APPLIED PHYSICS

MODULE-1 PART A

@Rajeshwari

1.) Relate the dependency of wavelength of matter waves on velocity and mass of material particle.?

A.) Louis de Broglie showed that the wavelength of a particle is equal to Planck's constant divided by the mass times the velocity of the particle.

$\lambda = h/p$

$\lambda = h/mv$

De Broglie's Equation

$$\lambda = \frac{h}{mv}$$

Where
λ = wavelength in meters
v = the velocity in meters/sec
m = the mass in kilograms
h = Plancks's constant in J/Hz

2.) Write an expression for de-Broglie wavelength in terms of momentum and kinetic energy.

A.) De Broglie's hypothesis of matter waves postulates that any particle of matter that has linear momentum is also a wave. The wavelength of a matter wave associated with a particle is inversely proportional to the magnitude of the particle's linear momentum.

$$E_K = \frac{p^2}{2m} \qquad \text{where} \begin{bmatrix} E_K = \text{Kinetic energy} \\ p = \text{momentum} \\ m = \text{mass of the particle} \end{bmatrix}$$

de-Broglie wavelength, $\lambda = \dfrac{h}{p}$

...where [h = Planck's constant

$$\therefore \quad \lambda = \frac{h}{\sqrt{2mE_K}}$$

$\because$ Both the particles have the same de-Broglie wavelength ...[Given

$$\therefore \quad \frac{h}{\sqrt{2m_e E_{Ke}}} = \frac{h}{\sqrt{2m_\alpha E_{K\alpha}}}$$

$$\text{or} \quad \frac{m_e}{m_\alpha} = \frac{E_{K\alpha}}{E_{Ke}} \qquad \text{where} \begin{bmatrix} m_e = \text{mass of electron} \\ m_\alpha = \text{mass of } \alpha\text{ - particle} \\ E_{Ke} = \text{K.E. of electron} \\ E_{K\alpha} = \text{K.E. of } \alpha\text{ - particle} \end{bmatrix}$$

As $m_\alpha > m_e$ $\qquad \therefore$ K.$E_{Ke} > E_{K\alpha}$

3.) Explain the conception of light behaving both as a particle and wave.

A.) Light exhibits wave nature when we try to explain the phenomenon of Interference and diffraction. It behaves like a particle while explaining Compton effect and photo electric effect. However, there has never been an experiment able to capture both natures of light at the same time; the closest we have come is seeing either wave or particle, but always at different times.

4.) Justify the statement that Heisenberg's uncertainty principle is a direct consequence of dual nature of Wave matter.

A.) Heisenberg's Uncertainty Principle states that both the momentum and position of a particle cannot be determined simultaneously. The Dual Nature of matter however, mentions that matter possesses properties of both particle and a wave. The Heisenberg uncertainty principle is a direct consequence of de-Broglie theory only..

5.) Prove that matter waves travel with a velocity greater than velocity of light Also justify it.

A)

Velocity of matter wave Greater than Speed of light.

$$V_p = \nu \lambda \cdot$$

$$E = h\nu \qquad E = mc^2 \quad \text{— (I)}$$

$$\nu = mc^2/h \quad \text{— (II)}$$

$$\lambda = \frac{h}{p} = \frac{h}{mv} \quad \text{— (III)}$$

$$V_p = \frac{\cancel{m}c^2}{\cancel{h}} \frac{\cancel{h}}{\cancel{m}v} = \frac{c^2}{v}$$

$$\boxed{V_p \cdot v = c^2} \qquad v < c \qquad V_p > c$$

Properties of matter wave $\boxed{\lambda = \frac{h}{mv}}$ (I) $\lambda \propto \frac{1}{m}$ (II) $\lambda \propto \frac{1}{v}$

6.) Write one dimensional time independent Schrodinger equation associated with matter wave.

$$E = K.E + P.E$$

$$E = \frac{1}{2}mv^2 + V$$

or

$$E = \frac{1}{2}\frac{m^2}{m}v^2 + V$$

=>

$$E = \frac{P^2}{2m} + V$$

on multiplying both sides by ψ

$$E\psi = \left(\frac{P^2}{2m} + V\right)\psi$$

on putting values of $P\psi$ from equation (12)

$$E\psi = \frac{1}{2m} \times (-i\text{h})^2 \frac{\partial^2\psi}{\partial x^2} + V\psi$$

or

$$E\psi = \frac{-\text{h}^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi$$

=>

$$\frac{\text{h}^2}{2m}\frac{\partial^2\psi}{\partial x^2} + (E - V)\psi = 0$$

=>

$$\boxed{\frac{\partial^2\psi}{\partial x^2} + \frac{2m}{\text{h}^2}(E - V)\psi = 0} \qquad (13)$$

This is time independent Schrodinger's equation for one-dimension motion of particle. For three-dimensional motion

$$\left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2}\right) + \frac{2m}{\text{h}^2}(E - V)\psi = 0 \qquad (14)$$

7.) Explain the feature of wave function which connects the particle nature and wave nature of matter wave.

⑦ Wave function

→ The quantity with which Quantum mechanics is concerned is the wave function of a body.

→ Wave function, ψ is a quantity associated with a moving particle it is a complex quantity.

→ $|\psi|^2$ is proportional to the probability of finding a particle at a particular point at a particular time.

It is the probability density

$$|\psi|^2 = \psi^* \psi$$

→ ψ is the probability amplitude

Normalization

$|\psi|^2$ is the probability density.

The probability of finding the particle within an element of vol dx

$$|\psi|^2 dx$$

Since the particle is definetely be some where so

$$\int_{-\infty}^{\infty} |\psi|^2 dx = 1$$

A wave-function that obeys this equation is said to be normalized.

8.) Describe behavior of matter waves by giving any two of its properties.

A.) The Wave Character of Matter. That is, light, which had always been regarded as a wave, also has properties typical of particles, a condition known as wave–particle duality (a principle that matter and energy have properties typical of both waves and particles). v is the velocity of the particle.

1. Matter waves are not electromagnetic in nature
2. Matter-wave represents the probability of finding a particle in space.
3. Matter waves are independent of the charge on the material particle.
4. Electron microscope works on the basis of de-Broglie waves.
5. Matter waves can propagate in a vacuum, hence they are not mechanical wave.
6. The number of de-Broglie waves associated with nth orbital is n.
7. The phase velocity of matter waves can be greater than the speed of light

9.) Write expressions for wave function and energy of a particle in three dimensional square well box of infinite potential

A.) $E = \frac{n^2\pi^2\hbar^2}{2mL^2}$

Energy for 3D Well

$$\frac{-\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\Psi$$
$$= -\left({k_x}^2+{k_y}^2+{k_z}^2\right)\left(\frac{-\hbar^2}{2m}\right)\Psi = E\Psi$$

- We then get $E = \frac{\pi^2\hbar^2}{2m}\left(\frac{{n_x}^2}{{L_x}^2}+\frac{{n_y}^2}{{L_y}^2}+\frac{{n_z}^2}{{L_z}^2}\right)$
- Note $\frac{\pi^2\hbar^2}{2m} = \frac{h^2}{8m}$
- If well is a cube, $L_x=L_y=L_z=L$

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2}\right)=E\psi(x,y,z)$$

- **The Schrödinger equation in 3D**
- **V=0 (free particle)**

$$-\frac{\hbar^2}{2m}\nabla^2\psi=E\psi(x,y,z)$$

$$\nabla^2=\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2}$$

Laplacian Operator

$$\Psi_{n_x,n_y,n_z}(x,y,z)=\psi_{n_x}(x)\psi_{n_y}(y)\psi_{n_z}(z)$$

Wavefunction

$$\psi_{n_x}(x)=\sqrt{\frac{2}{a}}\sin\left(\frac{n_x\pi}{a}x\right)$$

$$\psi_{n_y}(y)=\sqrt{\frac{2}{b}}\sin\left(\frac{n_y\pi}{b}y\right)$$

$$\psi_{n_z}(z)=\sqrt{\frac{2}{c}}\sin\left(\frac{n_z\pi}{c}z\right)$$

$$\psi_{n_x n_y n_z}(x,y,z)=\sqrt{\frac{8}{L_x L_y L_z}}\sin\frac{n_x\pi x}{L_x}\cdot\sin\frac{n_y\pi y}{L_y}\cdot\sin\frac{n_z\pi z}{L_z},$$

10.) Write expressions for Eigen function and Eigen values for a particle in one dimensional square well box of infinite potential.

if $n=2$

$$E_2=\frac{4h^2}{8mL^2}$$

$$E_2=\frac{h^2}{2mL^2}=4E_1$$

if $n=3$.

$$E_3=\frac{9h^2}{8mL^2}=9E_1$$

$\therefore E_1:E_2:E_3 \Rightarrow 1:4:9\ldots$

Expression for eigen function i.e., wave function is $\Psi(x)=\sqrt{2/L}\sin\frac{n\pi}{L}x$

11.) What are the limitations of wave function to be a solution of second order differential equation associated with material particle?

A.) The function must be single valued. It must have a finite value or it must be normalized. It has continuous first derivative on the indicated interval. The wave function must be square integrable. In order to avoid multiple probability values, Ψ must take a single value at each position and time.

12.) Discuss about Normalization condition as postulated by Max Born?

A.) The equation of a second order differential equation in quantum mechanics, the solution of which is the wave-function that describes an electron.

Max Born interpreted the modulus of the calculated wave-function as the probability of finding an electron at a specific space and time.

Now since, this mathematical wave-function had to be viewed as a probability distribution, the equation had to incorporate the fact, that the maximum probability of finding an electron over the entire space has to be 1. This is where a new quantity i.e. "normalization constant" was introduced that took care of this.

P.S. This process of integrating the mod square of the wave-function over the entirety of space and setting it to 1 is called normalization. (Which provides us with the normalization constant). Below is an example of normalization of a wave-function of an electron in an infinite potential well (1D). **REFER Q7 as well**

Standing Wave Function

The wave function can thus be written

$$\boxed{\Psi_n(x) = A_n \sin\left(\frac{n\pi x}{L}\right)}$$

The constant A_n is determined by normalization condition

$$\boxed{\int \Psi_n^2 dx = \int A_n^2 \sin^2\left(\frac{n\pi x}{L}\right) dx = 1}$$

The result of evaluating the integral and solving for A_n is independent from n. $\boxed{A_n = \sqrt{\frac{2}{L}}}$

The normalized wave function for a particle in a box are thus

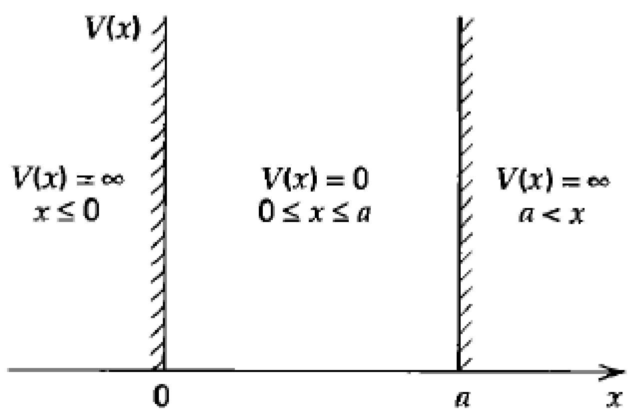
$$\boxed{\Psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)}$$

13.) What is the Schrodinger's interpretation of complex and not observable wave function?

A.) The wave function represents the motion of an atomic particle. It is a function of position and time. It is a complex function as given by Schrodinger and it has no physical significance as it is not Observable quantity. According to Heisenberg the probable value cannot be negative. So, wave function can not determine the position of the particle but it is in some way an index of the presence of the particle. As the wave function is complex we multiply it with the complex conjugate at relate it to the probability of finding the particle.

14.) How energy of a particle confined in a potential box is related to the width of the box?

A.) Energy levels, and hence the minimum kinetic energy of the particle in a box is inversely proportional to the mass and the square of the box width, in qualitative agreement.



Expression for Energy is $E = \frac{n^2\pi^2\hbar^2}{2mL^2}$

15.) Write about probability density of moving material particle as explained by Born and Schrodinger.

15.) $P(x)dx = |\psi(x,t)|^2 dx$

$P(x) = |\psi(x,t)|^2$ (probability density function)

$\Rightarrow$ $\psi(x,t)$ itself is not a measurable quantity but $|\psi(x,t)|^2$ is measurable & equal to the probability per unit length (probability density) of finding the particle in the interval dx about the point x explained by Born.

$|\psi|^2$ is the probability density.

(i) The probability of finding the particle within an element of vol dx

$$|\psi|^2 dx$$

(ii) Since, the particle is definetely be somewhere so

$\int_{-\infty}^{\infty} |\psi|^2 dx = 1$ which is explained by Schrodinger

16.) What is the minimum energy possessed by the particle in an infinitely deep potential well?

A.) The particle in a box has zero energy, it will be at rest inside the well and it violates the Heisenberg's Uncertainty Principle. Thus, the minimum energy possessed by a particle is not equal to zero.

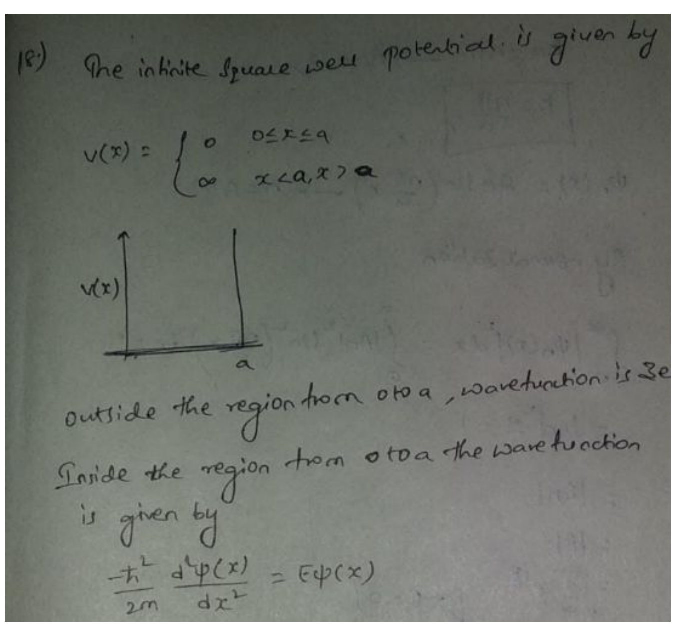
$$E = \frac{\pi^2\hbar^2}{2mL^2} \text{ for n=1}$$

Inside the well there is no potential energy, and the particle is trapped inside the well by “walls” of infinite potential energy. This has solutions of E=∞, which is impossible (no particle can have infinite energy) or ψ=0. Since ψ=0, the particle can never be found outside of the well.

17.) Discuss about the nature of the walls of the box in which a particle is bound.

A.) In quantum mechanics, the particle in a box model (also known as the infinite potential well or the infinite square well) describes a particle free to move in a small space surrounded by impenetrable barriers. In classical systems, for example, a particle trapped inside a large box can move at any speed within the box and it is no more likely to be found at one position than another. However, when the well becomes very narrow (on the scale of a few nanometers), quantum effects become important. The particle may only occupy certain positive energy levels. Likewise, it can never have zero energy, meaning that the particle can never "sit still". Additionally, it is more likely to be found at certain positions than at others, depending on its energy level. The particle may never be detected at certain positions, known as spatial nodes.

18.) What happens to the wave function associated with a particle in an infinitely deep potential well.


18.) The infinite square well potential is given by

$$v(x) = \begin{cases} 0 & 0 \le x \le a \\ \infty & x < a, x > a \end{cases}$$

Outside the region from 0 to a, wavefunction is ze

Inside the region from 0 to a the wavefunction is given by

$$\frac{-\hbar^2}{2m} \frac{d^2\psi(x)}{dx^2} = E\psi(x)$$

$\because V(x) \to 0$

$\psi(x) = Ae^{ikx} + Be^{-kx}$ } By differentiating we get the free particle solutions.

$k = \frac{\sqrt{2mE}}{\hbar}$

Applying boundaries, we get

$\psi(0) = Ae^{ik0} + Be^{-ik0}$

$= A + B$.

$B = -A$

$\psi(x) = Ae^{ikx} - Ae^{-ikx} = A\sin(kx)$

$\psi(a) = A\sin(ka) = 0$

$ka = n\pi \to n = 0, 1, 2, \ldots$

$\boxed{k = \frac{n\pi}{a}}$

$\psi_n(x) = A_n \sin\left(\frac{n\pi}{a}x\right) \to n = 0, 1, 2 \ldots$

By normalization

$\int_{-\infty}^{\infty} |\psi_n(x)|^2 dx = \int_0^a |A_n|^2 \sin^2\left(\frac{n\pi}{a}x\right) dx = 1$

$= |A_n|^2 \frac{a}{2} = 1$

$= |A_n|$

$= |A|$

$= \sqrt{\frac{2}{a}} e^{i\beta}$ $[\because \beta = 0]$

$\boxed{|A| = \sqrt{\frac{2}{a}}}$

So, it finally gives.

$\boxed{\psi_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi}{a}x\right) \to n = 1, 2, 3 \ldots}$

19.) What is the boundary condition for normalized wave function?

A.) The wave function must be finite, single-valued and continuous. At the boundary this is ensured by requiring the magnitude and the first derivative be equal.

20.) Define square well potential associated with a bound electron moving along one dimension.

A.

$$V(x) = 0, for\ 0 < x < L \text{———————} \mathbf{(1)}$$

$$V(x) = \infty, for\ x > L$$

For the 1-dimensional case on the *x*-axis, the time-independent Schrödinger equation can be written as:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x)\psi = E\psi \quad (1)$$

where

$\hbar = \frac{h}{2\pi}$,

h is Planck's constant,

m is the mass of the particle,

ψ is the (complex valued) wavefunction that we want to find,

$V(x)$ is a function describing the potential energy at each point *x*, and

E is the energy, a real number, sometimes called eigenenergy.

MODULE 2 PART -B

@Rajeshwari

1.) Compare a particle with a wave and discuss about dual nature of radiation.

	PARTICLE		WAVE
1.	A particle occupies a well-defined position in space i.e a particle is **localized** in space e.g. a grain of sand, a cricket ball etc.	1.	a wave is spread out in space e.g. on throwing a stone in a pond of water, the waves start moving out in the form of concentric circles. Similarly, the sound of the speaker reaches everybody in the audience. Thus a wave is **delocalized** in space.
2.	When a particular space is occupied by one particle, the same space cannot be occupied simultaneously by any other particle. In other words, particles do **not interfere**.	2.	Two or more waves can coexist in the same region of space and hence **interfere**.
3.	When a number of particles are present in a given region of space, their **total value is equal to their sum** i.e it is neither less nor more.	3.	When a number of waves are present in a given region of space, due to interference, the **resultant wave can be larger or smaller** than the individual waves i.e. interference may be constructive or destructive.

The photoelectric effect could be explained considering that radiations consist of small packets of energy called quanta. These packets of energy can be treated as particles. On the other hand, radiations exhibit a phenomenon of interference and diffraction which indicated that they possess wave nature. So it may be concluded that electromagnetic radiations possess dual nature.

- Particle nature
- Wave nature

2.) Enlist physical significance of wave function according to Schrodinger and Max – Born interpretation.

The Schrödinger equation describes the evolution of the quantum state of a single- or many-body system in the case that the particle number is strictly conserved. The meaning of the wave function is that its square is the probability distribution to find the particles at a position. What remained unclear was the meaning of the wave function that appeared in Schrödinger's equation. In 1926, Born submitted two papers in which he formulated the quantum mechanical description of collision processes and found that in the case of the scattering of a particle by a potential, the wave function at a particular spatiotemporal location should be interpreted as the probability amplitude of finding the particle at that specific space-time point. **Refer Part A Q7 as well**

3.) Matter waves are new kind of waves. Justify this concept by discussing different properties of matter waves.

A.) Electromagnetic waves carry electromagnetic radiant energy in free space. While matter waves carry particles that constitute it to propagate energy. **Also Refer Part A Q8**

- Matter waves are not electromagnetic in nature
- Matter-wave represents the probability of finding a particle in space.
- Matter waves are independent of the charge on the material particle.
- Electron microscope works on the basis of de-Broglie waves.
- Matter waves can propagate in a vacuum, hence they are not mechanical wave.
- The number of de-Broglie waves associated with nth orbital is n.
- The phase velocity of matter waves can be greater than the speed of light.
- The number of de-Broglie waves associated with the nth orbital electron is n.

4.) Using Planck's and Einstein's theory of radiation, Show that the wavelength associated with an electron of mass 'm' and kinetic energy 'E' is given by $\frac{h}{\sqrt{2mE}}$.

Kinetic energy of a particle $E = \frac{1}{2}mv^2$

or $mv = \sqrt{2mE}$

de-Broglie wavelength associated with the particle is $\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mE}}$

For a given value of E, $\lambda = \frac{1}{\sqrt{m}}$

Mass of electron < mass of proton

So, electron has greater de-Broglie wavelength.

5.) Determine an expression for the wavelength associated with an electron, accelerated by a potential V.

Expression for de Broglie Wavelength associated with Accelerated Electrons The de Broglie wavelength associated with electrons of momentum p is given by

$$\lambda = \frac{h}{p} = \frac{h}{mv} \quad ...(i)$$

where m is mass and v is velocity of electron. If Ek is the kinetic energy of electron, then

$$E_K = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{p}{m}\right)^2 = \frac{p^2}{2m} \quad \left(\text{since } p = mv \Rightarrow v = \frac{p}{m}\right)$$

$$\Rightarrow \quad p = \sqrt{2mE_K}$$

$$\therefore \text{ Equation } (i) \text{ gives } \lambda = \frac{h}{\sqrt{2mE_K}} \quad ...(ii)$$

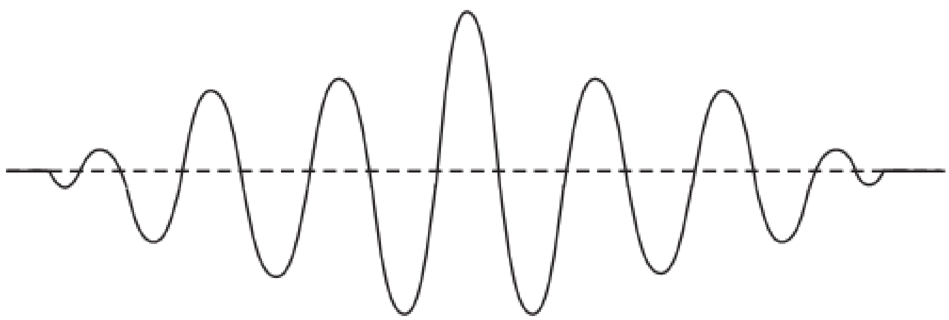
If V volt is accelerating potential of electron, then Kinetic energy,

$$E_K = eV$$

$\therefore$ Equation (ii) gives

$$\lambda = \frac{h}{\sqrt{2meV}} \qquad ...(iii)$$

This is the required expression for de Broglie wavelength associated with electron accelerated to potential of V volt. The diagram of wave packet describing the motion of a moving electron is shown here



6.) Why matter waves are observed for particles of atomic or nuclear size?

Matter waves are observed only for atomic particles, they cannot be observed for macroscopic bodies as wavelength becomes insignificant because of larger mass even at low velocities. So, the wave length of the matter wave will be inversely proportional to mass of the particle. So, lesser the mass greater will be the wave length. (i.e.) smaller the size of the particle the smaller will be its mass then greater will be its wavelength. If the size of particle is large then the mass will be large and the wave length of matter waves will be negligible.

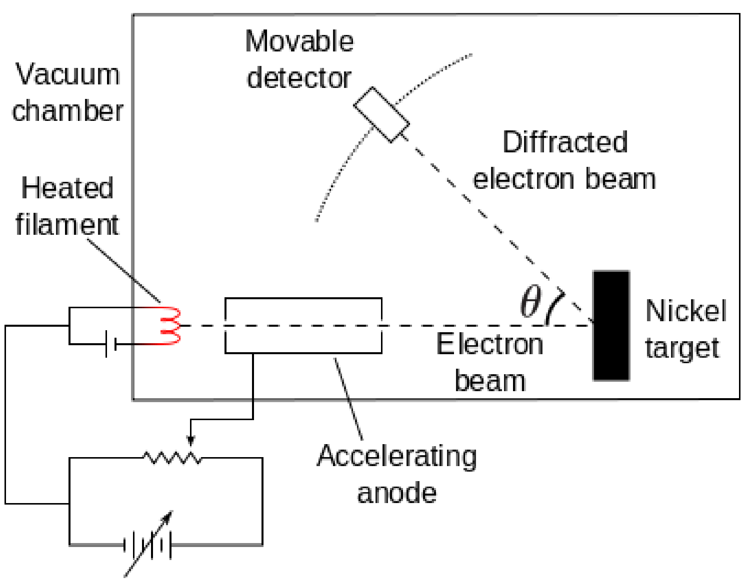
7.) Explain the difference between a matter wave and an electromagnetic wave?

MATTER WAVE	EM WAVE
Matter wave is associated with a particle.	Oscillating charged particle gives rise to electromagnetic wave.
Wavelength depends on the mass of the particle and its velocity $\lambda= h/mv$	Wavelength depends on the energy of the photon $\lambda= hC/E$
Can travel with a velocity greater than the velocity of light.	Travels with velocity of light.
Matter wave is not electromagnetic wave.	Electric field and magnetic field oscillate perpendicular to each other.
Matter wave require medium for propagation	Electromagnetic waves do not require medium i.e., they travel in vacuum also.

8.) Describe Davisson Germer experiment with a neat diagram and explain how it established the proof for wave nature of electrons?

A.) Initial atomic models proposed by scientists could only explain the particle nature of electrons but failed to explain the properties related to their wave nature. C.J. Davisson and L.H. Germer in the year 1927 carried out an experiment, popularly known as Davisson Germer's experiment to explain the wave nature of electrons through electron diffraction. In this article, we will learn about the observations and conclusions of the experiment.

Setup of Davisson Germer Experiment



Observations of Davisson Germer experiment:
From this experiment, we can derive the below observations:

- We obtained the variation of the intensity (I) of the scattered electrons by changing the angle of scattering, θ.
- By changing the accelerating potential difference, the accelerated voltage was varied from 44V to 68 V.
- With the intensity (I) of the scattered electron for an accelerating voltage of 54V at a scattering angle $\theta = 50^\circ$, we could see a strong peak in the intensity.
- This peak was the result of constructive interference of the electrons scattered from different layers of the regularly spaced atoms of the crystals.

Co-relating Davisson Germer experiment and de Broglie relation:
According to de Broglie,

$\lambda = h/p$ $\qquad$ $\lambda = (1.22754\surd) = 0.167$nm

Λ = wavelength associated with electrons

Thus, Davisson Germer experiment confirms the wave nature of electrons and the de Broglie relation.

The Davisson and Germer experiment showed that electron beams can undergo diffraction when passed through the atomic crystals. This shows that the wave nature of electrons as waves can exhibit interference and diffraction.

9.) Considering dual nature of electron, derive Schrodinger's time independent wave equation for the motion of an electron.

A. Schrodinger’s time-independent wave equation describes the standing waves. Sometimes the potential energy of the particle does not depend upon time, and the potential energy is only the function of position. In such cases, the behavior of the particle is expressed in terms of Schrodinger’s time-independent wave equation.

According to classical mechanics, the total energy of the particle is

$$E = K.E + P.E$$

$$E = \frac{1}{2} mv^2 + V$$

or $$E = \frac{1}{2} \frac{m^2}{m} v^2 + V$$

=> $$E = \frac{P^2}{2m} + V$$

on multiplying both sides by ψ

$$E\psi = \left(\frac{P^2}{2m} + V \right) \psi$$

on putting values of $P\psi$ from equation (12)

$$E\psi = \frac{1}{2m} \times (-i\hbar)^2 \frac{\partial^2 \psi}{\partial x^2} + V\psi$$

or $$E\psi = \frac{-\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V\psi$$

=> $$\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + (E - V)\psi = 0$$

=> $$\boxed{\frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\hbar^2}(E - V)\psi = 0} \qquad (13)$$

This is time independent Schrodinger's equation for one-dimension motion of particle. For three-dimensional motion

$$\left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} \right) + \frac{2m}{\hbar^2}(E - V)\psi = 0 \qquad (14)$$

10.) Assuming that a particle of mass m is confined in a field free region between impenetrable walls in infinite height at x=0 and x=a, show that the permitted energy levels of a particle are given by n^2*h^2 / 8 m a^2

(10 A:)

We know that, the Schrodinger equation for a particle in a box.

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{(\hbar)^2} E\psi = 0$$

We know that

$$\lambda = \frac{h}{p} = \frac{2\pi}{k}$$

$$k = \frac{2\pi}{h} \cdot p$$

$$k = \frac{p}{\hbar} = \frac{\sqrt{2m_0E}}{\hbar}$$

$$\boxed{k^2 = \frac{2m_0E}{(\hbar)^2}} \rightarrow (1)$$

So $$\frac{\partial^2 \psi}{\partial x^2} + k^2(\psi) = 0 \quad - (2)$$

$$\psi(x) = A\sin kx + B\cos kx \quad - (3)$$

Boundary conditions at x?0 and x?a

$$\psi(0) = B$$

$$\boxed{B > 0}$$

So $$\boxed{\psi(x) = A\sin kx}$$

So $\psi = 0$ when $x = a$

$$\psi(a) = A\sin k(a)$$

$$0 = A\sin k(a) \qquad (A \neq 0)$$

$$ka = n\pi$$

$$\boxed{k = \frac{n\pi}{a}} \quad - (4)$$

from (1) & (4)

$$\frac{n^2\pi^2}{a^2} = \frac{2m_0E}{(\hbar)^2}$$

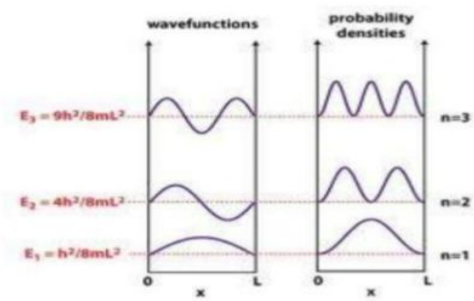
$$E = \frac{n^2\pi^2}{a^2} \left(\frac{(\hbar)^2}{2m_0} \right)$$

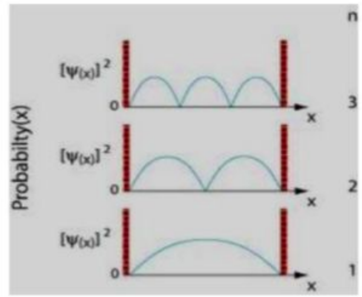
$$\cancel{= \frac{h^2\pi^2}{a^2} \left(\frac{h^2}{2m_0} \right)}$$

$$= \frac{n^2\pi^2}{a^2} \left(\frac{h^2}{2m_0 4\pi^2} \right)$$

$$\boxed{E = \frac{n^2h^2}{8m_0a^2}}$$

11.) Discuss the results from the Eigen values, Eigen functions and probability density for a particle in a one dimensional potential box of infinite height. Also sketch the figures





✓ **Energy quantization: It is not possible for the particle to have any arbitrary definite energy. Instead only discrete definite energy levels are allowed.**

✓ **Zero-point Energy: the lowest possible energy level of the particle, called the zero-point energy, is non-zero.**

✓ **Spatial-nodes: in contrast to classical mechanics the Schrodinger equation predicts that for some energy levels there are nodes, implying positions at which the particle can never be found.**

11 sol

Wave function is given by

$$\psi_r(x) = A \sin \frac{n\pi x}{L}$$

Using normal factor,

$$\int_{-\infty}^{\infty} |\psi_r(x)|^2 dx = 1$$

$$A^2 \int_0^L \sin^2 \frac{n\pi x}{L} dx = 1$$

$$A^2 \left(\frac{L}{2}\right) = 1 \Rightarrow A = \sqrt{\frac{2}{L}}$$

The normal eigen function of particle are

$$\boxed{\psi_r(x) = \sqrt{\frac{2}{L}} \sin \frac{\pi n x}{L}}$$

12.) Show that the energies of a particle confined between two rigid walls of infinite potential are quantized.

12.) Refer Question (10) & (11)

we know that

$$\boxed{E = \frac{n^2h^2}{8mL^2}}$$

i) Zero point energy

$$\boxed{E = \frac{h^2}{8mL^2}}$$

If $E \rightarrow 0$ $\Delta p \rightarrow 0$

So, $\Delta x \rightarrow \infty$.

But here $\Delta x_{max} = L$

So, zero point energy doesn't obeys the heisenberg uncertainity principle so, all the energy values are not possible for particle in potential well. So, energy is Quantised

13.) What are de Broglie matter waves? Derive expression for de Broglie wavelength associated with a particle having mass m and velocity v.

A.) De Broglie equation states that a matter can act as waves much like light and radiation, which also behave as waves and particles. ... Therefore, if we look at every moving particle whether it is microscopic or macroscopic it will have a wavelength

$$E = mc^2 \quad \text{— (1)}$$

$$E = h\nu \quad \text{— (2)}$$

From eqn (1) & (2), we have

$$h\nu = mc^2 \qquad \left[\nu = \frac{c}{\lambda}\right]$$

$$\frac{hc}{\lambda} = mc^2$$

$$\lambda = \frac{h}{mc}$$

$$\boxed{\lambda = \frac{h}{p}}$$

14.) Discuss different phenomenon's that show the behavior of light radiation interacting with matter.

A.) When light interacts with matter it can do one of several things, depending on its wavelength and what kind of matter it encounters: it can be transmitted, reflected, refracted, diffracted, adsorbed or scattered. When light passes through a sufficiently-thin slit it will diffract and spread. Black Body Radiation (where radiation takes place due to presence of black body) and Crompton Effect (where scattering of photon occurs by charged particle) are also few phenomenons that show similar behavior.

15.) Write major differences between classical mechanics and quantum mechanics

- Classical mechanics describes the motion of macroscopic objects such as spacecraft, planets, stars, and galaxies. The classical mechanics (as known as Newtonian mechanics) provides extremely accurate results as long as the domain of study is restricted to large objects and the speeds involved do not approach the speed of light. The classical theories are simple, but this branch of mechanics cannot be applied to extremely small particles moving at very high speed, as the results may turn inaccurate.
- Quantum Mechanics has much more complicated theories than classical mechanics, but provides accurate results for particles of even very small sizes. Quantum Mechanics handles the wave-particle duality of atoms and molecules.

Classical Mechanics	Quantum Mechanics
(i) It deals with macroscopic particles.	(i) It deals with microscopic particles.
(ii)It is based upon Newton's laws of motion	(ii) It takes into account Heisenberg's uncertainty principle and de Broglie concept of dual nature of matter (particle nature and wave nature)
(iii) It is based on Maxwell's electromagnetic wave theory according to which any amount of energy may be emitted or absorbed continuously.	(iii) It is based on Planck's quantum theory according to which only discrete values of energy are emitted or absorbed.
(iv) The state of a system is defined by specifying all the forces acting on the particles as well as their positions and velocities (moment). The future state then can be predicted with certainty.	(iv) It gives probabilities of finding the particles at various locations in space.

16.) Differentiate between ψ and | ψ|2?

- ψ is a wave function and refers to the amplitude of electron wave i.e. probability amplitude. It has got no physical significance. The wave function ψ may be positive, negative or imaginary.
- $[\psi]^2$ is known as probability density and determines the probability of finding an electron at a point within the atom. This means that if:
  - it is zero, the probability of finding an electron at that point is negligible.
  - $[\psi]^2$ is high, the probability of finding an electron is high

17.) Highlight the conditions for an acceptable wave function.

A.)

For a wave function to be acceptable over a specified interval, it must satisfy the following conditions:

(i) The function must be single-valued,

(ii) It is to be normalized (It must have a finite value),

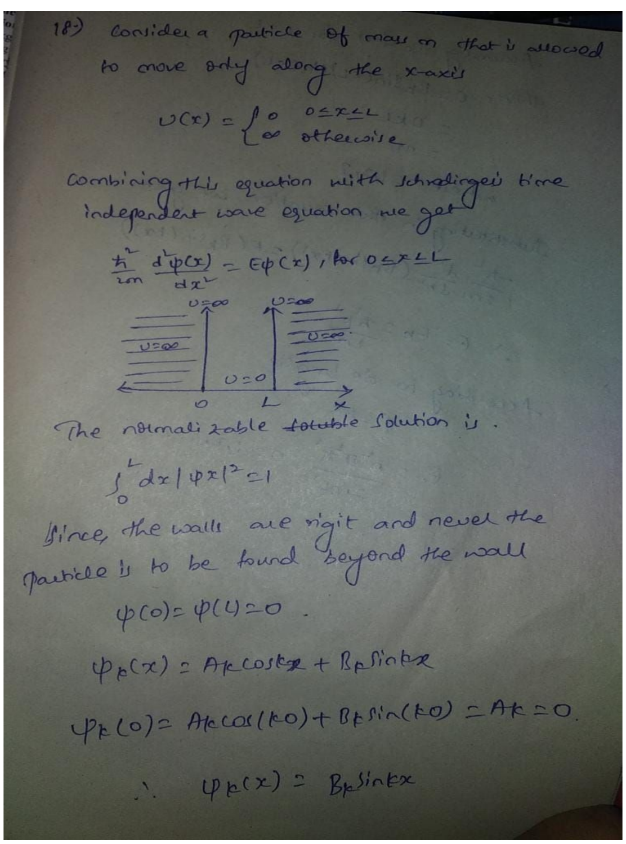
(iii) It must be continuous in the given interval.

(iv) It has continuous first derivative on the indicated interval.

(v) The boundary conditions must be satisfied by the wavefunction.

18.) How do you predict the energy of a particle in closed box from classical theory and quantum theory?

A. Classically, a particle in a box can have any positive energy, But, quantum theory shows that the particle is restricted to take only a certain discrete values. a particle can have zero energy, but quantum theory shows that the particle can have an energy called as zero-point energy which is non-zero.



18.) Consider a particle of mass m that is allowed to move only along the x-axis

$$U(x) = \begin{cases} 0 & 0 \le x \le L \\ \infty & \text{otherwise} \end{cases}$$

Combining this equation with Schrodinger's time independent wave equation we get

$$\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} = E\psi(x), \text{ for } 0 \le x \le L$$

The normalizable ~~soluble~~ solution is.

$$\int_0^L dx\,|\psi x|^2 = 1$$

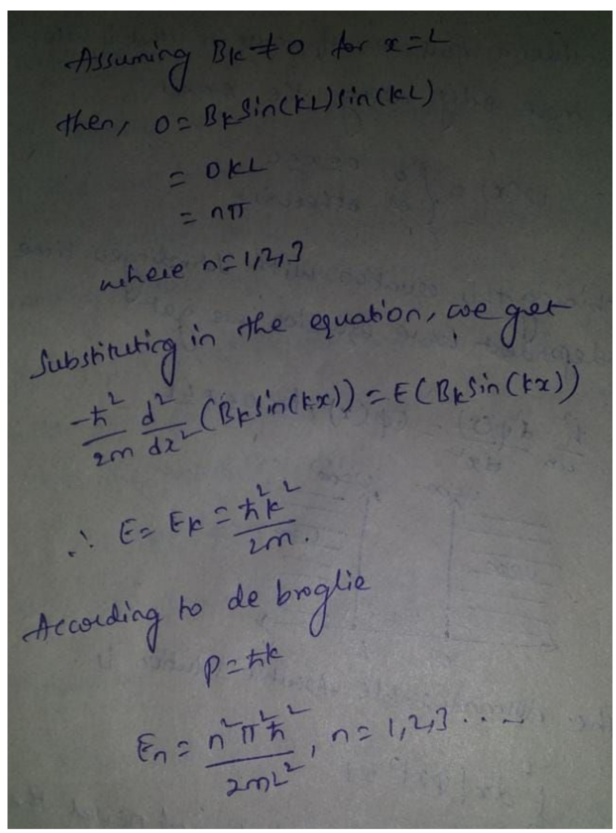
Since the walls are rigit and never the particle is to be found beyond the wall

$$\psi(0) = \psi(L) = 0.$$

$$\psi_k(x) = A_k \cos kx + B_k \sin kx$$

$$\psi_k(0) = A_k \cos(k0) + B_k \sin(k0) = A_k = 0.$$

$$\therefore \psi_k(x) = B_k \sin kx$$



Assuming $B_k \neq 0$ for $x = L$

then, $0 = B_k \sin(kL) \sin(kL)$

$= 0kL$

$= n\pi$

where $n = 1, 2, 3$

Substituting in the equation, we get

$$\frac{-\hbar^2}{2m}\frac{d^2}{dx^2}(B_k \sin(kx)) = E(B_k \sin(kx))$$

$$\therefore E = E_k = \frac{\hbar^2 k^2}{2m}.$$

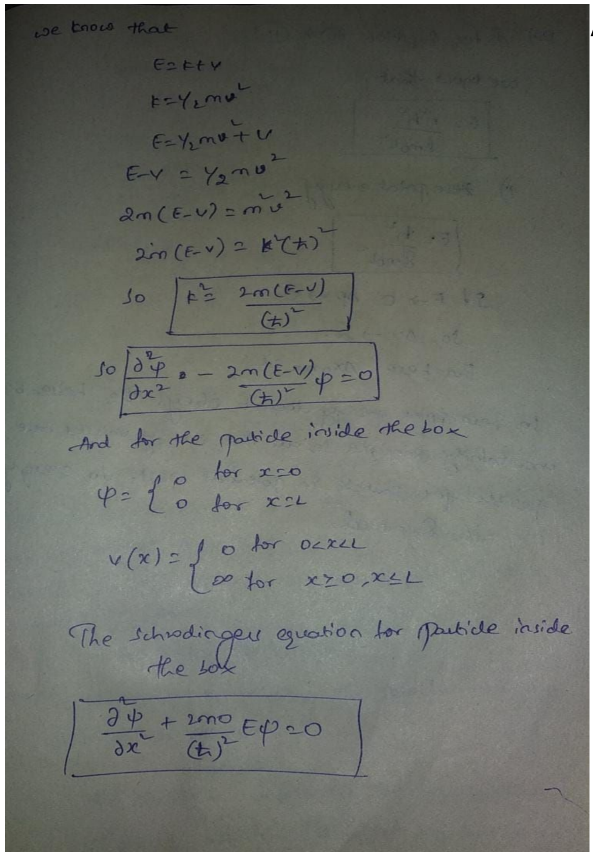
According to de broglie

$$p = \hbar k$$

$$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}, \quad n = 1, 2, 3 \ldots$$

19.) Starting from the wave equation and introducing energy and momentum of particle, obtain n expressions for on dimensional Schrodinger's equation of free particle

A.)


we know that

$$E = K + V$$
$$K = \frac{1}{2} m v^2$$
$$E = \frac{1}{2} m v^2 + V$$
$$E - V = \frac{1}{2} m v^2$$
$$2m(E - V) = m^2 v^2$$
$$2m(E - V) = k^2 (\hbar)^2$$

so $\boxed{k^2 = \frac{2m(E-V)}{(\hbar)^2}}$

so $\boxed{\frac{\partial^2 \varphi}{\partial x^2} - \frac{2m(E-V)}{(\hbar)^2} \varphi = 0}$

And for the particle inside the box

$$\varphi = \begin{cases} 0 & \text{for } x = 0 \\ 0 & \text{for } x = L \end{cases}$$

$$v(x) = \begin{cases} 0 & \text{for } 0 < x < L \\ \infty & \text{for } x \geq 0, x \leq L \end{cases}$$

The schrodinger equation for particle inside the box

$$\boxed{\frac{\partial \varphi}{\partial x^2} + \frac{2m}{(\hbar)^2} E \varphi = 0}$$

20.) Enlighten different laws of quantum physics that lead to different interpretation of energy.?

A.) There are many laws of quantum physics which leads to different interpretation some of them are

- Copenhagen interpretation.
- Quantum information theories.
- Relational quantum mechanics.
- QBism.
- Many worlds.
- Consistent histories.
- Ensemble interpretation.
- De Broglie–Bohm theory

PART C

@Bharath

1. **Calculate the velocity and kinetic energy of an electron of wavelength 0.21nm.**

Solution:

de- Broglie wavelength $\lambda = \dfrac{h}{m\upsilon}$

$$\upsilon = \frac{h}{m\lambda}$$

$$\upsilon = \frac{6.626 \times 10^{-34}}{9.1 \times 10^{-31} \times 2.1 \times 10^{-10}}$$

$= 34.67 \times 10^5$ m/s

Kinetic energy of electron

$$E = \frac{1}{2} m\upsilon^2$$

$$= \frac{1}{2} \times 9.1 \times 10^{-31} \times 34.67 \times 34.67 \times 10^{10}$$

$= 0.5469 \times 10^{-17}$ J

$$= \frac{4.1927 \times 10^{-17}}{1.6 \times 10^{-19}} eV$$

= 341.82eV

2. Calculate the de Broglie wavelength associated with a proton moving with a velocity

of 1/10 of velocity of light. (Mass of proton = 1.674 x 10^{-27} kg).

Solution:

de- Broglie wavelength $\lambda = \dfrac{h}{m\upsilon}$

$$\lambda = \frac{6.626 \times 10^{-34}}{1.674 \times 10^{-27} \times \frac{1}{10} \times 3 \times 10^{8}}$$

$= 1.31 \times 10^{-14}$ m

3. Calculate the wavelength of an electron raised to a potential 15kV.

Solution: de-Broglie wavelength

$$\lambda = \frac{12.26}{\sqrt{V}} \text{ A}^{o}$$

$$= \frac{12.26}{\sqrt{15000}}$$

$$= \frac{12.26}{122.47}$$

$= 0.1\text{A}^{o}$

4. **If the kinetic energy of the neutron is 0.025eV calculate its de-Broglie wavelength (mass of neutron =1.674 x 10^{-27} Kg)**

Solution: Kinetic energy of neutron

$$E = \frac{1}{2} m\upsilon^{2} = 0.025\text{eV}$$

$= 0.025 \times 1.6\text{x}10^{-19}$ J

$$\upsilon = \left(\frac{2\, X\, 0.025 \times 1.6 \times 10^{-19}}{1.674 \times 10^{-27}} \right)^{\frac{1}{2}} \left(0.04779 \times 10^{8} \right)^{\frac{1}{2}}$$

$0.2186\text{x}10^{4}$m/s

$\therefore d$e- Broglie wavelength $\lambda = \frac{h}{m\upsilon}$

$$\lambda = \frac{6.626 \times 10^{-34}}{1.67 \times 4\, 10^{-27} \times 0.2186 \times 10^{4}}$$

=0.181nm

5. **Calculate the wavelength of an electron, if the kinetic energy of the neutron is 0.025 eV**

Solution:

Yet again, the de Broglie wavelength is given by $\gamma = \frac{h}{p} = \frac{h}{mv}$.

For a free electron , m_n= 9.1x10^-31

$$KE = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{KE \times 2}{m}}$$

$$\gamma = \frac{h}{\sqrt{KE \times 2m}}$$

Substitute the values to get the solution

6. Calculate the wavelength of an electron raised to a potential 1600V.

Solution: de-Broglie wavelength

$$\lambda = \frac{12.26}{\sqrt{V}} \text{ A}^0$$

$$= \frac{12.26}{\sqrt{1600}}$$

$$= \frac{12.26}{40}$$

$$= 0.3065\text{A}^0$$

7. **Calculate the energies that can be possessed by a particle of mass 8.50 x10^{-31}kg which is placed in an infinite potential box of width 10^{-9}cm.**

Solution: The possible energies of a particle in an infinite potential box of width L is given by $E_n = \frac{n^2h^2}{8mL^2}$

m =8.50 x 10^{-31}Kg

L =1 x 10^{-11}m

h=6.626× 10^{-34}J-s

For ground state n=1

$$E_1 = \frac{\left(6.626 \times 10^{-34}\right)^2}{8\left(8.50 \times 10^{-31}\right)\left(1 \times 10^{-11}\right)^2}$$

=6.456 × 10^{-16} joule

For first excited state, E_2 = 4 x 6.4456 x 10^{-16}

= 25.8268 x10^{-16}Joule

8. Find the lowest energy of an electron confined in a square box of side 0.1nm.

Solution: The possible energies of a particle in an infinite potential box of width L is given by $E_n = \frac{n^2 h^2}{8 m L^2}$

m =9.1 x 10^{-31}Kg

L =0.1 x 10^{-9}m

h=6.626× 10^{-34}J-s

For lowest energy n=1

$$E_1 = \frac{\left(6.626 \times 10^{-34}\right)^2}{8\left(9.1 \times 10^{-31}\right)\left(0.1 \times 10^{-9}\right)^2}$$

=60.307 × 10^{-19} joule

9. Electrons are accelerated by 344 volts and are reflected from a crystal. The first reflection maximum occurs when the glancing angle is 60^0 . Determine the spacing of the crystal.

Solution: $v=344\,V\,;\Theta=60^0 \quad 2\,d\sin\Theta=n\lambda$

$$\lambda=\frac{12.24}{\sqrt{344}}$$

As n=1 here, the above values can be substituted in $2\,d\sin\Theta=n\lambda$ to get the solution.

10. An electron is bound in 1-dimensional infinite well of width 1 x 10^{-10} m. Find the energy values of ground state and first two excited states.

Solution: The possible energies of a particle in an infinite potential box of width L is given by $E_n = \frac{n^2 h^2}{8mL^2}$

$m = 9.1 \times 10^{-31}$ Kg

$L = 1 \times 10^{-10}$ m

$h = 6.626 \times 10^{-34}$ J-s

For ground state n=1

$$E_1 = \frac{(6.626 \times 10^{-34})^2}{8(9.1 \times 10^{-31})(10^{-10})^2}$$

$= 0.6031 \times 10^{-17}$ joule

For first excited state, $E_2 = 4 \times 0.6031 \times 10^{-17}$

$= 2.412 \times 10^{-17}$ Joule

For second excited state, $E_3 = 9 \times 0.6031 \times 10^{-17}$

$= 5.428 \times 10^{-17}$ Joule