JULY 2021

P&S MODULE 4 SOLUTIONS

HANDWRITTEN

PRANAV | PRANEETH | VISHAL

UJJWAL | VISHNU

PART-A

1. List out the different types of sampling methods.

A: Sample with replacement: A finite population, which is sampled with replacement, can theoritically be considered infinite since samples of any size can be drawn without exhausting the population.

Sample without replacement: If each ~~number~~ member cannot be choosen more than once is called sample without replacement.

2. State the definition of population? Give an example.

Population is the set or collection or total number of the objects, animate or inanimate, actual or hypothetical under study. Thus, mainly population consists of set of numbers or observations etc, which are of interest.

Example:
The population of a country includes all people currently within that country.

3. State the definition of sample? Give an example.

A finite sub-set of the population is known as Sample. Size of the sample is denoted by n.

Example: 300 Netflix customers.

where, $n = 300$.

4. State the definitions of parameter and statistic.

A. Parameters: Statistical measures or constants obtained from the population are known as population parameters or simply parameters.

Statistic: A number describing a sample is called statistic (eg. sample mean).

5. Find the value of correction factor if $n=5$ and $N=200$

A. Given,

$n = 5$

$N = 200$

$$CF = \sqrt{\frac{N-n}{N-1}}$$

$$= \sqrt{\frac{195}{199}}$$

$$= 0.98.$$

6. State the definition of standard error of a statistic.

The standard error of a statistic is the standard deviation of its sampling distribution.

7 Find out How many different sample of size $n=2$ can be chosen from a finite population of size 25.

A. Given,

$N = 25$

$n = 2.$

$$a = {}^{N}C_{n}$$

$$a = {}^{25}C_{2}$$

$$a = 300.$$

8. Find the standard error and probable error of sample size 14 and ~~coefficient 0.74~~ correlation coefficient 0.74

A. $n = 14$

$r = 0.74$

$$SE = \frac{1 - r^2}{\sqrt{n}}$$

$$= \frac{1 - (0.74)^2}{\sqrt{14}}$$

$S.E = 0.8536$

$P.E = 0.6745 \times S.E$

$P.E = 0.5757$

9. If the population consists of four members 1, 5, 6, 8 Find How many samples of size three can be drawn with replacement?

Given population :- {1, 3, 6, 8}

Total no. of samples of size 3 with replacement

$N^n = 4^3 = 64$

10. The mean weekly wages of workers are with standard deviation of rupees 4. A sample 625 is selected. Find the standard error of the mean.

A.

$n = 625$

$\sigma = 4$

$$SEM = \frac{\sigma}{\sqrt{n}}$$

$$= \frac{4}{\sqrt{625}}$$

$$= \frac{4}{25} = 0.16$$

11. List out the difference between large and small samples with examples

large sampling: If $n \geq 30$ the sampling is said to be large sampling.

Small sampling: If $n < 30$ then the sampling is said to be small sampling.

12. In a manufacturing company out of 100 goods 25 are top quality. Find sample proportion.

$X = 25$
$n = 100$

$$\text{Sample proportion} = \frac{X}{n} = \frac{25}{100}$$
$$= 0.25.$$

13. Find the confidence interval for single mean if mean of sample size of 400 is 40, standing deviation is 10.

Given,

$\bar{x} = 40, \quad s = 10, \quad n = 400$

$$\text{confidence interval} = \left(\bar{x} - z_\alpha \frac{s}{\sqrt{n}}, \bar{x} + z_\alpha \frac{s}{\sqrt{n}}\right)$$
$$= (40 - 0.98, 40 + 0.98)$$
$$= (39.02, 40.98$$

14. Find the confidence interval for a single proportion if 18 goods are defective from a sample of 200 goods.

14) $$\text{Confidence Interval} = \left(\bar{x} - Z_\alpha \frac{\sigma}{\sqrt{n}}, \bar{x} + Z_\alpha \frac{\sigma}{\sqrt{n}}\right]$$

$n = 200 \; ; \; p = \frac{18}{200} = 0.09 \; ; \; q = 0.91$

$\text{Mean } (\sigma) = np = 18$

$SD\ (\sqrt{n}) = \sqrt{npq} = \sqrt{16.37} = 4.047$

$C.I = [18 - 0.56, 18 + 0.56)$

$= [17.44, 18.56]$

15. State the Formula of standard error of Sample proportion.

$$\sqrt{\frac{P(1-P)}{n}}$$

where,

P = proportion of successes

n = sample size

16. In a manufacturing company out of 200goods 80 were faulty. Find the sample proportion

A- $x = 80$

$n = 200$

$$\text{Sample proportion} = \frac{x}{n}$$

$$= \frac{80}{200}$$

$$= 0.4$$

17. Find the sample proportion in one day production of 400 articles only 50 are top quality.

$x = 50$

$n = 400$

$$\text{Sample proportion} = \frac{x}{n}$$

$$= \frac{50}{400}$$

$$= 0.125$$

18. Stabe the formula for difference of means in large samples.

$$Z = \frac{\bar{x_1} - \bar{x_2}}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

19. State the formula of test statistic for difference of proportions in large sample.

$$\frac{P_1 - P_2}{\sqrt{Pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = z$$

20. Find the confidence interval for mean if mean of sample size of 144 is 150, standard deviation is 2.

Given, $\bar{x} = 150$, $s = 2$, $n = 144$

confidence internal $= \left(x - 2 \times \frac{s}{\sqrt{n}}, \bar{x} + 2 \times \frac{s}{\sqrt{n}}\right)$

$= (150 - 0.32, 150 + 0.32)$

$= (149.68, 150.32.$

P&S Module 4 Part B Solutions

Questions and Answers are seperate here.

PART-B (LONG ANSWER QUESTIONS)				
1	A population consists of ranks of five students based on their performance in a physical test namely 2,3,6,8 and 11. Consider all possible samples of size two which can be drawn with replacement from This population. Calculate The mean of the population. The standard deviation of the population. The mean of the sampling distribution of means. The standard deviation of the sampling distribution of means.	Apply	Learner to recall the concept of sampling distribution of means and explain the parameters related to sampling distribution of means under with replacement and hence use them to calculate the required values.	CO 7
2	A population consists of ranks of six students based on their performance in a physical test namely 5, 10, 14, 18, 13, 24. Consider all possible samples of size two which can be drawn without replacement from This population. Calculate The mean of the population. The standard deviation of the population. The mean of the sampling distribution of means. The standard deviation of the sampling distribution of means.	Apply	Learner to recall the concept of sampling distribution of means and explain the parameters related to sampling distribution of means under without replacement and hence use them to calculate the required values.	CO 7
3	A population consists of ranks of six students based on their performance in a physical test namely 4, 8, 12, 16, 20, 24. Consider all possible samples of size two which can be drawn without replacement from This population. Calculate The mean of the population. The standard deviation of the population. The mean of the sampling distribution of means. he standard deviation of the sampling distribution of means.	Apply	Learner to recall the concept of sampling distribution of means and explain the parameters related to sampling distribution of means under without replacement and hence use them to calculate the required values.	CO 7
4	A population consists of ranks of six students based on their performance in a physical test. Samples of size 2 are taken from the population 1, 2, 3, 4, 5, 6. Which can be drawn with replacement? Calculate The mean of the population. The standard deviation of the population. The mean of the sampling distribution of means. The standard deviation of the sampling distribution of means.	Apply	Learner to recall the concept of sampling distribution of means and explain the parameters related to sampling distribution of means under with replacement and hence use them to calculate the required values.	CO 7
5	A population consists of ranks of five students based on their performance in a physical test. Samples of size 2 are taken from the population 3, 6, 9, 15 27. Which can be drawn with replacement? Calculate i) The mean of the population ii) The standard deviation of the population iii) The mean of the sampling distribution of means iv) The standard deviation of the sampling distribution of means.	Apply	Learner to recall the concept of sampling distribution of means and explain the parameters related to sampling distribution of means under with replacement and hence use them to calculate the required values.	CO 7
6	A population consists of ranks of five students based on their performance in a physical test. If the population is 3, 6, 9, 15, 27. List all possible samples of size 3 that can be taken without replacement from the finite population. Calculate the mean of each of the sampling distribution of means. Calculate the standard deviation of sampling distribution of means.	Apply	Learner to recall the concept of sampling distribution of means and explain the parameters related to sampling distribution of means under without replacement and hence use them to calculate the required values.	CO 7
7	The mean height of students in a college is 155 cm and standard deviation is 15. Estimate the probability that the mean height of 36 students is less than 157 cm.	Apply	Learner to recall the statement of central limit theorem and Relate it to the normality and calculate the required probabilities by using the concept of central limit theorem.	CO 5
8	A random sample of size 100 is taken from an infinite population having the mean 76 and the variance 256. Estimate the probability that $\bar{x}$ will be between 75 and 78.	Apply	Learner to recall the statement of central limit theorem and Relate it to the normality and calculate the required probabilities by using the concept of central limit theorem	CO 5

<table>
<tr><td>9</td><td>The mean of certain normal population is equal to the standard error of the mean of the samples of 64 from that distribution. Calculate the probability that the mean of the sample size 36 will be negative.</td><td>Apply</td><td>Learner to recall the statement of central limit theorem and Relate it to the normality and calculate the required probabilities by using the concept of central limit theorem</td><td>CO 5</td></tr>
<tr><td>10</td><td>A random sample of size 64 is taken from a normal population with $\mu = 51.4$ and $\sigma = 68$. Estimate the probability that the mean of the sample will i) exceed 52.9 ii) fall between 50.5 and 52.3 iii) be less than 50.6.</td><td>Apply</td><td>Learner to recall the statement of central limit theorem and Relate it to the normality and calculate the required probabilities by using the concept of central limit theorem</td><td>CO 5</td></tr>
<tr><td>11</td><td>A sample of 400 items is taken from a population whose standard deviation is 10. The mean of sample is 40. Examine whether the sample has come from a population with mean 38 also calculate 95% confidence interval for the population.</td><td>Apply</td><td>Learner to recall the procedure of testing of hypothesis and select the suitable test statistic formula and compare the calculated test statistic value with the tabulated value to draw the inference.</td><td>CO 8,11</td></tr>
<tr><td>12</td><td>The means of two large samples of sizes 1000 and 2000 members are 67.5 inches and 68.0 inches respectively. Can the samples be regarded as drawn from the same population of S.D 2.5 inches?</td><td>Apply</td><td>Learner to recall the procedure of testing of hypothesis and select the suitable test statistic formula and compare the calculated test statistic value with the tabulated value to draw the inference.</td><td>CO 8, CO 11</td></tr>
<tr><td>13</td><td>An ambulance service claims that it takes on the average 8.9 minutes to reach its destination in emergency calls. To check on This claim the agency which issues license to Ambulance service has then timed on fifty emergency calls getting a mean of 9.2 minutes with 1.6 minutes. Examine the claim at 5% LOS</td><td>Apply</td><td>Learner to recall the procedure of testing of hypothesis and select the suitable test statistic formula and compare the calculated test statistic value with the tabulated value to draw the inference.</td><td>CO 8, CO 11</td></tr>
<tr><td>14</td><td>According to norms established for a mechanical aptitude test, the persons who are 18 years have an average weight of 73.2 with S.D 8.6 if 40 randomly selected persons have average 76.7 Examine the truth value of the hypothesis $H_0 : \mu = 73.2$ against alternative hypothesis: $\mu > 73.2$.</td><td>Apply</td><td>Learner to recall the procedure of testing of hypothesis and select the suitable test statistic formula and compare the calculated test statistic value with the tabulated value to draw the inference.</td><td>CO 8, CO 11</td></tr>
<tr><td>15</td><td>A sample of 100 electric bulbs produced by manufacturer 'A' showed a mean life time of 1190 hours and s .d. of 90 hours A sample of 75 bulbs produced by manufacturer 'B' Showed a mean life time of 1230 hours with s.d. of 120 hrs. Examine whether there is any difference between the mean life times of the two brands at a significance level of 0.05.</td><td>Apply</td><td>Learner to recall the procedure of testing of hypothesis and select the suitable test statistic formula and compare the calculated test statistic value with the tabulated value to draw the inference.</td><td>CO 8, CO 11</td></tr>
<tr><td>16</td><td>On the basis of their total scores, 200 candidates of a civil service examination are divided into two groups; the first group is 30% and the remaining 70%. Consider the first question of the examination among the first group, 40 had the correct answer. Whereas among the second group, 80 had the correct answer. On the basis of these results, can one conclude that the first question is not good at discriminating ability of the type being examined here.</td><td>Apply</td><td>Learner to recall the procedure of testing of hypothesis and select the suitable test statistic formula and compare the calculated test statistic value with the tabulated value to draw the inference.</td><td>CO 8</td></tr>
<tr><td>17</td><td>A cigarette manufacturing firm claims that brand A line of cigarettes outsells its brand B by 8%. if it is found that 42 out of a sample of 200 smokers prefer brand A and 18 out of another sample of 100 smokers prefer brand B. Examine whether 8% difference is a valid claim.</td><td>Apply</td><td>Learner to recall the procedure of testing of hypothesis and select the suitable test statistic formula and compare the calculated test statistic value with the tabulated value to draw the inference.</td><td>CO 8</td></tr>
<tr><td>18</td><td>If 48 out of 400 persons in rural area possessed 'cell' phones while 120 out of 500 in urban area. Can it be accepted that the proportion of 'cell' phones in the rural area and Urban area is same or not. Use 5% of level of significance.</td><td>Apply</td><td>Learner to recall the procedure of testing of hypothesis and select the suitable test statistic formula and compare the calculated test statistic value with the tabulated value to draw the inference.</td><td>CO 8</td></tr>
<tr><td>19</td><td>Samples of students were drawn from two universities and from their weights in kilograms mean and S.D are calculated and shown below make a large sample Examine the significance of difference between means.
<table>
<tr><th></th><th>Mean</th><th>Standard Deviation</th><th>Sample Size</th></tr>
<tr><td>University A</td><td>55</td><td>10</td><td>400</td></tr>
<tr><td>University B</td><td>57</td><td>15</td><td>100</td></tr>
</table></td><td>Apply</td><td>Learner to recall the procedure of testing of hypothesis and select the suitable test statistic formula and compare the calculated test statistic value with the tabulated value to draw the inference.</td><td>CO 8, CO 11</td></tr>
<tr><td>20</td><td>In a big city 325 men out of 600 men were found to be smokers. Does This information support the conclusion that the majority of men in This city are smokers?</td><td>Apply</td><td>Learner to recall the procedure of testing of hypothesis and select the suitable test statistic formula and compare the calculated test statistic value with the tabulated value to draw the inference.</td><td>CO 8</td></tr>
</table>

1) $N=5$, $n=2$

(i) Mean of the population

$$\mu = \sum \frac{x_i}{N} = \frac{2+3+6+8+11}{5} = \frac{30}{5} = 6$$

(ii) Variance of the population

$$\sigma^2 = \sum \frac{(x_i-\bar{x})^2}{N} = \frac{(2-6)^2+(3-6)^2+(6-6)^2+(8-6)^2+(11-6)^2}{5}$$

$$= 10.8$$

$$\sigma = \sqrt{10.8} = 3.29$$

Sampling with replacement (infinite population):

The total no of samples with replacement is

$$N^n = 5^2 = 25$$

There are 25 samples that can be drawn

$$\begin{bmatrix} (2,2) & (2,3) & (2,6) & (2,8) & (2,11) \\ (3,2) & (3,3) & (3,6) & (3,8) & (3,11) \\ (6,2) & (6,3) & (6,6) & (6,8) & (6,11) \\ (8,2) & (8,3) & (8,6) & (8,8) & (8,11) \\ (11,2) & (11,3) & (11,6) & (11,8) & (11,11) \end{bmatrix}$$

The sample means are

$$\begin{bmatrix} 2 & 2.5 & 4 & 5 & 6.5 \\ 2.5 & 3 & 4.5 & 5.5 & 7 \\ 4 & 4.5 & 6 & 7 & 8.5 \\ 5 & 5.5 & 7 & 8 & 9.5 \\ 6.5 & 7 & 8.5 & 9.5 & 11 \end{bmatrix}$$

(iii) Mean of the sampling distribution of means is

$$\mu_{\bar{x}} = \frac{2+2.5+4+5+6.5+\cdots+11}{25}$$

$$= 6$$

(iv) standard deviation of the sampling distribution of means

$$\sigma_{\bar{x}}^2 = \frac{(2-6)^2+(2.5-6)^2+\cdots+(11-6)^2}{25}$$

$$= 5.40$$

$$\sigma_{\bar{x}} = \sqrt{5.40} = \boxed{2.32}$$

2) $N=6$, $n=2$

(i) Mean of the population

$$\mu = \frac{\sum x_i}{N} = \frac{5+10+14+18+13+24}{6} = 14$$

(ii) Variance of the population

$$\sigma^2 = \frac{\sum(x_i-\bar{x})^2}{N} = \frac{(5-14)^2+(10-14)^2+(14-14)^2+(18-14)^2+(13-14)^2+(24-14)^2}{6}$$

$$= 35.66$$

$$\sigma = \sqrt{35.66} = 5.97$$

Sampling without replacement (finite population):

The total numbers of samples without replacement is $^{N}C_n = {^6C_2} = 15$

There are 15 Samples that can be drawn

$$\begin{bmatrix} (5,10) & (5,14) & (5,18) & (5,13) & (5,24) \\ (10,14) & (10,18) & (10,13) & (10,24) & \\ (14,18) & (14,13) & (14,24) & & \\ (18,13) & (18,24) & & & \\ (13,24) & & & & \end{bmatrix}$$

The sample means are

$$\begin{bmatrix} 7.5 & 9.5 & 11.5 & 9 & 14.5 \\ 12 & 14 & 11.5 & 17 & \\ 16 & 13.5 & 19 & & \\ 15.5 & 21 & & & \\ 18.5 & & & & \end{bmatrix}$$

The mean of the sampling distribution of means is

$$\mu_{\bar{x}} = \frac{7.5 + 9.5 + 11.5 + 9 + \cdots + 18.5}{15}$$

$$= \frac{210}{15} = 14$$

The standard deviation of the sampling distribution of means

$\sigma^2_{\bar{x}} = 14.266$

$\sigma_{\bar{x}} = 3.77$

3) Same method as 2^{nd} q

$\mu = 14$ $\quad \sigma = 3.29$ $\quad \mu_{\bar{x}} = 14$ $\quad \sigma_{\bar{x}} = 4.32$

$\sigma^2 = 46.67$ $\quad {}^N C_n = {}^6 C_2 = 15$ $\quad \sigma^2_{\bar{x}} = 18.67$

4) ~~Same as 2nd and 3rd q~~ [with replacement]

Same as 1st q

Total no of samples with replacement is

$6^2 = 36$

Remaining process is same

5) Same as 1st q

Total no of samples with replacement is

$5^2 = 25$

Remaining process is same

6) Same as 2nd q

$N=5, n=3$

Total no of samples without replacement is $5C_3 = 10$

Remaining process is same

7) μ = Mean of the population = Mean height of students = 155 cm

$\sigma = 15$

$n = 36$

$\bar{x} = 157$

$$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{157-155}{\frac{15}{\sqrt{36}}} = \frac{2}{\frac{15}{6}}$$

$$= \frac{12}{15} = \boxed{0.8}$$

$\therefore P(\bar{x} \le 157) = P(z < 0.8) = 0.5 + P(0 \le z \le 0.8)$

$= 0.5 + 0.2881 = 0.7881$

Thus the probability that the mean height of 36 students is less than

$157 = 0.7881$

8) ~~[illegible]~~ $n=100$

$\mu=76$ $\sigma=16$

$$z=\frac{(\bar{x}-76)10}{16}$$

Probability lies between 75 and 78

$P(75\le x\le 78)= P(-0.625\le x\le 1.25)$

$=|A(0.625)+A(1.25)|$

$=0.2324+0.3944$

$=0.6268$

9) Question incomplete

10) $n=64$

$\mu=51.4$

$\sigma=6.8$

(i) exceed 52.9 $=P(\bar{x}\ge 52.9)$

$$z=\frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}=\frac{52.9-51.4}{\frac{6.8}{8}}=1.76$$

$P(\bar{x}>52.9)=P(z>1.76)$

$=0.5-P(0<z<1.76)$

$=0.5-0.4608=0.0392$

(ii) $P(\bar{x}$ fall between 50.5 and 52.3)

$P(50.5<\bar{x}<52.3)=P(\bar{x}_1<\bar{x}<\bar{x}_2)$

$$z_1 = \frac{\bar{x}_1 - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{50.5 - 51.4}{0.85} = -1.06$$

$$z_2 = \frac{\bar{x}_2 - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{52.3 - 51.4}{0.85} = 1.06$$

$P(50.5 < \bar{x} < 52.3) = P(-1.06 < z < 1.06)$

$= 2[0.3554) = \underline{0.7108}$

(iii) $P(\bar{x}$ less than $50.6) = P(\bar{x} < 50.6)$

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{50.6 - 51.4}{\frac{6.8}{\sqrt{64}}} = -0.94$$

$P(z < -0.94) = 0.5 - 0.3264$

$= \underline{0.1736}$

11) $n = 400, \bar{x} = 40, \mu = 38, \sigma = 10$

Null Hypothesis $H_0: \mu = 38$

Alternate Hypothesis $H_1: \mu \neq 38$ (Two-Tailed Test)

level of significance $\alpha = 0.05$ $(z_\alpha = 1.96)$

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{38 - 40}{\frac{10}{20}} = \boxed{-4}$$

Conclusion: Since $z > z_\alpha$, we reject H_0

$\therefore$ 95% confidence interval is $\left(\bar{x} - 1.96 \cdot \frac{\sigma}{\sqrt{n}}, \bar{x} + 1.96 \cdot \frac{\sigma}{\sqrt{n}}\right)$

$$\left(40 - \frac{1.96(10)}{20},\ 40 + \frac{1.96(10)}{20}\right)$$

$$(39.02,\ 40.98)$$

12) Let μ_1 and μ_2 be the means of two populations

$n_1 = 1000$, $n_2 = 2000$, $\bar{x}_1 = 67.5$ inches, $\bar{x}_2 = 68$ inches

Population S.D, $\sigma = 2.5$ inches

Null Hypothesis H_0: The samples have been drawn from the same Population of S.D 2.5 inches

i.e $H_0: \mu_1 = \mu_2$

Alternate Hypothesis H_1: $\mu_1 \neq \mu_2$ (Two-Tailed Test)

Level of significance $\alpha = 0.05$ ($z_\alpha = 1.96$)

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sigma\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{67.5 - 68}{\sqrt{(2.5)^2\left(\frac{1}{1000} + \frac{1}{2000}\right)}} = \frac{-0.5}{0.0968} = -5.16$$

Conclusion: Since $z > z_\alpha$ we reject H_0. Hence we conclude that the samples are not drawn from the same population of SD 2.5 inches

13) $\bar{x} = 9.2$, $\mu = 8.9$, $\sigma = 1.6$, $n = 50$

Null Hypothesis H_0: $\mu_1 = \mu_2$

Alternate Hypothesis (H_1): $\mu_1 \neq \mu_2$

$$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{9.2 - 8.9}{\frac{1.6}{\sqrt{50}}} = 1.325$$

$\alpha = 5\%$ or 0.05 so $Z_\alpha = 1.96$

$Z < Z_\alpha$ so null hypothesis is accepted

$\therefore$ The claim is acceptable at 5% LOS

14) $\mu = 73.2$, $\sigma = 8.6$ $n = 40$ $\bar{x} = 76.7$

let $\alpha = 0.01$ so $Z_\alpha = 2.33$

Null Hypothesis H_0 : $\mu = 73.2$

Alternative Hypothesis H_1: $\mu > 73.2$

$$Z = \frac{76.7 - 73.2}{\frac{8.6}{\sqrt{40}}} = 2.57$$

Since $2.57 > 2.33$ we reject the null Hypothesis

16) $n_1 = 60$, $n_2 = 140$, $x_1 = 40$, $x_2 = 80$

$P_1 = \frac{x_1}{n_1} = \frac{2}{3} = 0.667$ $\qquad P_2 = \frac{x_2}{n_2} = \frac{80}{140} = 0.571$

$$P = \frac{n_1 P_1 + n_2 P_2}{n_1 + n_2} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{120}{200} = 0.6$$

$q = 1 - p = 1 - 0.6 = 0.4$

Null Hypothesis H_0: $P_1 = P_2$

Alternative Hypothesis. H_1: $P_1 \neq P_2$

level of significance $\alpha = 0.05$ (assumed) $\quad z_\alpha = 1.96$

$$z = \frac{P_1 - P_2}{\sqrt{pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$= \frac{0.667 - 0.571}{\sqrt{(0.6)(0.4)\left(\frac{1}{60} + \frac{1}{40}\right)}} = \frac{0.096}{0.1} = 0.96$$

$z < z_\alpha$ so we accept the null Hypothesis.

$\therefore$ The first question is good enough in discriminating the ability of the students of both grps.

17) $n_1 = 200 \qquad x_1 = 42$

$n_2 = 100 \qquad x_2 = 18$

$p_1 = \frac{x_1}{n_1} = 0.21 \qquad p_2 = \frac{x_2}{n_2} = 0.18$

$P_1 - P_2 = 8\% = 0.08$

Null Hypothesis $H_0 : P_1 - P_2 = 0.08$

Alternate Hypothesis $H_1 : P_1 - P_2 \neq 0.08$ [Two tailed test]

$\alpha = 0.05$ [$z_\alpha = 1.96$]

$$z = \frac{(p_1 - p_2) - (P_1 - P_2)}{\sqrt{pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$p = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{42 + 18}{200 + 100} = 0.2$

$q = 0.8$

$$= \frac{(0.21 - 0.18) - 0.08}{\sqrt{(0.2)(0.8)\left(\frac{1}{200} + \frac{1}{100}\right)}}$$

Scanned with CamScanner

$= -1.02$

$z < z_\alpha$ so we accept null Hypothesis H_0

Hence we conclude that 8% difference in the sale of two brands is a valid claim.

19) $n_1 = 400$, $n_2 = 100$, $\bar{x}_1 = 55$, $\bar{x}_2 = 57$, $S_1 = 10$, $S_2 = 15$

Null Hypothesis (H_0): $\bar{x}_1 = \bar{x}_2$

Alternative Hypothesis (H_1): $\bar{x}_1 \neq \bar{x}_2$

$\alpha = 0.05 \qquad Z_\alpha = 1.96$

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} = \frac{55-57}{\sqrt{\frac{100}{400} + \frac{225}{100}}} = -1.26$$

$|Z| = 1.26$

$|Z| < Z_\alpha$

$\therefore$ We accept the Null Hypothesis

15) **Solution.** We have

$$n_1 = 100,\ \bar{x}_1 = 1190,\ \sigma_1 = 90$$
$$n_2 = 75,\ \bar{x}_2 = 1230,\ \sigma_2 = 120.$$

Therefore, the test statistics is

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{1190 - 1230}{\sqrt{\frac{90^2}{100} + \frac{120^2}{75}}}$$
$$= -\frac{40}{\sqrt{81 + 192}} = -\frac{40}{16.523} = -2.42.$$

Since $|Z| = 2.42 > 1.96$, there is a difference between the mean life time of the two brands at a significant level of 5%.

P&S Module 4 Part C Solutions

1	Let S= {1, 5, 6, 8}, Calculate the probability distribution of the sample mean for random sample of size 2 drawn without replacement. Calculate i) The mean of the population. ii) The standard deviation of the population. iii) The mean of the sampling distribution of means. iv) The standard deviation of the sampling distribution of means.

1) Given,

$S = \{1, 5, 6, 8\} \Rightarrow N = 4$

The probability distribution:-

sample	(1, 5)	(1, 6)	(1, 8)	(5, 6)	(5, 8)	(6, 8)
	3	3.5	4.5	5.5	6.5	7

Given,

$n = 2$

$\therefore$ Total no. of samples $= {}^{N}C_{n} = {}^{4}C_{2} = 6$

i) Mean of the population (μ):-

$$\mu = \frac{\Sigma x_i}{N} = \frac{1+5+6+8}{4} = 5$$

ii) To find standard deviation (σ) of the population.

$$\text{variance } (\sigma^2) = \frac{\Sigma(x_i - \mu)^2}{N} = \frac{16+0+1+9}{4} = 6.5$$

$$\text{Standard deviation } (\sigma) = \sqrt{6.5} = 2.54$$

iii) Mean of the sampling distribution ($\mu_{\bar{x}}$):-

$$\mu_{\bar{x}} = \frac{3+3.5+4.5+5.5+6.5+7}{6} = 5$$

iv) To find standard deviation of the sampling distribution

$$\text{variance } (\sigma_{\bar{x}}^2) = \frac{\sigma^2}{n}\left(\frac{N-n}{N-1}\right) = \frac{6.5}{2} \times \frac{2}{3} = 2.17$$

$\Rightarrow$ Standard deviation $(\sigma_{\bar{x}}) = \sqrt{2.17} = 1.47$

2	Samples of size 2 are taken from the population 1, 2, 3, 4, 5, 6. Which can be drawn without replacement? Calculate i) The mean of the population. ii) The standard deviation of the population. iii) The mean of the sampling distribution of means.

2) Given $n=2$, population $= \{1,2,3,4,5,6\}$, $N=6$

Total no. of sample $= N_{c_n} = 6_{C_2} = 15$

i) Mean of the sample $(\mu) = \frac{\sum x_i}{N}$

$$\Rightarrow \mu = \frac{1+2+3+4+5+6}{6} = 3 5$$

ii) To Find SD(σ):-

$$\text{variance } (\sigma^2) = \frac{\sum (x_a - \mu)^2}{N} = \frac{6.25+2.25+0.25+0.25+2.25+6.25}{6}$$

$$\sigma^2 = 2.91$$

$$S.D(\sigma) = \sqrt{2.91} = 1.7$$

The sampling distribution is:-

$\{(1,2), (1,3), (1,4), (1,5), (1,6), (2,3), (2,4), (2,5), (2,6), (3,4), (3,5), (3,6), (4,5), (4,6), (5,6)\}$

iii) Means of each samplerg distribution (U_x):-

$$U_x = \frac{\sum x_i}{N_{c_n}} = 3.5.$$

iv) To find : $0 \leq(\sigma_x)$:-

$$\text{variance } (\sigma_x^2) = \frac{\sigma^2}{n}\left(\frac{N-n}{N-1}\right) = \frac{2.91}{2} \times \frac{4}{5} = 1.164$$

$$\Rightarrow S.D(\sigma_x) = \sqrt{1.164} = 1.07$$

3	A normal population has a mean of 0.1 and standard deviation of 2.1. Calculate the probability that mean of a sample of size 900 will be negative.

3) Given,

$\mu = 0.1,\ \sigma = 2.1,\ n = 900,\ P(x < 0) = ?$

Now, $z = \dfrac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \dfrac{0 - 0.1}{\frac{2.1}{\sqrt{900}}} = \dfrac{-0.1}{0.07} = -1.42$

$\Rightarrow P(z \leq -1.42) = P(-\infty \leq z \leq -1.42)$

$= P(-\infty \leq z \leq 0) - P(-1.42 \leq z \leq 0)$

$= 0.5 - P(0 \leq z \leq 1.42)$

$= 0.5 - 0.4222$

$= 0.0778$

$\therefore$ The probability is 0.0778.

4	A random sample of size 64 is taken from an infinite population having the mean 45 and the standard deviation 8. Calculate probability that x will be between 46 and 47.5.

4) Given, $n = 64,\ \mu = 45,\ \sigma = 8,\ P(46 \leq x < 47.5) = ?$

Now, $z_1 = \dfrac{\bar{x}_1 - \mu}{\frac{\sigma}{\sqrt{n}}} = \dfrac{46 - 45}{\frac{8}{\sqrt{64}}} = 1$

$z_2 = \dfrac{\bar{x}_2 - \mu}{\frac{\sigma}{\sqrt{n}}} = \dfrac{47.5 - 45}{\frac{8}{\sqrt{64}}} = 2.5$

$P(1 \leq z \leq 2.5) = P(0 \leq z \leq 2.5) - P(0 \leq z \leq 1)$

$= 0.4938 + 0.3413$

$= 0.8351$

$\therefore$ The probability is 0.8351

5	If a 1-gallon can of paint covers on an average 513 square feet with a standard deviation of 31.5 square feet, Calculate the probability that the mean area covered by a sample of 40 of these 1-gallon cans will be anywhere from 510 to 520 square feet?

5)

Given,

$\mu = 513,\ \sigma = 31.5,\ n = 40.\ P(510 < x < 520) = ?$

$$Z_1 = \frac{\bar{x}_1 - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{510 - 513}{\frac{31.5}{\sqrt{40}}} = \frac{-3}{4.98} = -0.6$$

$$Z_2 = \frac{\bar{x}_2 - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{520 - 513}{\frac{31.5}{\sqrt{40}}} = \frac{7}{4.98} = 1.4$$

$$\Rightarrow P(-0.6 \leq Z \leq 1.4) = P(-0.6 \leq Z \leq 0) + P(0 \leq Z \leq 1.4)$$
$$= P(0 \leq Z \leq 0.6) + P(0 \leq Z \leq 1.4)$$
$$= 0.2258 + 0.4192$$
$$= 0.645$$

$\therefore$ The probability is 0.645.

6	A sample of 900 members has mean of 3.4 and S.D of 2.61.is This sample has been taken from a large population mean 3.25 and S.D 2. 61? Also calculate 95% confidence interval.

6. Given,

$n = 900$, $\bar{x}_n = 3.4$ $s = 2.61$, $\mu = 3.25$, $\sigma = 2.61$

Step - 1

Null hypothesis (H_0) : $\mu = 3.25$

Step-2

Alternative hypothesis (H_1) : $\mu \neq 3.25$ (Two tailed)

Step-3

level of significance :- $\alpha = 0.05$

$z_\alpha = 1.96$

step.4

Test statistic:

$$z = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{3.4 - 3.25}{\frac{2.61}{\sqrt{900}}}$$

$$= 1.72$$

$$|z| = 1.72$$

Step - 5 : conclusion:

$$|z| < z_\alpha$$

∴ We accept the Null hypothesis

confidence interbal :- $\left(\bar{x} \cdot z_\alpha \frac{s}{\sqrt{n}},\ \bar{x} + z_\alpha \frac{s}{\sqrt{n}}\right)$

$= (3.4 - 0.17052, 3.4 + 0.17052)$

$= (3.22948, 3.57052)$

7	It is claimed that a random sample of 49 tires has a mean life of 15200 kms This sample was taken from population whose mean is 15150 kms and S.D is 1200 km Examine the truth value of the claim at 0.05 level of significant.

7. $n = 49$

$\bar{x} = 15{,}200$

$\mu = 15150$

$\sigma = 1200$

$\alpha = 0.05$

Step-I : Null Hypothesis : $\mu = 15150$

Step-II : Alternative Hypothesis : $\mu \neq 15150$

Step-III : level of significance : $\alpha = ~~1200~~ \; 0.05$

$Z_\alpha = 1.96$, at $\alpha = 0.05$

Step-IV : Test statistics:

$$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

$$Z = \frac{15200 - 15{,}150}{\frac{1200}{\sqrt{49}}}$$

$$Z = \frac{150}{\frac{1200}{7}}$$

$$= 0.875$$

$$|Z| = 0.875$$

Step V : conclusion :

$$\underset{0.875}{|Z|} < \underset{1.96}{Z_\alpha}$$

Hence, Null hypothesis is accepted.

8	A manufacturer claims that at least 95% of the equipment which he supplied to a factory conformed to specifications. An examination of sample of 200 pieces of equipment received 18 were faulty Examine the truth value of the claim at 0.05 level.

8. Given,

$n = 200, \quad x = 18$

$$p = \frac{x}{n} = \frac{18}{200} =$$

$$= 0.09$$

$\hat{P} = \cancel{Q}(P \gtrless 0.95).0.95$

$Q = 1 - 0.95$

$Q = 0.05$

$\therefore \alpha = 0.05$

Step. 1

Null hypothesis: $\hat{P} = 0.95 \quad (P > 0.95)$

Step-2

Alternative hypothesis: $P < 0.95$.

Step-3

level of significance: $\alpha = 0.05$

$Z_\alpha = -1.645$ at $\alpha = 0.05$

Step 4

Test statistic:

$$Z = \frac{p - \hat{P}}{\sqrt{\frac{\hat{P}\alpha}{n}}}$$

$$Z = \frac{0.09 - 0.95}{\sqrt{\frac{0.95 \times 0.05}{200}}}$$

$$Z = -55.8$$

$$|Z| = 55.8$$

Step- 5: Conclusion

$|Z| > Z_\alpha$

$\therefore$ We reject Null Hypothesis

9	Among the items produced by a factory out of 500, 15 were defective. In another sample of 400, 20 were defective Examine whether there is any significant difference between two proportions at 5% level.

9) $P_1 = \frac{15}{500} = 0.03, \quad P_2 = \frac{20}{400} = 0.05, \quad \alpha = 0.05$

$P = \frac{x_1 + x_2}{n_1 + n_2} = \quad = 0.03$

$Q = 1 - P$

$= 1 - 0.03$

$= 0.97$

3 Step 1: Null hypothesis (H_0): $P_1 = P_2$

Step 2: Alternative hypothesis (H_1): $P_1 \neq P_2$

Step 3: level of Significance: $\alpha = 0.05$, $Z_\alpha = 1.96$

Step 4: Test statistic: $Z = \frac{P_1 - P_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$

$= \frac{-0.02}{0.011} = -1.81$

$Z = -1.81$

$|Z| = 1.81$

$|Z| < Z_\alpha$

10	A manufacturer produced 20 defective articles in a batch of 400. After overhauled it produced 10 defectives in a batch of 300 Examine whether the machine being improved after over hauling or not.

10.

Given,

$x_1 = 20$ $\qquad$ $x_2 = 10$

$n_1 = 400$ $\qquad$ $n_2 = 300$

$$\hat{P}_1 = \frac{x_1}{n_1} = \frac{20}{400} = 0.05 \qquad \hat{P}_2 = \frac{x_2}{n_2} = \frac{10}{300} = 0.0333$$

$$P = \frac{x_1 + x_2}{n_1 + n_2}$$

$$= \frac{20+10}{400+300} = \frac{30}{700}$$

$\therefore P = 0.0429$ & $Q = 1 - P \Rightarrow 0.9571$

Step-1: Null Hypothesis: The machine has not improved:

$: P_1 = P_2 \Rightarrow P_1 - P_2 = 0$

Step-2: Alternative Hypothesis: The machine has improved:

$: P_1 > P_2 \Rightarrow P_1 - P_2 \doteq 0$

Step-3: level of significance. 5%: $\alpha = 0.01$

$Z_\alpha = \cancel{2.32}$ at $\alpha = 0.01$.
2.326

Step-4: Test Statistics

$$Z = \frac{\hat{P}_1 - \hat{P}_2 - (P_1 - P_2)}{\sqrt{\frac{PQ}{n_1} + \frac{PQ}{n_2}}}$$

$$Z = \frac{0.05 - 0.0333 - (0)}{\sqrt{\frac{0.0429(0.9571)}{400} + \frac{0.0429(0.9571)}{300}}}$$

$|Z| = 1.098 \approx 1.1$

Step-5: conclusion

$|Z| < Z_\alpha$

1.1 $\qquad$ 2.326

Hence, $\cancel{H_0}$ null hypothesis is Accepted

P-value

$= 0.5 - \left(\text{Normal area for } |Z|\right)$

$= 0.5 - 0.3643$

$\therefore P = 0.1357$

$P > \alpha$

$\therefore$ The machine hasn't improved