25/6/23

Module - IV

TOC
Tare

Push down Automata

According to chomsky herirachy

R.G → Type 3 → FSM → Finite amount of Info

CFG → Type 2 → PSD → Infinite amount of Info
↓
Stack.
↓ operations ⟨ Push / pop

3 Components

Input tape → Input string
finite control → Push/pop
Stack → Elements

Push Down Automata = finite state Machine + stack.
↓ 5 Tuple ↓ 2 Tuple
<=> 7 Tuple.

Definition of Pushdown Automata:

7 Tuple Notation

$(Q, \Sigma, \delta, \Gamma, q_0, Z_0, F)$

Q → No. of states. Σ = Alphabet with input symbols

δ → Transition function (Triple Tuple) → $\{(q_n, \alpha) \ldots\}$

$\delta(q, a, x)$ q → current state
↓
Instantaneous Description (IDP)

a → input to be processed
x → Top of stack
q_n → new state
α → Top of stack to be replaced of x otherwords string of stack symbols

Γ → Stack symbols

q_0 → Initial state

Z_0 → Initial top of stack. $\boxed{Z_0}$

F → final state

Example of PDA

(Q) Construct PDA for language = $\{a^n b^n / n \geq 1\}$

Sol $L = \{ab, aabb, aaabbb \ldots\}$

$\delta(q, a, y)$ → Top of stack; → Input to Proce; q ↓ current state

⊢ turnstile notation (q', α) → string of stack symbols in stack; q' → New state

$\delta(q_0, a, z_0) = (q_0 a z_0)$

| a | a | a | b | b | b |

$a\ z_0$ push

a → Push

b → POP

unchange.

$(q', \epsilon) \rightarrow$ Pop

$\delta(q_0, a, a) = \quad aaz_0 \quad = (q_0, aaz_0)$

$\delta(q_0, a, a) = \quad aaaz_0 \quad = (q_0, aaaz_0)$

$\delta(q_0, b, a) = (q_1, \epsilon) = \quad aaaz_0$

$\delta(q_1, b, a) = (q_1, \epsilon) =$

| a | a | a | b | b | b | ϵ |

$\delta(q_1, b, a) = (q_1, \epsilon) =$

$\delta(q_1, \epsilon, z_0) = (q_2, \epsilon) = \sqcup$ } Empty stack.

$F = \{q_2\}$ $Q = \{q_0, q_1, q_2\}$ $\Sigma = \{a, b\}$ $q_0 \rightarrow$ Initial state

$\Gamma = \{z_0, a\}$ $z_0 \rightarrow$ Top of stack.

P = {{

Example 2

Sol.

$P = \{ \{q_0, q_1, q_2\}, \{a, b\}, \{z_0, a\}, \delta, \{q_0\}, \{z_0\}, \{q_2\} \}$

Q Σ Γ δ q_0 z_0 F

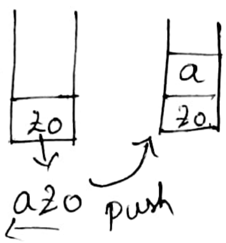
Example 2: Give a PDA to accept the following Language $L = \{a^n b^{2n} / n \geq 1\}$ by final state

Sol: $L = \{aa, bbbb \ldots\}$

$L = \{abb, aabbbb \ldots\}$

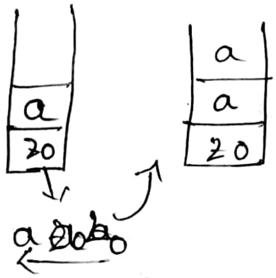
1st b No change
2nd b pop a
3rd b No change
4th b pop a
ε

$\delta(q_0, a, z_0) =$

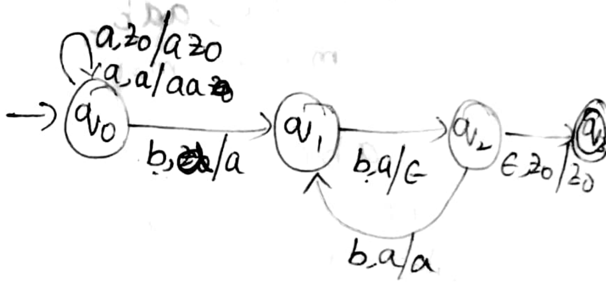


$= (q_0, az_0)$

$\delta(q_0, a, a) =$



$\Rightarrow (q_0, aaz_0)$



$\delta(q_0, b, a\cancel{az_0}) = (q_1, a z_0$

$\delta(q_1, b, a) = (q_2, \epsilon)$

$\delta(q_2, b, a) = (q_1, a z_0$

$\delta(q_1, b, a) = (q_2, \epsilon)$ ×

$\delta(q_2, \epsilon, z_0) = (q_3, z_0)$

$P = (\{q_0, q_1, q_2, q_3\}, \{a, b\}, \{a, z_0\}, \delta, q_0, z_0, q_3)$.

G 1, 3, 6, 9

H 0, 3, 4, 5, 8, 9

J 0, 1, 2, 3, 5, 6, 9

K 0, 4, 5, 9

L 2, 9

M 4, 7,

N 1, 2, 3, 5, 7,

P 4, 8,

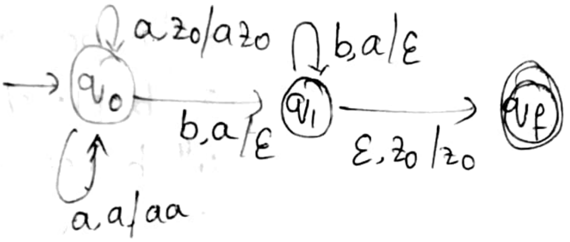
Le 15, 16, 17,

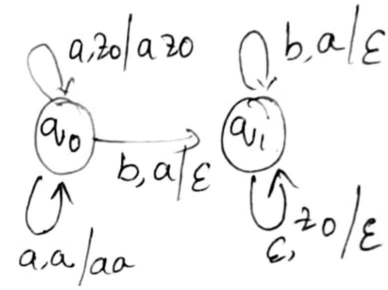
Push Down Automata Acceptance

There are two different ways to define PDA acceptab

1. Final State Acceptability:
2. Empty state Acceptability:

$a^n b^n$





Example 3

$L = \{a^n b^m c^n / n \geq 1, m \geq 1\}$

$L = \{abc, aabcc, aabbcc, aaabbbccc \ldots\}$
$m, n = 1$ $\quad m = 1, n = 2$ $\quad m = 2, n = 2$ $\quad m = 2, n = 3$

abc

no. of a's = no. of c's

a	a	b	c	c	ε

a → push (a)
b → unchange
c → Pop (a)

$\delta(q_0, a, z_0) = \delta(q_0, a z_0)$

$\delta(q_0, a, a) = \delta(q_0, aa z_0)$

$\delta(q_0, b, a) = \delta(q_1, aa z_0)$

$\delta(q_1, c, a) = \delta(q_2, \epsilon)$

$\delta(q_2, c, a) = \delta(q_2, \epsilon)$ $\boxed{z_0}$

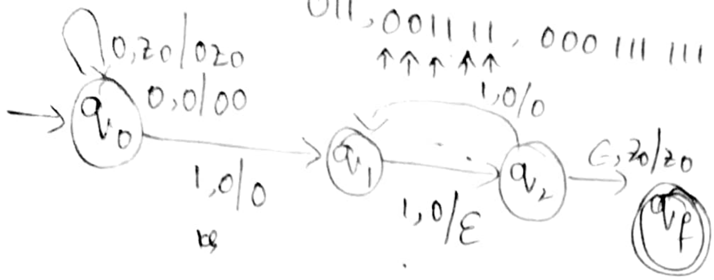
$\boxed{\delta(q_2, \epsilon, z_0) = \delta(q_3, \epsilon)}$ → End.

$Q = \{q_0, q_1, q_2, q_3\}, \Sigma = \{a, b, c\}$ $\Gamma = \{a, z_0\}$ q_0, z_0, q_3

Example 4

$L = \{0^n, 1^{2n}, n > 0\}$

011, 001111, 000111111



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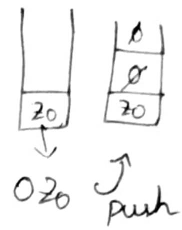
01 ×



$\delta(q_0, 0, Z_0) \vdash q_0, 0Z_0$

$\delta(q_0, 0, 0) \vdash q_0, 00$

$\delta(q_0, 1, 0) \vdash q_1, 0$

$\delta(q_1, 1, 0) \vdash q_2, \epsilon$

$\delta(q_2, 1, 0) \vdash q_1, 0$

$\delta(q_2, \epsilon, Z_0) \vdash q_f, Z_0$

Equivalence of Acceptance by Final and Acceptance by Empty stack for Push down automata.

* PDA's that accept by final state
  - PDA = P
  - Language accepted by P.
    - denoted by L(P) by final state
    - * $\{w \mid (q_0, w, Z_0) \vdash^* (q, \epsilon, A)\}$ s.t., $q \in F$

Check List

- Entire i/p should be exhausted?
- in a final state

* PDA's that accept by Empty Stack:

Language accepted by is denoted by P by $N(P)$ is.

$\{ w \mid (q_0, w, z_0) \vdash^* (q, \varepsilon, \varepsilon) \}$ for any $q \in Q$.

Check List

- input Exhausted?
- is Stack Empty.

(Q) Does a PDA that accepts by Empty Stack need any final state specified in the design.

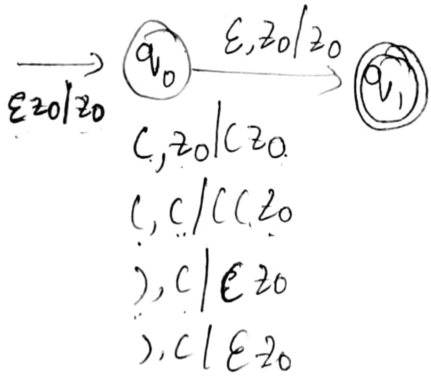
PDA that accepts by final state when '(' is poped by if we get ')'

P_F: $($, z_0 / $(z_0$

$($, $($ / $(($

$)$, $($ / ε

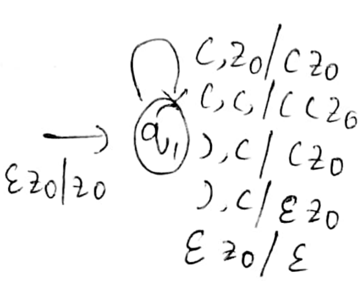
PDA = $\{ Q, \Sigma = \{ (,) \}, \Gamma, z_0, q_0, q_f \}$



(()))

An Equivalent PDA that accepts by Empty stack.

(()))



eg. a) → 1 symbol (2') = 2

→ push
pop
pop

((( ))) ( )) ( ) ⇒ Example 2

P_F ⇐ PDA accepting by final state

$P_F = (Q_F, \Sigma, \Gamma, \delta, q_0, Z_0, F)$

P_N ⇐ PDA accepting by Empty stack

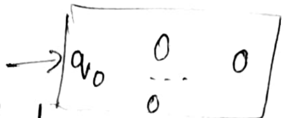
$P_N = (Q_N, \Sigma, \Gamma, \delta, q_0, Z_0)$

Theorem ($P_N \Rightarrow P_F$) for every P_N there exists a P_F st
$L(P_F) = L(P_N)$

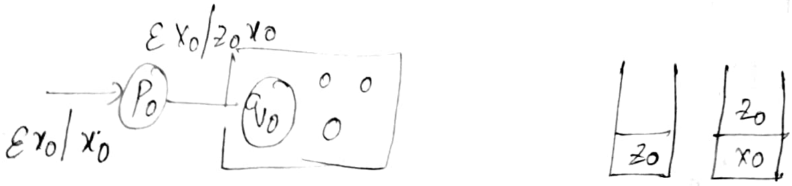
($P_F \Rightarrow P_N$) for every P_F there exists a P_N st
$L(P_F) = L(P_N)$

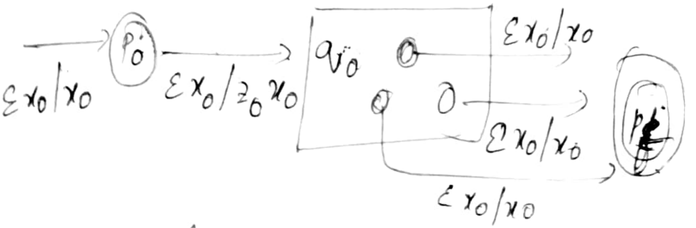
{ P_N == P_F Construction }

* Whenever P_N's stack becomes empty, make P_F go to a final state without consuming any aditional symbol



* To detect empty stack in P_N: P_F pushes a new stack symbol X_0 (not in Γ of P_N) initially before simulating P_N





$P_F = (Q_N \cup \{P_0, P_f\}, \Sigma, \Gamma \cup \{X_0\}, \delta, P_0, X_0, \{P_f\})$

Example "(" ")" Matching Parenthesis

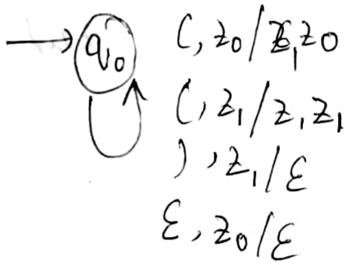
$P_N: \{\{q_0\}, \{(,)\}, \{z_0, z_1\}, \delta, q_0, z_0\}$ Empty stack.

$P_N: \delta(q_0, (, z_0) = \{(q_0, z_1 z_0)\}$

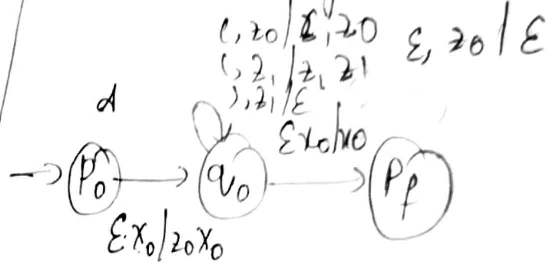
$\delta(q_0, (, z_1) = \{(q_0, z_1 z_1)\}$

$\delta(q_0,), z_1) = \{(q_0, \varepsilon)\}$

$\delta(q_0, \varepsilon, z_0) = \{(q_0, \varepsilon)\}$



Acceptance by final state



$\delta(P_0, \varepsilon, x_0) = \{(q_0, z_0)\}$

$\delta(q_0, (, z_0) = \{(q_0, z_1 z_0)\}$

$\delta(q_0, (, z_1) = \{q_0, z_1 z_1\}$

$\delta(q_0,), z_1) = \{(q_0, \varepsilon)\}$

$\delta(q_0, \varepsilon, z_0) = \{(q_0, \varepsilon)\}$

$\delta(q_0, \varepsilon, x_0) = \{q_f, x_0)\}$

How to Convert an Final state PDA into an Empty stack PDA?

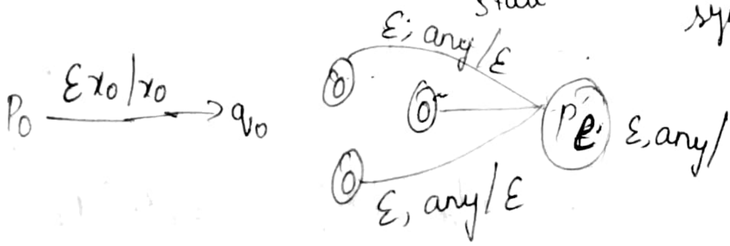
Main idea:

Whenever P_f reaches a final state, just make ε-transition into a new end state. Clear out the stack and accept

→ to address this, add a new Start symbol x_0 (NOT in Γ of P_f)

$P_N = (Q \cup \{P_0, P_e\}, \Sigma, \Gamma \cup \{x_0\}, \delta, P_0, x_0)$ → top

P_e ↓ Empty state; $\{x_0\}$ ↓ New symbol; P_0 ↓ Initial ~~Extra~~ state.



Convertion of CFG to PDA

@ Bhanu Priya.

Ex:

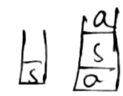
$$\left.\begin{aligned} S &\to asa \\ S &\to bsb \\ S &\to c \end{aligned}\right\} \text{CFG}$$

$S \Rightarrow NT$
$a, b, c \Rightarrow$ Terminal.

Design Push Down Automata.
↳ Push & Pop to accept string.

Sol: Procedure: write production rules.
then pop elements

① $\delta(q_0, \epsilon, \epsilon) = (q_0, \epsilon)$

② $\delta(q_0, \epsilon, s) \Rightarrow (q_0, asa)$ 

③ $\delta(q_0, \epsilon, s) \Rightarrow (q_0, bsb)$

④ $\delta(q_0, \epsilon, s) \Rightarrow (q_0, c)$

* write all productions

⑤ $\delta(q_0, a, a) \Rightarrow (q_1, \epsilon)$

⑥ $\delta(q_1, b, b) \Rightarrow (q_2, \epsilon)$

⑦ $\delta(q, c, c) \Rightarrow (q_3, \epsilon)$

Transition table

S.No.	state	un read i/p	stack	transition No.
1.	q_0	abbcbba	ϵ	1
2.	q_0	abbcbba	S	1
3.	q_0	~~a~~bbcbba	~~a~~sa	2
4.	q_1	bbcbba	Sa	5
5.	q_0	~~b~~bcbba	bsba	3
6.	q_2	bcbba	Sba	6
7.	q_0	~~b~~cbba	~~b~~sbba	3

8.	q_2	cbba	sbba	6
9.	q_0	~~c~~bba	~~c~~bba	4
10	q_3	~~b~~ba	~~b~~ba	~~7~~
11	q_2	~~b~~a	~~b~~a	6
12.	q_1	~~a~~	~~a~~	5

abba

abb, abba

$L = \{ abbbba, aaaa \ldots \}$

bb

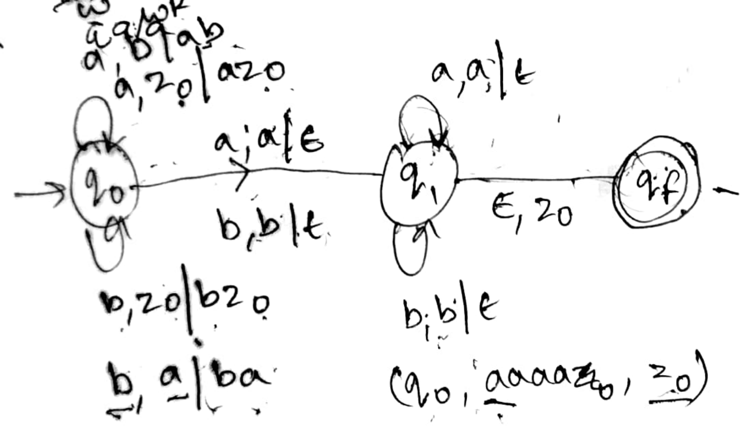
aaaa

$w \; w^R$

$L = \{ww^R \ldots \}$ $\Sigma = a, b$

$L = \{abba, baab$

NDPDA



aa|aa

$(q_0, aaaa, Z_0)$

$\vdash (q_0, aaa, aZ_0)$

$\vdash (q_0, aa, aaZ_0)$ $\quad$ $\vdash (q_1, aa, \epsilon)$ X

$(q_0, a, aaaZ_0)$ $\quad$ (q_1, a, aZ_0)

$(q_0, \epsilon, aaaaZ_0)$ X $\quad$ (q_1, ϵ, Z_0) X $\quad$ (q_1, ϵ, aaZ_0) X $\quad$ (q_1, ϵ, Z_0)

$\downarrow$

q_f

Push down Automata (PDA)

ε-NFA + Stack

Input → Finite State control → Accept / Reject

stack

Definition:

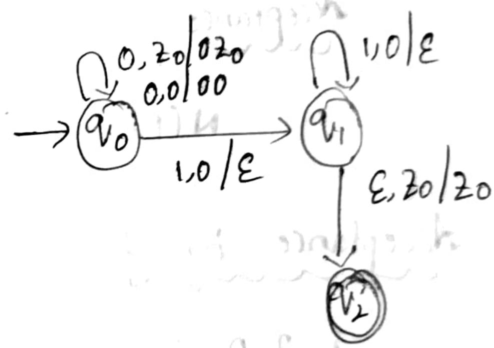
PDA $P = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$

δ – Transition function

$\delta(q, a, x) = (p, \gamma)$

Construct a PDA for $L = \{0^n 1^n \mid n \geq 1\}$

Sol: $L = \{01, 0011, 000111 \ldots\}$



000111ε

0
0
0
Z_0

$\delta(q_0, 0, Z_0) = (q_0, 0Z_0)$

$\delta(q_0, 0, 0) = (q_0, 00)$

No Need to write a transition (0 should be in stack)

$\delta(q_0, 1, 0) = (q_1, \varepsilon)$

$\delta(q_1, 1, 0) = (q_1, \varepsilon)$

$\cancel{\delta(q_1, 1, 0)}$ = No Need to write a transition. (0 will be poped)

$\delta(q_1, \varepsilon, Z_0) = (q_2, Z_0)$

PDA $P = (\{q_0, q_1, q_2\}, \{0,1\}, \{0, Z_0\}, \delta, q_0, Z_0, \{q_2\})$

Instantaneous Description of PDA

If $\delta(q, a, x) = (p, \alpha)$

$(q, aw, x\beta) \vdash (p, w, \alpha\beta)$

0011

$\delta(q_0, 0, Z_0) = (q_0, 0Z_0)$

0011

$(q_0, 0011, z_0) \vdash (q_0, 011, 0z_0)$

$\vdash (q_0, 11, 00z_0)$

$\vdash (q_1, 1, 0z_0)$ pop *

$\vdash (q_1, \varepsilon, z_0)$

$\vdash (q_2, \varepsilon, z_0)$

Acceptance by final state

$L(P) = \{ w / (q_0, w, z_0) \vdash^* (q_f, \varepsilon, \alpha)$ → any thing

Acceptance by Empty stack.

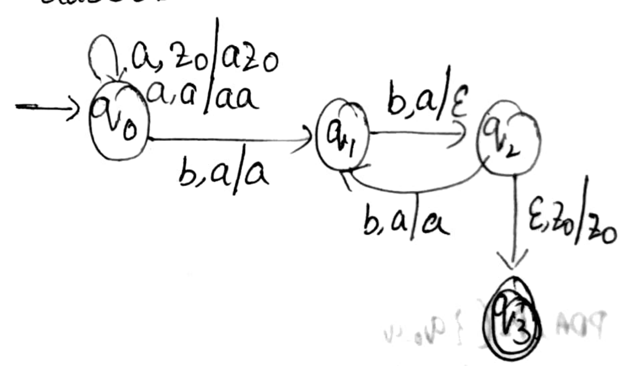
$N(P) = \{ w / (q_0, w, z_0) \vdash^* (q, \varepsilon, \varepsilon)$ → Empty stack.

Acceptance by final state:

$L = \{a^n, b^{2n} \mid n \geq 1\}$

Sol: {abb, aabbbb - - - -}

aabbbb



$\delta(q_0, a, z_0) = (q_0, az_0)$

$\delta(q_0, a, a) = (q_0, aa)$

$\delta(q_0, b, a) = (q_1, a)$

$\delta(q_1, b, a) = (q_2, \varepsilon)$

$\delta(q_2, b, a) = (q_1, a)$ Repeat

~~δ(q₁,~~ No. No Need to do

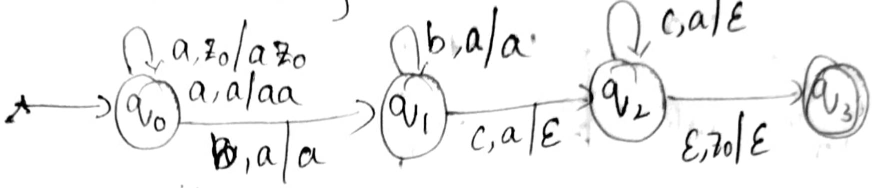
$\delta(q_2, \varepsilon, z_0) = (q_3, z_0)$

PDA $P = \{(q_0, q_1, q_2, q_3\}, \{a, b\}, \{z_0, a\}, \delta, q_0, z_0, q_3\}$.

$L = \{a^n \cdot b^m \cdot c^n \mid n \geq 1,\ m \geq 1\}$

No. of a's & c's should be Equal.

Sol: $L = \{aa\,bbb\,cc \ldots\}$



$\delta(q_0, a, Z_0) = (q_0, aZ_0)$

$\delta(q_0, a, a) = (q_0, aa)$

$\delta(q_0, b, a) = (q_1, a)$ No change because of b.

$\delta(q_1, b, a) = (q_1, a)$

$\delta(q_1, c, a) = (q_2, \epsilon)$

$\delta(q_2, c, a) = (q_2, \epsilon)$

$\delta(q_2, \epsilon, Z_0) = (q_3, \epsilon)$ → Empty stack.

PDA $P = \{(q_0, q_1, q_2, q_3), \{a, b, c\}, \{a, Z_0\}, \delta, q_0, Z_0, q_3\}$

G0, G3, G5, 6, ~~7~~,

~~1~~, 3, 5, ~~7~~, 9,

1, ~~7~~, 3, 4 5, 6,

K0, 5, ~~6~~ 8, 9,

L2, 9

M3, 4, 5, 8, 9,

N2, 3, 4, 5, ~~6~~,

P0, ~~7~~, 4,

Le11, 16, 18, 19,

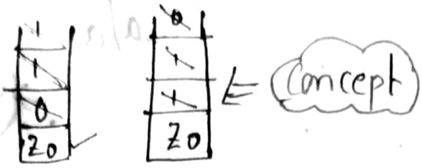
construct Non-deterministic PDA for

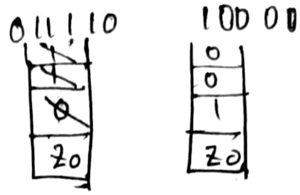
$L = \{ ww^R / w \text{ is in } (0+1)^+ \}$

011 110

$\underset{w}{110}$ $\underset{w^R}{011}$

011110





$\delta(q_0, 0, Z_0) = (q_0, 0Z_0)$

$\delta(q_0, 1, Z_0) = (q_0, 1Z_0)$

$\delta(q_0, \phi, 0) = (q_0, 10Z_0)$

$\delta(q_0, 0, 0) = (q_0, 00Z_0)$

$\delta(q_0, 0, 1) = (q_0, 01Z_0)$

$\delta(q_0, 1, \phi) = (q_0, 11Z_0)$.

2) $\delta(q_0, \varepsilon, 0) = (q_1, 0)$

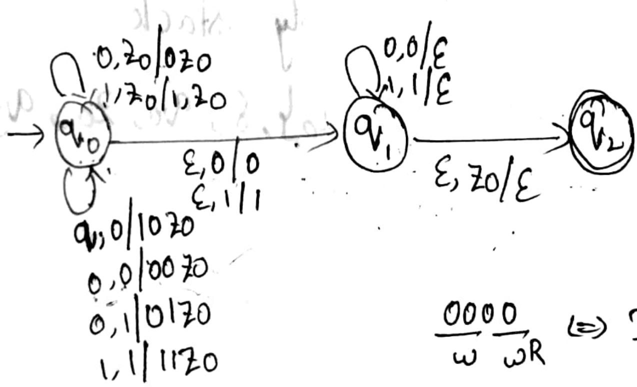
$\delta(q_0, \varepsilon, 1) = (q_1, \phi)$

3) $\delta(q_1, 0, 0) = (q_1, \varepsilon)$

$\delta(q_1, 1, 1) = (q_1, \varepsilon)$

4) $\delta(q_1, \varepsilon, Z_0) = (q_2, \varepsilon)$

PAD $P = \{(q_0, q_1, q_2)\ (0,1),\ \{0,1,Z_0\}\ .\delta, q_0, Z_0, \{q_2\}\}$



$\underset{w}{00}\underset{w^R}{00}$ ⟺ IDP

$(q_0, 0000, Z_0) \vdash (q_0, 000, 0Z_0)$

$(q_0, 00, 00Z_0)$ $(q_1, 000, 0Z_0)$

$(q_0, 0, 000Z_0)$ $(q_1, 00, 00Z_0)$ $(q_1, 00, Z_0)$

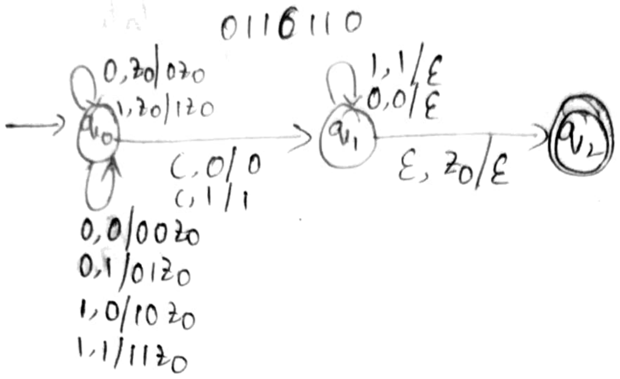
$(q_0, \varepsilon, 0000Z_0)$ $(q_1, 0, 000Z_0)$ $(q_1, 0, 0Z_0)$ $(q_2, 00, \varepsilon)$ ×

$(q_1, \varepsilon, 0000Z_0)$ × $(q_1, \varepsilon, 00Z_0)$ × (q_1, ε, Z_0)

$(q_2, \varepsilon, \varepsilon)$

Deterministic Push down Automata

$L = \{wcw^R / w \text{ is in } (0+1)^*\}$



H5 J0, J1, J5, J8, K0, K2, K5, L9, M4, N

G3, 6, 9,
2, 3, 4, 5,
1, 3, 5,
K0, 2, 5,
2, Le
M
N2
P0, 4

Convertion of CFG to PDA

$S \to AB$
$A \to 0S / 0$
$B \to 1S / 1$

$A \to \alpha$
$\delta(q, \epsilon, A) = (q, \alpha)$
$\delta(q, a, a) = (q, \epsilon)$

Sol: ① $S \to AB$, $\delta(q, \epsilon, S) = \delta(q, AB)$
$A \to 0S$, $\delta(q, \epsilon, A) = (q, 0S)$
$A \to 0$, $\delta(q, \epsilon, 0) = (q, 0)$
$B \to 1S$, $\delta(q, \epsilon\, B) = (q, \cancel{0}S)$
$B \to 1$, $\delta(q, \epsilon, B) = (q, 1)$

Convert PDA to CFG

Def $m = (Q, \Sigma, \Gamma, \delta, q_0, z_0, \phi)$ ↳ final state

$m = (\{q_0, q_1\}, \{a, b\}, \{a, z_0\}, \delta, q_0\ z_0, \phi)$

$\delta(q_0, a, z_0) = (q_0, a z_0)$ $\quad \delta(q_0, a, a) = (q_0, aa)$

$\delta(q_0, b, a) = (q_1, a)$ $\quad \delta(q_1, b, a) = (q_1, a)$

$\delta(q_1, a, a) = (q_1, \epsilon)$ $\quad \delta(q_1, \epsilon, z_0) = (q_1, \epsilon)$

$G = (V, T, P, S)$

Sol: $V = (S, [q_0 a q_0], [q_0 a q_1]\ [q_1 a q_0]\ [q_1 a q_1]$

$[q_0 z_0 q_0]\ [q_0 z_0 q_1]\ [q_1 z_0 q_0]\ [q_1 z_0 q_1])$

$T = \{a, b\}$ $\quad S \to [q_0 z_0 q_0]$

$S \to [q_0 z_0 q_1]$

$\delta(q_0, a, z_0) = (q_0, \overrightarrow{a z_0})$ 2 symbols

4 productions

$[q_0 z_0 q_0] \to a [q_0 a q_0] [q_0 z_0 q_0]$

$[q_0 z_0 q_0] \to a [q_0 a q_1] [q_1 z_0 q_0]$

$[q_0 z_0 q_1] \to a [q_0 a q_0] [q_0 z_0 q_1]$

$[q_0 z_0 q_1] \to a [q_0 a q_1] [q_1 z_0 q_1]$

$\delta(q_0, a, a) = (q_0, \overrightarrow{aa})$ 2 symbols.

productions

$[q_0 a q_0] \to a [q_0 a q_0] [q_0 a q_0]$

$[q_0 a q_0] \to a [q_0 a q_1] [q_1 a q_0]$

$[q_0 a q_1] \to a [q_0 a q_0] [q_0 a q_1]$

$[q_0 a q_1] \to a [q_0 a q_1] [q_1 a q_1]$

* $\delta(q_0, b, a) = (q_1, a)$ → 1 symbol $(2^1) = 2$

$[q_0 a q_0] \rightarrow b[q_1 a q_0]$

$[q_0 a q_1] \rightarrow b[q_1 a q_1]$

* $\delta(q_1, b, a) = (q_1, a)$

$[q_1 a q_0] \rightarrow b[q_1 a q_0]$

$[q_1 a q_1] \rightarrow b[q_1 a q_1]$

* $\delta(q_1, a, a) = (q_1, \epsilon)$ → when ϵ only one production

$[q_1, a, q_1] \rightarrow a \; \cancel{[q_1, \;]}$

* $\delta(q_1, \epsilon, Z_0) = (q_1, \epsilon)$

$[q_1, Z_0, q_1] \rightarrow \epsilon$

$S \rightarrow AA \mid a$
$A \rightarrow SA \mid b$

CFG to PDA.

NT : S, A

T : a b

$\delta(q_0, \epsilon, S) = (q_0, AA)$
$\delta(q_0, \epsilon, S) = (q_0, a)$
$\delta(q_0, \epsilon, A) = (q_0, SA)$
$\delta(q_0, \epsilon, A) = (q_0, b)$
$\delta(q_0, a, a) = (q_0, \epsilon)$
$\delta(q_0, b, b) = (q_0, \epsilon)$

abbabb

$(q_0, abbabb, S) \vdash (q_0, abbabb, AA)$
$\vdash (q_0, abbabb, SAA)$
$\vdash (q_0, abbabb, aAA)$
$\vdash (q_0, \cancel{a}bbabb, \cancel{b}A)$
$\vdash (q_0, babb, SA)$
$\vdash (q_0, \cancel{b}abb, AAA)$
$\vdash (q_0, \cancel{b}abb, bAA)$
$\vdash (q_0, abb, SAA)$
$\vdash (q_0, \cancel{a}bb, aAA)$
$\vdash (q_0, bb, AA)$
$\vdash (q_0, \cancel{b}_{\epsilon}, \cancel{b}_{\epsilon})$