4A. Describe CFG for $\Sigma = \{a, b\}$

i) All strings having atleast 2a's

ii) All possible strings not containing 3b's.

(i) $\Sigma = \{a, b\}$

$L = \{aa, aaab, aaaaabb \ldots\}$

Regular Expression : There must be min 2a's

a   a

Before and after a, there can be any number of a's and b's.

$\therefore$

$R.E = (a+b)^* a (a+b)^* a (a+b)^*$

$P : S \to A a A a A$

$A \to aA \mid bA \mid \epsilon$

$\therefore G = V, T, P, S$

$= \{(S, A), (a, b), P, S\}$

(ii) No triple b's

$w = aaababbaa$

$S \to Sa \mid Sab \mid Sabb \mid b \mid bb \mid \epsilon$

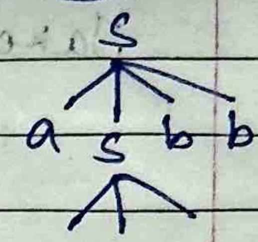
1A $L = \{a^n b^{2n} ; n \geq 1\}$

Sol: $S \to aSbb$

$S \to abb$.

For each a, there should be 2 b's and this production must repeat recursively

aabbbb



To end the recursion, we form a production with a and 2 b's.

2B $L = \{a^i b^j \mid i < 2j\}$

→ The string should start with a, followed by b

$S \to AB$

A and B together will satisfy the condition $i < 2j$

$A \to aAa \mid \blacksquare \mid a \mid \epsilon$

This production generates sequence of a's

$B \to Bb \mid \blacksquare \mid b \mid \epsilon$

generates sequence of b's.

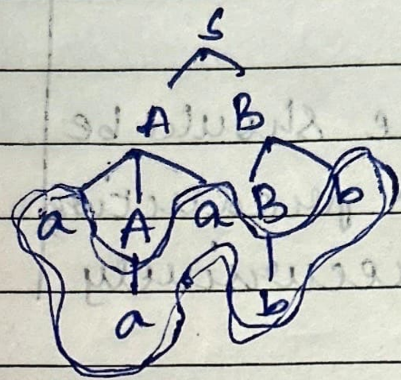
Take a string where $j = 2$

then $i < 4$

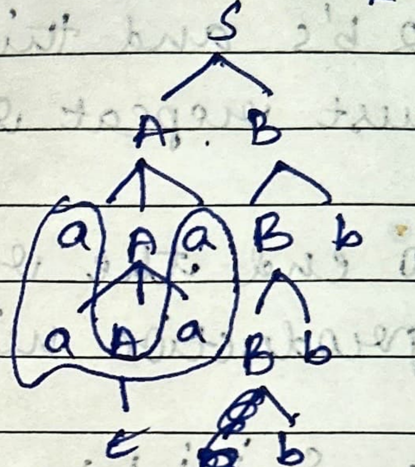
aaabb

$j = 3$

then $i < 5$



aaaaabbb



∴ Production :

$S \rightarrow AB$

$A \rightarrow aAa \mid a \mid \epsilon$

$B \rightarrow Bb \mid b \mid \epsilon$.

19B $L = \{0^i 1^j 0^k : j > i + k\}$

$S \rightarrow ABC$

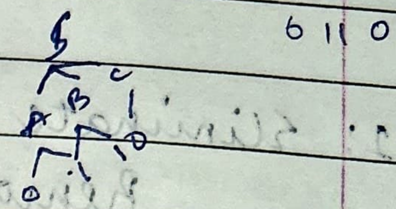
$A \rightarrow 0A \mid \epsilon$

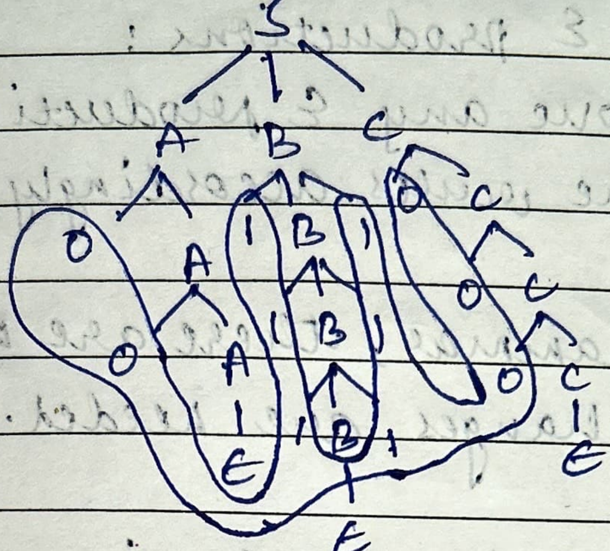
$B \rightarrow 1B1 \mid \epsilon$

$C \rightarrow 0C \mid \epsilon$

check for $i = 2$, $k = 3$ then $j > 5$

00111111000





Minimization of CFG includes 3 steps

→ Elimination of useless symbols

Useless symbols are the non-Terminals which are not derived from the starting symbol and if the symbol doesn't produce set of terminals.

→ Elimination of ϵ production

→ Elimination of unit production.

Minimization:

Useless: N.T should not be derived from S, if it doesnt produce terminals

Unit: N.T → N.T

ε: Replace all the symbols with ε.

Doubt

6.A $S \to Ab \mid Bb$
$A \to a \mid aS \mid Baa$
$B \to b \mid bS \mid aBB$.

Step 1: Eliminate ε productions:
Remove any ε productions and update the rules accordingly.

In this grammar, there are no ε-produc. so no changes are needed.

Step 2: Eliminate unit productions:
→ Unit productions are productions that derive a non-terminal to another non-terminal [A → B].

In this grammar, there are no unit productions.

Step 3: Eliminate useless symbols:
→ ~~Remove any symbols (Terminals and non-terminals) that do not contribute to generating any string in the language.~~

Useless symbols are symbols i.e. Non-terminals which are not derived from start symbol and if the Non-terminal doesn't produce a string of terminals.

$S \to Ab$ $\quad S \to Bb$ $\quad A \to a$ ✓ $\quad A \to aS$

$S \to ab$ ✓ $\quad S \to bb$ ✓ $\quad A \to aAb$

$A \to Baa$ ✓ $\quad B \to b$ ✓ $\quad A \to aab$ ✓

$A \to baa$ $\quad B \to bS \Rightarrow B \to bBb$

$B \to aBB$ $\quad B \to bbb$ ✓

$B \to abb$ ✓.

(4B) $S \to AbA$

$A \to Aa \mid \epsilon$.

Step1: Eliminate ϵ productions

Here, $A \to \epsilon$.

To eliminate $A \to \epsilon$, replace all productions of A with ϵ.

$S \to AbA \mid bA \mid Ab \mid b$

$A \to Aa \mid a$

Step2: Eliminate unit productions.

Here, there are no unit productions

Step 3: Eliminate Useless Symbols:

$S \to AbA \mid bA \mid Ab \mid b$

$A \to Aa \mid a$

$S \to AbA$ $\to aba$ ✓ $S \to bA$ $\to ba$ ✓ $S \to Ab$ $\to ab$ ✓ $S \to b$ ✓

$A \to Aa$ $\to aa$ ✓ $A \to a$ ✓

∴ There are no useless Symbols

5B $S \to aSa$

$S \to bSb$

$S \to a \mid b \mid \epsilon$

Step 1: Eliminate ϵ:

$S \to \epsilon$

Replace S with ϵ.

$S \to aSa \mid aa$

$S \to bSb \mid bb$

$S \to a \mid b$

Step2: Remove unit productions.
There are no unit productions

Step3: Remove Useless Symbols:
There are no useless Symbols.

(6B) $S \rightarrow A0 \mid B$
$A \rightarrow 0 \mid 12 \mid B.$

Step1: ϵ:
No ϵ

Step2: ~~...~~ Useless Productions:
$S \rightarrow B$
$A \rightarrow B$ is useless because
there are no productions from B.

After removing:
$S \rightarrow A0$
$A \rightarrow 0 \mid 12.$

Step3: Useless: No useless productions.

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(13B) $S \to aS_1b$
$S_1 \to aS_1b \mid \epsilon$.

Step 1 ϵ :
Replace $S_1 = \epsilon$ in all productions

$S \to aS_1b \mid ab$
$S_1 \to aS_1b \mid ab$.

Step 2 : Useless : No useless

$S \to aS_1b$ $\quad$ $S \to ab$ ✓
$\to aabb$ ✓

$S_1 \to aS_1b$ $\quad$ $S_1 \to ab$ ✓
$\to aabb$ ✓

Step 3 Unit - No unit productions

(18B) $A \to aBb \mid bBa$
$B \to aB \mid bB \mid \epsilon$.

ϵ : $\underline{B \to \epsilon}$
$A \to aBb \mid bBa \mid ab \mid ba$
$B \to aB \mid bB \mid a \mid b$

(20B) $S \to ABCa \mid bD$

$A \to BC \mid b$

$B \to b \mid \epsilon$

$C \to \epsilon$

$D \to d.$

ϵ: $B \to \epsilon$

$S \to ABCa \mid bD \mid ACa$

$A \to BC \mid b \mid C$

$B \to b$

$C \to \epsilon$

$D \to d.$

$C \to \epsilon$

$S \to ABCa \mid bD \mid ACa \mid ABa \mid Aa$

$A \to BC \mid b \mid C \mid B$

$B \to b$

$D \to d$

~~[illegible]~~.

Useless: $A \to C$ is useless, because there is no production from C.

$S \to ABCa$ is useless because

$S \to ACa$ " "

$A \to BC$ " "

After Eliminating:

$S \to bD \mid ABa \mid Aa$
$A \to b \mid B$
$B \to b$
$D \to d.$

Unit: $A \to B$ is unit production

$A \to B$, $B \to b$
$\therefore A \to b.$

After Eliminating:

$S \to bD \mid ABa \mid Aa$
$A \to b$
$B \to d$
$D \to d.$

22B $S \to aAa$
$A \to Sb \mid bcc \mid DaA$
$C \to abb \mid DD$
$E \to ac$
$D \to aDa$

Useless:

✓ $S \rightarrow aAa$

$\rightarrow abccaa$ ✓

✓ $A \rightarrow Sb$

$\rightarrow aAab$

$\rightarrow abccab$ ✓

✗ $A \rightarrow DaA$

$\rightarrow aDaabcc$

✓ $C \rightarrow abb$

✗ $C \rightarrow DD$

$\rightarrow aDaaDa$

✓ $E \rightarrow ac$

✗ $D \rightarrow aDa$

After Elimination:

$S \rightarrow aAa$

$A \rightarrow Sb \mid bcc$

$C \rightarrow abb$

$E \rightarrow ac$

But C, E are not reachable

$\therefore$ After Elimination,

$$\boxed{\begin{array}{l} S \rightarrow aAa \\ A \rightarrow Sb \mid bcc \end{array}}$$

Chomskey Normal form (CNF)

↳ It should be of form $A \to BC$, $A \to a$

i.e 2 N.T or 1 Terminal

(8A) $S \to AaA \mid CA \mid BaB$

$A \to aaBa \mid CDA \mid aa \mid DC$

$B \to bB \mid bAB \mid bb \mid aS$

$C \to Ca \mid bC \mid D$

$D \to bD \mid A$.

Remove Unit Productions:

$C \to D$

$C \to bD$

$D \to A$

∴ $S \to AaA \mid CA \mid BaB$

$A \to aaBa \mid CDA \mid aa \mid DC$

$B \to bB \mid bAB \mid bb \mid aS$

$C \to Ca \mid bC \mid D$

$D \to bD \mid aa$

Remove Useless Symbols:

$S \to CA$ ×

$A \to CDA$ ×

$A \to DC$ ×

$C \to Ca \mid bC$ ×

8(A) $S \to AaA \mid CA \mid BaB$

$A \to aaBa \mid CDA \mid aa \mid DC$

$B \to bB \mid bAB \mid bb \mid aS$

$C \to Ca \mid bC \mid D$

$D \to bD \mid A.$

Unit Symbols:

$D \to A.$

$D \to A$, $A \to aa$ $\therefore D \to aa.$

$C \to D$

$C \to D$, $D \to aa$ $\therefore C \to aa$

After Removing:

$S \to AaA \mid CA \mid BaB$

$A \to aaBa \mid CDA \mid aa \mid DC$

$B \to bB \mid bAB \mid bb \mid aS$

$C \to Ca \mid bC \mid aa$

$D \to bD \mid aa.$

Let $P \to a$

$Q \to b$, then $S \to CA \mid APA \mid BPB$

$A \to DC \mid PP \mid PBP \mid CDA$

$B \to QQ \mid PS \mid QB \mid QAB$

$C \to PP \mid CP \mid QC$

$D \to PP \mid QD$

Let $P_1 \to PA$

$P_2 \to PB$

$S \to CA \mid AP_1 \mid BP_2$

$A \to DC \mid PP \mid P_2P \mid CDA$

$B \to QQ \mid PS \mid QB \mid QAB$

$C \to PP \mid CP \mid QC$

$D \to PP \mid QD.$

$R_1 \to DA$

$R_2 \to AB$

$S \to CA \mid AP_1 \mid BP_2$

$A \to DC \mid PP \mid P_2P \mid CR_1$

$B \to QQ \mid PS \mid QB \mid QR_2$

$C \to PP \mid CP \mid QC$

$D \to PP \mid QD.$

$R_1 \to DA$

$R_2 \to AB$

$P_1 \to PA$

$P_2 \to PB$

$P \to a$

$Q \to b$

$A \to AB$
$A \to a$

(1B) $S \to aSa \mid aa$
$S \to bSb \mid bb$
$S \to a \mid b$.

Sol: It is in reduced form.

Let $P \to a$
$Q \to b$.

$S \to PSP \mid PP$
$S \to QSQ \mid QQ$
$S \to P \mid Q$
$P \to a$
$Q \to b$.

Let $R = SP$
$Z = SQ$.

$S \to PR \mid PP$
$S \to QZ \mid QQ$
$S \to P \mid Q$
$P \to a$
$Q \to b$
$R \to SP$
$Z = SQ$

Remove unit productions
$S \to P$, $S \to Q$.
$S \to P$, $P \to a$, $\Rightarrow S \to a$
$S \to Q$, $Q \to b \Rightarrow S \to b$.

$S \to PR \mid PP$
$S \to QZ \mid QQ$
$S \to a \mid b$
$P \to a$
$Q \to b$
$R \to SP$
$Z = SQ$.

Pumping Lemma for CFG:

Let L be a CFL. Let 'n' be an integer constant. Select string 'z' from L such that $|z| \geq n$

Divide z into 5 parts $uvwxy$ such that

$|vwx| \leq n$

$|vx| \geq 1$

for $i \geq 0$, uv^iwx^iy is in L.

(12B) $L = \{a^n b^n c^n \mid n \geq 1\}$

Let L is a CFL.

$L = \{abc, aabbcc, aaabbbccc \ldots\}$

Let $n = 3$.

$z = aaabbbccc$

$|z| \geq 3$

$\therefore |z| \geq n$.

Let $u = aa$

$v = a$

$w = b$

$x = b$

$y = bccc$

$|vwx| \leq n \Rightarrow |abb| \leq n$ ✓

$|vx| \geq 1$

$|ab| \geq 1$ ✓

then $i \geq 0$, uv^iwx^iy in L.

$i=0 \Rightarrow aa(a)^0 b (b)^0 bccc$

$= aabbccc \notin L \Rightarrow$ Contradiction.

$\therefore$ It is not CFG

Ambiguous CFG:

It is the condition in which the input string made using the grammar posseses 2 or more LMD

...}

11B) $S \rightarrow iCtSeS \mid iCtS \mid a$

$C \rightarrow b$.

$w = ibtibtibtaea$.

$S \rightarrow iCtSeS$	$S \rightarrow iCtS$
$\rightarrow ibtSeS$	$\rightarrow ibtS$
$\rightarrow ibtiCtSeS$	$\rightarrow ibtiCtS$
$\rightarrow ibtibtiCtSeS$	$\rightarrow ibtibtS$
$\rightarrow ibtibtibtaea$	$\rightarrow ibtibtiCtSeS$
	$\rightarrow ibtibtibtaea$