Module-4 (Part-B)

1) Define the NPDA (Non deterministic PDA) and (DPDA (Deterministic PDA)) equivalent? Illustrate with an example.

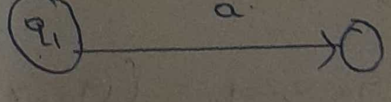
Sol:

DPDA	NPDA
It is less powerful than NPDA	It is more powerful than DPDA
It is possible to Convert every DPDA to Corresponding NPDA.	It is not possible to Convert every NPDA to a Corresponding DPDA
The language accepted by DPDA is a Subset of the language accepted by NPDA.	The language accepted by NPDA is not a Subset of the language accepted by DPDA
The language accepted by DPDA is Called DCFL (Deterministic Context-free Language) which is a Subset of NCFL (Non-deterministic Context-Free language) accepted by NPDA.	The language accepted by NPDA is Called NCFL (Non-deterministic Context Free language)

DPDA :- For every input with the Current state, there is only one move.

$M = (Q, \Sigma, \Gamma, q_0, Z, F, \delta)$

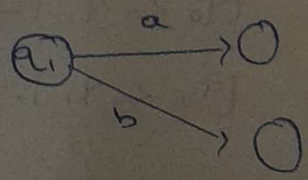
$\delta : Q \times \Sigma \times \Gamma \rightarrow Q \times \Gamma$



NPDA :- For every input with the Current state, we have multiple moves.

$M = (Q, \Sigma, \Gamma, q_0, Z, F, \delta)$

$\delta : Q \times (\Sigma \cup \epsilon) \times \Gamma \rightarrow 2^{Q} \times \Gamma^{*}$



2) Describe the grammar for the following PDA.

$M = (\{q_0, q_1\}, \{0,1\}, \{x, z_0\}, q_0, z_0, \phi)$ and where δ is given by

$\delta(q_0, 0, z_0) = \{(q_0, xz_0)\}$

$\delta(q_0, 0, x) = \{(q_0, xx)\}$

$\delta(q_0, 1, x) = \{(q_1, e)\}$

$\delta(q_1, 1, x) = \{(q_1, e)\}$

$\delta(q_1, e, x) = \{(q_1, e)\}$

$\delta(q_1, e, z_0) = \{(q_1, e)\}$

Sol:

$M = (Q, \Sigma, \Gamma, \delta --)$

$G = (V, T, P, S)$

$V = (S, [q_0 x q_0], [q_0 x q_1], [q_1 x q_0] [q_1 x q_1]$
$[q_0 z_0 q_0] [q_0 z_0 q_1] [q_1 z_0 q_0] (q_1 z_0 q_1])$

$T = \{0, 1\}$

$S \rightarrow$ ~~q0xq0~~ $[q_0 z_0 q_0]$

$S \rightarrow [q_0 z_0 q_1]$

Consider,

$\delta(q_0, 0, z_0) = \{(q_0, xz_0)\}$ 2-Symbols $2^n = 4$ [ ]

$[q_0, z_0\, q_0] \rightarrow 0 [q_0 x q_0] [q_0 z_0 q_0]$

$[q_0 z_0 q_0] \rightarrow 0 [q_0 x q_0] [q_0 z_0 q_0]$

$[q_0 z_0 q_1] \rightarrow 0 [q_0 x q_0] [q_0 z_0 q_1]$

$[q_0 z_0 q_1] \rightarrow 0 [q_0 x q_1] [q_1 z_0 q_1]$

Consider,

$\delta(q_0, 0, x) = \{(q_0, xx)\}$

$[q_0 \, x \, q_0] \rightarrow 0[q_0 \, x \, q_0] \; [q_0 \, x \, q_0]$

$[q_0 \, x \, q_0] \rightarrow 0[q_0 \, x \, q_1] \; [q_1 \, x \, q_0]$

$[q_0 \, x \, q_1] \rightarrow 0[q_0 \, x \, q_0] \; [q_0 \, x \, q_1]$

$[q_0 \, x \, q_1] \rightarrow 0[q_0 \, x \, q_1] \; [q_1 \, x \, q_1]$

Consider,

$\delta(q_0, 1, x) = \{(q_1, e)\}$

$[q_0 x q_0] \rightarrow (1)$

Consider,

$\delta(q_1, 1, x) = \{(q_1, e)\}$

$[q_1 x q_0] \rightarrow 1$

Consider,

$\delta(q_1, \epsilon, x) = \{(q_1, e)\}$

$[q_1 x q_0] \rightarrow \varepsilon$

Consider,

$\delta(q_1 \epsilon z_0) = \{(q_1, \epsilon)\}$

$[q_1 z_0 q_0] \rightarrow \epsilon$

3) Describe PDA for string of form $a^n b^{2n}$ $(n \geq 1)$

Sol:
$M = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$

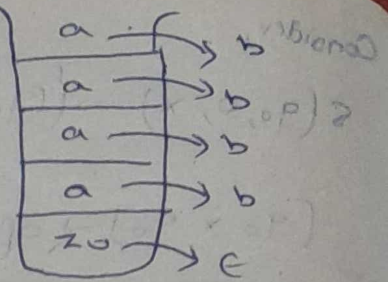
$L = \{abb, aabbbb, ---\}$

a	a	b	b	b	b	ϵ

- If there is one input a, we push 2 a's into the stack.
- If there is input symbol b, we perform pop operation.

$\delta(q_0, a, z_0) = (q_0, aaz_0)$

↑ top of stack



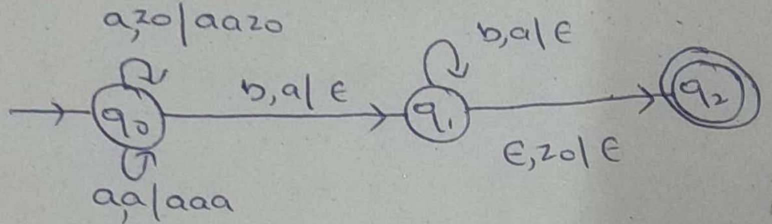
$\delta(q_0, a, a) = (q_0, aaa)$

$\delta(q_0, b, a) = (q_1, \epsilon)$

$\delta(q_1, b, a) = (q_1, \epsilon)$

$\delta(q_1, \epsilon, z_0) = (q_2, \epsilon)$

Transition diagram,



$M = (\{q_0, q_1, q_2\}, \{a, b\}, \{z_0, a\}, \delta, q_0, z_0, q_2)$

4) Define PDA mathematically. With a neat diagram explain the working of a Turing machine.

Sol: Push down Automata is a way to implement a CFG in the same way we design DFA for a regular grammar. A DFA can remember a finite amount of information, but a PDA can remember an infinite amount of information.

Formal defination of PDA:-

The PDA can be defined as a collection of 7 components:

Q: the finite set of states

Σ: the input sets

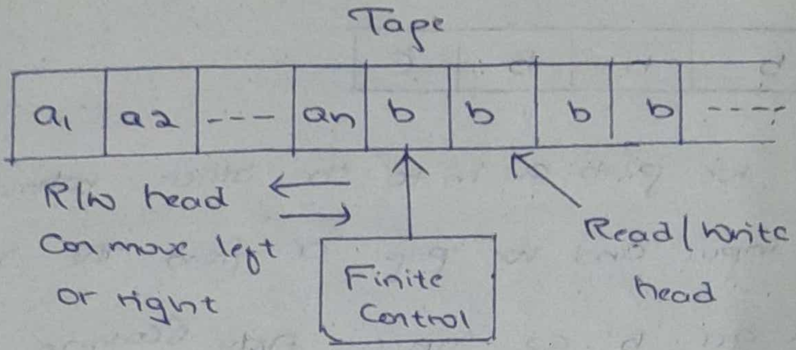
Γ: a stack symbol which can be pushed and popped from the stack.

q_0 : the initial State

Z : a start Symbol which is in Γ

F : a finite set of final States.

δ : A mapping function which is used for moving from Current state to next state.



A Turing machine (TM) is a mathematical model which Consists of an infinite length tape divided into cells on which input is given. It Consists of a head which reads the input tape. A State register stores the state of the Turning machine. After reading an input Symbol, it is replaced with another Symbol, its internal state is changed and it moves from one Cell to the right or left. If the TM reaches the final state, the input String is accepted, otherwise rejected.

A TM can be formally described as a 7 tuple,

$(Q, X, \Sigma, \delta, q_0, B, F)$ where,

Q : Finite set of States.

X : Tape Alphabet

Σ : input alphabet

δ : Transition function : $\delta : Q \times X \rightarrow Q \times X \times \{$left-shift, right-shift$\}$

q_0 : Initial State

B : blank Symbol

F : Set of final States.

5) Describe the PDA that accepts the language :

$\{a^m b^n \mid n > m\}$

Sol: $M = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$

$L = \{b, abb, aabbb \text{---}\}$

a	a	b	b	b	∈

- In this case we push a into the stack whenever we will get 'a' as input and we pop 'a' from the stack whenever we will get 'b' as input. After scanning all inputs if the input tape has still some no. of b's and stack is empty then we can say the string is accepted otherwise rejected.

$\delta(q_0, a, z_0) = (q_0, az_0)$

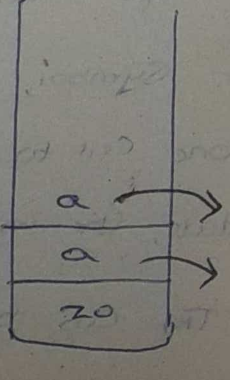
$\delta(q_0, a, a) = (q_0, aa)$

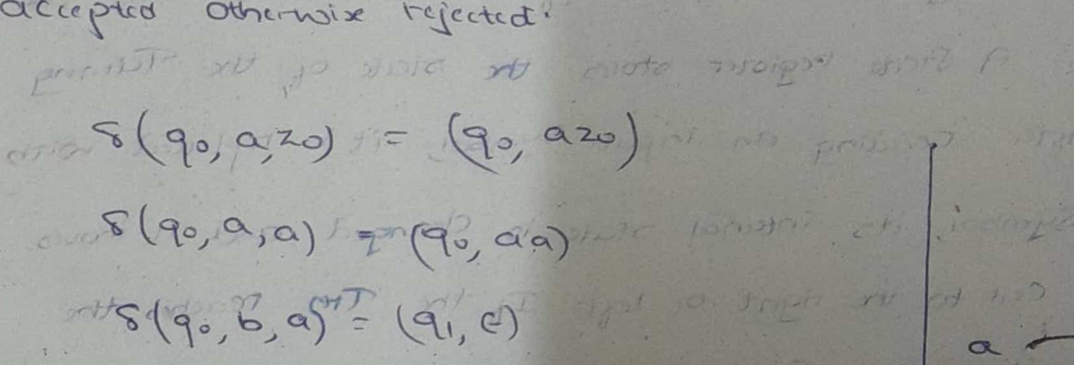
$\delta(q_0, b, a) = (q_1, \epsilon)$

$\delta(q_1, b, a) = (q_1, \epsilon)$

$\delta(q_1, b, z_0) = (q_2, \epsilon)$

$\delta(q_2, \epsilon, z_0) = (q_3, \epsilon)$





6) Describe a PDA for the following grammar,

$S \to 0A$

$A \to 0AB \mid 1$

$B \to 1$

Sol. $S \to 0A$, $\delta(q, \epsilon, S) = (q, 0A)$

$A \to 0AB$, $\delta(q, \epsilon, A) = (q, 0AB)$

$A \to 1$, $\delta(q, \epsilon, A) = (q, 1)$

$B \to 1$, $\delta(q, \epsilon, B) = (q, 1)$

$\delta(q, 0, 0) = (q, \epsilon)$

$\delta(q, 1, 1) = (q, \epsilon)$

7) Convert the following PDA to CFG

$M = (\{q_0, q_1\}, \{a, b\}, \{x, z_0, za\}, \delta, q_0, z_0, \phi)$ and where δ is

given by,

$\delta(q_0, a, z_0) = (q_0, zz)$

$\delta(q_0, a, z) = (q_0, zz_0)$

$\delta(q_0, b, z) = (q_1, \epsilon)$

$\delta(q_1, b, z) = (q_1, \epsilon)$

$\delta(q_1, \epsilon, z_0) = (q_1, \epsilon)$

Sol: $M = (Q, \Sigma, \Gamma, \delta, ---)$

$G = (V, T, P, S)$

$V = (S, [q_0 x q_0]\ [q_0 z_0 q_0]\ [q_0 za q_0]$

$[q_0 x q_1]\ [q_0 z_0 q_1]\ [q_0 za q_1]$

$[q_1 x q_1]\ [q_1 z_0 q_1]\ [q_1 za q_1]$

$[q_1 x q_0]\ [q_1 z_0 q_0]\ [q_1 za q_0])$

$T = \{a, b\}$

$S \to [q_0 z_0 q_0]$

$S \to [q_0 z_0 q_1]$

Consider,

$\delta(q_0, a, z_0) = (q_0, zz)$

$[q_0 z_0 q_0] = a[q_0 z q_0]\ [q_0 z q_0]$

$[q_0 z_0 q_0] = a[q_0 z q_1]\ [q_1 z q_0]$

$[q_0 z_0 q_1] = a[q_0 z q_0]\ [q_0 z q_1]$

$[q_0 z_0 q_1] = a[q_0 z q_1]\ [q_1 z q_1]$

Consider,

$\delta(q_0, a, z) = (q_0, zz_0)$

$[q_0 z q_0] = a[q_0 z q_0]\ [q_0 z_0 q_0]$

$[q_0 z q_0] = a[q_0 z q_1]\ [q_1 z_0 q_0]$

$[q_0 z q_1] = a[q_0 z q_0]\ [q_0 z_0 q_1]$

$[q_0 z q_1] = a[q_0 z q_0]\ [q_1 z_0 q_1]$

Consider,

$\delta(q_0, b, z) = (q_1, \epsilon)$

$[q_0 z q_1] = b$

Consider,

$\delta(q_1, b, z) = (q_1, \epsilon)$

$[q_1 z q_1] = b$

Consider,

$\delta(q_1 \epsilon z_0) = (q_1 \epsilon)$

$[q_1 z_0 q_1] = \epsilon$

8) Describe the PDA mathematically. Describe the PDA for the following language $L = \{w \mid w \text{ of form } a^n b^n\}$

Sol. PDA :- Refer part-B (4th Question)

Given,

$$L = \{w \mid w \text{ of form } a^n b^n\} \quad n \geq 1$$

$$L = \{ab, aabb, aaabbb, \dots\}$$

- This can be achieved by pushing 'a's into the stack when the input symbol is 'a'
- Then we have to pop from the stack when input symbol is 'b'.
- Finally at the end of the string if nothing is left in the stack then we can declare that language is accepted, otherwise rejected.

a	a	b	b	ϵ

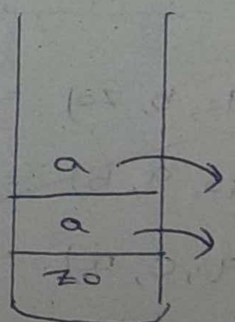
$\delta(q_0, a, z_0) = (q_0, az_0)$

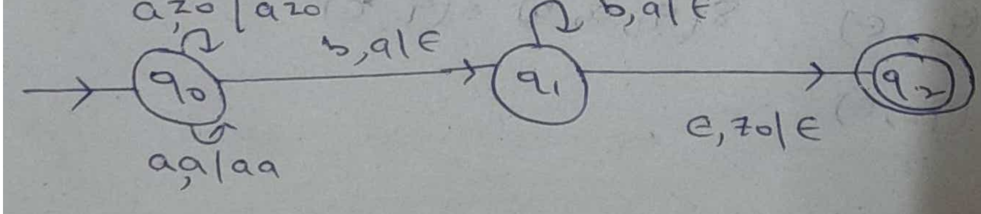
$\delta(q_0, a, a) = (q_0, aa)$

$\delta(q_0, b, a) = (q_1, \epsilon)$

$\delta(q_1, b, a) = (q_1, \epsilon)$

$\delta(q_1, \epsilon, z_0) = (q_2, \epsilon)$





Q) Describe PDA for the language

$L = \{xcx^r \mid x \in \{a,b\}^*\}$ and trace it for String 'bacab'.

Sol: Given,

$M = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$

$L = \{$bacab, abcba, aabcbaa, --- abacaba, ---$\}$

- We have to push the elements until the input symbol is c
- When the input symbol is c, do nothing.
- After the input symbol c, pop all the elements from the stack.
- If the stack is empty, the language is accepted, otherwise rejected.

Consider the String 'bacab'

b	a	c	a	b	ϵ

$\delta(q_0, b, z_0) = (q_0, bz_0)$ $\quad$ $\delta(q_0, a, z_0) = (q_0, az_0)$

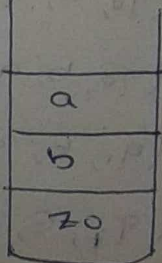
$\delta(q_0, a, b) = (q_0, ab)$ $\quad$ $\delta(q_0, b, a) = (q_0, ba)$

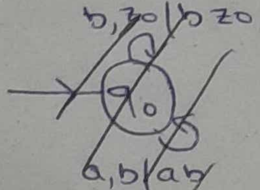
$\delta(q_0, c, a) = (q_1, a)$ $\quad$ $\delta(q_0, c, b) = (q_1, b)$

$\delta(q_1, a, a) = (q_1, \epsilon)$ $\quad$ $\delta(q_1, b, b) = (q_1, \epsilon)$

$\delta(q_1, b, b) = (q_1, \epsilon)$ $\quad$ $\delta(q_1, a, a) = (q_1, \epsilon)$

$\delta(q_1, \epsilon, z_0) = (q_2, \epsilon)$





$\rightarrow q_0$

$\delta(q_0, a, z_0) = (q_0, a z_0)$

$\delta(q_0, b, z_0) = (q_0, b z_0)$

$\delta(q_0, a, a) = (q_0, aa)$

$\delta(q_0, a, b) = (q_0, ab)$

$\delta(q_0, b, a) = (q_0, ba)$

$\delta(q_0, b, b) = (q_0, bb)$

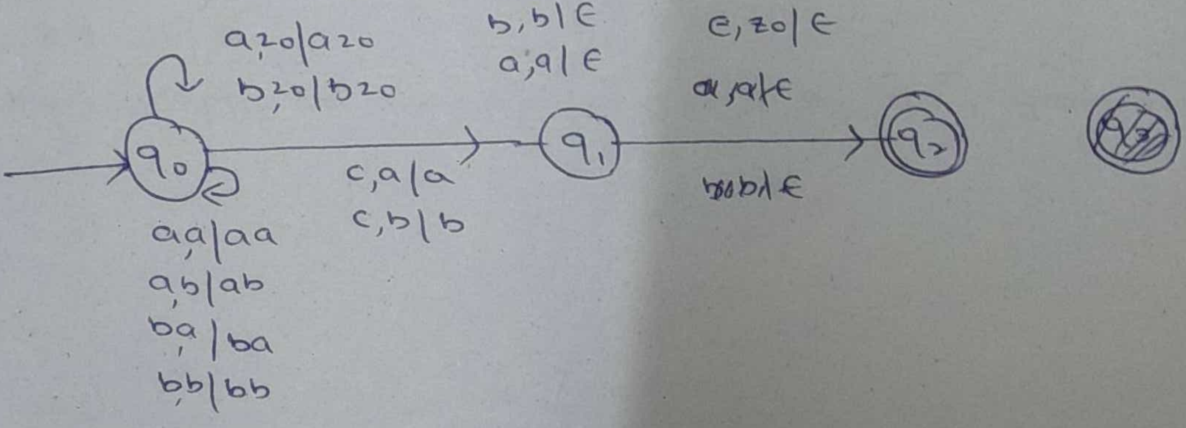
$\delta(q_0, c, a) = (q_1, a)$

$\delta(q_0, c, b) = (q_1, b)$

$\delta(q_1, a, a) = (q_1, \epsilon)$

$\delta(q_1, b, b) = (q_1, \epsilon)$

$\delta(q_1, \epsilon, z_0) = (q_2, \epsilon)$



$M = (\{q_0, q_1, q_2\}, \{a, b\}, \{a, b\}, \delta, q_0, z_0, q_2)$

10) Describe the pushdown automaton A is specified by

$M = (\{q_0, q_1\}, \{a, b\}, \{x, z\}, \delta, q_0, z, \phi)$ and where δ

Contains the following transitions,

14) Convert the following Context free grammar to push down Automata.

$S \to aAbB$

$A \to aB \to a$

$B \to b$

Verify the String aab accepted by equivalent PDA.

Sol:

15) Describe DPDA for $L = a^n b^n$ where $n \geq 1$

Sol:

$M = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$

$L = \{ab, aabb, aaabbb \text{---}\}$

- If there is input symbol 'a', we push into the stack.
- If the input symbol is 'b', we pop the elements from Stack.

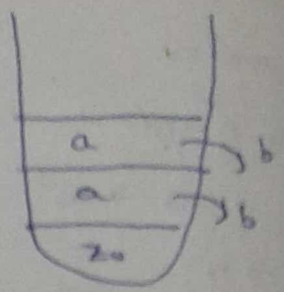
$\delta(q_0, a, z_0) = (q_0, az_0)$

$\delta(q_0, a, a) = (q_0, aa)$

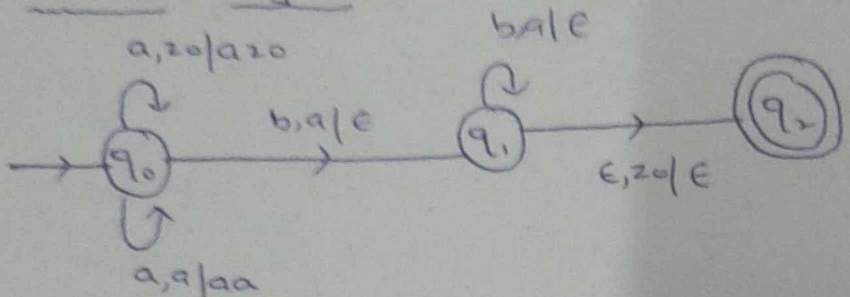
$\delta(q_0, b, a) = (q_1, \epsilon)$

$\delta(q_1, b, a) = (q_1, \epsilon)$

$\delta(q_1, \epsilon, z_0) = (q_2, \epsilon)$



Transition diagram :-



$M = (\{q_0, q_1, q_2\}, \{a,b\}, \{z_0, a\}, \delta, q_0, z_0, q_2)$

16) Describe PDA accepts PDA for the language $L = \{ww^R \mid w \in \{a,b\}^*\}$ such that $L = L(M)$

Sol: Given,

$L = \{abba, abbbba, aaaaaa \text{ ---}\}$

Here, we have two situations,

i) When the end symbol of w is equal to start symbol of w^R we need to start poping.

ii) For the third string 'aaaaaa', we don't the centre, so we have two situations, so we use NPDA here

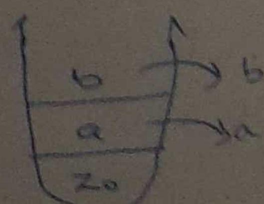
$\delta(q_0, a, z_0) = (q_0, az_0)$

$\delta(q_0, b, z_0) = (q_0, bz_0)$

$\delta(q_0, a, b) = (q_0, ab)$

$\delta(q_0, b, a) = (q_0, ba)$

$\delta(q_0, a, a) = (q_0, aa)$

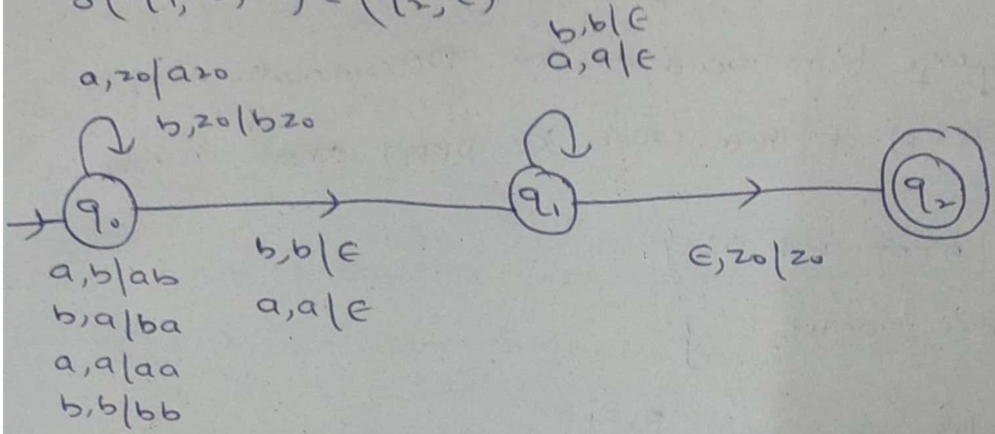


$\delta(q_0, b, b) = (q_0, bb)$

$\delta(q_0, a, a) = (q_1, \epsilon)$

$\delta(q_0, b, b) = (q_1, \epsilon)$

$\delta(q_1, \epsilon, z_0) = (q_2, \epsilon)$



17) Illustrate PDA m for the language $L = \{x \in \{a,b\}^* \mid n_a(x) = n_b(x)\}$

Sol: Given,

$L = \{\epsilon, ab, ba, aabb, abab, bbaa, \ldots\}$

$\delta(q_0, a, z_0) = (q_0, az_0)$

$\delta(q_0, b, z_0) = (q_0, bz_0)$

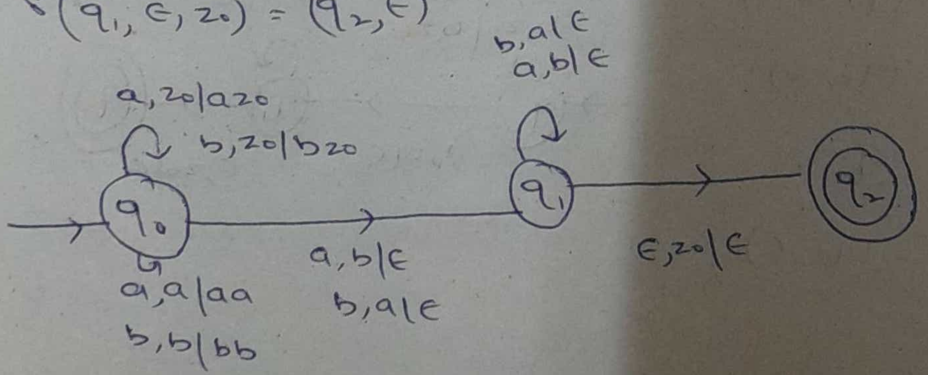
$\delta(q_0, a, a) = (q_0, aa)$

$\delta(q_0, b, b) = (q_0, bb)$

$\delta(q_0, a, b) = (q_1, \epsilon)$

$\delta(q_0, b, a) = (q_1, \epsilon)$

$\delta(q_1, \epsilon, z_0) = (q_2, \epsilon)$



$PDA = (\{q_0, q_1, q_2\}, \{a,b\}, \{z_0, a, b\}, \delta, q_0, z_0, q_2)$

18) Show that the below languages are deterministic Context free languages

$L_1 = \{0^n 1^m \mid n = m \text{ and } n \geq 1\}$

$L_2 = \{0^n 1^m \mid n = 2m \text{ and } n \geq 1\}$

Sol: A language L is said to be Deterministic Context-Free language (DCFL) if there exists a DPDA that accepts it.

i) $L_1 = \{0^n 1^m \mid n = m \text{ and } n \geq 1\}$

$L_1 = \{01, 0011, 000111, \ldots\}$

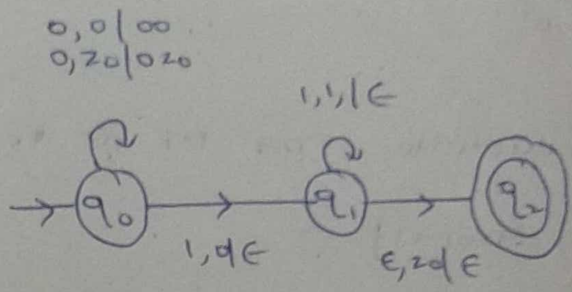
$\delta(q_0, 0, z_0) = (q_0, 0z_0)$

$\delta(q_0, 0, 0) = (q_0, 00)$

$\delta(q_0, 1, 0) = (q_1, \epsilon)$

$\delta(q_0, 1, 1) = (q_1, \epsilon)$

$\delta(q_1, \epsilon, z_0) = (q_2, \epsilon)$



ii) $L_2 = \{0^n 1^m \mid n = 2m \text{ and } n \geq 1\}$

$L_2 = \{001, 000011, 000000111, \ldots\}$

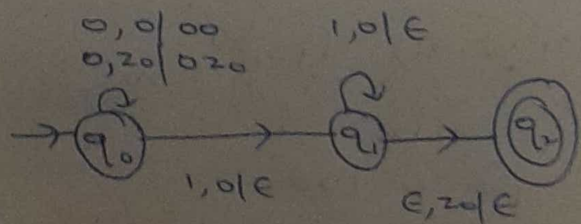
Here, when the input symbol is 0, we push into stack and when input symbol is 1, we pop the two 0's from the stack. At end, if the stack is empty, the language is accepted.

$\delta(q_0, 0, z_0) = (q_0, 0z_0)$

$\delta(q_0, 0, 0) = (q_0, 00)$

$\delta(q_0, 1, 0) = (q_1, \epsilon)$

$\delta(q_1, \epsilon, z_0) = (q_2, \epsilon)$



19) Describe deterministic Context free languages and deterministic pushdown Automata.

Sol: **Deterministic Context free languages (DCFL):-**

A DCFL is a Subset of Context-free languages that can be recognized and generated by a DPDA. A Context free language is a language that can be generated by a CFG, where production rules define how non-terminals symbols can be replaced with Sequences of terminals and non-terminals.

DCFLs have the property that, for a given input String there is only one possible valid derivation or parse Tree.

Deterministic pushdown Automata (DPDA):-

The DPDA operates on an input alphabet and a stack alphabet and uses transitions to move between states while reading the input Symbols and manipulating the Stack.

There is only one possible transition in DPDA - It is represented as 7 tuple $(Q, \Sigma, \Gamma, \delta, q_0, z, F)$

20) Describe PDA that recognizes the language

$L = \{x = x^R : x \in \{a,b\}^+\}$

Sol: Given, $L = \{x = x^R : x \in \{a,b\}^+\}$

$L = \{aaaa, aba, abba, ---\}$

$\delta(q_0, a, z_0) = (q_0, az_0)$

$\delta(q_0, b, z_0) = (q_0, bz_0)$

$\delta(q_0, a, a) = (q_0, aa)$

$\delta(q_0, b, b) = (q_0, bb)$

$\delta(q_0, b, a) = (q_0, ba)$

$\delta(q_0, a, b) = (q_0, ab)$

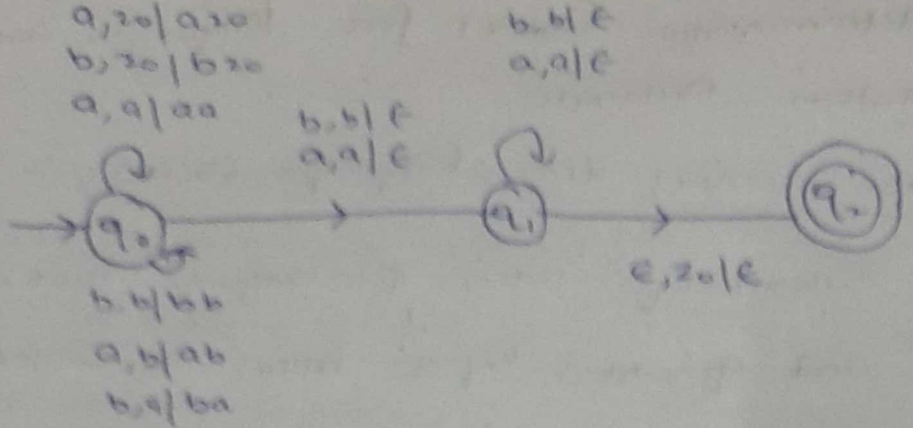
$\delta(q_0, a, a) = (q_1, \epsilon)$

$\delta(q_0, b, b) = (q_1, \epsilon)$

$\delta(q_1, a, a) = (q_2, \epsilon)$

$\delta(q_1, b, b) = (q_2, \epsilon)$

$\delta(q_2, \epsilon, z_0) = (q_3, z_0)$



Part-A

1) Construct PDA for equal number of x's and y's eg: xyyxxy.

Sol: Equal number of x and equal number of y.

(Refer part-B 17th Question)

2) Construct NDPDA for $L = \{w + w^R \mid w \in (x+y)^*\}$

Sol:

3) Convert the following PDA to CFG

i) $\delta(q_0, 0, z_0) = (q_0, x z_0)$

ii) $\delta(q_0, 0, x) = (q_0, xx)$

iii) $\delta(q_0, 1, x) = (q_1, \epsilon)$

iv) $\delta(q_1, 1, x) = (q_1, \epsilon)$

v) $\delta(q_1, \epsilon, x) = (q_1, \epsilon)$

vi) $\delta(q_1, \epsilon, z_0) = (q_1, \epsilon)$

Sol:

i) $\delta(q_0, 0, z_0) = (q_0, x z_0)$

$[q_0, z_0\ q_0] = 0[q_0\ x\ q_0]\ [q_0\ z_0\ q_0]$

$[q_0, z_0\ q_0] = 0[q_0\ x\ q_1]\ [q_1\ z_0\ q_0]$

$[q_0\ z_0\ q_1] = 0[q_0\ x\ q_0]\ [q_0\ z_0\ q_1]$

$[q_0\ z_0\ q_1] = 0[q_0\ x\ q_1]\ [q_1\ z_0\ q_1]$

ii) $\delta(q_0, 0, x) = (q_0, xx)$

$[q_0\ x\ q_0] = 0[q_0\ x\ q_0]\ [q_0\ x\ q_0]$

$[q_0\ x\ q_0] = 0[q_0\ x\ q_1]\ [q_1\ x\ q_0]$

$[q_0\ x\ q_1] = 0[q_0\ x\ q_0]\ [q_0\ x\ q_1]$

$[q_0\ x\ q_1] = 0[q_0\ x\ q_1]\ [q_1\ x\ q_1]$

iii) $\delta(q_0, 1, x) = (q_1, \epsilon)$

$[q_0\ x\ q_1] = 1$

iv) $\delta(q_1, 1, x) = (q_1, \epsilon)$

$[q_1\ x\ q_1] = 1$

v) $\delta(q_1, \epsilon, x) = (q_1, \epsilon)$

$[q_1\ x\ q_1] = \epsilon$

vi) $\delta(q_1, \epsilon, z_0) = (q_1, \epsilon)$

$[q_1\ z_0\ q_1] = \epsilon$

4) Construct DPDA for $L = \{ w \# w^R \mid w \in (x+y)^* \}$.

Sol:

5) Construct pushdown automata for the following languages. Acceptance either by empty stack or by final state.

a) $\{a^n b^m a^n \mid m,n \in N\}$

b) $\{a^n b^m c^m \mid m,n \in N\}$

c) $\{a^i b^j c^k \mid i,j,k \in N,\ i>j\}$

d) $\{a^i b^j c^k \mid i,j,k \in N,\ i+j=k\}$

e) $\{a^i b^j c^k \mid i,j,k \in N,\ i+k=j\}$

f) $\{a^n b^m \mid n \le m \le 2n\}$

g) $PAL = \{w \in \{a,b\}^* \mid mir(w) = w\}$

h) $\{w_1 c w_2 c \ldots c w_k c x \mid x, w_1, \ldots, w_k \in \{a,b\}^*, k \in N,$
$x = mir(w_j)$ for some $j\}$

i) $\{w \in \{a,b\}^* \mid \#a(w) \neq \#b(w)\}$, $\#a(w)$ represents the number of a's in w

j) $\{w \in \{a,b\}^*\ \#a(w) = 2\#b(w)\}$

a) $L = \{a^n b^m a^n \mid m,n \in N\}$

$L = \{aba,\ abba,\ aabbaa,\ ---\}$

$\delta(q_0, a, z_0) = (q_0, az_0)$

$\delta(q_0, a, a) = (q_0, aa)$

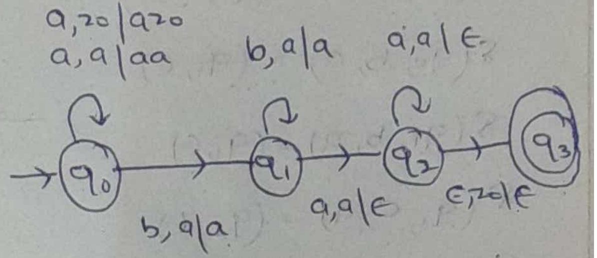
$\delta(q_0, b, a) = (q_1, a)$

$\delta(q_1, b, a) = (q_1, a)$

$\delta(q_1, a, a) = (q_2, \epsilon)$

$\delta(q_2, a, a) = (q_2, \epsilon)$

$\delta(q_2, \epsilon, z_0) = (q_3, \epsilon)$



b) $L = \{a^n b^m c^m \mid m,n \in N\}$

$\delta(q_0, a, z_0) = (q_0, z_0).$

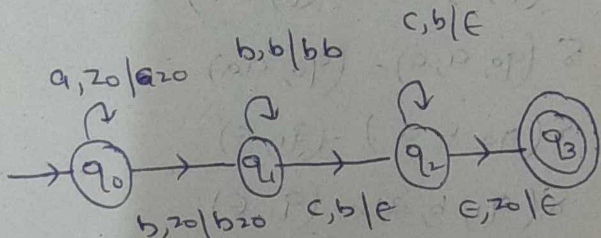
$\delta(q_0, b, z_0) = (q_1, bz_0)$

$\delta(q_1, b, b) = (q_1, bb)$

$\delta(q_1, c, b) = (q_2, \epsilon)$

$\delta(q_2, c, b) = (q_2, \epsilon)$

$\delta(q_2, \epsilon, z_0) = (q_3, z_0)$



d) $\{a^i b^j c^k \mid i,j,k \in N,\ i+j=k\}$

$\delta(q_0, a, z_0) = (q_0, az_0)$

$\delta(q_0, a, a) = (q_0, aa)$

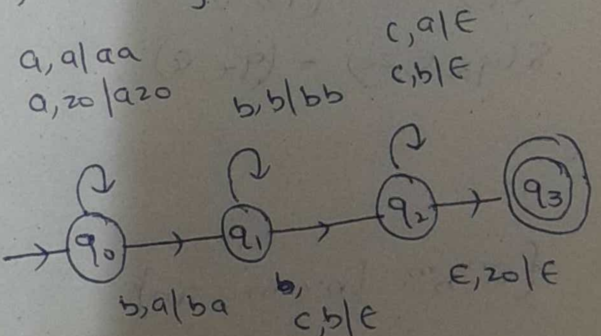
$\delta(q_0, b, a) = (q_1, ba)$

$\delta(q_1, b, b) = (q_1, bb)$

$\delta(q_1, c, b) = (q_2, \epsilon)$

$\delta(q_1, c, a) = (q_2, \epsilon)$

$\delta(q_2, \epsilon, z_0) = (q_3, \epsilon)$



c) $L = \{a^i b^j c^k \mid i,j,k \in N,\ i > j\}$

$\delta(q_0, a, z_0) = (q_0, az_0)$

$\delta(q_0, a, a) = (q_0, aa)$

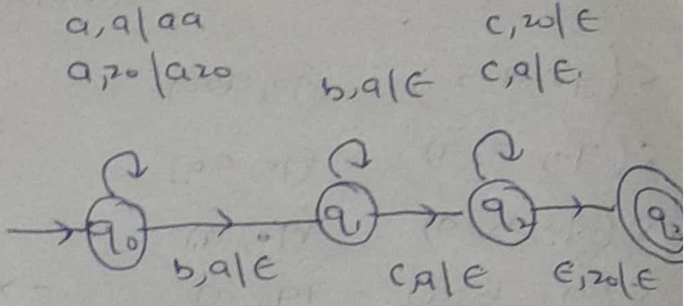
$\delta(q_0, b, a) = (q_1, \epsilon)$

$\delta(q_1, b, a) = (q_1, \epsilon)$

$\delta(q_1, c, a) = (q_2, \epsilon)$

$\delta(q_1, c, z_0) = (q_2, \epsilon)$

$\delta(q_2, \epsilon, z_0) = (q_3, \epsilon)$



e) $L = \{a^i b^j c^k \mid i,j,k \in N,\ i+k = j\}$

$\delta(q_0, a, z_0) = (q_0, az_0)$

$\delta(q_0, a, a) = (q_0, aa)$

$\delta(q_0, b, a) = (q_1, \epsilon)$

$\delta(q_1, b, a) = (q_1, \epsilon)$

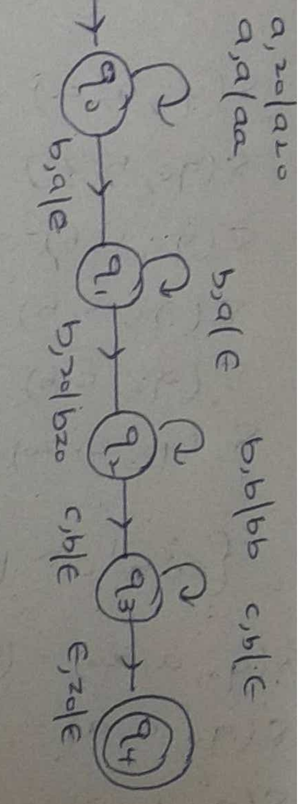
$\delta(q_1, b, b) = (q_2, bb)$

$\delta(q_1, b, z_0) = (q_2, bz_0)$

$\delta(q_2, c, b) = (q_3, \epsilon)$

$\delta(q_3, c, b) = (q_3, \epsilon)$

$\delta(q_3, \epsilon, z_0) = (q_4, \epsilon)$



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f) $\{a^n b^m \mid n \leq m \leq 2n\}$

$\delta(q_0, a, z_0) = (q_0, az_0)$

$\delta(q_0, a, a) = (q_0, aa)$

$\delta(q_0, \epsilon, a) = (q_1, \epsilon)$

$\delta(q_1, b, a) = (q_1, \epsilon)$

$\delta(q_1, b, a) = (q_2, ba)$

$\delta(q_2, b, a) = (q_1, \epsilon)$

$\delta(q_1, \epsilon, z_0) = (q_f, \epsilon)$