UNIT-V

Test of Hypothesis-II

PART-B

1. Producer of 'gutkha' claims that the nicotine content in his 'gutkha' on the average is 0.83 mg. Can this claim be accepted if a random sample of 8 'gutkhas' of this type have the nicotine contents of 2.0, 1.7, 2.1, 1.9, 2.2, 2.1, 2.0, 1.6 mg.

Sol:- Given $n=8$, $\mu=1.83$

i) Null hypothesis (H_0):- $\mu=1.83$

ii) Alternative hypothesis (H_1):- $\mu \neq 1.83$

iii) Level of significance:- 5%

$t_{\alpha/2}$ at $\nu=n-1$ d.o.f

$t_{0.025}$ at $\nu=8-1$ d.o.f $= 2.365$

iv) Test statistic :- $t = \dfrac{\bar{x}-\mu}{s/\sqrt{n-1}}$

$$\bar{x} = \frac{2.0+1.7+2.1+1.9+2.2+2.1+2.0+1.6}{8} = 1.95$$

$$(x-\bar{x})^2 = (2.0-1.95)^2+(1.7-1.95)^2+ \cdots (1.6-1.95)^2$$

$= 0.3$

$$s^2 = \frac{\Sigma(x_i-\bar{x})^2}{n-1} = \frac{0.3}{7} \Rightarrow s = 0.21$$

$$\therefore t = \frac{\bar{x}-\mu}{s/\sqrt{n-1}} = \frac{1.95-1.83}{\frac{0.21}{\sqrt{8-1}}} = 1.62 \quad \& \ |t| = 1.62$$

v) conclusion:- $|t| < t_{\alpha/2}$

$\therefore$ we accept null hypothesis.

2. A sample of 26 bulbs gives a mean life of 990 hrs with S.D of 20 hrs. The manufactures claims that the mean life of bulbs 1000 hrs. Is the sample not upto the standard?

Sol:- Here $n=26$, $\bar{x}=990$, $\mu=1000$, $S=20$

i, Null hypothesis (H_0):- $\mu=1000$

ii, Alternative hypothesis (H_1):- $\mu \neq 1000$

iii, Level of significance (α):- $t_{0.025}$ at 25 d.o.f $= 2.060$

iv, Test statistics- $t = \dfrac{\bar{x}-\mu}{S/\sqrt{n-1}} = \dfrac{990-1000}{\dfrac{20}{\sqrt{26-1}}} = 2.5$

$|t| = 2.5$

v, conclusion:- $|t| > t_{\alpha/2}$, we reject null hypothesis

3. A random sample of 10 boys had the following I.Q's 70, 120, 110, 101, 88, 83, 95, 98, 107, 100. Do the data support the assumption of population means I.Q of 100. Test at 5% level of significance?

Sol:- Given $n=10$, $\bar{x} = \dfrac{\Sigma x}{n} = \dfrac{70+120+110+101+88+83+95+98+107+100}{10}$

$= 97.2$

$\Sigma(x-\bar{x})^2 = (70-97.2)^2 + (120-97.2)^2 + \cdots + (100-97.2)^2$

$= 1833.60$

$s^2 = \dfrac{1}{n-1}\Sigma(x_i-\bar{x})^2 = \dfrac{1833.60}{9} \Rightarrow s = 14.27$

i. Null hypothesis (H_0) :- $\mu = 100$

ii. Alternative Hypothesis (H_1) :- $\mu \neq 100$

iii. Level of significance (α) :- $t_{0.025}$ at 9 d.o.f $= 2.262$

iv. Test statistics :- $t = \dfrac{\bar{x} - \mu}{\frac{s}{\sqrt{n-1}}} = \dfrac{97.2 - 100}{\frac{14.27}{\sqrt{10-1}}} = -0.62$

$|t| = 0.62$

v. conclusion :- $|t| < t_{\alpha/2}$

$\therefore$ we accept null hypothesis

4. The means of two random samples of sizes 9, 7 are 196.42 and 198.82. The sum of squares of deviations from their respective means are 26.94, 18.73. Can be samples be considered to have been the same population?

Sol:- Given $n_1 = 9$, $n_2 = 7$, $\bar{x}_1 = 196.42$, $\bar{x}_2 = 198.82$

$$\Sigma(x_i - \bar{x}_1)^2 = 26.96, \quad \Sigma(x_i - \bar{x}_2)^2 = 18.73$$

$$s^2 = \frac{\Sigma(x_i - \bar{x}_1)^2 + \Sigma(x_i - \bar{x}_2)^2}{n_1 + n_2 - 2} = \frac{26.96 + 18.73}{9+7-2} = 3.26$$

$\Rightarrow s = 1.81$

i. Null hypothesis (H_0) :- $\bar{x}_1 = \bar{x}_2$

ii. Alternative hypothesis (H_1) :- $\bar{x}_1 \neq \bar{x}_2$

iii. Level of significance :-

$t_{0.025}$ at 14 d.o.f $= 2.145$

iv. Test statistic :- $t = \dfrac{\bar{x}_1 - \bar{x}_2}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \dfrac{196.42 - 198.82}{1.81\sqrt{\frac{1}{9} + \frac{1}{7}}}$

$t = -2.63$ & $|t| = 2.63$

v) Conclusion:- $|t| > t_{\alpha/2}$

$\therefore$ we reject null hypothesis

5. In one sample of 8 observations the sum of squares of deviations of the sample values from the sample mean was 84.4 and another sample of 10 observations it was 102.6. Test whether there is any significant difference between two sample variances at 5% level of significance.

Sol:- Given $n_1 = 8$, $n_2 = 10$

$$\Sigma(x_i - \bar{x}_1)^2 = 84.4, \quad \Sigma(x_i - \bar{x}_2)^2 = 102.6$$

$$S_1^2 = \frac{1}{n_1 - 1} \Sigma(x_i - \bar{x}_1)^2 = \frac{1}{8-1}(84.4) = 12.057$$

$$S_2^2 = \frac{1}{n_2 - 1} \Sigma(x_i - \bar{x}_2)^2 = \frac{1}{10-1}(102.6) = 11.4$$

i) Null hypothesis (H_0):- $S_1^2 = S_2^2$

ii) Alternative hypothesis (H_1):- $S_1^2 \neq S_2^2$

iii) Level of significance:- $\alpha = 0.05$

F_α at $(n_1 - 1, n_2 - 1)$ d.o.f

$F_{0.05}$ at $(8-1, 10-1)$ d.o.f $= 3.39$

iv) Test statistic:- $F = \frac{S_1^2}{S_2^2} = \frac{12.057}{11.4} = 1.057$

v) conclusion:- $|F| < F_\alpha$, we accept null hypothesis

6. Two random samples gave the following results.

Sample	Size	Sample mean	Sum of squares of deviations from mean.
I	10	15	90
II	12	14	108

Test whether there is any

Q) Test whether the samples came from the same population or not?

Sol Given $n_1 = 10$, $n_2 = 12$

$\bar{x}_1 = 15$, $\bar{x}_2 = 14$

$\Sigma(x_i - \bar{x}_1)^2 = 90$, $\Sigma(x_i - \bar{x}_2)^2 = 108$

$$S_1^2 = \frac{1}{n_1 - 1} \Sigma(x_i - \bar{x}_1)^2 = \frac{1}{10-1}(90) = 10$$

$$S_2^2 = \frac{1}{n_2 - 1} \Sigma(x_i - \bar{x}_2)^2 = \frac{1}{12-1}(108) = 9.82$$

(i) Null hypothesis (H_0) :- $S_1^2 = S_2^2$

(ii) Alternative hypothesis (H_1) :- $S_1^2 \neq S_2^2$

(iii) Level of significance :- $\alpha = 0.05$

F_α at $(n_1 - 1, n_2 - 1)$ d.o.f

$F_{0.05}$ at $(10-1, 12-1)$ d.o.f $= 2.90$

(iv) Test statistic :- $F = \frac{S_1^2}{S_2^2} = \frac{10}{9.82} = 1.018$

(v) Conclusion :- $|F| < F_\alpha$, we accept null hypothesis

7. Two independent samples of items are given respectively had the following values.

Sample I	11	11	13	11	15	9	12	14
Sample II	9	11	10	13	9	8	10	-

Test whether there is any significant difference b/w their means?

Sol Given $n_1 = 8$, $n_2 = 7$

$$\bar{x}_1 = \frac{11 + 11 + \cdots + 14}{8} = 12, \quad \bar{x}_2 = \frac{9 + 11 + \cdots + 10}{7} = 10$$

$$\Sigma(x_i-\bar{x}_1)^2 = (11-12)^2+(11-12)^2+\cdots+(14-12)^2 = 26$$

$$\Sigma(x_i-\bar{x}_2)^2 = (9-10)^2+(11-10)^2+\cdots+(10-10)^2 = 16$$

$$S^2 = \frac{1}{n_1+n_2-2}\left[\Sigma(x_i-\bar{x}_1)^2+\Sigma(x_i-\bar{x}_2)^2\right]$$

$$= \frac{1}{8+7-2}\left[26+16\right] = 3.23$$

$$\Rightarrow S = 1.8$$

i, Null hypothesis (H_0) :- $\bar{x}_1 = \bar{x}_2$

ii, Alternative hypothesis (H_1) :- $\bar{x}_1 \neq \bar{x}_2$

iii, Level of significance :- $\alpha = 0.05$

t_α at (n_1+n_2-2) d.o.f

$t_{0.025}$ at $(8+7-2)$ d.o.f $= 2.160$

iv, Test statistic :- $t = \dfrac{\bar{x}_1-\bar{x}_2}{S\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \dfrac{12-10}{(1.8)\sqrt{\frac{1}{8}+\frac{1}{7}}}$

$t = 2.15$ & $|t| = 2.15$

v, Conclusion :- $|t| < t_{\alpha/2}$, we accept null hypothesis

8. Time taken by workers in performing a job by method 1 and method 2 is given below.

Method 1	20	16	27	23	22	26	–
Method 2	27	33	42	35	32	34	38

Does the data show that variances of time distribution from population which these samples are drawn do not differ significantly?

Sol: Given $n_1 = 6$, $n_2 = 7$

$$\bar{x}_1 = \frac{20+16+\cdots+26}{6} = 22.3$$

$$\bar{x}_2 = \frac{27+33+\cdots+38}{7} = \cancel{81.34}\ 34.4$$

$$\Sigma(x_i-\bar{x}_1)^2 = (20-22.3)^2+(16-22.3)^2+\cdots+(26-22.3)^2$$
$$= 81.34$$

$$\Sigma(x_i-\bar{x}_2)^2 = (27-34.4)^2+(33-34.4)^2+\cdots+(38-34.4)^2$$
$$= 183.72$$

$$s_1^2 = \frac{1}{n_1-1}\Sigma(x_i-\bar{x}_1)^2 = \frac{1}{5}(81.34) = 16.26$$

$$s_2^2 = \frac{1}{n_2-1}\Sigma(x_i-\bar{x}_2)^2 = \frac{1}{6}(183.72) = 22.9$$

i, Null hypothesis (H_0):- $s_1^2 = s_2^2$

ii, Alternative hypothesis (H_1):- $s_1^2 \neq s_2^2$

iii, Level of significance:- $\alpha = 0.05$

F_α at (n_1-1, n_2-1) d.o.f

$F_{0.05}$ at $(6-1, 7-1)$ d.o.f $= 4.39$

iv, Test Statistic:- $F = \frac{s_2^2}{s_1^2} = \frac{22.9}{16.26} = 1.37$ & $|F| = 1.37$

v, Conclusion:- $|F| < F_\alpha$, we accept null hypothesis

9. The no. of automobile accident per week in a certain area as follows: 12, 8, 20, 2, 14, 10, 15, 6, 9, 4. Are these frequencies in agreement with the belief that accidents were same in the during last 10 weeks.

Sol: Given $n=10$

Expected frequencies $E_i = \frac{12+8+\cdots+9+4}{10} = 10$

Observed Frequencies (O_i)	Expected frequencies (E_i)	$O_i - E_i$	$\frac{(O_i - E_i)^2}{E_i}$
12	10	2	0.4
8	10	-2	0.4
20	10	10	10
2	10	-8	6.4
14	10	4	1.6
10	10	0	0
15	10	5	2.5
6	10	-4	1.6
9	10	-1	0.1
4	10	-6	3.6

$\chi^2 = \Sigma \frac{(O_i - E_i)^2}{E_i} = 26.6$

i, Null hypothesis (H_0):- $O_i = E_i$

ii, Alternative hypothesis (H_1):- $O_i \neq E_i$

iii, Level of significance :- $\alpha = 0.05$

χ^2_α at $(n-1)$ d.o.f

$\chi^2_{0.05}$ at $10-1$ d.o.f $= 16.9$

iv, Test statistic :- $\chi^2 = \Sigma \frac{(O_i - E_i)^2}{E_i} = 26.6$

v, Conclusion :- $|\chi^2| > \chi^2_\alpha$, we reject null hypothesis

10. A die is thrown 264 times with the following results. Show that the die is unbiased

No. appeared on die	1	2	3	4	5	6
Frequency	40	32	28	58	54	52

Sol Given $n = 6$

Expected Frequencies $E_i = \frac{40+32+28+58+54+52}{6}$

$E_i = 44$

O_i	E_i	$O_i - E_i$	$\frac{(O_i-E_i)^2}{E_i}$
40	44	-4	0.3636
32	44	-12	3.2727
28	44	-16	5.8181
58	44	14	4.4545
54	44	10	2.2727
52	44	8	1.4545

$$\chi^2 = \sum \frac{(O_i-E_i)^2}{E_i} = 17.6362$$

i, Null hypothesis (H_0) :- $O_i = E_i$

ii, Alternative Hypothesis (H_1):- $O_i \neq E_i$

iii, Level of significance :-

$\alpha = 0.05$, χ^2_α at $(n-1)$ d.o.f

$\chi^2_{0.05}$ at 6-1 d.o.f = 11.07

iv, Test statistic:- $\chi^2 = \sum \frac{(O_i-E_i)^2}{E_i} = 17.6362$

v, conclusion :- $|\chi^2| > \chi^2_\alpha$, we ~~accep~~ reject null hypothesis

11. 200 digits were choosen at random from set of tables the frequency of the digits are

digit	0	1	2	3	4	5	6	7	8	9
Frequency	18	19	23	21	16	25	22	20	21	15

use chi-square test to asset the correctness of the hypothesis that the digits are distributed in equal number in the table.

Sol Given $n = 10$

$$E_i = \frac{18+19+23+21+16+25+22+20+21+15}{10}$$

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O_i	E_i	$(O_i-E_i)^2$	$\frac{(O_i-E_i)^2}{E_i}$
18	20	4	0.2
19	20	1	0.05
23	20	9	0.45
21	20	1	0.05
16	20	16	0.8
25	20	25	1.25
22	20	4	0.2
20	20	0	0
21	20	1	0.05
15	20	25	1.25

$$\chi^2 = \Sigma \frac{(O_i-E_i)^2}{E_i} = 4.3$$

i, Null hypothesis(H_0) :- $O_i = E_i$

ii, Alternative hypothesis (H_1) :- $O_i \neq E_i$

iii, Level of significance :- $\alpha = 0.05$

χ^2_α at $(n-1)$ d.o.f

$\chi^2_{0.05}$ at $(10-1)$ d.o.f $= 16.919$

iv Test statistic :- $\chi^2 = \Sigma \frac{(O_i-E_i)^2}{E_i} = 4.3$

v, Conclusion :- $|\chi^2| < \chi^2_\alpha$, we accept H_0.

12. Fit a Poisson distribution to the following data & test the goodness of fit at 0.05 level.

x	0	1	2	3	4	5	6	7
frequency	305	366	210	80	28	9	2	1

Sol:

x	f	fx
0	305	0
1	366	366
2	210	420
3	80	240
4	28	112
5	9	45
6	2	12
7	1	7

$N = \Sigma f = 1001$

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f} = \frac{1202}{1001} = 1.2$$

$\therefore \lambda = 1.2$

Expected frequencies $E_0 = N \cdot P(X=0) = N \cdot \frac{e^{-\lambda} \cdot \lambda^0}{0!} = 1001 \cdot \frac{e^{-1.2} \cdot (1.2)^0}{0!}$

$= 301.49$

E.F $E_1 = N P(X=1) = N \cdot \frac{e^{-\lambda} \lambda^1}{1!} = 1001 \cdot \frac{e^{-1.2} \cdot (1.2)^1}{1!} = 361.79$

$E_2 = N \cdot P(X=2) = N \cdot \frac{e^{-\lambda} \cdot \lambda^2}{2!} = 1001 \cdot \frac{e^{-(1.2)} (1.2)^2}{2!} = 217.07$

$E_3 = N \cdot P(X=3) = N \cdot \frac{e^{-\lambda} \lambda^3}{3!} = 1001 \cdot \frac{e^{-(1.2)} (1.2)^3}{3!} = 86.83$

$E_4 = N \cdot P(X=4) = N \cdot \frac{e^{-\lambda} \lambda^4}{4!} = 1001 \cdot \frac{e^{-1.2} (1.2)^4}{4!} = 26.04$

$E_5 = N \cdot P(X=5) = N \cdot \frac{e^{-\lambda} \lambda^5}{5!} = 1001 \cdot \frac{e^{-1.2} (1.2)^5}{5!} = 6.25$

$E_6 = N \cdot P(X=6) = N \cdot \frac{e^{-\lambda} \lambda^6}{6!} = 1001 \; \frac{e^{-1.2} (1.2)^6}{6!} = 1.25$

$E_7 = N \cdot P(X=7) = N \cdot \frac{e^{-\lambda} \lambda^7}{7!} = 1001 \cdot \frac{e^{-1.2} (1.2)^7}{7!} = 0.214$

O_i	E_i	$(O_i - E_i)^2$	$\frac{(O_i - E_i)^2}{E_i}$
305	301.49	12.320	0.041
366	361.79	17.7241	0.049
210	217.07	49.9849	0.230
80	86.83	46.6489	0.537
28	26.04	3.8416	0.147
9	6.25	7.5625	1.21
2	1.25	0.5625	0.45
1	0.214	0.61779	2.887

$$\chi^2 = \Sigma \frac{(O_i - E_i)^2}{E_i} = 5.551$$

i) Null Hypothesis (H_0) :- $O_i = E_i$

ii) Alternative Hypothesis (H_1) :- $O_i \neq E_i$

iii) Level of significance :-

$\alpha = 0.05$ $\quad \chi^2_\alpha$ at $(n-1)$ d.o.f

$\chi^2_{0.05}$ at 7 d.o.f $= 14.067$

iv, Test statistic :- $\chi^2 = \Sigma \frac{(O_i - E_i)^2}{E_i} = 5.551$

v, conclusion :- $|\chi^2| < \chi^2_\alpha$, we accept null hypothesis

13. Given below is the number of male births in 1000 families having 5 children

Male children	0	1	2	3	4	5
Number of families	40	300	250	200	30	180

Test whether the given data is consistent with the hypothesis that the binomial distribution holds if the chance of a male birth is equal to female birth.

Sol:

x	f	fx
0	40	0
1	300	300
2	250	500
3	200	600
4	30	120
5	180	900
	$N = \Sigma f = 1000$	$\Sigma fx = 2420$

$n = 6$

$mean = \frac{\Sigma fx}{\Sigma f} = \frac{2420}{1000} = 2.42$

$np = 2.42$

$6P = \frac{2.4}{6} = 0.4$

$q = 1 - P = 1 - 0.4 = 0.6$

Expected frequency $E_0 = NP(X=0) = N \cdot n_{C_x} P^x q^{n-x}$

$= 1000 \cdot 6_{C_0} (0.4)^0 (0.6)^{6-0}$

$= 46.656$

$E_1 = NP(X=1) = N \cancel{P} n_{C_x} P^x q^{n-x} = 1000 \cdot 6_{C_1} (0.4)^1 (0.6)^{6-1} = 186.62$

$E_2 = NP(X=2) = N \cdot n_{C_x} P^x q^{n-x} = 1000 \cdot 6_{C_2} (0.4)^2 (0.6)^{6-2} = 311.04$

$E_3 = NP(X=3) = N \cdot n_{C_x} P^x q^{n-x} = 1000 \cdot 6_{C_3} (0.4)^3 (0.6)^{6-3} = 276.48$

$E_4 = NP(X=4) = N \cdot n_{C_x} P^x q^{n-x} = 1000 \cdot 6_{C_4} (0.4)^4 (0.6)^{6-4} = 138.24$

$E_5 = NP(X=5) = N \cdot n_{C_x} P^x q^{n-x} = 1000 \cdot 6_{C_5} (0.4)^5 (0.6)^{6-5} = 36.864$

~~$E_6 = NP(X=6) = N \cdot n_{C_x} P^x q^{n-x} = 1000 \cdot 6_{C_6} (0.4)^6 (0.6)^{6-6}$~~

O_i	E_i	$(O_i-E_i)^2$	$\frac{(O_i-E_i)^2}{E_i}$
40	46.656	44.30	0.94
300	186.62	12855.02	68.88
250	311.04	3725.88	11.97
200	276.48	5849.19	21.15
30	138.24	11715.8	84.74
180	36.864	20489.05	555.80

$$\chi^2 = \sum \frac{(O_i-E_i)^2}{E_i} = 743.48$$

i, Null hypothesis (H_0) :- $O_i = E_i$

ii, Alternative hypothesis (H_1) :- $O_i \neq E_i$

iii, Level of significance :- $\alpha = 0.05$

χ^2_α at $(n-1)$ d.o.f

$\chi^2_{0.05}$ at $(6-1)$ d.o.f $= 11.070$

iv, Test statistic :- $\chi^2 = \sum \frac{(O_i-E_i)^2}{E_i} = 743.48$

v, Conclusion :- $|\chi^2| > \chi^2_\alpha$, we reject null hypothesis

14. 5 dice were ~~th~~ thrown 96 times the number of times showing 4, 5 or 6 obtain is given below.

x	0	1	2	3	4	5
Frequency	1	10	24	35	18	8

Fit a binomial distribution and test for goodness of fit ~~at~~ ~~ooo~~ level.

Sol:

x	f	fx
0	1	0
1	10	10
2	24	48
3	35	105
4	18	72
5	8	40
	$N=\sum f = 96$	$\sum fx = 275$

$$\text{Mean} = \frac{\sum fx}{\sum f} = \frac{275}{96} = 2.86$$

$\lambda = 2.86$

$n=6$, $np = \lambda$

$np = 2.86$

$\Rightarrow p = 2.86/6 = 0.4$

$q = 1-p = 0.6$

Expected frequency $(E_0) = NP(X=0) = N\,{}^nC_x\,p^x q^{n-x} = 96 \cdot {}^6C_0 (0.4)^0 (0.6)^{6-0}$
$= 4.47$

$E_1 = NP(X=1) = N\,{}^nC_x\,p^x q^{n-x} = 96\ {}^6C_1\ (0.4)^1 (0.6)^{6-1} = 17.91$

$E_2 = NP(X=2) = N\,{}^nC_x\,p^x q^{n-x} = 96\ {}^6C_2\ (0.4)^2 (0.6)^{6-2} = 29.85$

$E_3 = NP(X=3) = N\,{}^nC_x\,p^x q^{n-x} = 96\ {}^6C_3\ (0.4)^3 (0.6)^{6-3} = 26.54$

$E_4 = NP(X=4) = N\,{}^nC_x\,p^x q^{n-x} = 96\ {}^6C_4\ (0.4)^4 (0.6)^{6-4} = 13.27$

$E_5 = NP(X=5) = N\,{}^nC_x\,p^x q^{n-x} = 96\ {}^6C_5\ (0.4)^5 (0.6)^{6-5} = 3.53$

O_i	E_i	$(O_i-E_i)^2$	$\frac{(O_i-E_i)^2}{E_i}$
1	4.47	12.04	2.69
10	17.91	62.56	3.48
24	29.85	34.22	1.14
35	26.54	71.5	2.69
18	13.27	22.37	1.68
8	3.53	19.98	5.66

$$\chi^2 = \sum \frac{(O_i-E_i)^2}{E_i} = 17.34$$

i, Null hypothesis (H_0) :- $O_i = E_i$

ii, Alternative hypothesis (H_1) :- $O_i \neq E_i$

iii, Level of significance :- $\alpha = 0.05$

χ^2_α at $(n-1)$ d.o.f

$\chi^2_{0.05}$ at 5 d.o.f $= 11.070$

iv, Test statistic :- $\chi^2 = \sum \frac{(O_i-E_i)^2}{E_i} = 17.34$

v, Conclusion :- $|\chi^2| > \chi^2_\alpha$, we reject null hypothesis

$=$

15. The following is the distribution of the hourly number of trucks arriving at a company wear house.

Trucks per hour	0	1	2	3	4	5	6	7	8
Frequency	52	151	130	102	45	12	3	1	2

Fit a poisson distribution to the following table and test the goodness of fit at 0.05 level.

Sol:

x	f	fx
0	52	0
1	151	151
2	130	260
3	102	306
4	45	180
5	12	60
6	3	18
7	1	7
8	2	16
	$N=\Sigma f=498$	$\Sigma fx = 998$

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f} = \frac{998}{498} = 2$$

$$\text{Expected frequency } (E_0) = NP(x=0) = N\cdot\frac{e^{-\lambda}\cdot\lambda^0}{0!} = 498\cdot\frac{e^{-2}\cdot 2^0}{0!} = 67\cdot39$$

$$E_1 = N\cdot \cancel{e^{-\lambda}\lambda^1}\ P(X=1) = N\cdot\frac{e^{-\lambda}\cdot\lambda^1}{1!} = 498\cdot\frac{e^{-2}\cdot 2^1}{1!} = 134\cdot7$$

$$E_2 = N\cdot P(X=2) = N\,\frac{e^{-\lambda}\cdot\lambda^2}{2!} = 498\cdot\frac{e^{-2}\cdot 2^2}{2!} = 134\cdot7$$

$$E_3 = N\,P(X=3) = N\cdot\frac{e^{-\lambda}\cdot\lambda^3}{3!} = 498\cdot\frac{e^{-2}\cdot 2^3}{3!} = 89.86$$

$$E_4 = N\cdot P(X=4) = N\cdot\frac{e^{-\lambda}\cdot\lambda^4}{4!} = 498\cdot\frac{e^{-2}\cdot 2^4}{4!} = 44\cdot93$$

$$E_5 = N\cdot P(X=5) = N\cdot\frac{e^{-\lambda}\cdot\lambda^5}{5!} = 498\cdot\frac{e^{-2}\cdot 2^5}{5!} = 17\cdot97$$

$$E_6 = N\cdot P(X=6) = N\cdot\frac{e^{-\lambda}\cdot\lambda^6}{6!} = 498\cdot\frac{e^{-2}\cdot 2^6}{6!} = 5\cdot99$$

$$E_7 = N \cdot P(x=7) = N \cdot \frac{e^{-\lambda} \lambda^7}{7!} = 498 \cdot \frac{e^{-2} \cdot 2^7}{7!} = 1.711$$

$$E_8 = N \cdot P(x=8) = N \cdot \frac{e^{-\lambda} \lambda^8}{8!} = 498 \cdot \frac{e^{-2} \cdot 2^8}{8!} = 0.427$$

O_i	E_i	$(O_i - E_i)^2$	$\frac{(O_i - E_i)^2}{E_i}$
52	67.39	236.8	3.51
151	134.7	265.69	1.97
130	134.7	22.09	0.16
102	89.86	147.37	1.63
45	44.93	0.0049	0.0001
12	17.97	35.64	1.98
3	5.99	8.94	1.49
1	1.711	0.50	0.292
2	0.427	2.49	5.83

$$\chi^2 = \frac{(O_i - E_i)^2}{E_i} = 16.86$$

i, Null hypothesis (H_0) :- $O_i = E_i$

ii, Alternative hypothesis (H_1) :- $O_i \neq E_i$

iii, Level of significance :- $\alpha = 0.05$

$$\chi^2_\alpha \text{ at } (n-1) \text{ d.o.f}$$

$$\chi^2_{0.05} \text{ at } 8 \text{ d.o.f} = 15.507$$

iv, Test Statistic :- $\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i} = 16.86$

v, Conclusion :- $|\chi^2| > \chi^2_\alpha$, we reject null hypothesis

16. The average breaking strength of the steel rods is specified to be 18.5 thousand pounds. To test this sample of 14 rods were tested, the mean and S.D. obtained were 17.85 and 1.955 respectively. Is the result of experiment significant?

Sol: Given $n=14$, $\bar{x}=17.85$, $\mu=18.5$, $S=1.955$

i, Null hypothesis (H_0):- $\mu=18.5$

ii, Alternative hypothesis (H_1):- $\mu \neq 18.5$

iii, Level of significance:- $\alpha=0.05$

t_α at $n-1$ d.o.f

$t_{0.025}$ at $(14-1)$ d.o.f $= 2.160$

iv, Test statistic:- $t = \dfrac{\bar{x}-\mu}{S/\sqrt{n-1}} = \dfrac{17.85-18.5}{1.955/\sqrt{13}} = -1.199$

$\& \ |t| = 1.199$

v, Conclusion:- $|t| < t_{\alpha/2}$ we accept null hypothesis.

17. A group of 5 patients treated with medicine A weight 42, 39, 48, 60 & 41 kgs. Second group of 7 patients from the same hospital treated with medicine B weight 38, 42, 56, 64, 68, 69 and 62 kgs. Do you agree with the claim that medicine B increases the weight significantly.

Sol: Given $\bar{x}_1 = \dfrac{\cancel{38+4}\ 42+39+48+60+41}{4} = 230$

$\bar{x}_2 = \dfrac{38+42+56+64+68+69+62}{7} = 57$

$$\Sigma(x_i-\bar{x}_1)^2 = (42-46)^2+(39-46)^2+(48-46)^2 + (60-46)^2+(41-46)^2 = 290$$

$$\Sigma(x_i-\bar{x}_2)^2 = (38-57)^2+(42-57)^2+(56-57)^2 + (64-57)^2+(68-57)^2+(69-57)^2+(62-57)^2$$

$$s^2 = \frac{1}{n_1+n_2-2}\left[\Sigma(x_i-\bar{x}_1)^2+\Sigma(x_i-\bar{x}_2)^2\right]$$

$$= \frac{1}{5+7-2}[290+926]$$

$$s^2 = 121.6$$

$$\Rightarrow s = 11.03$$

i, Null hypothesis (H_0) :- $\bar{x}_1 = \bar{x}_2$

ii, Alternative hypothesis (H_1) :- $\bar{x}_1 \neq \bar{x}_2$

iii Level of Significance :- $\alpha = 0.05$

$t_{\alpha/2}$ at (n_1+n_2-2) d.o.f

$t_{0.025}$ at $(5+7-2)$ d.o.f $=$ ~~2.20~~ 2.228

iv, Test statistic :- $t = \dfrac{\bar{x}_1-\bar{x}_2}{S\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \dfrac{46-57}{(11.03)\sqrt{\frac{1}{5}+\frac{1}{7}}}$

$= -1.7$

$\& \ |t| = 1.7$

v, Conclusion :- $|t| < t_{\alpha/2}$ we accept null hypothesis

18. In one sample of 10 observations, the sum of the deviations of the sample values from sample mean was 120 and in the other sample of 12 observations it was 314. Test whether the difference is significant at 5% level.

Ar Given $n_1 = 10$, $n_2 = 12$

$\Sigma(x_i-\bar{x}_1)^2 = 120$, $\Sigma(x_i-\bar{x}_2)^2 = 314$

$$S_1^2 = \frac{1}{n_1-1} \Sigma (x_i-\bar{x}_1)^2 = \frac{1}{10-1} \times 120 = 13.3$$

$$S_2^2 = \frac{1}{n_2-1} \Sigma (x_i-\bar{x}_2)^2 = \frac{1}{12-1} \times 314 = 28.5$$

i. Null hypothesis (H_0):– $S_1^2 = S_2^2$

ii. Alternative hypothesis (H_1):– $S_1^2 \neq S_2^2$

iii. Level of significance:– $\alpha = 0.05$

F_α at (n_1-1, n_2-1) d.o.f

$F_{0.05}$ at $(9, 11)$ d.o.f is 2.90

iv. Test statistic :– $F = \frac{S_2^2}{S_1^2} = \frac{28.5}{13.3} = 2.14$, $|F| = 2.14$

v. Conclusion:– $|F| < F_\alpha$, we accept null hypothesis

19. The following table gives the classification of 1000 workers according to gender and nature of work. Test whether the nature of work is independent of the gender of the worker.

	stable	unstable	total
Male	40	20	60
Female	10	30	40
total	50	50	100

Sol:

Expected frequencies

	stable	unstable
Male	$\frac{50\times60}{100} = 30$	$\frac{50\times60}{100} = 30$
Female	$\frac{50\times40}{100} = 20$	$\frac{50\times40}{100} = 20$

O_i	E_i	$(O_i-E_i)^2$	$\frac{(O_i-E_i)^2}{E_i}$
40	30	100	3.333
20	30	100	3.333
10	20	100	5.0
30	20	100	5.0

$$\chi^2 = \Sigma \frac{(O_i-E_i)^2}{E_i} = 16.66$$

i, Null hypothesis (H_0):- $O_i = E_i$

ii, Alternative hypothesis (H_1):- $O_i \neq E_i$

iii, Level of significance:-

$\alpha = 0.05$ χ^2_α at $(r-1)(c-1)$ d.o.f

$\chi^2_{.05}$ at $(2-1)(2-1)$ d.o.f $= 3.84$

iv, Test statistic :- $\chi^2 = \Sigma \frac{(O_i-E_i)^2}{E_i} = 16.66$

v, Conclusion:- $|\chi^2| > \chi^2_\alpha$, we reject null hypothesis

20. The following random samples are measurements of the heat-producing capacity (in millions of calories per ton) of specimens of coal from two mines

Mine I	8260	8130	8350	8070	8340	
Mine II	7950	7890	7900	8140	7920	7840

Use the 0.05 level of significance to test whether it is reasonable to assume that the variances of the two populations are equal.

2) Here $n_1 = 5$, $n_2 = 6$

x	$(x-\bar{x})^2$	y	$(y-\bar{y})^2$
8260	900	7950	100
8180	10000	7890	2500
8350	14400	7900	1600
8070	25600	8140	40000
8340	12100	7920	400
		7840	10000
$\Sigma x = 41150$	$\Sigma(x-\bar{x})^2 = 63000$	$\Sigma y = 47640$	$\Sigma(y-\bar{y})^2 = 54600$

$$\bar{x} = \frac{\Sigma x}{n_1} = \frac{41150}{5} = 8230, \quad \bar{y} = \frac{\Sigma y}{n_2} = \frac{47640}{6} = 7940$$

$$\Sigma(x-\bar{x})^2 = 63000, \quad \Sigma(y-\bar{y})^2 = 54600$$

$$S_1^2 = \frac{1}{n_1 - 1}\Sigma(x-\bar{x})^2 = \frac{1}{5-1} \times 63000 = 15750$$

$$S_2^2 = \frac{1}{n_2 - 1}\Sigma(y-\bar{y})^2 = \frac{1}{6-1} \times 54600 = 10920$$

i, Null hypothesis (H_0) :— $S_1^2 = S_2^2$

ii, Alternative hypothesis (H_1) :— $S_1^2 \neq S_2^2$

iii, Level of significance $\alpha = 0.05$

F_α at $(n_1 - 1, n_2 - 1)$ d.o.f

$F_{0.05}$ at $(5-1, 6-1)$ d.o.f $= 5.19$

iv, Test Statistic :— $F = \frac{S_1^2}{S_2^2}$ $(\because S_1^2 > S_2^2)$

$$F = \frac{15750}{10920} = 1.44$$

v, Conclusion :— $|F| < F_\alpha$. we accept null hypothesis

PART - C

1. A mechanist making engine parts with axle diameter of 0.700 inch. A random sample of 10 parts shows a mean diameter of 0.742 inch with a S.D of 0.040 inch. Compute the statistic you would use to test whether the work is meeting the specifications.

Sol:- Given $n = 10$, $\bar{x} = 0.742$, $S = 0.040$, $\mu = 0.700$

i, Null hypothesis (H_0):- $\mu = 0.700$

ii, Alternative hypothesis (H_1):- $\mu \neq 0.700$

iii, Level of significance:- $\alpha = 0.05$

t_α at $(n-1)$ d.o.f

$t_{0.025}$ at $(10-1)$ d.o.f $= 2.262$

iv Test Statistic:- $t = \dfrac{\bar{x}-\mu}{S/\sqrt{n-1}} = \dfrac{0.742 - 0.700}{\frac{0.040}{\sqrt{10-1}}} = 3.15$

v, Conclusion:- $|t| > t_{\alpha/2}$, we reject null hypothesis

2. To examine the hypothesis that the husbands are more intelligent than the wives, an investigator took a sample of 10 couples and administered them a test which measures I.Q. The results are following

Husbands	117	105	97	105	123	109	86	78	103	107
wifes	106	98	87	104	116	95	90	69	108	85

Test the hypothesis with a reasonable test at the level of significance of 0.05

Sol: $n_1 = 10$, $n_2 = 10$

$$\bar{x}_1 = \frac{117 + 105 + 97 + \text{------} + 107}{10} = 103$$

$$\bar{x}_2 = \frac{106 + 98 + 87 + \text{----} + 85}{10} = 95.8$$

$$(x_i - \bar{x}_1)^2 = (117-103)^2 + (105-103)^2 + (97-103)^2 + \cdots + (107-103)^2$$

$$= 160.6$$

$$(x_i - \bar{x}_2)^2 = (106-95.8)^2 + (98-95.8)^2 + (87-95.8)^2 + \text{-----}$$

$$+ (85-95.8)^2$$

$$= 1679.6$$

$$S^2 = \frac{1}{n_1+n_2-2}\left[\Sigma(x_i-\bar{x}_1)^2 + \Sigma(x_i-\bar{x}_2)^2\right]$$

$$= \frac{1}{10+10-2}\left[1606 + 1679.6\right]$$

$$S^2 = 182.53$$

$$\Rightarrow S = 13.51$$

i. Null hypothesis (H_0):- $\bar{x}_1 = \bar{x}_2$

ii. Alternative hypothesis (H_1):- $\bar{x}_1 \neq \bar{x}_2$

iii. Level of significance:-

$t_{\alpha/2}$ at (n_1+n_2-2) d.o.f

$t_{0.025}$ at 18 d.o.f $= 2.101$

iv. Test statistic:- $t = \dfrac{\bar{x}_1 - \bar{x}_2}{S\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \dfrac{103-95.8}{13.51\sqrt{\frac{1}{10}+\frac{1}{10}}}$

$$= 1.019168$$

v. Conclusion:- $|t| < t_{\alpha/2}$, we accept null hypothesis

3. Two independent samples of 8 & 7 items respectively had the following values

Samp I	11	11	13	11	15	9	12	14
Sample II	9	11	10	13	9	8	10	-

Is the differece b/w the means of samples significant?

Sol:- Given $n_1 = 8$, $n_2 = 7$

$$\bar{x_1} = \frac{11+11+ --- +14}{8} = 12$$

$$\bar{x_2} = \frac{9+11+ ---- +10}{7} = 10$$

$$(x_i - \bar{x_1})^2 = (11-12)^2 + (11-12)^2 + --- + (14-12)^2 = 26$$

$$(x_i - \bar{x_2})^2 = (9-10)^2 + (11-10)^2 + ---- + (10-10)^2 = 16$$

$$S^2 = \frac{1}{n_1+n_2-2}\left[\Sigma (x_i - \bar{x_1})^2 + \Sigma (x_i - \bar{x_2})^2\right]$$

$$= \frac{1}{8+7-2}[26+16]$$

$$= 3.23$$

$$\Rightarrow S = 1.79$$

i, Null hypothesis (H_0):- $\bar{x_1} = \bar{x_2}$

ii, Alternative hypothesis (H_1):- $\bar{x_1} \neq \bar{x_2}$

iii, Level of significance:-

$t_{\alpha/2}$ at (n_1+n_2-2) d.o.f

$t_{0.025}$ at 13 d.o.f $= 2.160$

iv, test statistic:- $t = \dfrac{\bar{x_1} - \bar{x_2}}{S\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$

$$= \frac{12-10}{1.79\sqrt{\frac{1}{8}+\frac{1}{7}}} = 2.15$$

(v) Conclusion:- $|t| < t_{\alpha/2}$, we accept null hypothesis

4. Pumpkins were grown under two experimental conditions. Two random samples of 11 & 9 pumpkins. The sample standard deviation of their weights as 0.8 and 0.5 respectively. ~~Assuming~~ that the weight distributions are normal, test hypothesis that the true variances are equal.

Sol:- Given $n_1 = 11$, $n_2 = 9$

$s_1 = 0.8$, $s_2 = 0.5$

~~$\frac{s_1^2 ...}{n_1 - 1}$~~

(i) Null hypothesis:- $s_1^2 = s_2^2$

(ii) Alternative hypothesis:- $s_1^2 \neq s_2^2$

(iii) Level of significance:- $\alpha = 0.05$

$(n_1 - 1, n_2 - 1)$ d.o.f

F_α at (n_1-1, n_2-1) d.o.f $= 3.35$

$F_{0.05}$ at $(11-1, 9-1)$ d.o.f

(iv) Test statistic:- $F = \frac{s_1^2}{s_2^2} = \frac{(0.8)^2}{(0.5)} = 2.56$

$\& \ |F| = 2.56$

(v) conclusion:- $|F| < F_\alpha$, we accept null hypothesis

5. From the following data, find whether there is any significant linking in the habit of taking soft drinks among the categories of employees.

Soft drinks	Clerks	Teachers	Officers
Pepsi	10	25	65
Thumsup	15	30	65
Fanta	50	60	30

Sol: Expected Frequencies (Ei)

Soft drinks	Clerks	Teachers	Officers
Pepsi	$\frac{75\times100}{350}=21.4$	$\frac{115\times100}{350}=32.9$	$\frac{160\times100}{350}=45.71$
Thumsup	$\frac{75\times110}{350}=23.6$	$\frac{115\times110}{350}=36.1$	$\frac{160\times110}{350}=50.3$
Fanta	$\frac{75\times140}{350}=30$	$\frac{115\times140}{350}=46$	$\frac{160\times140}{350}=64$

O_i	E_i	$(O_i-E_i)^2$	$\frac{(O_i-E_i)^2}{E_i}$
10	21.4	129.96	6.073
25	32.9	62.41	1.897
65	45.7	372.49	8.151
15	23.6	73.96	3.134
30	36.1	37.21	1.031
65	50.3	216.09	4.3
50	30	400	13.333
60	46	196	4.261
30	64	1156	18.062

$$\chi^2 = \Sigma \frac{(O_i-E_i)^2}{E_i} = 60.2425$$

i. Null hypothesis :- $O_i = E_i$

ii. Alternative hypothesis :- $O_i \neq E_i$

iii. Level of significance :- $\alpha = 0.05$

χ^2_α at $\vartheta = (r-1)(c-1)$ d.o.f

$\chi^2_{0.05}$ at $\vartheta = (3-1)(3-1)$ d.o.f $= 9.488$

iv. Test statistic :- $\chi^2 = \Sigma \frac{(O_i - E_i)^2}{E_i} = 60.2425$

v. Conclusion :- $|\chi^2| > \chi^2_\alpha$, we reject null hypothesis

6. In an investigation on the machine performance, the following results are obtained.

	No. of units inspected	No. of defective
Machine-I	375	17
Machine-II	450	22

Sol: Expected Given

	No. of units inspected	No. of defective	total
Machine I	375	17	392
Machine II	450	22	472
total	825	39	864

Expected frequencies

	No. of units inspected	No. of defective
machine I	$\frac{825 \times 392}{864} = 374.3$	$\frac{39 \times 392}{864} = 17.69$
machine II	$\frac{825 \times 472}{864} = 450.69$	$\frac{39 \times 472}{864} = 21.3$

O_i	E_i	$(O_i-E_i)^2$	$\frac{(O_i-E_i)^2}{E_i}$
375	374.3	0.49	0.00131
17	17.69	0.4761	0.0269
450	450.69	0.4761	0.0011
22	21.3	0.49	0.023

$$\chi^2 = \Sigma \frac{(O_i-E_i)^2}{E_i} = 0.05231$$

i, Null hypothesis :- $O_i = E_i$

ii, Alternative hypothesis :- $O_i \neq E_i$

iii, Level of significance :-

$$\chi^2_\alpha \text{ at } \nu = (r-1)(c-1) \text{ d.o.f}$$

$$\chi^2_{0.05} \text{ at } \nu = (2-1)(2-1) \text{ d.o.f} = 3.84$$

iv, Test statistic : $\chi^2 = \Sigma \frac{(O_i-E_i)^2}{E_i} = 0.052$

$$|\chi^2| = 0.052$$

v, conclusion :- $|\chi^2| < \chi^2_\alpha$, we accept null hypothesis

7. A survey of 240 families with 4 children each revealed the following distribution.

Male births	4	3	2	1	0
No. of families	10	55	105	58	12

Test whether the male & female births are equally popular.

Sol: Given $p = q = 1/2$, $N = 240$, $n = 4$

Expected frequencies $E = N \cdot {}^nC_x \, p^x q^{n-x}$

$$E_0 = 240 \times {}^4C_0 (1/2)^0 (1/2)^{4-0} = 15$$

$$E_1 = 240 \times {}^4C_1 (1/2)^1 (1/2)^{4-1} = 60$$

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$E_2 = 240 \times 4c_2 (1/2)^2 (1/2)^{4-2} = 90$

$E_3 = 240 \times 4c_3 (1/2)^3 (1/2)^{4-3} = 60$

$E_4 = 240 \times 4c_4 (1/2)^4 (1/2)^{4-4} = 15$

O_i	E_i	$(O_i - E_i)^2$	$\frac{(O_i-E_i)^2}{E_i}$
10	15	25	1.67
55	60	25	0.42
105	90	225	2.5
58	60	4	0.07
12	15	9	0.6

$$\chi^2 = \Sigma \frac{(O_i-E_i)^2}{E_i} = 5.26$$

i, Null hypothesis :- $O_i = E_i$

ii, Alternative hypothesis :- $O_i \neq E_i$

iii, Level of significance :- $\alpha = 0.05$

χ^2_α at $\nu = n-1$ d.o.f

$\chi^2_{0.05}$ at $\nu = 5-1$ d.o.f $= 9.488$

iv, Test statistic :- $\chi^2 = \Sigma \frac{(O_i-E_i)^2}{E_i} = 5.26$. $|\chi^2| = 5.26$

v, Conclusion :- $|\chi^2| < \chi^2_\alpha$, we accept null hypothesis

8. Samples of students were drawn from two universities and from their weights in kilograms mean and S.D are calculated and shown below make a large sample test to the significance of difference b/w means.

	Mean	S.D	Sample Size
University A	55	10	10
University B	57	15	20

Sol Given $\bar{x}_1 = 55$, $\bar{x}_2 = 57$

$n_1 = 10$, $n_2 = 20$

$S_1 = 10$, $S_2 = 15$

$$S^2 = \frac{1}{n_1+n_2-2}\left[n_1 S_1^2 + n_2 S_2^2\right]$$

$$= \frac{1}{10+20-2}\left[10\times(10)^2 + 20\times(15)^2\right]$$

$S^2 = 196.42$

$\Rightarrow S = 14.01$

i, Null hypothesis :- $\bar{x}_1 = \bar{x}_2$

ii, Alternative hypothesis :- $\bar{x}_1 \neq \bar{x}_2$

iii Level of significance :- $\alpha = 0.05$

$t_{\alpha/2}$ at $\nu = n_1+n_2-2$ d.o.f

$t_{0.05/2}$ at $\nu = 10+20-2$ d.o.f

$t_{0.025}$ at $\nu = 28$ d.o.f $= 2.048$

iv, Test statistic :- $t = \dfrac{\bar{x}_1 - \bar{x}_2}{S\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \dfrac{55-57}{(4.01)\sqrt{\frac{1}{10}+\frac{1}{20}}}$

$= -1.28$

$|t| = 1.28$

v, Conclusion :- $|t| < t_{\alpha/2}$, we accept null hypothesis

=

9. The measurements of the output of two units have given the following results. Assuming that both samples have been obtained from the normal populations at 10% significant level, test whether the two populations have the same variance.

Unit-A	14.1	10.1	14.7	13.7	14.0
Unit-B	14.0	14.5	13.7	12.7	14.1

Given $n_1 = 5,\ n_2 = 5$

$$\bar{x}_1 = \frac{\Sigma x_i}{n_1} = \frac{14.1 + 10.1 + 14.7 + 13.7 + 14.0}{5} = 13.32$$

$$\bar{x}_2 = \frac{\Sigma x_i}{n_2} = \frac{14.0 + 14.5 + 13.7 + 12.7 + 14.1}{5} = 13.8$$

$$\Sigma(x_i - \bar{x}_1)^2 = (14.1-13.32)^2 + (10.1-13.32)^2 + \cdots + (14.0-13.32)^2 = 13.488$$

$$\Sigma(x_i - \bar{x}_2)^2 = (14.0-13.8)^2 + (14.5-13.8)^2 + \cdots + (14.1-13.8)^2 = 1.84$$

$$S_1^2 = \frac{1}{n_1-1}\Sigma(x_i-\bar{x}_1)^2 = \frac{1}{5-1} \times 13.488 = 3.372$$

$$S_2^2 = \frac{1}{n_2-1}\Sigma(x_i-\bar{x}_2)^2 = \frac{1}{5-1} \times 1.84 = 0.46$$

i. Null hypothesis :- $S_1^2 = S_2^2$

ii. Alternative hypothesis :- $S_1^2 \neq S_2^2$

iii Level of Significance :- $\alpha = 0.05$

F_α at $(\nu_1, \nu_2) = (n_1-1, n_2-1)$ d.o.f

$F_{0.05}$ at $(5-1, 5-1)$ d.o.f $= 6.39$

iv Test statistic :- $F = \frac{S_1^2}{S_2^2} = \frac{3.372}{0.46} = 7.33,\ |F| = 7.33$

v. Conclusion :- $|F| > F_\alpha$, we reject null hypothesis

10. The nicotine in milligrams of two samples of tobacco were found to be as follows. Test the hypothesis for the difference b/w means at 0.05 level.

Sample - A	24	27	26	23	25	—
Sample - B	29	30	30	31	24	36

Sol: Given $n_1 = 5,\ n_2 = 6$

$$\bar{x}_1 = \frac{\Sigma x_i}{n} = \frac{24+27+26+23+25}{5} = 25$$

$$\bar{x}_2 = \frac{\Sigma x_i}{n} = \frac{29+30+30+31+24+36}{6} = 30$$

$$\Sigma(x_i - \bar{x}_1)^2 = (24-25)^2 + (27-25)^2 + \cdots + (25-25)^2 = 10$$

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$$\Sigma(x_i - \bar{x}_2)^2 = (27-30)^2 + (30-30)^2 + \cdots + (36-30)^2 = 74$$

$$S^2 = \frac{1}{n_1+n_2-2}\left[\Sigma(x_i-\bar{x}_1)^2 + \Sigma(x_i-\bar{x}_2)^2\right]$$

$$= \frac{1}{5+6-2}\left[10+74\right]$$

$$S^2 = 9.33$$

$$\Rightarrow S = 3.049$$

(i) Null hypothesis :- $\bar{x}_1 = \bar{x}_2$

(ii) Alternative hypothesis :- $\bar{x}_1 \neq \bar{x}_2$

(iii) Level of significance $\alpha = 0.05$

$t_{\alpha/2}$ at $\vartheta = n_1+n_2-2$ d.o.f

$t_{0.025}$ at $\vartheta = 5+6-2 = 9$ d.o.f $= 2.262$

(iv) Test statistics :- $t = \dfrac{\bar{x}_1 - \bar{x}_2}{S\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \dfrac{25-30}{(3.049)\sqrt{\frac{1}{5}+\frac{1}{6}}}$

$$= -2.71$$

$$|t| = 2.71$$

(v) conclusion :- $|t| > t_{\alpha/2}$, we reject null hypothesis