

Part B

①

Cook's theorem states that any NP problem can be converted to SAT in polynomial time.

Cook's theorem implies that any NP problem is at most polynomially harder than SAT.

this means that if we find a way to solve SAT in polynomial time then we will be able to solve any NP problem in polynomial time.

For example:

suppose SAT can be converted into problem D in polynomial time.

Now take any NP problem P_0 . we know that we can convert it into SAT in polynomial time, and we know that we can convert SAT into D in polynomial time.

so, we can convert P_0 into D and so, D will be NP complete.

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Deterministic

For a particular
input the
computer will give
always same
output

Can solve in
Polynomial
time

can determine the
next step of
execution

non-deterministic

For particular
input the computer will
give different output
on different execution

can't solve in
polynomial time

can't determine the
next step of execution
due to more than
one path an
Algorithm can take

③ write any sorting algo you know

④A

(4A)

P① 3D

P②

Algorithm DKP (P, W, n, m, x, y)

$\{ w: 20, P: 20;$

for $i=1$ to n do

$\{ x[i] := \text{choice}(0,1);$

$w := w + x[i] * w[i]; P := P + x[i] * P[i];$

$\}$

if $(w > m) \text{ or } (P < x)$ then Failure();

else Success();

$\}$

(5)

= Every decision problem solved by determining

'P' can also be solved using NP.

But every NP will not be a 'P' because

we know $P \subseteq NP$ But when $P \neq NP$

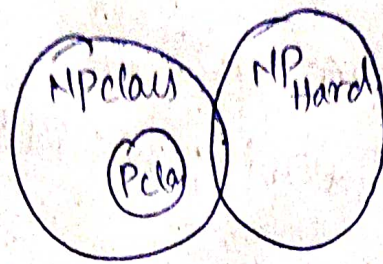
many problems in NP can be proved

if $P = NP$.

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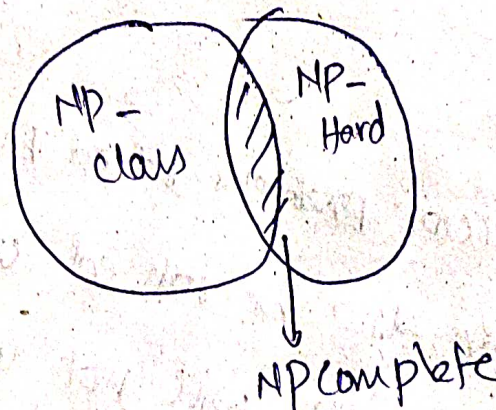
NP Hard

Every problem in NP class can be reduced into other set using polynomial time then it is called NP-Hard



NP complete

- the group of problems which are both NP and NP-Hard are called NP Complete



③

- (110) (10) Refer
- (120) (102) Refer
- (130) Any searching and sorting algorithm you like
- (140) (104) refer
- (150) (105) refer
- (160) (106) refer

~~170~~

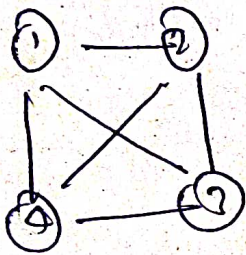
9A) we know that to prove any problem $P \geq L_2$ as a NP-hard problem first we have to consider an another problem which is NP-hard ' L_1 '. And then we have to find a reducibility Relation between these two and then we can say that ' L_2 ' is also NP-hard and so the ' P ' also.

So, as we want to prove 'cliques decision' problem as NP-hard now let us consider a NP-hard problem we consider SAT problem in this case.

④

clique decision problem

First before understanding what is meant by a clique decision problem first we will understand what is meant by a complete graph:



A complete graph is a graph who have their ^{all} adjacent vertices joined with the vertex we are considering that means simply each and every vertex should be connected to each and every other vertex in the graph.

now
And let us discuss about what is meant by a
clique. A clique is a subgraph in some other
graph which can form a complete graph.

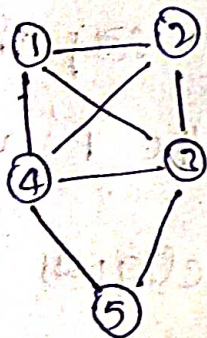
that means a clique is a subgraph in a ~~main~~ graph
which forms a complete graph.

And now let us discuss about clique decision problem

we know that if any problem has yes/no type
of answer is referred to as decision problem.

For example

let us consider



Here the total graph is not
a complete graph and but
the sub-graph 1-2-3-4
form a complete graph so

it has a clique 'K' of '4' and the subgraph

2-3-4 is also ~~not~~ complete graph.

so it also have clique of size '3'. And

if we consider 4-5 subgraph it is also

a complete graph so it also have a clique
of size '2'.

So here we can say that here we have
in this graph

* * Cliques of size = 4, 3, 2

And let us prove cliques ~~decision~~ decision problem

as NP-hard

we know that to prove any problem

$P = L_2$ as a NP-hard problem first
we have to consider an another problem
which is NP-hard ' L_1 '. And then we have to
find a reducibility relation between these
two. and then we can say that ' L_2 '
is also NP-hard and so the ' P ' also.

so, as we want to prove cliques decision
problem as NP-hard now let us consider
a NP-hard problem we consider SAT problem
in this case

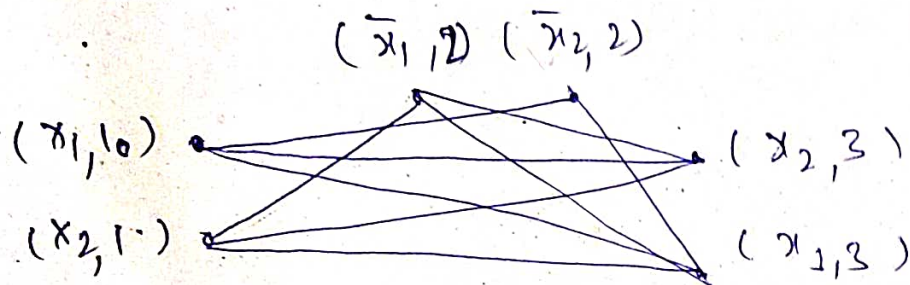
Now let us take a satisfiability formula with three variables x_1, x_2, x_3

$$F = (x_1 \vee x_2) \wedge (\bar{x}_1 \vee \bar{x}_2) \wedge (x_2 \vee x_3)$$

$\quad \quad \quad c_1 \quad \quad \quad c_2 \quad \quad \quad c_3$

Now we have to prepare a graph of having a clique of size '3'.

Now let us draw the vertices.



So, Here we can see that we have clique of size '3'. $(x_2, \bar{x}_1, \bar{x}_2)$

So, check formula.

$$(x_1 \vee x_2) \wedge (\bar{x}_1 \vee \bar{x}_2) \wedge (x_2 \vee x_3)$$

$$(0 \vee 1) \wedge (1 \vee 1) \wedge (1 \vee 0)$$

$$1 \wedge 1 \wedge 1 \geq 1$$

So, true.

Hence it satisfies so we can now say that cliques decision problem is also NP-hard problem.