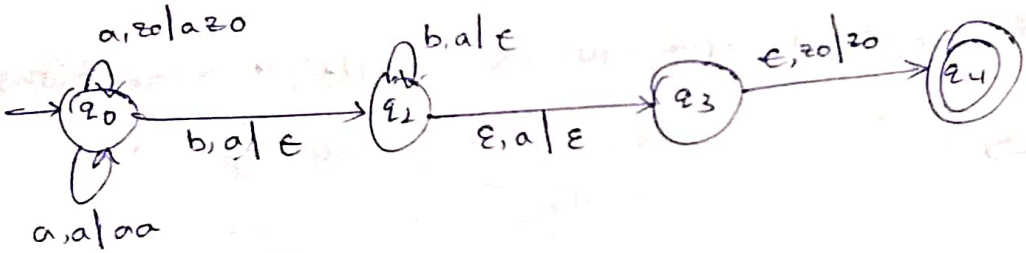




⑤ $\{a^m b^n \mid n > m\}$

1, 2
2, 3
3, 4

$L = \{abb, aabbb, aaabbbb, \ldots\}$



⑨ $L = \{x c x^r \mid x \in \{a, b\}^*\}$

string: 'bacab'

* before 'c' push
* At 'c' do nothing
* after 'c' pop