

APPLIED PHYSICS

MODULE 2

PART A

@K. Manisha and Ujjwal Acharya

1. What do you mean by periodic potential associated with an electron moving in a one-dimensional crystal lattice?

Sol. Crystalline solid consists of a lattice which is composed of a large number of ionic cores at regular intervals and the conduction electrons move throughout the lattice. The periodic potential associated with an electron moving in a one-dimensional crystal lattice is expressed in Bloch's Theorem. The solution $\psi(x)$ is given by:

$$\begin{aligned} V(x) &= V(x+a) \\ \text{'a' periodicity of potential. The schrodinger wave equation of moving} \\ \text{electron is} \\ \frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2} [E - V(x)]\psi &= 0 \longrightarrow \textcircled{1} \\ \text{the solution of eq. (1)} \\ \psi(x) &= e^{ikx} u_k(x) \longrightarrow \textcircled{2} \\ \text{where } u_k(x) &= u_k(x+a) \longrightarrow \textcircled{3} \\ \text{Eq. (3) represents periodic function \& e^{ikx} represents plane} \\ \text{wave. The above statement is known as Bloch's Theorem.} \\ \text{and eq. (2) is known as Bloch function.} \end{aligned}$$

2. Explain a metallic solid and draw its band diagram to explain its electronic behavior.

Sol. Metallic solids are composed of metal cations held together by a delocalized "sea" of valence electrons. Because their electrons are mobile, metallic solids are good conductors of heat and electricity.

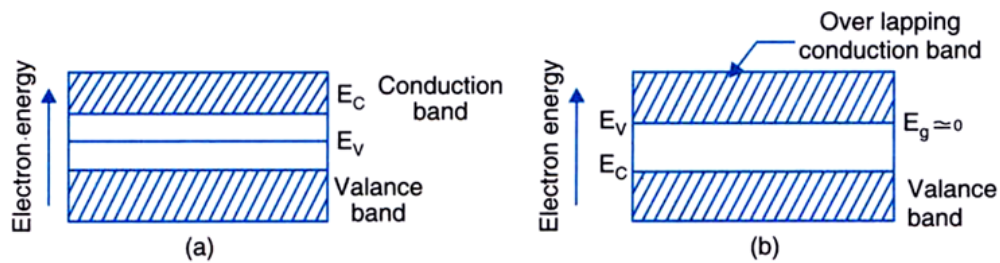
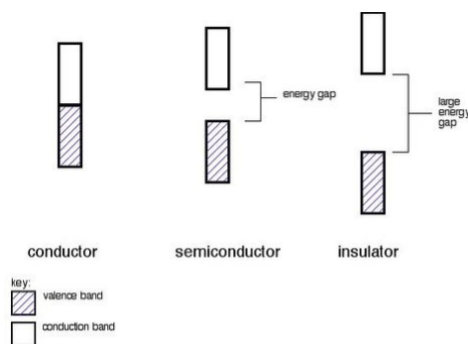


Fig. (f) Metals

3. Justify that the crystalline solids are classified into conductors, semiconductors and insulators.



A. Conductors

Metals are conductors. There is no band gap between their valence and conduction bands, since they overlap. There is a continuous availability of electrons in these closely spaced orbitals.

B. Insulators

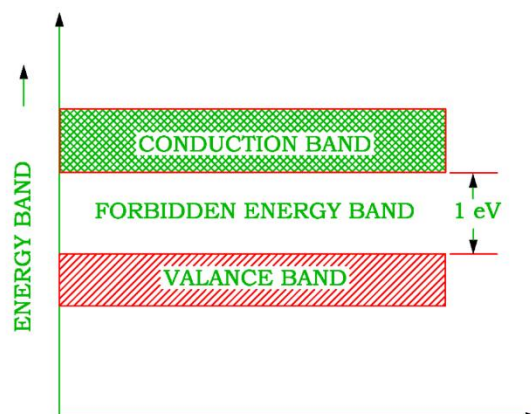
In insulators, the band gap between the valence band and the conduction band is so large that electrons cannot make the energy jump from the valence band to the conduction band.

C. Semiconductors

Semiconductors have a small energy gap between the valence band and the conduction band. Electrons can make the jump up to the conduction band, but not with the same ease as they do in conductors.

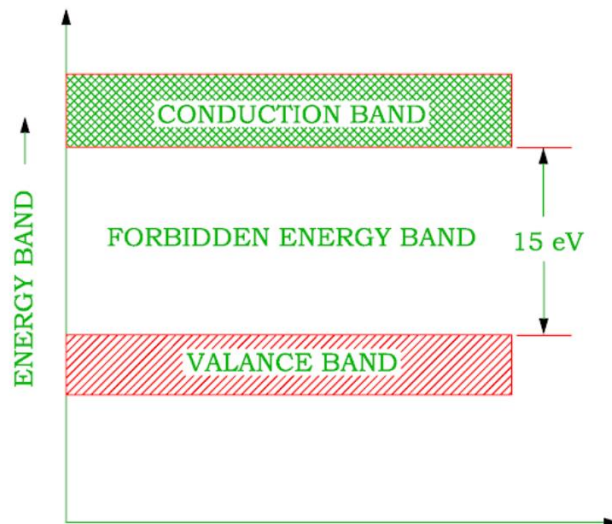
4. Illustrate electronic behavior of a semiconductor by drawing its band diagram.

Semiconductors are the materials which have a conductivity between conductors and non-conductors or insulators. Semiconductors can be compounds such as gallium arsenide or pure elements, such as germanium or silicon.



5. Outline the behavior of an insulator by sketching its band diagram.

Sol. Insulators are materials that hinder the free flow of electrons from one particle of the element to another. If we transfer some amount of charge to such an element at any point, the charge remains at the initial location and does not get distributed across the surface.



6. Distinguish semiconductors based on variation of conductivity in terms of temperature and doping.

Sol. **Based on Temperature:**

1. Extrinsic Semiconductors: Extrinsic semiconductors are semiconductors that are doped with specific impurities. The impurity modifies the electrical properties of the semiconductor and makes it more suitable for electronic devices such as diodes and transistors.

2. Intrinsic Conductors: Semiconductors that are chemically pure, in other words, free from impurities are termed intrinsic semiconductors. The number of holes and electrons is therefore determined by the properties of the material itself instead of the impurities.

Based on Doping:

1. n-type Semiconductors: When a pure semiconductor (Silicon or Germanium) is doped by pentavalent impurity (P, As, Sb, Bi) then, four electrons out of five valence electrons bond with the four electrons of Ge or Si. These are called n-type Semiconductors.

2. p-type Semiconductors: When a pure semiconductor is doped with a trivalent impurity (B, Al, In, Ga) then, the three valence electrons of the impurity bond with three of the four valence electrons of the semiconductor. These are called p-type Semiconductors.

7. Explore an intrinsic semiconductor by giving an example.

Sol. In intrinsic semiconductors the number of located electrons and the number of holes is equal $n = p$. The most common examples of the intrinsic semiconductors are silicon and germanium. Both of these are used frequently in manufacturing of Transistors and electronic products. Intrinsic and extrinsic semiconductor.

8. Give the expressions for carrier concentration of electrons and holes in intrinsic semiconductors.

Sol. The electron concentration in the conduction band is given as,

$$n = N_c e^{\frac{-(E_C - E_F)}{K_B T}}$$

The hole concentration in the valence band is given as,

$$p = N_v e^{\frac{-(E_F - E_V)}{K_B T}}$$

9. Write an expression for carrier concentration of electrons in n-type semiconductor.

Sol.

Let N_D is the donor concentration (no. of donor atoms per unit volume). Let it be written as

Or written as
$$= N_D \exp \left[\frac{-(E_F - E_D)}{K_B T} \right]$$

$$= N_D e^{\frac{-(E_F - E_D)}{K_B T}} \rightarrow (1)$$

The density of electrons in CB in pure semiconductor is given by

$$= 2 \left[\frac{2\pi m_e^* K_B T}{h^2} \right]^{\frac{3}{2}} e^{\frac{-(E_C - E_F)}{K_B T}} \rightarrow (2)$$

At very low temperatures the no. of electrons in CB must be equal to the no. of donor atoms per unit volume. Hence equating equations (1) & (2) we get

$$\begin{aligned} 2 \left[\frac{2\pi m_e^* K_B T}{h^2} \right]^{\frac{3}{2}} e^{\frac{-(E_C - E_F)}{K_B T}} &= N_D e^{\frac{-(E_F - E_D)}{K_B T}} \\ \frac{e^{\frac{-(E_C + E_F)}{K_B T}}}{e^{\frac{-(E_F + E_D)}{K_B T}}} &= \frac{N_D}{2 \left[\frac{2\pi m_e^* K_B T}{h^2} \right]^{\frac{3}{2}}} \\ e^{\frac{-E_C + E_F + E_F - E_D}{K_B T}} &= \frac{N_D}{2 \left[\frac{2\pi m_e^* K_B T}{h^2} \right]^{\frac{3}{2}}} \\ e^{\frac{2E_F - (E_C + E_D)}{K_B T}} &= \frac{N_D}{2 \left[\frac{2\pi m_e^* K_B T}{h^2} \right]^{\frac{3}{2}}} \end{aligned}$$

Taking logarithms on both sides we get

$$\frac{2E_F - (E_C + E_D)}{K_B T} = \log \left[\frac{N_D}{2 \left[\frac{2\pi m_e^* K_B T}{h^2} \right]^{\frac{3}{2}}} \right]$$

$$2E_F - (E_C + E_D) = K_B T \log \left[\frac{N_D}{2 \left[\frac{2\pi m_e^* K_B T}{h^2} \right]^{\frac{3}{2}}} \right]$$

$$E_F = \frac{(E_C + E_D)}{2} + \frac{K_B T}{2} \log \left[\frac{N_D}{2 \left[\frac{2\pi m_e^* K_B T}{h^2} \right]^{\frac{3}{2}}} \right] \quad \{ \text{At } T > 0K \} \rightarrow (3)$$

Case I: At $T = 0K$

$$E_F = \frac{(E_C + E_D)}{2}. \text{ That is } E_F \text{ lies between } E_C \text{ \& } E_D$$

Case II: At $T > 0K$. As temperature increases the Fermi level slightly shifts upwards towards CB, hence

$$E_F = \frac{(E_C + E_D)}{2} + \frac{K_B T}{2} \log \left[\frac{N_D}{2 \left[\frac{2\pi m_e^* K_B T}{h^2} \right]^{\frac{3}{2}}} \right]$$

10. Give an expression for carrier concentration of holes in p-type semiconductor?

Sol.

Let N_A is the acceptor concentration (no. of acceptor atoms per unit volume). Let it be written as

$$= N_A \exp \left[\frac{-(E_A - E_F)}{K_B T} \right]$$

Or written as

$$= N_A e^{\frac{(E_F - E_A)}{K_B T}} \rightarrow (1)$$

The density of holes in VB in pure semiconductor is given by

$$= 2 \left[\frac{2\pi m_p^* K_B T}{h^2} \right]^{\frac{3}{2}} e^{\frac{-(E_F - E_V)}{K_B T}} \rightarrow (2)$$

At very low temperatures the no. of holes in VB must be equal to the no. of acceptor atoms per unit volume. Hence equating equations (1) & (2) we get

$$\begin{aligned} 2 \left[\frac{2\pi m_p^* K_B T}{h^2} \right]^{\frac{3}{2}} e^{\frac{-(E_F - E_V)}{K_B T}} &= N_A e^{\frac{(E_F - E_A)}{K_B T}} \\ \frac{e^{\frac{(-E_F + E_V)}{K_B T}}}{e^{\frac{(E_F - E_A)}{K_B T}}} &= \frac{N_A}{2 \left[\frac{2\pi m_p^* K_B T}{h^2} \right]^{\frac{3}{2}}} \end{aligned}$$

$$e^{\frac{-E_F + E_V - E_F + E_A}{K_B T}} = \frac{N_A}{2 \left[\frac{2\pi m_p^* K_B T}{h^2} \right]^{\frac{3}{2}}}$$

$$e^{\frac{-2E_F + (E_V + E_A)}{K_B T}} = \frac{N_A}{2 \left[\frac{2\pi m_p^* K_B T}{h^2} \right]^{\frac{3}{2}}}$$

Taking logarithms on both sides we get

$$\frac{-2E_F + (E_V + E_A)}{K_B T} = \log \left[\frac{N_A}{2 \left[\frac{2\pi m_p^* K_B T}{h^2} \right]^{\frac{3}{2}}} \right]$$

$$-2E_F + (E_V + E_A) = K_B T \log \left[\frac{N_A}{2 \left[\frac{2\pi m_p^* K_B T}{h^2} \right]^{\frac{3}{2}}} \right]$$

$$E_F = \frac{(E_V + E_A)}{2} - \frac{K_B T}{2} \log \left[\frac{N_A}{2 \left[\frac{2\pi m_p^* K_B T}{h^2} \right]^{\frac{3}{2}}} \right] \quad \{ \text{At } T > 0K \} \rightarrow (3)$$

Case I: At $T = 0K$

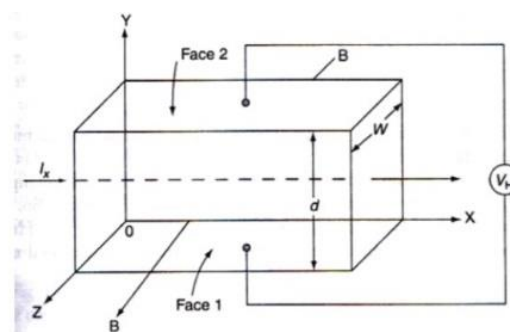
$$E_F = \frac{(E_V + E_A)}{2}. \text{ That is } E_F \text{ lies between } E_V \text{ \& } E_A$$

Case II: At $T > 0K$. As temperature increases the Fermi level slightly drops towards E_V , hence

$$E_F = \frac{(E_V + E_A)}{2} - \frac{K_B T}{2} \log \left[\frac{N_A}{2 \left[\frac{2\pi m_p^* K_B T}{h^2} \right]^{\frac{3}{2}}} \right]$$

11. Describe Hall effect using a proper diagram representing current, magnetic field and Hall voltage.

Sol. The presence of this measurable transverse voltage is called the Hall effect. Note that the direction of the current I in the diagram is that of conventional current, so that the motion of electrons is in the opposite direction.



semiconductor applied to current and magnetic field perpendicularly in Hall Effect

12. How does the Fermi level play a significant role in semiconductor?

Sol. The Fermi level represents the electron population at energy levels and consequently the conductivity of materials. Semiconductor doping and higher temperatures can greatly improve the conductivity of the pure semiconductor material.

13. Explain different mechanisms responsible for electrical resistance in metals.

Sol. Electrical resistance is opposition to the flow of current in an electric circuit: resistance converts electrical energy to thermal energy, and is similar in regard to mechanical friction.

The mechanisms are nowhere mentioned, if found please DM.

14. Explain about drift velocity and mobility associated with charge carriers in a semiconductor.

- The mobility of a charge carrier is defined as the drift velocity of the charge carrier per unit electric field. Thus, mobility (μ) = $E_v d$
- If q be the charge of carrier and τ be the relaxation time, then mobility $\mu = mq\tau$ where m be the mass of the charge carrier.

15. List out the failures of quantum free electron theory of solids.

Sol. 1. Classical free electron fails to explain the temperature dependence of electrical conductivity

2. It fails to explain the specific heat of a solid.

16. Enlighten about Root Mean Square velocity (RMS) and Relaxation time.

Sol. The root-mean square (RMS) velocity is the value of the square root of the sum of the squares of the stacking velocity values divided by the number of values.

Relaxation time is the time interval between two successive collisions of electrons in a conductor, when current flows.

17. Define mean collision time (τ) associated with moving electron

Sol. The typical time between collisions is very small, about 10^{-14} seconds. So that electrons are continually colliding with the ions but between collisions, the electric field is accelerating the electrons causing them to slowly drift through the wire.

18. Write about Mean free path as given by classical free electron theory of metals.

Sol. We can relate the mean free path to the size of the ions in the metal lattice. The number of ions in this volume and hence the number of collisions in the time 't' is ' $n_a \pi r^2 v t$ ' where ' n_a ' is the number of ions per unit volume. The total path length divided by the number of collisions is the mean free path λ .

$$\lambda = \frac{vt}{n_a \pi r^2 v t} = \frac{1}{n_a \pi r^2}$$

19. Discuss about formation of a hole in a semiconductor.

Sol. Holes are formed when electrons in atoms move out of the valence band (the outermost shell of the atom that is completely filled with electrons) into the conduction band (the area in an atom where electrons can escape easily), which happens everywhere in a semiconductor.

20. How do you interpret the idea of conduction taking place due to movement of holes in a semiconductor

Sol. The cause of electrical conduction in semiconductors is due to the movement of the holes in the valence band along with the movement of the electrons in the conduction band.

When electric field is applied, then as a result the electrons will start moving in the conduction band in the direction opposite to that of the electric field. The current produced at that time is known as hole current. In other words, we can say that two types of currents are mainly working in the semiconductor. One is the hole current that is due to the movement of the holes in the valence band and other is the electronic current which is caused by the movement of the electrons in the conduction band. The total resultant current in the semiconductor is the sum of the both the currents.

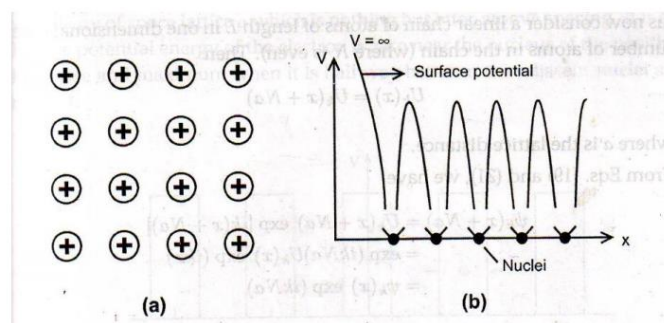
PART B

@ B. Jyoshna Rama Devika, Gade Mary Sannihitha, Ujjwal, Vishal, Nishant

1. Summarize Bloch's theorem? Demonstrate in detail the motion of an electron in a periodic potential.

A. Crystalline solid consists of a lattice which is composed of a large number of ionic cores at regular intervals and the conduction electrons move throughout the lattice.

Let us consider the picture of the lattice in only one dimension, i.e., only an array of ionic cores along x-axis. If we plot the potential energy of a conduction electron as a position in the lattice, the variation of potential energy is as shown in figure. The potential is minimum at the positive ion sites and maximum between the two ions.



(a) periodic positive ion cores inside metallic crystals. b) One dimensional periodic potential in crystal.

The one dimension Schrodinger equation corresponding to this can be written as

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2m}{h^2}[E-V(x)] \psi = 0 \quad \rightarrow (1)$$

The periodic potential $V(x)$ may be defined by means of the lattice constant 'a' as

$$V(x) = V(x+a) \quad \rightarrow (2)$$

Bloch considered the solution as

$$\psi_k(x) = \exp(ikx)U_k(x) \quad \rightarrow (3)$$

Eqn (2) is known as Bloch function. $U_k(x)$ is periodic with the periodicity of the crystal lattice.

The free electron wave is modulated by periodic function $U_k(x)$ is periodic with the periodicity of the crystal lattice. The free electron wave is modulated by periodic function $U_k(x)$.

For a linear chain of atoms of length 'L' in one dimensional case with 'N' (= even) number of atoms in the chain,

$$U_k(x) = U_k(x+Na) \quad \rightarrow (4)$$

From eqn (3) and eqn (4)

$$\begin{aligned} \psi_k(x+na) &= U_k(x+Na) e^{[ik(x+Na)]} \\ &= e^{(ikNa)} U_k(x) e^{(ikx)} \\ &= \psi_k(x) e^{(ikNa)} \end{aligned} \quad \rightarrow (5)$$

This is referred to as Bloch condition.

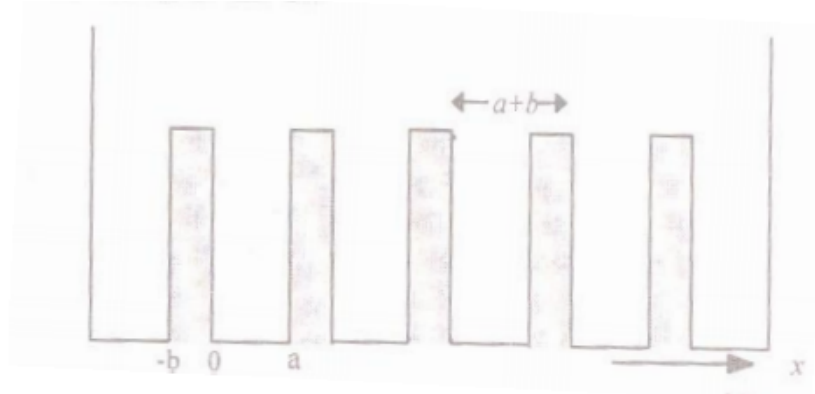
Now,

$$\begin{aligned} \psi_k(x+Na)\psi_k^*(x+Na) &= \psi_k(x)e^{(ikNa)}\psi_k^*(x)e^{(-ikNa)} \\ &= \psi_k(x)\psi_k^*(x)e^{(0)} \\ \psi_k(x+Na)\psi_k^*(x+Na) &= \psi_k(x)\psi_k^*(x) \end{aligned} \quad \rightarrow (6)$$

This means that the electron is not located around any particular atom and the probability of finding the electron is same throughout the crystal.

2. Using Kronig-Penny model, show that the energy spectrum of an electron contains a number of allowed energy bands separated by forbidden bands.

A. The periodic potential assumed by Kronig and Penny is shown in Figure. i.e., a series of rectangular wells of width a and are placed at a separation of b . in the regions where $0 < x < a$, the potential energy is zero and in regions such as $-b < x < 0$, the potential energy is V_0 .



One dimensional periodic potential assumed by Kronig and Penny

The main features of the model and its predictions can be explained qualitatively

Main features of the model

A. Schrodinger equation:

The dynamical behavior of electrons in the Kronig-Penny model is represented by the following Schrodinger equation,

$$\frac{d^2\psi}{dx^2} + \left[\frac{2m}{h^2}\right] E\psi = 0 \quad \text{for } 0 < x < a$$

And

$$\frac{d^2\psi}{dx^2} + \left[\frac{2m}{h^2}\right] (E - V_0)\psi = 0 \quad \text{for } -b < x < 0 \quad \rightarrow (1)$$

Let us assume that total energy 'E' of the electron under consideration is less than V_0 .

$$\text{Further, let us substitute } \alpha^2 = \frac{2mE}{h^2} \quad \text{and} \quad \beta^2 = \frac{2m}{h^2}(V_0 - E) \quad \rightarrow (2)$$

Where α and β are real quantities.

Now Eq(1) becomes

$$\frac{d^2\psi}{dx^2} + \alpha^2\psi = 0, \quad \text{for } 0 < x < a$$

And

$$\frac{d^2\psi}{dx^2} - \beta^2\psi = 0, \quad \text{for } -b < x < 0 \quad \rightarrow (3)$$

These equations can be solved with the help of block theorem. The final solution of eq (3) is given in the form of the following condition.

$$P \frac{\sin \alpha a}{\alpha a} + \cos \alpha a = \cos k a \quad \rightarrow (4)$$

Where $P = \frac{mb}{h^2} V_0 a$ is scattering power of the potential barrier and V_0 is barrier strength. That means, eq (3) will have a solution only when the condition (4) is satisfied.

3. Explain in detail the origin of energy band formation in solids that lead to the classification of materials based on conductivity.

A.

Origin of Energy band formation in solids

In an isolated atom, the electrons are tightly bound and have discrete, sharp energy levels

When two identical atoms are brought closer, the outermost orbits of these atoms overlap and interact.

When the wave functions of the electrons of the different atoms begin to overlap considerably, the energy levels split into two.

If more atoms are brought together, more levels are formed and for a solid of N atoms, each of the energy levels of an atom splits into N levels of energy [Figure].

The levels are so close together that they form an almost continuous band. The width of this band depends on the degree of overlap of the electrons of adjacent atoms and is largest for the outermost atomic electrons.

In a solid, many atoms are brought together that the split energy levels form a set of energy bands of very closely spaced levels with forbidden energy gaps between them.

The band corresponding to outermost orbit is called conduction band and the next band is called valence band. The gap between these two allowed bands is called forbidden energy gap or band gap. According to the width of the gap between the bands and band occupation by electrons all solids can be classified broadly into three groups namely, conductors, semiconductors and Insulators.

4. Distinguish between intrinsic and extrinsic semiconductors. Indicate on an energy level diagram, the conduction and valence bands, donor and acceptor levels for intrinsic and extrinsic semiconductors.

A. On the basis of band theory, solids can be broadly classified into three categories, viz, insulators, semiconductors and conductors. Their band structures can be as shown in figure.

Insulators:

1. In case of insulators, the forbidden gap is very wide. Due to this fact electrons cannot jump from valence band to conduction band.

2. They have completely filled valence band and completely empty conduction band.

3. The resistivity of insulators is very high.

4. Insulators are bad conductors of electricity

Semiconductors:

1. In semiconductors, the band gap is very small (0.7 eV for germanium and 1.1 eV for silicon).

2. At 0K, there are no electrons in the conduction band and the valence band is completely filled. As the temperature increases, electrons from the valence band jump into conduction band.

3. The resistivity varies from 10^{-14} to $10^7 \Omega \text{ meter}$.

4. They have electrical properties between those of insulators and conductors.

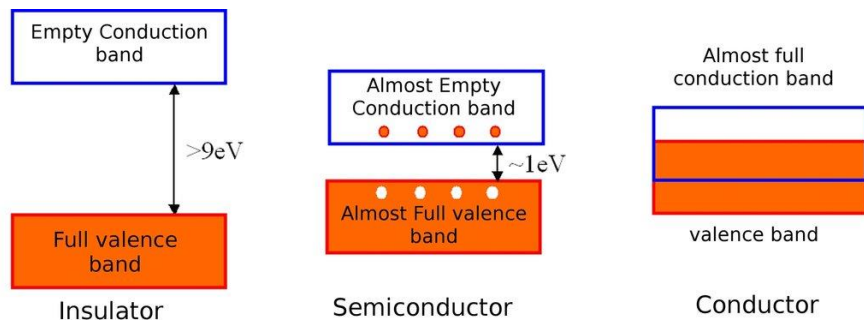
Conductors:

1. In case of conductors, there is no forbidden gap and the valence band conduction band overlaps each other.

2. Plenty of free electrons are available for electrical conduction.

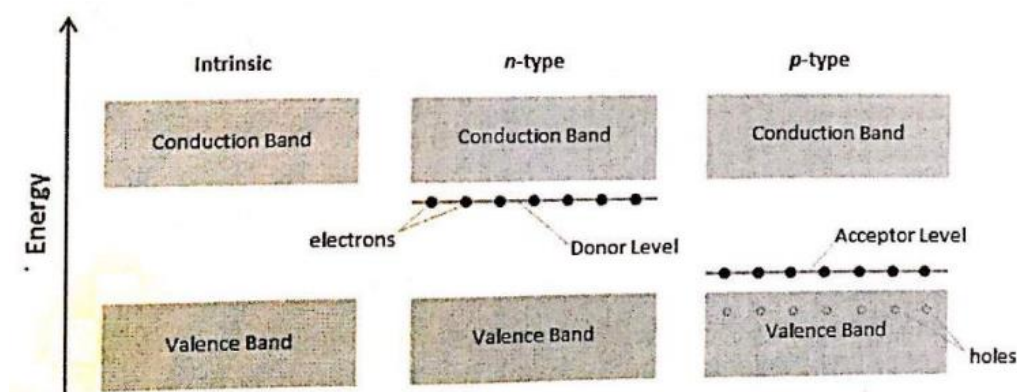
3. They possess very low resistivity and very high conductivity values.

4. Metals like copper, iron etc. are best examples of conductors.



5. Develop a mathematical expression for intrinsic carrier concentration and hence prove that the Fermi level lies at the middle for an intrinsic semiconductor.

A.



S.No	Intrinsic Semiconductor	Extrinsic Semiconductor
1.	Semiconductor in a pure form is called intrinsic semiconductor.	Semiconductor which are doped with impurity is called extrinsic semiconductor
2.	Here the charge carriers are produced only due to thermal agitation.	Here the charge carriers are produced due to impurities and may also be produced due to thermal agitation.
3.	They have low electrical conductivity.	They have high electrical conductivity.
4.	They have low operating temperature.	They have high operating temperature.
5.	At 0K, Fermi level exactly lies between conduction band and valence band.	At 0K, Fermi level exactly lies closer to conduction band in "n" type semiconductor and lies near valence band in "p" type semiconductor.
	Examples: Si, Ge, etc.	Examples: Si and Ge doped with Al, In, P, As etc

6. Obtain an expression for carrier concentration of n- type semiconductor.

A.

Let N_D is the donor concentration (no. of donor atoms per unit volume). Let it be written as

$$\begin{aligned} \text{Or written as} \quad &= N_D \exp \left[\frac{-(E_F - E_D)}{K_B T} \right] \\ &= N_D e^{\frac{-(E_F - E_D)}{K_B T}} \rightarrow (1) \end{aligned}$$

The density of electrons in CB in pure semiconductor is given by

$$= 2 \left[\frac{2\pi m_e^* K_B T}{h^2} \right]^{\frac{3}{2}} e^{\frac{-(E_C - E_F)}{K_B T}} \rightarrow (2)$$

At very low temperatures the no. of electrons in CB must be equal to the no. of donor atoms per unit volume. Hence equating equations (1) & (2) we get

$$2 \left[\frac{2\pi m_e^* K_B T}{h^2} \right]^{\frac{3}{2}} e^{\frac{-(E_C - E_F)}{K_B T}} = N_D e^{\frac{-(E_F - E_D)}{K_B T}}$$

$$\frac{e^{\frac{-(E_C + E_F)}{K_B T}}}{e^{\frac{-(E_F + E_D)}{K_B T}}} = \frac{N_D}{2 \left[\frac{2\pi m_e^* K_B T}{h^2} \right]^{\frac{3}{2}}}$$

$$e^{\frac{-E_C + E_F + E_F - E_D}{K_B T}} = \frac{N_D}{2 \left[\frac{2\pi m_e^* K_B T}{h^2} \right]^{\frac{3}{2}}}$$

$$e^{\frac{2E_F - (E_C + E_D)}{K_B T}} = \frac{N_D}{2 \left[\frac{2\pi m_e^* K_B T}{h^2} \right]^{\frac{3}{2}}}$$

Taking logarithms on both sides we get

$$\frac{2E_F - (E_C + E_D)}{K_B T} = \log \left[\frac{N_D}{2 \left[\frac{2\pi m_e^* K_B T}{h^2} \right]^{\frac{3}{2}}} \right]$$

$$2E_F - (E_C + E_D) = K_B T \log \left[\frac{N_D}{2 \left[\frac{2\pi m_e^* K_B T}{h^2} \right]^{\frac{3}{2}}} \right]$$

$$E_F = \frac{(E_C + E_D)}{2} + \frac{K_B T}{2} \log \left[\frac{N_D}{2 \left[\frac{2\pi m_e^* K_B T}{h^2} \right]^{\frac{3}{2}}} \right] \quad \{\text{At } T > 0K\} \rightarrow (3)$$

Case I: At $T = 0K$

$$E_F = \frac{(E_C + E_D)}{2}. \text{ That is } E_F \text{ lies between } E_C \text{ \& } E_D$$

Case II: At $T > 0K$. As temperature increases the Fermi level slightly shifts upwards towards CB, hence

$$E_F = \frac{(E_C + E_D)}{2} + \frac{K_B T}{2} \log \left[\frac{N_D}{2 \left[\frac{2\pi m_e^* K_B T}{h^2} \right]^{\frac{3}{2}}} \right]$$

7. Derive an expression for carrier concentration of p- type semiconductor.

A.

Let N_A is the acceptor concentration (no. of acceptor atoms per unit volume). Let it be written as

$$= N_A \exp \left[\frac{-(E_A - E_F)}{K_B T} \right]$$

Or written as

$$= N_A e^{\frac{(E_F - E_A)}{K_B T}} \rightarrow (1)$$

The density of holes in VB in pure semiconductor is given by

$$= 2 \left[\frac{2\pi m_p^* K_B T}{h^2} \right]^{\frac{3}{2}} e^{\frac{-(E_F - E_V)}{K_B T}} \rightarrow (2)$$

At very low temperatures the no. of holes in VB must be equal to the no. of acceptor atoms per unit volume. Hence equating equations (1) & (2) we get

$$2 \left[\frac{2\pi m_p^* K_B T}{h^2} \right]^{\frac{3}{2}} e^{\frac{-(E_F - E_V)}{K_B T}} = N_A e^{\frac{(E_F - E_A)}{K_B T}}$$

$$\frac{e^{\frac{(-E_F + E_V)}{K_B T}}}{e^{\frac{(E_F - E_A)}{K_B T}}} = \frac{N_A}{2 \left[\frac{2\pi m_p^* K_B T}{h^2} \right]^{\frac{3}{2}}}$$

$$e^{\frac{-E_F + E_V - E_F + E_A}{K_B T}} = \frac{N_A}{2 \left[\frac{2\pi m_p^* K_B T}{h^2} \right]^{\frac{3}{2}}}$$

$$e^{\frac{-2E_F + (E_V + E_A)}{K_B T}} = \frac{N_A}{2 \left[\frac{2\pi m_p^* K_B T}{h^2} \right]^{\frac{3}{2}}}$$

Taking logarithms on both sides we get

$$\frac{-2E_F + (E_V + E_A)}{K_B T} = \log \left[\frac{N_A}{2 \left[\frac{2\pi m_p^* K_B T}{h^2} \right]^{\frac{3}{2}}} \right]$$

$$-2E_F + (E_V + E_A) = K_B T \log \left[\frac{N_A}{2 \left[\frac{2\pi m_p^* K_B T}{h^2} \right]^{\frac{3}{2}}} \right]$$

$$E_F = \frac{(E_V + E_A)}{2} - \frac{K_B T}{2} \log \left[\frac{N_A}{2 \left[\frac{2\pi m_p^* K_B T}{h^2} \right]^{\frac{3}{2}}} \right] \{ \text{At } T > 0K \} \rightarrow (3)$$

Case I: At $T = 0K$

$$E_F = \frac{(E_V + E_A)}{2}. \text{ That is } E_F \text{ lies between } E_V \text{ \& } E_A$$

Case II: At $T > 0K$. As temperature increases the Fermi level slightly drops towards VB, hence

$$E_F = \frac{(E_V + E_A)}{2} - \frac{K_B T}{2} \log \left[\frac{N_A}{2 \left[\frac{2\pi m_p^* K_B T}{h^2} \right]^{\frac{3}{2}}} \right]$$

8. Illustrate the dependence of Fermi level on carrier-concentration and temperature in n-type and p-type semiconductors.

A. Variation of Fermi level E_F with respect to temperature in N -type:

As temperature increases the electrons in the Donor level ' E_D ' moves into CB leaving holes. Also The Fermi level slightly shifts upwards towards CB. If further increase of temperature is done, the hole concentration also increases with respect to electron concentration. Hence it reaches again the concentration of $n = p$ i.e. an intrinsic semiconductor. At last the Fermi level drops in the middle of the energy gap (or) Forbidden band gap, indicating a pure semiconductor that is

towards 'E_i'. This happens only for very high temperatures as shown in the figure 8 below.

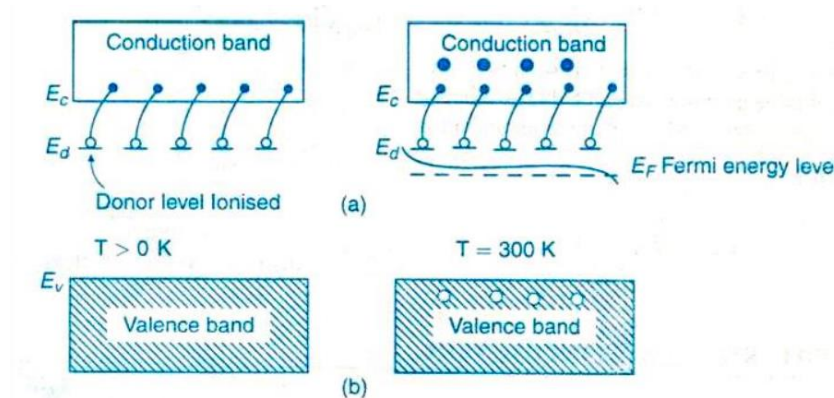
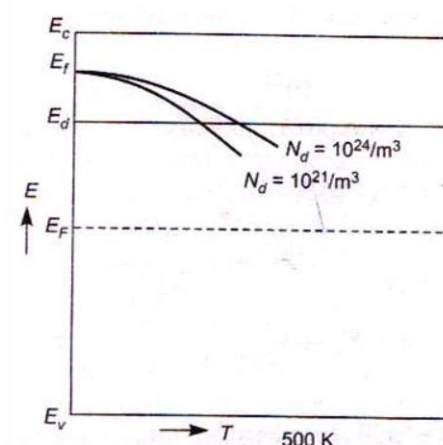


Fig.8 Variation of Fermi level wrt temperature in n- type semiconductor

Variation of E_F with respect to donor concentration

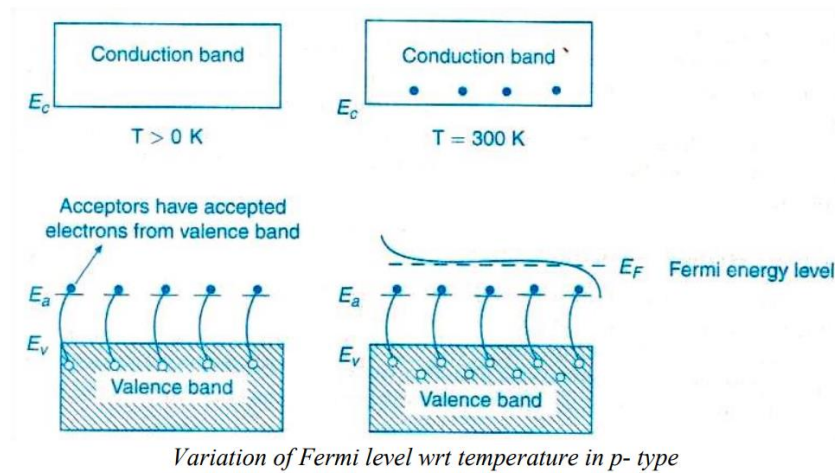
As the donor concentration increases the Fermi level decreases (lowers) as in case of intrinsic semiconductor E_i. This is shown in the figure below



Variation of Fermi level with temperature for different donor concentrations in an n-type semiconductor

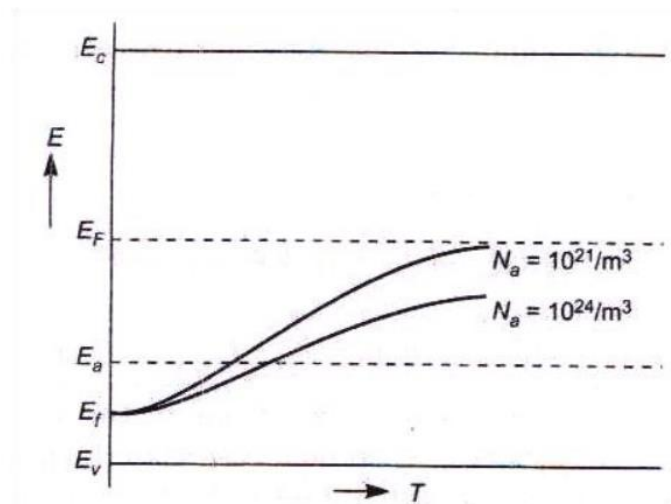
Variation of Fermi level E_F with respect to temperature in P- type semiconductor:

As temperature increases the Fermi level E_F slightly drops towards VB. For further increase of high temperatures, the electron concentration also increases with respect to hole concentration. Hence a condition is reached such that $n = p$ i.e. it becomes an intrinsic or pure semiconductor. Hence the Fermi level increases and reaches to intrinsic level E_i as in case of pure semiconductor. This is shown in the figure below.



Variation of Fermi level with respect to acceptor concentration:

Also, as the acceptor concentration increases, we find that Fermi level E_F reaches (increases) towards intrinsic level E_i as in case of pure or intrinsic semiconductor. This is shown in the figure below



Variation of Fermi level with temperature for different acceptor concentrations in a p-type

9. Demonstrate in detail Hall effect and obtain an expression for Hall coefficient. List out the uses of Hall effect.

A. Hall effect is defined as the production of a voltage difference across an electrical conductor which is transverse to an electric current and with respect to an applied magnetic field it is perpendicular to the current. It is also defined as the field developed across the conductor and Hall voltage is the corresponding potential difference.

Here is the expression for Hall Coefficient:

$$R_H = \frac{p\mu_H^2 - n\mu_e^2}{e(p\mu_H + n\mu_e)} \therefore R_H = \frac{(p - nb^2)}{e(p + nb)^2}$$

Where,

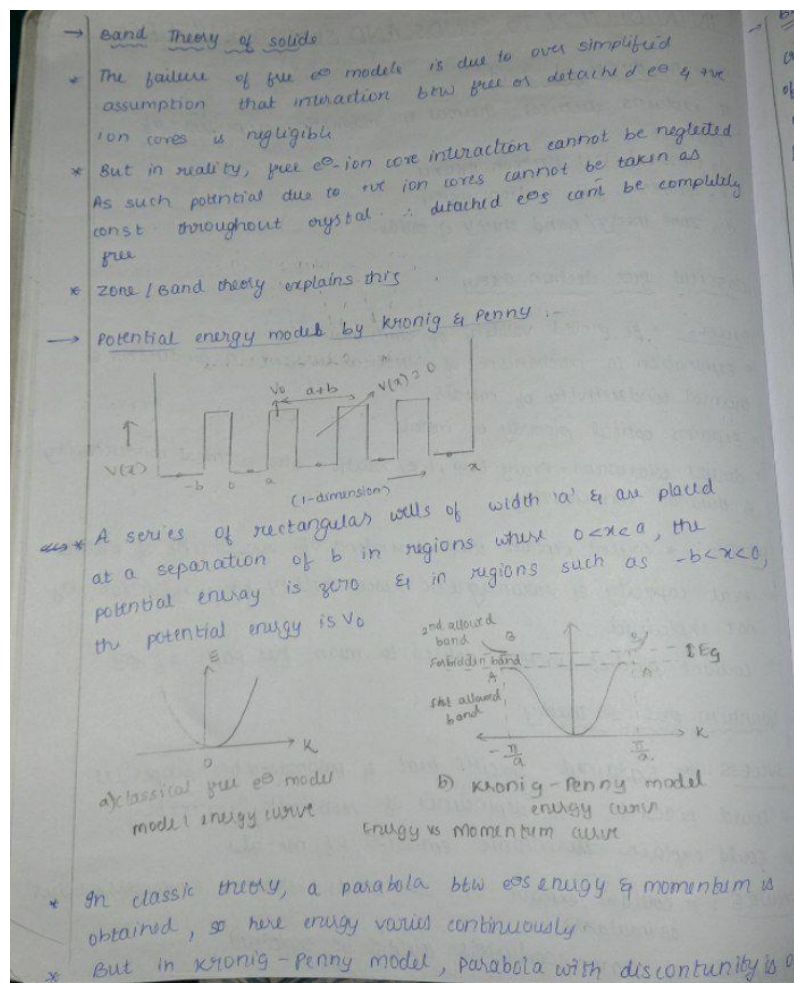
$$b = \frac{\mu_e}{\mu_H}$$

- n is electron concentration
- p is hole concentration
- μ_e is the mobility of electron
- μ_H is the mobility of the hole
- e is an elementary charge

Hall effect finds many applications:

- It is used to determine if the given material is a semiconductor or insulator.
- It is used to measure the magnetic field and is known as a magnetometer
- They find applications in position sensing as they are immune to water, mud, dust, and dirt.
- They are used in integrated circuits as Hall effect sensors.

10. Interpret the graphical representation of Kronig-Penny model. Extend the conclusions drawn from the graph.



conclusions of graph

* The energy spectrum consists of alternative regions of allowed & vacant bands. Forbidden band implies that energy levels that lie in this region are not ~~allowed~~ occupied by e⁻s.

The allowed (shaded) bands are narrowest for low values of energy & become broader as energy rises, the unallowed (~~forbidden~~) bands becoming narrower.

a) For $P > 0$ (extreme left), the whole energy spectrum is quasi continuous. That is all allowed bands are joined together forming an almost continuum.

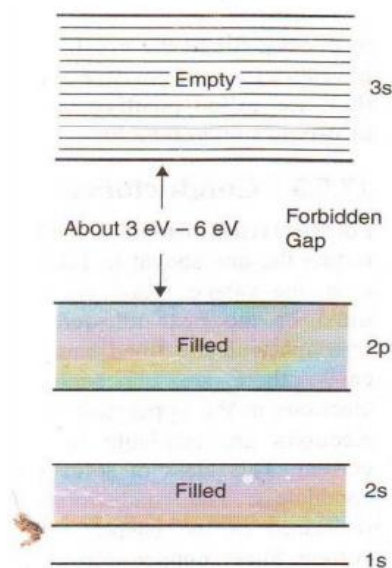
b) However, the width of a particular allowed band decreases with rise in value of P . As $P \rightarrow \infty$, the allowed energy bands compress into simple energy levels & thus result in a line spectrum.

11) With neat energy band diagrams, classify the materials into conductors, insulators and semiconductors.

Ans:

Insulators:

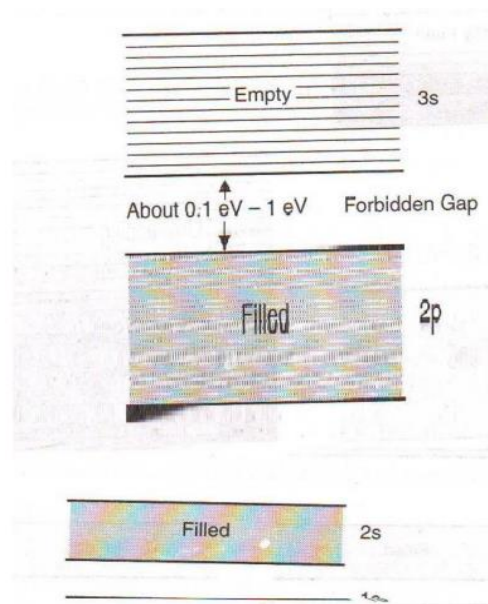
1. In case of insulators, the forbidden gap is very wide. Due to this fact electrons cannot jump from valence band to conduction band.
2. They have completely filled valence band and completely empty conduction band.
3. The resistivity of insulators is very high.
4. Insulators are bad conductors of electricity.



Valence and conduction bands of insulator separated by large band gap

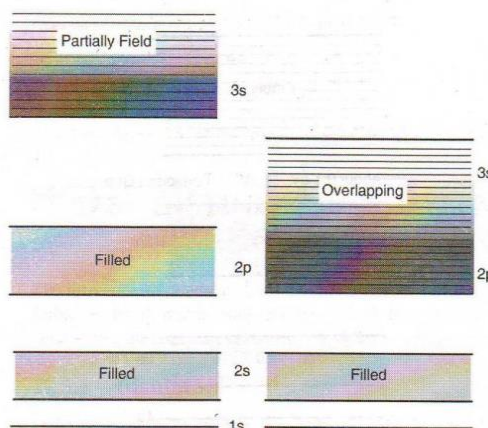
Semiconductors:

1. In semiconductors, the band gap is very small (0.7 eV for germanium and 1.1 eV for silicon).
2. At 0K, there are no electrons in the conduction band and the valence band is completely filled. As the temperature increases, electrons from the valence band jump into conduction band.
3. The resistivity varies from 10^{-14} to $10^7 \Omega \text{ meter}$.
4. They have electrical properties between those of insulators and conductors.



Conductors:

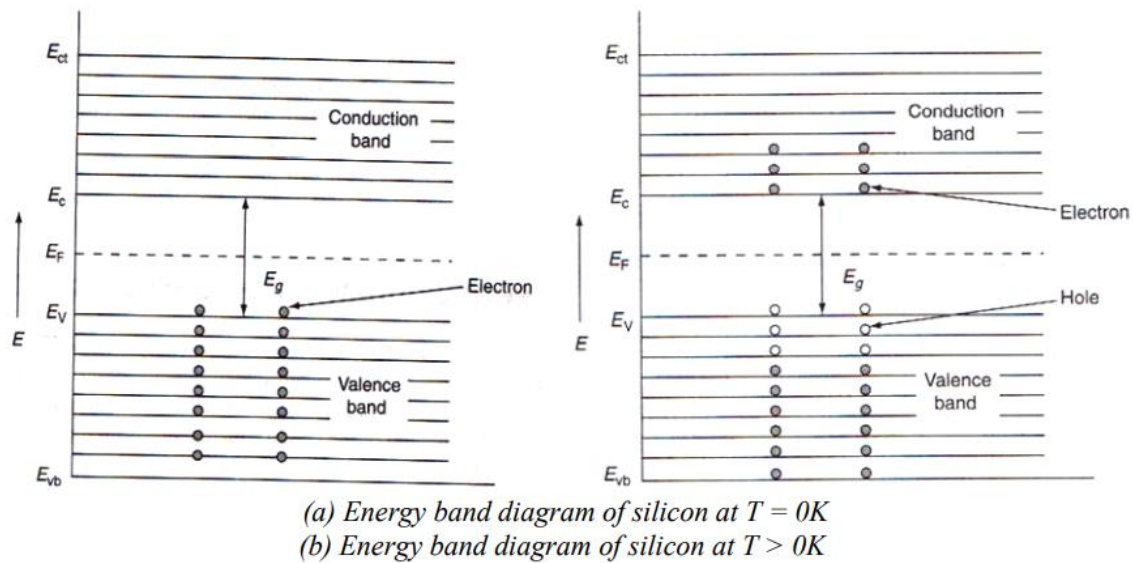
1. In case of conductors, there is no forbidden gap and the valence band and conduction band overlap each other.
2. Plenty of free electrons are available for electrical conduction.
3. They possess very low resistivity and very high conductivity values.
4. Metals like copper, iron etc. are best examples of conductors.



12) Derive an expression for the electron concentration in the conduction band of an intrinsic semiconductor.

Ans:

Definition: The no. of free electrons per unit volume of the conduction band of a given intrinsic semiconductor is called electron concentration, represented by 'n'.



Derivation: Let the no. of free electrons per unit volume of the semiconductor having energies E and $E + dE$ in CB is represented by $n(E) dE$. It is obtained by multiplying the density of energy states $Z_C(E) d(E)$ [No. of energy states per unit volume] and Fermi – Dirac distribution function for the Probability of occupation of electrons $F_C(E)$

Therefore $n(E) dE = [Z_C(E) d(E)] [F_C(E)] \rightarrow (1)$

Where $Z_v(E) d(E)$ = Density of energy states

$F_h(E)$ = Probability of occupation of electrons given by Fermi – Dirac function

The total no. of electrons in CB per unit volume between the energies E_C to E_{ct} is given by integrating equation (1) with limits E_C to E_{ct}

$$n = \int_{E_C}^{E_{ct}} n(E) dE \rightarrow (2)$$

But equation (2) can be written as

$$n = \int_{E_C}^{\infty} n(E) dE + \int_{\infty}^{E_{ct}} n(E) dE \rightarrow (3)$$

$$n = \int_{E_C}^{\infty} n(E) dE - \int_{E_{ct}}^{\infty} n(E) dE \rightarrow (4)$$

In equation (4) the second term vanishes (disappears).

Since, above E_{ct} electrons do not present. Hence equation (4) becomes

$$n = \int_{E_C}^{\infty} n(E) dE$$

$$n = \int_{E_C}^{\infty} [Z_C(E) dE] \times [F_C(E)] \rightarrow (5)$$

{Since from equation (1)}

But $[F_C(E)]$ is Fermi – Dirac distribution function;

$$F_C(E) = \frac{1}{1 + e^{\frac{E - E_F}{k_B T}}} \rightarrow (6)$$

Here $E > E_F$, i. e. $e^{\frac{E - E_F}{k_B T}} \gg 1$

Hence '1' can be neglected in equation (6)

$$F_C(E) = \frac{1}{e^{\frac{E-E_F}{K_B T}}}$$

$$F_C(E) = e^{\frac{E_F-E}{K_B T}} \rightarrow (7)$$

Also the density of electrons

$$Z_C(E) d(E) = \frac{\pi}{2} \left[\frac{8m_e^*}{h^2} \right]^{\frac{3}{2}} (E)^{\frac{1}{2}} dE \rightarrow (8)$$

Here $E > E_C$. Since E_C is the minimum energy state in CB. Hence equation (8) becomes

$$Z_C(E) d(E) = \frac{\pi}{2} \left[\frac{8m_e^*}{h^2} \right]^{\frac{3}{2}} (E - E_C)^{\frac{1}{2}} dE \rightarrow (9)$$

Substituting equations (7) & (9) in equation (5) we get

$$n = \int_{E_C}^{\infty} \frac{\pi}{2} \left[\frac{8m_e^*}{h^2} \right]^{\frac{3}{2}} (E - E_C)^{\frac{1}{2}} e^{\frac{E_F-E}{K_B T}} dE$$

$$n = \frac{\pi}{2} \left[\frac{8m_e^*}{h^2} \right]^{\frac{3}{2}} \int_{E_C}^{\infty} (E - E_C)^{\frac{1}{2}} e^{\frac{E_F-E}{K_B T}} dE \rightarrow (10)$$

Let $\varepsilon = E - E_C$

$d\varepsilon = dE$ { E_C is constant } . The limits are $\varepsilon = 0$ to $\varepsilon = \infty$

Hence equation (10) can be written as

$$n = \frac{\pi}{2} \left[\frac{8m_e^*}{h^2} \right]^{\frac{3}{2}} \int_{\varepsilon=0}^{\infty} (\varepsilon)^{\frac{1}{2}} e^{\frac{E_F-(\varepsilon+E_C)}{K_B T}} d\varepsilon$$

$$n = \frac{\pi}{2} \left[\frac{8m_e^*}{h^2} \right]^{\frac{3}{2}} e^{\frac{(E_F-E_C)}{K_B T}} \int_{\varepsilon=0}^{\infty} (\varepsilon)^{\frac{1}{2}} e^{\frac{-\varepsilon}{K_B T}} d\varepsilon \rightarrow (11)$$

In equation (11) But $\int_{\varepsilon=0}^{\infty} (\varepsilon)^{\frac{1}{2}} e^{\frac{-\varepsilon}{K_B T}} d\varepsilon = \frac{\sqrt{\pi}}{2} (k_B T)^{\frac{3}{2}} \rightarrow (12)$

Substituting (12) in (11) we get

$$n = \frac{\pi}{2} \left[\frac{8m_e^*}{h^2} \right]^{\frac{3}{2}} e^{\frac{(E_F-E_C)}{K_B T}} \frac{\sqrt{\pi}}{2} (k_B T)^{\frac{3}{2}}$$

$$n = \frac{1}{4} \left[\frac{8\pi m_e^* K_B T}{h^2} \right]^{\frac{3}{2}} e^{\frac{(E_F-E_C)}{K_B T}}$$

$$n = \frac{8}{4} \left[\frac{2\pi m_e^* K_B T}{h^2} \right]^{\frac{3}{2}} e^{\frac{(E_F-E_C)}{K_B T}}$$

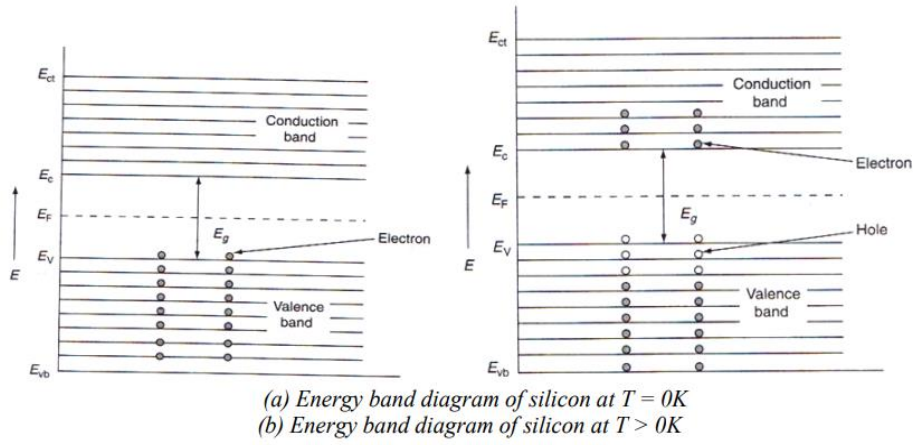
$$n = 2 \left[\frac{2\pi m_e^* K_B T}{h^2} \right]^{\frac{3}{2}} e^{\frac{-(E_C-E_F)}{K_B T}} \rightarrow (13)$$

{ Here $N_C = 2 \left[\frac{2\pi m_e^* K_B T}{h^2} \right]^{\frac{3}{2}}$ } Therefore $n = N_C e^{\frac{-(E_C-E_F)}{K_B T}}$

13) Infer an expression for hole concentration in the valence band of an intrinsic semiconductor. Ans:

Definition: The number of holes per unit volume of the valence band of a given intrinsic semiconductor is called hole concentration, represented by p .

Derivation: Let the number of holes per unit volume of the semiconductor having energies $E, E + dE$ in VB is represented by $p(E) dE$. It is obtained by multiplying the density of energy states $Z_V(E) d(E)$ [No. of energy states per unit volume] and Fermi – Dirac distribution function for the Probability of occupation of holes $F_h(E)$.



Therefore $p(E) dE = [Z_v(E) d(E)] [F_h(E)] \rightarrow (1)$

Where $Z_v(E) d(E)$ = Density of energy states.

$F_h(E)$ = Hole probability given by Fermi – Dirac function

The total no. of holes in VB per unit volume between the energies E_{vb} to E_v is given by integrating equation (1) with limits E_{vb} to E_v

$$p = \int_{E_{vb}}^{E_v} p(E) dE \rightarrow (2)$$

But equation (2) can be written as

$$p = \int_{E_{vb}}^{-\infty} p(E) dE + \int_{-\infty}^{E_v} p(E) dE \rightarrow (3)$$

$$p = - \int_{-\infty}^{E_{vb}} p(E) dE + \int_{-\infty}^{E_v} p(E) dE \rightarrow (4)$$

In equation (4) the first term vanishes (disappears).

Since, below E_{vb} holes do not present. Hence equation (4) becomes

$$p = \int_{-\infty}^{E_v} p(E) dE$$

$$p = \int_{-\infty}^{E_v} [Z_v(E) dE] \times F_h(E) \rightarrow (5)$$

{Since from equation (1)}

But $[F_h(E)]$ is Fermi – Dirac distribution function;

$$F_h(E) = 1 - F_c(E) \rightarrow (6)$$

$$= 1 - \frac{1}{1 + e^{\frac{E - E_F}{k_B T}}}$$

$$\text{Simplifying; } F_h(E) = \frac{e^{\frac{E - E_F}{k_B T}}}{1 + e^{\frac{E - E_F}{k_B T}}}$$

Divide by $e^{\frac{E - E_F}{k_B T}}$ we get

$$F_h(E) = \frac{1}{1 + \frac{e^{\frac{E - E_F}{k_B T}}}{e^{\frac{E - E_F}{k_B T}}}}$$

$$F_h(E) = \frac{1}{1 + e^{\frac{E_F - E}{k_B T}}} \rightarrow (6)$$

Here $E_F > E$, i.e. $e^{\frac{E_F - E}{k_B T}} \gg 1$

Hence '1' can be neglected in equation (6)

$$F_h(E) = \frac{1}{e^{\frac{E_F - E}{K_B T}}}$$

$$F_h(E) = e^{\frac{E - E_F}{K_B T}} \rightarrow (7)$$

Also the density of holes

$$Z_v(E) d(E) = \frac{\pi}{2} \left[\frac{8m_p^*}{h^2} \right]^{\frac{3}{2}} (E)^{\frac{1}{2}} dE \rightarrow (8)$$

Here $E < E_v$. Since E_v is the maximum energy state in VB. Hence equation (8) becomes

$$Z_v(E) d(E) = \frac{\pi}{2} \left[\frac{8m_p^*}{h^2} \right]^{\frac{3}{2}} (E_v - E)^{\frac{1}{2}} dE \rightarrow (9)$$

Substituting equations (7) & (9) in equation (5) we get

$$p = \int_{-\infty}^{E_v} \frac{\pi}{2} \left[\frac{8m_p^*}{h^2} \right]^{\frac{3}{2}} (E_v - E)^{\frac{1}{2}} e^{\frac{E - E_F}{K_B T}} dE$$

$$p = \frac{\pi}{2} \left[\frac{8m_p^*}{h^2} \right]^{\frac{3}{2}} \int_{-\infty}^{E_v} (E_v - E)^{\frac{1}{2}} e^{\frac{E - E_F}{K_B T}} dE \rightarrow (10)$$

Let $\varepsilon = E_v - E$

$d\varepsilon = -dE$ { E_v is constant } . The limits are $\varepsilon = \infty$ to $\varepsilon = 0$

Hence equation (10) can be written as

$$p = \frac{\pi}{2} \left[\frac{8m_p^*}{h^2} \right]^{\frac{3}{2}} \int_{\varepsilon=\infty}^0 (\varepsilon)^{\frac{1}{2}} e^{\frac{(E_v - \varepsilon) - E_F}{K_B T}} d\varepsilon$$

$$p = \frac{\pi}{2} \left[\frac{8m_p^*}{h^2} \right]^{\frac{3}{2}} e^{\frac{(E_v - E_F)}{K_B T}} \int_{\varepsilon=0}^{\infty} (\varepsilon)^{\frac{1}{2}} e^{\frac{-\varepsilon}{K_B T}} d\varepsilon \rightarrow (11)$$

In equation (11) But $\int_{\varepsilon=0}^{\infty} (\varepsilon)^{\frac{1}{2}} e^{\frac{-\varepsilon}{K_B T}} d\varepsilon = \frac{\sqrt{\pi}}{2} (K_B T)^{\frac{3}{2}} \rightarrow (12)$

Substituting (12) in (11) we get

$$p = \frac{\pi}{2} \left[\frac{8m_p^*}{h^2} \right]^{\frac{3}{2}} e^{\frac{(E_v - E_F)}{K_B T}} \frac{\sqrt{\pi}}{2} (K_B T)^{\frac{3}{2}}$$

$$p = \frac{1}{4} \left[\frac{8\pi m_p^* K_B T}{h^2} \right]^{\frac{3}{2}} e^{\frac{(E_v - E_F)}{K_B T}}$$

$$p = \frac{8}{4} \left[\frac{2\pi m_p^* K_B T}{h^2} \right]^{\frac{3}{2}} e^{\frac{(E_v - E_F)}{K_B T}}$$

$$p = 2 \left[\frac{2\pi m_p^* K_B T}{h^2} \right]^{\frac{3}{2}} e^{\frac{-(E_F - E_v)}{K_B T}} \rightarrow (13)$$

$$\{ \text{Here } N_v = 2 \left[\frac{2\pi m_p^* K_B T}{h^2} \right]^{\frac{3}{2}} \}$$

$$p = N_v e^{\frac{-(E_F - E_v)}{K_B T}}$$

Fermi energy level in intrinsic semiconductor

At temperature T K, the electron concentration 'n' is equal to hole concentration 'p' in intrinsic semiconductor.

i.e.

$$n = p$$

$$2 \left[\frac{2\pi m_e^* K_B T}{h^2} \right]^{\frac{3}{2}} e^{\frac{-(E_C - E_F)}{K_B T}} = 2 \left[\frac{2\pi m_p^* K_B T}{h^2} \right]^{\frac{3}{2}} e^{\frac{-(E_F - E_v)}{K_B T}}$$

On simplifying we get

$$(m_e^*)^{\frac{3}{2}} e^{\frac{-(E_C - E_F)}{K_B T}} = (m_p^*)^{\frac{3}{2}} e^{\frac{-(E_F - E_v)}{K_B T}}$$

$$\frac{e^{\frac{-(E_C - E_F)}{K_B T}}}{e^{\frac{-(E_F - E_V)}{K_B T}}} = \frac{(m_p^*)^3}{(m_e^*)^3}$$

$$e^{\frac{E_C + E_F + E_F - E_V}{K_B T}} = \left[\frac{m_p^*}{m_e^*} \right]^3$$

$$e^{\frac{2E_F}{K_B T} - \frac{(E_C - E_V)}{K_B T}} = \left[\frac{m_p^*}{m_e^*} \right]^3$$

Taking logarithms on both sides we get

$$\frac{2E_F}{K_B T} - \frac{(E_C - E_V)}{K_B T} = \frac{3}{2} \ln \left[\frac{m_p^*}{m_e^*} \right]$$

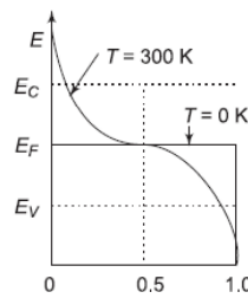
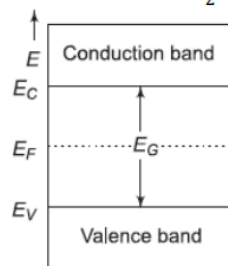
$$\frac{2E_F}{K_B T} = \frac{(E_C - E_V)}{K_B T} + \frac{3}{2} \ln \left[\frac{m_p^*}{m_e^*} \right]$$

$$E_F = \frac{(E_C + E_V)}{2} + \frac{3}{4} K_B T \ln \left[\frac{m_p^*}{m_e^*} \right] \quad \{ \text{At } T > 0K \}$$

$$\text{Let } T = 0K \quad E_F = \frac{(E_C + E_V)}{2}$$

This means E_F lies in the middle between $(E_C \& E_V)$ of the energy gap ' E_g '

As the temperature increases the electrons move from VB to CB. Also the Fermi level slightly rises upwards towards CB. Hence $E_F = \frac{(E_C + E_V)}{2} + \frac{3}{4} K_B T \ln \left[\frac{m_p^*}{m_e^*} \right]$. It is shown in the figure 6 below.

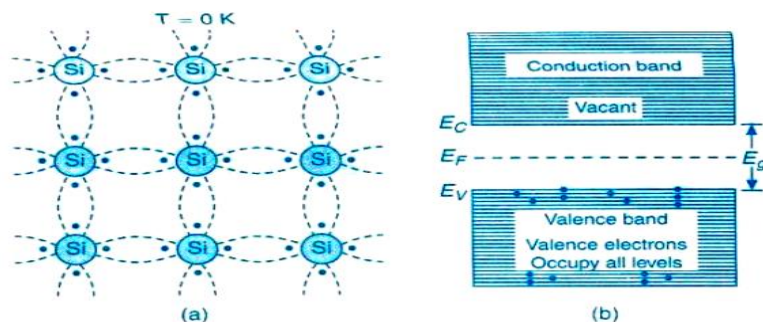


(a) Fermi level E_F at $T = 0K$ (b) Upward shift of E_F near E_C at $T > 0K$

14) Summarize an intrinsic semiconductor? Justify why an intrinsic semiconductor behaves as an insulator at 0K. Highlight 2D representations of the crystal of Silicon at $T = 0K$ and $T > 0K$.

Ans:

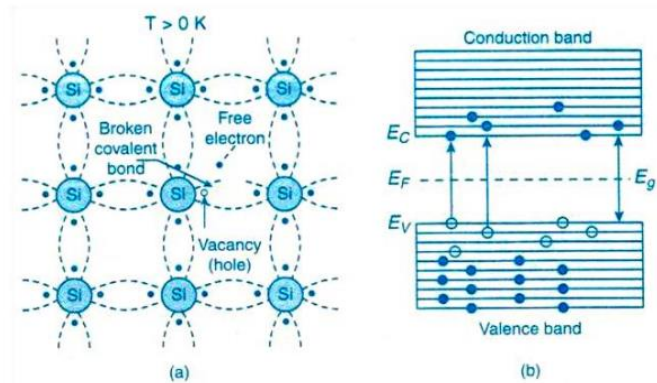
Definition: Pure germanium or silicon called an intrinsic semiconductor. Each atom possesses four valence electrons in outer most orbits. At $T = 0K$ a 2-D representation of the crystal of silicon & band diagram is shown in the figure.



Intrinsic silicon crystal at $T = 0K$ (a) 2-D representation of silicon crystal (c) Energy band diagram of intrinsic semiconductor

Explanation: At 0K, all the valence electrons of silicon atoms are in covalent bonds and their energies constitute a band of energies called valance band (VB). So, at 0K, VB is completely filled & conduction band (CB) is empty.

If we rise temperature ($T > 0K$), some of the electrons which are in covalent bonds break the bonds become free and move from VB to CB. The energy required should be greater than the energy gap of a semiconductor ($E > E_g$). The electron vacancy or deficiency created in VB is called holes. This is shown in the figure below.



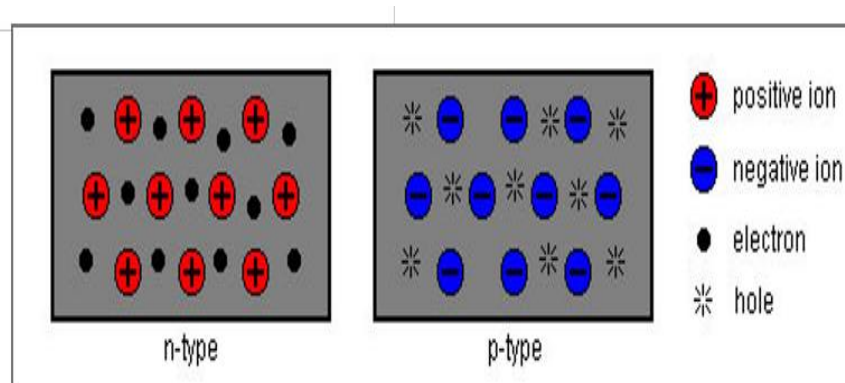
Silicon crystal at temperature above $0K$ (a) Due to thermal energy breaking of Covalent bonds take place (b) Energy band representation

15) Discuss about an extrinsic semiconductor? Distinguish between n-type and p-type semiconductors.

Ans:

Extrinsic semiconductor: The conductivity of an intrinsic semiconductor can be increased by adding small amounts of impurity atoms, such as IIIrd or Vth group atoms. The conductivity of silica is increased by 1000 times on adding 10 parts of boron per million part of silicon. The process of adding impurities is called doping and the impurity added is called dopant.

S.No	N-type semiconductors	P-type semiconductors
1.	N-type semiconductor is obtained by doping and intrinsic semiconductor with pentavalent impurity.	P-type semiconductor is obtained by doping and intrinsic semiconductor with trivalent impurity.
2.	Here electrons are majority carriers and holes are minority carriers.	Here holes are majority carriers and electrons are minority carriers
3.	It has donor energy levels very close to CB	It has acceptor energy levels very close to VB.
4.	When the temperature is increased, these semiconductors can easily donate an electron from donor energy level to the CB	When the temperature is increased, these semiconductors can easily accept an electron from VB to donor energy level .



16. Explain the significance of Fermi energy level. Mention its position in intrinsic and extrinsic semiconductors at 0 K.

Ans: It is important in determining the electrical and thermal properties of solids. The value of the Fermi level at absolute zero (-273.15°C) is called the Fermi energy and is a constant for each solid. The Fermi level changes as the solid is warmed and as electrons are added to or withdrawn from the solid.

At temperature T K, the electron concentration ' n ' is equal to hole concentration ' p ' in an intrinsic semiconductor i.e., $n = p$. Hence Fermi level lies in middle of Energy band gap.

In extrinsic semiconductor, the number of electrons in the conduction band and the number of holes in the valence band are not equal. Hence, the probability of occupation of energy levels in conduction band and valence band are not equal. Therefore, the Fermi level for the extrinsic semiconductor lies close to the conduction or valence band.

17) Develop the mathematical expression showing the variation of position of Fermi energy level in n-type semiconductor.

Ans: **PART B 8th Ans**

18) Derive mathematical expression showing the variation of position of Fermi energy level in p-type semiconductor.

Ans: **PART B 8th Ans**

19) Explain the classical free electron theory of metals. Also discuss its drawbacks.

Ans:

Postulates of Classical free electron theory:

1. The free electrons of a metal are having like the molecules of a gas in a container.
2. The free electrons are free to move in all possible directions about the whole volume of metals but in a particular direction, the resultant velocity of the free electron is zero.
3. In the absence of an electric field, the free electrons move in random directions making elastic collisions with no loss of energy.
4. When an external field is applied, the free electrons are slowly drifting towards the positive potential.
5. Since the electrons are assumed to be a perfect gas, they obey classical kinetic theory of gasses.
6. Classical free electrons in the metal obey Maxwell Boltzmann statistics.
7. The drift velocity ' v_d ' is the average velocity acquired by the free electron of a metal in a particular direction by the application of the electric field.
8. Relaxation time ' τ ' is defined as the time taken by the free electron to reach its equilibrium position from its disturbed position in the presence of electric field.
9. The collision time ' τ_c ' is the average time taken by a free electron between two successive collisions.
10. For an isotropic solid, like a metal $\tau = \tau_c$

Drawbacks of classical free electron theory:

- 1) It fails to explain the electric specific heat and the specific heat capacity of metals.
- 2) It fails to explain superconducting properties of metals.
- 3) It fails to explain new phenomena like photo-electric effect, Compton effect, Black body radiation, etc.
- 4) It fails to explain electrical conductivity of semiconductors or insulators.
- 5) The classical free electron model predicts the incorrect temperature dependence of σ .
- 6) It fails to give a correct mathematical expression for thermal conductivity.

7) Ferromagnetism could not be explained by this theory.

8) Susceptibility has greater theoretical value than the experimental value.

20) Discuss the assumptions made in quantum theory to overcome the drawbacks of free electron theory of metals.

Ans:

Assumptions (Postulates) of Quantum free electron theory:

1. In a metal the available free electrons are fully responsible for electrical conduction.

2. The electrons move in a constant potential inside the metal. They cannot come out from the metal surface have very high potential barrier.

3. Electrons have wave nature, the velocity and energy distribution for the electrons given by Fermi-Dirac distribution function.

4. The loss of energy due to interaction of the free electron with the other free electron

5. Electron's distributed various energy levels according to Pauli Exclusion Principle

PART C



1. For intrinsic semiconductor with band gap of $E_g=0.7\text{eV}$, calculate intrinsic carrier concentration at 37°C (300 K) if $m_e^* = m_p^* = m_0$ =rest mass of electron.

Given data :

$$E_g = 0.7 \text{ eV} = 0.7 \times 1.6 \times 10^{-19} \text{ Joules}$$

$$T = 37^\circ\text{C} = 300 \text{ K}$$

$$m_0 = 9.1 \times 10^{-31} \text{ kg}$$

$$K_B = 1.38 \times 10^{-23} \text{ J/K}$$

$$\text{Formula } n_i = 2 \left[\frac{2\pi m_0 K_B T}{h^2} \right]^{\frac{3}{2}} \exp \left(\frac{-E_g}{2k_B T} \right)$$

$$n_i = 2 \left[\frac{2 \times 3.14 \times 9.10 \times 10^{-31} \times 1.38 \times 10^{-23} \times 300}{(6.625 \times 10^{-34})^2} \right] \times \exp \left(\frac{-0.7 \times 1.6 \times 10^{-19}}{2 \times 1.38 \times 10^{-23} \times 300} \right)$$

$$n_i = 33.49 \times 10^{18} / \text{m}^3$$

2. The intrinsic carrier density at room temperature in Ge is $2.37 \times 10^{19} / \text{m}^3$. If the electron and hole mobilities are 0.38 and $0.18 \text{ m}^2\text{V}^{-1}\text{s}^{-1}$ respectively, calculate the resistivity.

Given data:

Intrinsic carrier concentration, $n_i = 2.37 \times 10^{19} / \text{m}^3$

Mobility of electron, $\mu_e = 0.38 \text{ m}^2\text{V}^{-1}\text{s}^{-1}$

Mobility of hole, $\mu_h = 0.18 \text{ m}^2\text{V}^{-1}\text{s}^{-1}$

Formula: Resistivity, $\rho =$

Conductivity, $\sigma_i = n_i e (\mu_e + \mu_h)$

$$= 2.37 \times 10^{19} \times 1.6 \times 10^{-19} (0.38 + 0.18) = 2.1235 \Omega^{-1}\text{m}^{-1}$$

Resistivity, $\rho = 1/\sigma = 1/2.1235 = 0.471 \Omega\text{m}$

3. What temperature would the Fermi level E_F is shifted by 15% from middle of forbidden gap (E_g)? Given $E_g = 1.2\text{eV}$, effective mass of holes is 5 times that of electrons.

Given data: Energy gap, $E_g = 1.2\text{ eV} = 1.2 \times 1.6 \times 10^{-19}\text{ J}$
At temperature T , $m_h^*/m_e^* = 5$

We know that the Fermi energy is

$$E_F = \frac{E_C + E_V}{2} + \frac{3kT}{4} \ln\left(\frac{m_h^*}{m_e^*}\right)$$

For 0 K, E_F lies between the band gap. Hence, E_F is given by

$$E_F = E_V + 0.6\text{ eV}$$

$$(E_v + 0.6) \text{ eV} = \frac{E_c + E_v}{2} (\because T = 0)$$

Let Fermi level shift by 15% at temperature T, i.e., 0.18 eV

i.e.,
$$(E_v + 0.78) \text{ eV} = \frac{E_c + E_v}{2} + \frac{3kT}{4} \ln 5$$

Subtracting the above equation, we get

$$0.18 \text{ eV} = \frac{3kT}{4} \ln 5 \quad E_F = \frac{E_c + E_v}{2} + \frac{3kT}{4} \log\left(\frac{m_h^*}{m_e^*}\right)$$

Simplying, we get

$$\begin{aligned} T &= \frac{4 \times 0.288 \times 10^{-19}}{3 \times 1.38 \times 10^{-23} \times \ln 5} \\ &= \frac{1.152 \times 10^{-19}}{6.663 \times 10^{-23}} = 1729 \text{ K} \end{aligned}$$

Therefore, the temperature at which Fermi level is shifted 10% is 116K.

4. For silicon $E_g=1.12\text{eV}$. Find E_F at 300K if $m_e^* = 0.12m_o$ & $m_p^* = 0.28 m_o$.

Given data:

$$E_g=1.12\text{eV}; T = 300\text{K}; m_e^* = 0.12m_o \text{ \& } m_p^* = 0.28 m_o$$

$$k_B = \frac{1.38 \times 10^{-23}}{1.61 \times 10^{-19}} \quad \text{eV}k^{-1} \quad [\text{Joules to eV}]$$

$$E_F = \frac{E_C + E_V}{2} + \frac{3}{4} K_B T \ln \left(\frac{m_p^*}{m_e^*} \right)$$

$$\frac{E_C + E_V}{2} = E_F \quad (\text{or}) \quad E_C - E_V = E_g$$

$$E_F = \frac{E_C + E_V}{2} \quad (\text{or}) \quad \frac{E_C + E_V}{2} = \frac{E_g}{2}$$

$$E_F = \frac{1.12}{2} + \frac{3}{4} \times \frac{1.38 \times 10^{-23} \times 300}{1.6 \times 10^{-19}} \ln \left(\frac{0.28m_o}{0.12m_o} \right); E_F = 0.576\text{eV}.$$

5. In a Hall experiment, a current of 25 A is passed through a long foil of silver which is 0.1 mm thick and 3 cm wide. If the magnetic field of flux density 0.14 Wb/m² is applied perpendicular to the foil, calculate the Hall voltage developed and estimate the mobility of electrons in silver. The conductivity of silver is $6.8 \times 10^7 \Omega^{-1}\text{m}^{-1}$ and the Hall coefficient is $-8.4 \times 10^{-11} \text{ m}^3/\text{coulomb}$.

Given data:

Current, $I = 25 \text{ A}$

Magnetic field, $B = 0.14 \text{ T}$

Thickness of the slab, $t = 0.1 \times 10^{-3} \text{ m}$

Hall coefficient, $R_H = -8.4 \times 10^{-11} \text{ m}^3/\text{coulomb}$

Conductivity of silver, $\sigma = 6.8 \times 10^7 \Omega^{-1}\text{m}^{-1}$

$$\text{Hall voltage, } V_H = \frac{R_H B I}{d} = \frac{8.4 \times 10^{-11} \times 0.14 \times 25}{0.1 \times 10^{-3}} = 29.4 \times 10^{-6} \text{ V} = 29.4 \mu\text{V}$$

$$\text{Mobility, } \mu = R_H \sigma = 8.4 \times 10^{-11} \times 6.8 \times 10^7 = 57.7 \times 10^{-4} \text{ m}^2 \text{V}^{-1}$$

6. Simulate Hall voltage developed across the width of the slab of a metallic slab carrying a current of 30A subjected to a magnetic field of 1.75T. The magnetic field is perpendicular to the plane of the slab and to the current. The thickness of the slab is 0.35cm. The concentration of free electrons in the metal is 6.55×10^{28} electrons/m³.

Given data:

Current, $I = 30 \text{ A}$

Magnetic field, $B = 1.75 \text{ T}$

Thickness of the slab, $t = 0.35 \times 10^{-2} \text{ m}$

Concentration of electrons, $n = 6.55 \times 10^{28} / \text{m}^3$

The Hall voltage is given by

$$V_H = \frac{R_H BI}{d}$$

$$V_H = \frac{BI}{ned}$$

Substituting the value, we get

$$\begin{aligned} V_H &= \frac{1.75 \times 30}{6.55 \times 10^{28} \times 1.6 \times 10^{-19} \times 0.35 \times 10^{-2}} \\ &= 14.313 \times 10^{-7} \text{ volt} \end{aligned}$$

The Hall voltage is 1.431×10^{-6} V $1.431 \mu\text{V}$.

7. Evaluate the value of carrier concentration, if the R_H of a specimen is $3.66 \times 10^{-4} \text{ m}^3 \text{ C}^{-1}$.

Given data: $R_H = 3.66 \times 10^{-4} \text{ m}^3 \text{ C}^{-1}$

Formula:- $R_H = \frac{1}{ne}$; Density 'n' = $\frac{1}{R_H e} = \frac{1}{3.66 \times 10^{-4} \times 1.6 \times 10^{-19}}$

$$n = 1.708 \times 10^{22} \text{ atoms/m}^3$$

8. Calculate the density of charge carriers of semiconductor, given the Hall efficient is $-6.85 \times 10^{-5} \text{ m}^3/\text{Coulomb}$.

Given data:

$$R_H = -6.85 \times 10^{-5} \text{ m}^3/\text{coulomb}$$

$$\text{Density of charge carriers, } n = \frac{1}{R_H e}$$

$$n = \frac{1}{6.85 \times 10^{-5} \times 1.6 \times 10^{-19}}$$

$$n = 9.124 \times 10^{22} \text{ atoms/m}^3$$

9. A silicon plate of thickness 1 mm, breadth 10 mm and length 100 mm is placed in a magnetic field of 0.5 wb/m^2 acting perpendicular to its thickness. If 10^{-2} A current flows along its length, obtain the Hall voltage developed if the Hall coefficient is $3.66 \times 10^{-4} \text{ m}^3/\text{coulomb}$.

Solution:

$$\text{Hall coefficient } R_H = \frac{V_H t}{IB}$$

$$\begin{aligned}\text{Hence, } V_H &= \frac{R_H IB}{t} \\ &= \frac{3.66 \times 10^{-4} \times 10^{-2} \times 0.5}{1 \times 10^{-3}} \\ &= 1.83 \times 10^{-3} \text{ V} = 1.83 \mu\text{V}\end{aligned}$$

10. For a semiconductor, the Hall coefficient is $-3.7 \times 10^{-6} \text{ m}^3/\text{coulomb}$ and electrical conductivity is $250 \text{ m}^{-1}\Omega^{-1}$. Calculate the density and mobility of charge carriers.

Given data:

$$R_H = -3.7 \times 10^{-6} \text{ m}^3/\text{coulomb}$$

$$\sigma = 250 \text{ m}^{-1}\Omega^{-1}$$

Density of charge carriers 'n' = ?

Mobility μ = ?

We know Hall coefficient $R_H = \frac{1}{ne}$

$$n = \frac{1}{R_H e} = \frac{1}{3.7 \times 10^{-6} \times 1.6 \times 10^{-19}} = 1.689 \times 10^{22} \text{ atoms/m}^3$$

Conductivity $\sigma = ne\mu$

$$\text{Mobility } \mu = \frac{\sigma}{ne} = \frac{250 \times 1}{1.689 \times 10^{22} \times 1.6 \times 10^{-19}} = 92.5 \times 10^{-3} \text{ m}^2 \text{V}^{-1} \text{s}^{-1}$$