Part-A

1) **Correlation Coefficient:**

The correlation coefficient is a statistical measure of strength of the relationship between the relative moments of two variables.

2) There are 4 types of correlation:

1) Positive and Negative correlation.
2) Simple and multiple correlation.
3) Partial and total correlation.
4) Linear and non-linear correlation.

3) Given,

$n=12, \ \sigma_x = 2.5, \ \sigma_y = 3.6$

$\sum xy = 64$

$$r = \frac{\sum xy}{n \cdot \sigma_x \sigma_y} = \frac{64}{12 \times 2.5 \times 3.6}$$

$$r = \frac{64}{108} = 0.59259$$

4)

Rank correlation $= \rho = 1 - \frac{6\sum D^2}{N(N^2-1)}$

Where, ρ = Rank coefficient of correlation

D^2 = Sum of squares of differences of two ranks $= \sum(x-y)^2$

N = No. of paired observations

5) **Properties of correlation coefficient:**

1) The coefficient of correlation lies b/w -1 and 1, symbolically we can write it as $-1 \leq r \leq 1$ (or) $|r| \leq 1$

2) The coefficient of correlation is independent of the change of origin of scale of measurement.

3) If $r = 1$, correlation is perfect and +ve correlation.

4) If $r = -1$, correlation is perfect and -ve correlation

5) If $r = 0$, then there is no relationship between the variable.

6) Given,

$\sum xy = 216$, $\sum x^2 = 102$, $\sum y^2 = 471$

$$r = \frac{\sum xy}{\sqrt{\sum x^2 \sum y^2}} = \frac{216}{\sqrt{102 \times 471}}$$

$$r = \frac{216}{219.184} = 0.98547$$

7) Given,

$n = 10$, $\sigma_x = 5.4$, $\sigma_y = 6.2$

$\sum xy = 66$

$$r = \frac{\sum xy}{n \times \sigma_x \times \sigma_y} = \frac{66}{10 \times 5.4 \times 6.2}$$

$$r = \frac{66}{334.8} = 0.1971$$

8) Properties of rank correlation:

1) The value of ρ lies between -1 and 1 then this is a perfect rank correlation.

2) If $\rho = 1$, there is complete agreement in the order if the ranks and the direction of the rank is same

3) If $\rho = -1$, then there is complete disagreement in the order of ranks and they are in apposite direction.

10) Given,

$$N = 8 \; ; \; \Sigma x = 544 \; ; \; \Sigma y = 552$$

$$\Sigma xy = 37560$$

$$\text{Covariance}(x, y) = \frac{1}{n}\left(\Sigma xy - \frac{(\Sigma x)(\Sigma y)}{n}\right)$$

$$\text{cov}(x, y) = \frac{1}{8}\left(37560 - \frac{(544)(552)}{8}\right)$$

$$= \frac{1}{8}(37560 - 37536)$$

$$\therefore \text{cov}(x, y) = 24/8 = 3.$$

9) Given, $b_{xy} = 0.85$; $b_{yx} = 0.89$, $\sigma_x = 3$

i)

$$r = \pm\sqrt{b_{xy} \times b_{yx}}$$

$$r = \sqrt{0.85 \times 0.89}$$

$$r = \sqrt{0.7565}$$

$$r = 0.87$$

ii)

$$b_{yx} = r(\sigma_y/\sigma_x)$$

$$0.89 = 0.87(\sigma_y/3)$$

$$\sigma_y = \frac{0.89 \times 3}{0.87}$$

$$\sigma_y = 2.61$$

∵ These formulas are based on regression, which is not yet taught for some sections, so just remember these formulas for CIE-1.

Part-B:-

1)

Mathematics	Statistics	x (Rank of maths)	y (Rank of statistics)	D=x−y	D^2
85	93	2	1	1	1
60	75	4	3	1	1
73	65	3	4	−1	1
40	50	5	5	0	0
50	80	1	2	−1	1

$\Sigma D^2 = 4$

$$\rho = 1 - \frac{6(\Sigma D^2)}{n(n^2-1)}$$

$$\rho = 1 - \frac{6(4)}{5(24)} = \frac{4}{5}$$

$$\rho = 0.8.$$

Given coefficient of correlation is a perfect rank correlation.

2)

x	y	X=(x−x̄)	Y=(y−ȳ)	XY	X^2	Y^2
12	14	2	3.572	7.144	4	12.759
9	8	−1	−2.428	2.428	1	5.895
8	6	−2	−4.428	8.856	4	19.607
10	9	0	−1.428	0	0	2.039
11	11	1	0.572	0.572	1	0.327
13	12	3	1.572	4.716	9	2.471
7	13	−3	2.572	−7.716	9	6.615

$\Sigma x = 70$ $\Sigma y = 73$ $\Sigma xy = 16$ $\Sigma x^2 = 28$ $\Sigma y^2 = 49.983$

$$\bar{x} = \frac{\Sigma x_i}{n} = \frac{70}{7} = 10$$

$$\bar{y} = \frac{\Sigma y_i}{n} = \frac{73}{7} = 10.428$$

$$r = \frac{\Sigma xy}{\sqrt{\Sigma x^2 \Sigma y^2}} = \frac{16}{\sqrt{28 \times 50}} = 0.5092$$

$$\therefore r = 0.5092.$$

3)

A (x)	S (y)	X	Y	XY	X^2	Y^2
45	35	-16	-31	496	256	961
70	90	9	24	216	81	576
65	70	4	4	16	16	16
30	40	-31	-26	806	961	676
90	95	29	29	841	841	841
40	40	-21	-26	546	441	676
50	80	-11	14	-154	121	196
75	80	14	14	196	196	196
85	180	24	14	336	576	196
60	50	-1	-16	16	1	256

$\Sigma x_i = 610$ $\Sigma y_i = 660$ $\Sigma xy = 3315$ $\Sigma x^2 = 3490$ $\Sigma y^2 = 4590$

$\bar{x} = \frac{\Sigma x_i}{n} = \frac{610}{10} = 61$

$\bar{y} = \frac{\Sigma y_i}{n} = \frac{660}{10} = 66.$

$r = \frac{\Sigma xy}{\sqrt{\Sigma x^2 \Sigma y^2}} = \frac{3315}{\sqrt{3490 \times 4590}} = \frac{3315}{4002.38}$

$r = 0.828.$ ∴ Given correlation coeff is perfect b/w two variably

Write Part-A 8th question answer for properties of rank correlation.

4) Write part-A 5th question answer for properties correlation coefficient.

x	y	X	Y	XY	X^2	Y^2
100	98	0.56	2.56	1.43	0.31	6.55
101	99	1.56	3.56	5.55	2.43	12.67
102	99	2.56	3.56	9.11	6.55	12.67
102	97	2.56	1.56	3.99	6.55	2.43
100	95	0.56	-0.44	-0.24	0.31	0.19
99	92	-0.44	-3.44	1.51	0.19	11.83
97	95	-2.44	-0.44	1.07	5.95	0.19
98	94	-1.44	-1.44	2.07	2.07	2.07
96	90	-3.44	-5.44	18.71	11.83	29.59

$\Sigma x = 895$ $\Sigma y = 859$ $\Sigma xy = 43.2$ $\Sigma x^2 = 36.19$ $\Sigma y^2 = 78.2$

$\bar{x} = \frac{\sum x_i}{n} = \frac{895}{9} = 99.44$

$\bar{y} = \frac{\sum y_i}{n} = \frac{859}{9} = 95.44$

$r = \frac{\sum XY}{\sqrt{\sum X^2 \sum Y^2}} = \frac{43.2}{\sqrt{36.19 \times 78.2}} = \frac{43.2}{53.198}$

$r = 0.812$

Given coefficient of correlation is perfect, between two variables.

5) Write part-A 8th question answer for the properties of rank correlation

x	y	X	Y	XY	X^2	Y^2
15	85	-14	-32.25	451.5	196	1040.0625
18	93	-11	-24.25	266.75	121	588.0625
20	95	-9	-22.25	200.25	81	495.0625
24	105	-5	-12.25	61.25	25	150.0625
30	120	1	2.75	2.75	1	7.5625
35	130	6	12.75	76.5	36	162.5625
40	150	11	32.75	360.25	121	1072.5625
50	160	21	42.75	897.75	441	1827.5625

$\sum x = 232$ $\sum y = 938$ $\sum XY = 2316.5$ $\sum X^2 = 1022$ $\sum Y^2 = 5343.5$

$\bar{x} = \frac{\sum x_i}{n} = \frac{232}{8} = 29$

$\bar{y} = \frac{\sum y_i}{n} = \frac{938}{8} = 117.25$

$r = \frac{\sum XY}{\sqrt{\sum X^2 \sum Y^2}} = \frac{2316.5}{2336.89} = 0.99127$

6)

Age (x)	Mid value (m)	$x = \frac{m-A}{10}$	x^2	$\frac{\text{Blind}}{x \times 1000} \times 100000 =$ Blind Per lakh (y)	y.	y^2	xy
0 - 10	5	-4	16	$\frac{55}{100000} \times 100000 = 55$	-130.375	16997.64	521.5
10 - 20	15	-3	9	$\frac{40}{60000} \times 100000 = 67$	-118.375	14012.64	355.125
20 - 30	25	-2	4	$\frac{40}{40000} \times 100000 = 100$	- 85.375	7288.89	170.75
30 - 40	35	-1	1	$\frac{40}{36000} \times 100000 = 111$	-74.375	5531.64	74.375
40 - 50	45 A	0	0	$\frac{36}{24000} \times 100000 = 150$	-35.375	1251.39	0
50 - 60	55	1	1	$\frac{22}{11000} \times 100000 = 200$	14.625	213.89	14.625
60 - 70	65	2	4	$\frac{18}{6000} \times 100000 = 300$	114.625	13138.89	229.25
70 - 80	75	3	9	$\frac{15}{3000} \times 100000 = 500$	314.625	98988.89	943.875.

$\sum x^2 = 44$ $\quad \sum y = 1483$ $\quad \sum y^2 = 157423.87$

$\sum xy = 2309.5$

$$\bar{y} = \frac{\sum y_i}{n} = \frac{1483}{8} = 185.375.$$

$$r = \frac{\sum xy}{\sqrt{\sum x^2 \sum y^2}} = \frac{2309.5}{\sqrt{44 \times 157423.87}}$$

$$r = \frac{2309.5}{2631.853} = 0.8775.$$

Write Part-A 5th question Answer for properties of Correlation coeff.

7) Write Part-A 8th question for properties of rank correlation.

S(x)	M(y)	D(x-y)	D^2
1	2	-1	1
2	4	-2	4
3	1	2	4
4	5	-1	1
5	3	2	4
6	9	-3	9
7	7	0	0
8	10	-2	4
9	6	3	9
10	8	2	4

$\sum D^2 = 40$

$$\rho = 1 - \frac{6(\sum D^2)}{N(N^2-1)} = 1 - \frac{6 \times 4\cancel{0}}{99\cancel{0}} = \frac{75}{99}$$

$\rho = 0.7575$

10) Write Part-A - 8th question Answer for properties of rank correlation.

X	Rank (x)	Y	Rank (y)	D = (x-y)	D^2
48	8	13	5.5	2.5	6.25
33	6	13	5.5	0.5	0.25
40	7	24	10	-3	9
9	1	6	2.5	- 1.5	2.25
16	3	15	7	- 4	16
16	3	4	1	2	4
65	10	20	9	1	1
24	5	9	4	1	1
16	3	6	2.5	0.5	0.25
57	9	19	8	1	1.

$\Sigma D^2 = 41$

[Rank of 16 in x is 3 because 16 is repeated three times so the ranks 2, 3, 4 average is 3, and it will the rank for 16.

similarly, rank of 6 in y is 2.5 because 6 is repeated two times so, the ranks of 6 are 2 and 3, it's average is 2.5.]

This is just an explination, no need to write in exam.

$$\rho = 1 - \frac{6\left[\Sigma D^2 + \frac{1}{12}(m^3-m) + \frac{1}{2}(m^3-m) + \frac{1}{12}(m^3-m)\right]}{N(N^2-1)}$$

$\because$ 16 is repeated 3 times in x, hence $m=3$, and 13, 6 are repeated twice in y, hence $m=2$.

$$\rho = 1 - \frac{6\left[41 + \frac{1}{12}(3^3-3) + \frac{1}{12}(2^3-2) + \frac{1}{12}(2^3-2)\right]}{990}$$

$$\rho = 1 - \frac{6(41 + 2 + \frac{1}{2} + \frac{1}{2})}{990}$$

$$\rho = 1 - \frac{6(44)}{990}$$

$$\rho = 1 - 0.2667$$

$$\rho = 0.733.$$

8) Write part-A 8th question answer for properties of rank correlation.

M (x)	S (y)	D = (x-y)	D^2
1	1	0	0
2	10	-8	64
3	3	0	0
4	4	0	0
5	5	0	0
6	7	-1	1
7	2	5	25
8	6	2	4
9	8	1	1
10	11	-1	1
11	15	-4	16
12	9	3	9
13	14	-1	1
14	12	2	4
15	16	-1	1
16	13	3	9

$\Sigma D^2 = 136$

$$\rho = 1 - \frac{6(\Sigma D^2)}{n(n^2-1)}$$

$$\rho = 1 - \frac{6(136)}{16 \times 255}$$

$$\rho = 1 - \frac{816}{4080} = 1 - 0.2$$

$$\rho = 0.8.$$

9) Write Part-A 5th question answer for properties of correlation coefficient.

F	S	x	y	D = (x-y)	D^2
65	68	8	5.5	2.5	6.25
63	66	10	8.5	1.5	2.25
67	68	6	5.5	0.5	0.25
64	65	9	10.5	-1.5	2.25
68	69	4.5	3	1.5	2.25
62	66	11	8.5	2.5	6.25
70	68	2	5.5	-3.5	12.25
66	65	7	10.5	-3.5	12.25
68	71	4.5	1	3.5	12.25
69	68	3	5.5	-2.5	6.25
71	70	1	2	-1	1

$\sum D^2 = 63.5$

In x, 68 repeated 2 times so $m=2$

In y, 68 is repeated 4 times so, $m=4$

66 is repeated 2 times so, $m=2$

65 is repeated 2 times so, $m=2$

$$\rho = 1 - \frac{6\left(\sum D^2 + \frac{1}{12}(m^3-m) + \frac{1}{12}(m^3-m) + \frac{1}{12}(m^3-m) + \frac{1}{12}(m^3-m)\right)}{n(n^2-1)}$$

$$\rho = 1 - \frac{6\left(63.5 + \frac{1}{12}\left(2^3-2+4^3-4+2^3-2+2^3-2\right)\right)}{11(120)}$$

$$\rho = 1 - \frac{6(63.5+6.5)}{1320} = 1 - 0.31818$$

$$\rho = 0.68182.$$

Part - C

1) For properties of correlation - write the answer of Part-A - 5th Q.

x	y	X	Y	XY	X^2	Y^2
10	13	-9	-5	45	81	25
12	18	-7	0	0	49	0
18	12	-1	-6	6	1	36
24	25	5	7	35	25	49
23	30	4	12	48	16	144
27	10	8	-8	-64	64	64

$\Sigma x = 114$ $\Sigma y = 108$ $\Sigma xy =$ $\Sigma x^2 = 236$ $\Sigma y^2 = 318$

$$\bar{x} = \frac{\Sigma x_i}{n} = \frac{114}{6} = 19$$

$$\bar{y} = \frac{\Sigma y_i}{n} = \frac{108}{6} = 18$$

$$r = \frac{\Sigma xy}{\sqrt{\Sigma x^2 \ \Sigma y^2}} = \frac{70}{\sqrt{236 \times 318}} = \frac{70}{273.9489}$$

$$r = 0.255$$

2) For the properties of rank correlation - write part-A 8th Q Answer.

Rank-A (x)	Rank-B (y)	Rank-C (z)	D_{xy}	D_{yz}	D_{xz}	D_{xy}^2	D_{yz}^2	D_{xz}^2
1	3	6	-2	-3	-5	4	9	25
6	5	4	1	1	2	1	1	4
5	8	9	-3	-1	-4	9	1	16
10	4	8	6	-4	2	36	16	4
3	7	1	-4	6	2	16	36	4
2	10	2	-8	8	0	64	64	0
4	2	3	2	-1	1	4	1	1
9	1	10	8	-9	-1	64	81	1
7	6	5	1	1	2	1	1	4
8	9	7	-1	2	1	1	4	1

$\Sigma D_{xy}^2 = 200$ $\Sigma D_{yz}^2 = 214$ $\Sigma D_{xz}^2 = 60$

$$\rho_{xy} = 1 - \frac{6(\sum D_{xy}^2)}{n(n^2-1)} = 1 - \frac{6(200)}{990} = -\frac{7}{33} = -0.2121$$

$$\rho_{yz} = 1 - \frac{6(\sum D_{yz}^2)}{n(n^2-1)} = 1 - \frac{6(214)}{990} = -\frac{49}{165} = -0.2969$$

$$\rho_{xz} = 1 - \frac{6(\sum D_{xz}^2)}{n(n^2-1)} = 1 - \frac{6(60)}{990} = \frac{7}{11} = 0.6363.$$

$\because \rho_{xz}$ is maximum, we conclude that the pair of judges A, C has the nearest approch to common linkings in music.

3) For the properties of rank correlation – Write part-A 8th Q answer.

x	y	x	y	$d = (x-y)$	d^2
68	62	4	5	-1	1
64	58	6	7	-1	1
75	68	2.5	3.5	-1	1
50	45	9	10	-1	1
64	81	6	1	5	25
80	60	1	6	-5	25
75	68	2.5	3.5	-1	1
40	48	10	9	1	1
55	50	8	8	0	0
64	70	6	2	4	16

$\sum D^2 = 72$

$$\rho = 1 - \frac{6\left(\sum D^2 + \frac{1}{12}(2^3-2) + \frac{1}{12}(2^3-2)\right) + \frac{1}{12}(3^3-3)}{990}$$

$$\rho = 1 - \frac{6(72 + 1/2 + 1/2 + 2)}{990} = 1 - \frac{6(75)}{990}$$

$$\rho = 0.5454$$

$\because$ 75 in x repeated 2 times so $m=2$

64 repeated 3 times so $m=3$.

68 in repeated 2 times so $m=2$

4) <u>Proof:-</u> Let x and y be the derivations of x and y series from their mean.

Let σ_x and σ_y be their respective standard deviations

Let $\sum\left(\frac{x}{\sigma_x}+\frac{y}{\sigma_y}\right)^2 = \sum\left(\frac{x^2}{\sigma_x^2}+\frac{y^2}{\sigma_y^2}+\frac{2xy}{\sigma_x\sigma_y}\right) = \frac{\sum x^2}{\sigma_x^2}+\frac{\sum y^2}{\sigma_y^2}+\frac{2\cdot\sum xy}{\sigma_x\sigma_y} \rightarrow ①$

But $\frac{\sum x^2}{\sigma_x^2} = N$, Similarly $\frac{\sum y^2}{\sigma_y} = N \rightarrow ②$

$\because r = \frac{\sum xy}{N\sigma_x\sigma_y} \Rightarrow Nr = \frac{\sum xy}{\sigma_x\sigma_y} \rightarrow ③$

From ①, ② and ③

$$\sum\left[\frac{x}{\sigma_x}+\frac{y}{\sigma_y}\right]^2 = N+N+2Nr = 2N+2Nr$$

$$= 2N(1+r).$$

But $\left(\frac{x}{\sigma_x}+\frac{y}{\sigma_y}\right)^2$ is the sum of squares of real quantities and as such it cannot be negative

$$2N(1+r) \geq 0 \Rightarrow 1+r \geq 0;$$

$$r \geq -1 \rightarrow ④$$

Hence r cannot be less than -1

Similarly, by expanding $\sum\left(\frac{x}{\sigma_x}-\frac{y}{\sigma_y}\right)^2$ it can be shown that

$\sum\left(\frac{x}{\sigma_x}-\frac{y}{\sigma_y}\right)^2 = 2N(1-r)$, this can't be negative

so. $2N(1-r) \geq 0$

$1-r \geq 0, \Rightarrow r \leq 1 \rightarrow 5$

$\therefore$ From ④ & ⑤

$-1 \leq r \leq 1$, Hence proved.

5)

X	Y	$D=(x-y)$	D^2
1	10	-9	81
2	7	-5	25
3	2	1	1
4	6	-2	4
5	4	1	1
6	8	-2	4
7	3	4	16
8	1	7	49
9	11	-2	4
10	15	-5	25
11	9	2	4
12	5	7	49
13	14	-1	1
14	12	2	4
15	13.	2	4

$\sum D^2 = 272.$

$$\rho = 1 - \frac{6(\sum D^2)}{15(224)} = 1 - \frac{6(272)}{15(224)}$$

$$\rho = 1 - 0.4857$$

$$\rho = 0.5143$$