Module-2

Part-A:-

1) $P = 20\% = \frac{20}{100} = \frac{1}{5}$

$n = 5$

$p + q = 1$

P = defective (success)

q = Non-defective.

$q = 1 - P$

$q = 1 - \frac{20}{100} = \frac{80}{100}$

$q = \frac{4}{5}$

2) No. of misprints $(x) = 2$

$$P(X=x) = \frac{e^{-\lambda} \cdot \lambda^x}{x!}$$

$$P(X=2) = \frac{e^{-4} \cdot \lambda^2}{2!}$$

$$P(X=2) = \frac{0.0183 \times \cancel{16}^{8}}{\cancel{2}}$$

$$P(X=2) = 0.1464$$

3) $P = 50\% = \frac{50}{100} = \frac{1}{2}$

$q = 1 - P = \frac{1}{2}$; $n = 18$

$x = 10$

$$P(X=10) = {}^{n}C_x \, p^x q^{n-x}$$

$$P(X=10) = {}^{18}C_{10} \cdot \left(\frac{1}{2}\right)^{10} \left(\frac{1}{2}\right)^{8}$$

$$P(X=10) = \frac{43758}{2^{18}} = 0.167$$

4) (i) $n = 400$; $P = 0.2$

$q = 1 - P = 1 - 0.2 = 0.8$

$\text{mean} = np = 400 \times 0.2 = 80$

(ii) Standard deviation $= \sqrt{npq}$

$= \sqrt{400 \times 0.2 \times 0.8}$

$= \sqrt{64}$

$S.D = 8.$

5) **Properties of Binomial distribution:**

(i) The number of observation 'n' is fixed.

(ii) Each observation is independent

(iii) Each observation represents one of two outcomes (Success or failure.)

(iv) The probability of success 'p' is the same for each outcome.

6) $n = 4$; $P = 0.5$

Standard deviation $= \sqrt{npq}$

$P + q = 1$; $q = 1 - P$

$q = 1 - 0.5 = 0.5$

$S.D = \sqrt{4 \times 0.5 \times 0.5}$

$S.D = \sqrt{1} = 1$

7) Properties of poisson distribution:

(a) n, the number trails is very large i.e., when $n \to \infty$

(b) P, the probability of success for each trail is very small i.e., $P \to 0$.

c) $Mean = np = \lambda$ is finite and equals to variance.

d) Range of the variable is from 0 to ∞.

8) Given,

$$mean = np = 4$$

$$Variance = npq = 3$$

$$q = \frac{3}{np} = \frac{3}{4}$$

$$P + q = 1$$

$$P = 1 - q$$

$$P = 1 - 3/4 = 1/4$$

$$npq = 3$$

$$n\left(\tfrac{1}{4}\right)\left(\tfrac{3}{4}\right) = \cancel{3}$$

$$n = 16$$

Binomial distribution

$$P(X = x) = {}^{16}C_x \cdot P^x \cdot q^{16-x}$$

where $x = 0, 1, 2, 3, \ldots\ldots, 16$

9) Given,

$$\mu = 2$$

$$\sigma^2 = 0.1$$

$$\sigma = 0.316$$

$$Z = \frac{x-\mu}{\sigma} = \frac{x-2}{0.31622}$$

$$P(|x-2| \geq 0.01)$$

$$P(-0.01 \geq x-2 \geq 0.01)$$

$$P(1.99 \geq x \geq 2.01)$$

$$P(-2.01 \leq x \leq -1.99)$$

$$P\left(\frac{-2.01-2}{0.31622} \leq Z \leq \frac{-1.99-2}{0.31622}\right)$$

$$P(-12.68$$

9) Given, $\mu = 2$; $\sigma^2 = 0.1$

$$\sigma = 0.31622$$

$$Z = \frac{x-\mu}{\sigma} = \frac{x-2}{0.316227766}$$

$$P(|x-2| \geq 0.01)$$

$$P(x-2 \geq 0.01) = P(x-2 \geq -0.01)$$

$$P(x \geq 2.01) = P\left(Z \geq \frac{2.01-2}{0.316227766}\right)$$

$$= P(Z \geq 0.0316)$$

$$= 0.5 - A(0.03)$$

$$= 0.5 - 0.0120$$

$$= 0.488.$$

10) $P(x=1) = 24\,(P=3)$

$\lambda = ?$

$$\frac{\cancel{e^{-\lambda}} \cdot \lambda}{1!} = \cancel{24}^{4} \cdot \frac{\cancel{e^{-\lambda}} \cdot \lambda^{\cancel{3}}}{\cancel{3!}}$$

$\lambda^2 = 1/4$

$\lambda = 1/2$

$\therefore$ Mean $= \lambda = 1/2$

12) Same as 5th question and

Recurrance relation:

$P(X=x) = {}^nC_x\, P^x q^{n-x} \quad \rightarrow ①$

$P(X=x+1) = {}^nC_{x+1} \cdot P^{x+1} q^{n-(x+1)} \quad \rightarrow ②$

solving $\frac{②}{①}$

$$\Rightarrow \frac{P(x=x+1)}{P(x=x)} = \frac{{}^nC_{x+1} \cdot P^{x+1} \cdot q^{n-(x+1)}}{{}^nC_x\, P^x\, q^{n-x}}$$

$$= \frac{\frac{\cancel{n!}}{(n-x-1)!\,(x+1)!} \cdot P^{x+1} q^{n-(x+1)}}{\frac{\cancel{n!}}{(n-x)!\,x!}\, P^x \cdot q^{n-x}}$$

$$\frac{P(x=x+1)}{P(x=x)} = \left(\frac{n-x}{x+1}\right) \cdot \frac{P}{q}$$

$$\boxed{P(x=x+1) = \left(\frac{n-x}{x+1}\right) \cdot \frac{P}{q} \cdot P(x=x)}$$

11) Properties of Normal distribution:

(a) The mean, median and mode are equal.

(b) The curve is symmetric at the centre. (i.e., around the mean μ)

(c) Exactly half of the values are to the left of centre and exactly half the values are to the right.

(d) Total area under the curve is 1.

13) mean $= np = 4$

Variance $= npq = 4/3$

$q = \frac{4}{3np} = 1/3$

$P+q = 1$; $q = 1-P$; $P = 1-q$

$P = 1 - 1/3 = 2/3$

$np = 4$

$n = 4/P = \frac{4}{2/3} = 6$

$P(x=1) = {}^6C_1 \cdot P^1 q^{6-1}$

$P(x=1) = \frac{6!}{5!} \cdot \left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^5$

$P(x=1) = \frac{12}{3^6} = \frac{4}{3^5}$

14) Given $n=8$,

Probability of getting 5 or 6 is $\frac{2}{6}$

$$P = \frac{2}{6} = \frac{1}{3}$$

$$q = 1 - P = 1 - \frac{1}{3} = \frac{2}{3}.$$

$$\text{mean} = np = 8 \cdot \frac{1}{3} = \frac{8}{3}.$$

15) Given, $\lambda = 6$

$$P(X=4) = \frac{e^{-6} \cdot 6^4}{4!}$$

$$P(X=4) = 0.1338526.$$

16) <u>Properties of Normal Curve:</u>

(a) All Normal curves have the same general bell shape.

(b) The total area under the curve is always 1.

(c) The curve is symmetric with respect to a vertical line that passes through the peak of the curve.

(d) The curve is centered at mean μ which coincides with the median and the mode and is located at the point beneath the peak of the curve

[We can answer the same points of Q.11 for this question]

17) $\text{mean} = np = \lambda$

$\text{Variance} = V(x) = \lambda$

18) $\text{Mode} \Rightarrow P(x+1) = \left(\frac{n-x}{x+1}\right)\left(\frac{p}{q}\right) \cdot P(x)$

19) $\text{mean} = np$

$\text{Variance} = npq$

20) Same answer as 7th question.

<u>Recurrence relation:</u>

$$P(X=x) = \frac{e^{-\lambda} \cdot \lambda^x}{x!} \longrightarrow ①$$

$$P(X=x+1) = \frac{e^{-\lambda} \cdot \lambda^{x+1}}{(x+1)!} \longrightarrow ②$$

$$\frac{②}{①} = \frac{P(X=x+1)}{P(X=x)} = \frac{\cancel{e^{-\lambda}} \cdot \lambda^{x+1}}{(x+1)\cancel{!}} \Big/ \frac{\cancel{e^{-\lambda}} \cdot \lambda^x}{\cancel{x!}}$$

$$P(X=x+1) = \frac{\lambda}{(x+1)} \cdot P(X=x).$$

Part - **B**

1) Given, Total $=20$; defective $=5$

$$n=10 ; \quad P=\frac{5}{20}=\frac{1}{4}$$

$$q=1-P=1-\frac{5}{20}=\frac{3}{4}$$

i) $P(X=0)={}^{10}C_0\left(\frac{1}{4}\right)^0\left(\frac{3}{4}\right)^{10}=(0.75)^{10}$

$=0.05631$

ii) $P(X=1)={}^{10}C_1\left(\frac{1}{4}\right)^1\left(\frac{3}{4}\right)^9=10\times0.25\times(0.75)^9$

$=0.1877$

iii) $P(X=2)={}^{10}C_2\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^8=45\times(0.25)^2\times(0.75)^8$

$=0.281$

iv) $P(0<x<4)=P(X=1)+P(X=2)+P(X=3)$

$=0.1877+0.281+{}^{10}C_3\left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right)^7$

$=0.1877+0.281+0.250$

$=0.7189.$

2) Given, $\lambda=1.5$

$$P(X=x)=\frac{e^{-\lambda}\cdot\lambda^x}{x!}$$

i) No demand $=P(X=0)$

$=\frac{e^{-1.5}\cdot(1.5)^0}{0!}=e^{-1.5}$

$=0.2231$

ii) Demand refused

$P(X\geq2)=-\left(\frac{e^{-1.5}\cdot(1.5)^2}{2!}\right)+1$

$=1-\left[0.2231+e^{-1.5}\cdot1.5+\right.$

ii) Demand is refused

$P(x\geq2)=1-P(X\leq2)$

$=1-[P(X=0)+P(X=1)+P(X=2))$

$=1-\left[0.2231+e^{-1.5}\cdot1.5+\frac{e^{-1.5}\cdot(1.5)^2}{2!}\right]$

$=1-[0.2231+0.3346+0.2509)$

$=1-0.8086$

$=0.1914.$

3) Given, $\lambda=2.5$

i) 4 or fewer cells

$P(x\leq4)=1-[P(X=0)+P(X=1)+P(X=2)$
$P(X=4)+P(X=3)]$

$=1-$

3) Given,

$\lambda = 2.5$

i) 4 or Fewer cells

$$P(X \leq 4) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4)$$

$$= e^{-2.5}\left(\frac{(2.5)^0}{0!} + \frac{(2.5)^1}{1!} + \frac{(2.5)^2}{2!} + \frac{(2.5)^3}{3!} + \frac{(2.5)^4}{4!}\right)$$

$$= 0.0821(1 + 2.5 + 3.125 + 2.6041 + 1.6076)$$

$$= 0.0821(10.8567)$$

$$P(X \leq 4) = 0.8911$$

ii) more than 6 balls

$$P(x > 6) = 1 - \left(P(x=0) + P(x=1) + P(x=2) + P(x=3) + P(x=4) + P(x=5)\right) + P(x=6))$$

$$= 1 - \left(0.8911 + \left(0.0821 \times (2.5)^5/5!\right)\right) + \left(0.0821 + (2.5)^6/6!\right))$$

$$= 1 - (0.8911 + 0.0668 + 0.0278)$$

$$= 1 - (0.985)$$

$$= 0.0144.$$

4)

x	0	1	2	3	4	5	6	7	Total
f	305	365	210	80	28	9	2	1	1000

$n = 7$

$N = \sum f_i = 1000$

$$\mu = \frac{\sum f_i x_i}{\sum f_i} = \frac{0(305) + 1(365) + 2(210) + 3(80) + 4(28) + 5(9) + 6(2) + 7(1)}{1000}$$

$$\mu = \frac{1201}{1000} = \lambda = 1.201.$$

$P(x=0) = \dfrac{e^{-1.2}(1.2)^0}{0!} = 0.30089$

$F = N \cdot P_i(x) = 300.89$

$P(x=1) = \dfrac{e^{-1.2}(1.2)^1}{1!} = 0.36136$

$F = N \cdot P_i(x) = 361.36$

$P(x=2) = \dfrac{e^{-1.2}(1.2)^2}{2!} = 0.217002$

$F = N \cdot P_i(x) = 217.002$

$P(x=3) = \dfrac{e^{-1.2} \cdot (1.2)^3}{3!} = 0.08687$

$F = N \cdot P_i(x) = 86.87$

$P(x=4) = \dfrac{e^{-1.2} \cdot (1.2)^4}{4!} = 0.02608$

$F = N \cdot P_i(x) = 26.08$

$P(x=5) = \dfrac{e^{-1.2} \cdot (1.2)^5}{5!} = 0.00626$

$F = 6.26.$

$P(x=6) = \dfrac{e^{-1.2} \cdot (1.2)^6}{6!} = 0.00125$

$F = 1.25$

$P(x=6) = \dfrac{e^{-1.2} \cdot (1.2)^7}{7!} = 0.00021$

$F = 0.21$

$\sum F = 999.922 \approx 1000.00$

5) Mean $\mu = 1$

$S.D = \sigma = 3$

(i) $P(3.43 \le x \le 6.19)$

$x_1 = 3.43 \quad ; \quad x_2 = 6.19$

$z_1 = \dfrac{x_1 - \mu}{\sigma} = \dfrac{3.43 - 1}{3} = 0.81$

$z_2 = \dfrac{x_2 - \mu}{\sigma} = \dfrac{6.19 - 1}{3} = 1.73.$

$P(0.81 \le z \le 1.73) = |A(z_2) - A(z_1)|$

$= |A(1.73) - A(0.81)|$

$= |0.4582 - 0.2910|$

$= 0.1692$

(ii) $P(-1.43 \le x \le 6.19)$

$x_1 = -1.43 \quad ; x_2 = 6.19$

$z_1 = \dfrac{-1.43 - 1}{3} = -\dfrac{2.43}{3} = -0.81$

$z_2 = \dfrac{6.19 - 1}{3} = \dfrac{5.19}{3} = 1.73$

$P(-0.81 \le z \le 1.73) = |A(z_2) + A(z_1)|$

$= 0.4582 + 0.2910$

$= 0.7492.$

6) $\mu = 30$

$S.D = 5$

i) $26 \le x \le 40$

$x_1 = 26$; $x_2 = 40$

$z_1 = \frac{x_1 - \mu}{\sigma} = \frac{26-30}{5} = \frac{-4}{5} = -0.8$

$z_2 = \frac{x_2 - \mu}{\sigma} = \frac{40-30}{5} = \frac{10}{5} = 2$

$P(x_1 \le x \le x_2) = P(z_1 \le x \le z_2)$

$= P(-0.8 \le x \le 2)$

$= |A(z_2) + A(z_1)|$

$= 0.4772 + 0.2881$

$= 0.7653$

ii) $P(x \ge 45)$

$x = 45 \Rightarrow Z = \frac{x-\mu}{\sigma} = \frac{45-30}{5}$

$= \frac{15}{5} = 3$

$P(x \ge 45) = P(z \ge 3)$

$= |0.5 - A(z_1)|$

$= |0.5 - A(3)|$

$= |0.5 - 0.4987|$

$= 0.0013.$

7)

x	0	1	2	3	4
f	5	22	65	60	8

$n = 4$; $N = \sum f_i = 160$

$\mu = \text{Mean} = \frac{\sum f_i x_i}{N} = \frac{0 + 22(1) + 2(65) + 3(60) + 4(8)}{160} = \frac{364}{160}$

$= 2.275$

$np = 2.275 \Rightarrow P = \frac{2.275}{n} = \frac{2.275}{4}$

$P = 0.57$

$q = 1 - p = 0.43$

$P(X=0) = {}^4C_0 (0.57)^0 (0.43)^4$

$= 0.0341$

$F = N \cdot P_i(x)$

$= 160 \times 0.0341$

$= 5.456$

$P(X=1) = {}^4C_1 (0.57)^1 (0.43)^3$

$= 0.1812$

$F = 160 \times 0.1812$

$= 28.992.$

$P(X=2) = {}^4C_2 (0.57)^2 (0.43)^2$

$= 0.36044$

$F = 160 \times 0.36044$

$= 57.6704.$

$P(X=3) = {}^4C_3 (0.57)^3 (0.43)^1$

$= 0.3185$

$F = 160 \times 0.3185 = 50.96$

$P(X=4) = {}^4C_4 (0.57)^4 (0.43)^0$

$= 0.1055$

$F = 160 \times 0.1055$

$= 0.844$

8) $\mu = 75$ kg ; $\sigma = 7$kg

i) $P(60 \le x \le 78)$

$z_1 = \dfrac{x_1 - \mu}{\sigma} = \dfrac{60 - 75}{7} = \dfrac{-15}{7} = -2.14$

$z_2 = \dfrac{x_2 - \mu}{\sigma} = \dfrac{78 - 75}{7} = \dfrac{3}{7} = 0.42$

$P(-2.14 \le z \le 0.42) = |A(z_2) + A(z_1)|$

$= 0.1628 + 0.4838$

$= 0.6466$

No. of students $= 500 \times 0.6466$

$= 323.3$

$\simeq 323.$

ii) $P(x \ge 92)$

$z = \dfrac{92 - 75}{7} = \dfrac{17}{7} = 2.42.$

$P(x > 2.42) = |0.5 - A(z)| = |0.5 - 4922|$

$= 0.0078.$

No. of students $= 500 \times 0.0078$

$= 3.9$

$= 4$

9) $\mu = 34.5$

$\sigma = 16.5$

No. of students = 1000   i) $P(30 \le x \le 60)$

$z_1 = \frac{x_1 - \mu}{\sigma} = \frac{30 - 34.5}{16.5}$; $z_2 = \frac{x_2 - \mu}{\sigma} = \frac{60 - 34.5}{16.5} = 1.54.$

$= -0.27$

$P(-0.27 \le z \le 1.54) = |A(z_2) + A(z_1)|$

$= |0.4382 + 0.1064|$

$= 0.5446.$

No. of students $= 0.5446 \times 1000 = 544.6 = 545$

10) $\mu = 68$; $\sigma = 3$

i) $P(x > 72)$

$z = \frac{x - \mu}{\sigma} = \frac{72 - 68}{3} = \frac{4}{3} = 1.33$

$P(x > 72) = P(z > 1.33) = |0.5 - A(z_1)|$

$= |0.5 - 0.4082|$

$= 0.0918$

No. of Students $= 300 \times 0.0918$

$= 27.54$

$= 28.$

ii) $P(x \le 64)$

$z = \frac{64 - 68}{3} = \frac{-4}{3} = -1.33$

$P(z \le -1.33) = |0.5 - A(z_1)| = |0.5 - 0.4082)| = 0.0918$

No. of Students $= 0.0918 \times 300 = 27.54 = 28$ students

11) Equal probabilities for boys and girls

$P(B) = P(G) = \frac{1}{2}$

$n = 5.$

Probability distribution $P(X=x) = n_{C_x} p^x q^{n-x} = 5_{C_x}\left(\frac{1}{2}\right)^x\left(\frac{1}{2}\right)^{5-x} = 5_{C_x}\left(\frac{1}{2}\right)^5$

a) 3 boys. $\Rightarrow P(X=3) = 5_{C_3}\left(\frac{1}{2}\right)^5 = 10 \times \frac{1}{32} = \frac{5}{16}$

No. of families with three boys $= \frac{5}{16} \times 800 = 250$

b) 5 girls $=0$ boys

$P(X=0) = 5_{C_0}\left(\frac{1}{5}\right)^5 = \frac{1}{2^5} = \frac{1}{32}$

No. of families with 5 girls $= \frac{1}{32} \times 800 = 25$

c) $P(2 \le x \le 3) = P(X=2) + P(x=3)$

$= 5_{C_2}\left(\frac{1}{2}\right)^5 + 5_{C_3}\left(\frac{1}{2}\right)^5$

$= \left(\frac{1}{2}\right)^5 (20)$

$= \frac{5}{8}$

No. of families with either 2 or 3 boys $= \frac{5}{8} \times 800 = 500$

12) $P(X=1) = \frac{3}{2}\left(P(X=3)\right)$

$\frac{e^{-\lambda} \cdot \lambda^1}{1!} = \frac{3}{2} \frac{e^{-\lambda} \cdot \lambda^3}{3!}$

$\lambda^2 = 4$

$\lambda = 2.$

i) $P(x \geq 1) = 1 - (P(x=0) + P(x=1))$

$= 1 - \left[e^{-2} + \frac{e^{-2} \cdot (2)^1}{1!}\right]$

$= 1 - [e^{-2} + 2e^{-2})$

$= 1 - (3e^{-2})$

$= 1 - 0.40600$

$= 0.5939.$

ii) $P(X \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$

$= 0.40600 + \frac{e^{-2} \cdot 2^2}{2!} + \frac{e^{-2} \cdot 2^3}{3!}$

$= 0.40600 + 0.2706 + 0.1799$

$= 0.8565.$

iii) $P(2 \leq x \leq 5) = P(x=2) + P(x=3) + P(x=4) + P(X=5)$

$= 0.2706 + 0.1799 + \frac{e^{-2} 2^4}{4!} + \frac{e^{-2} \cdot 2^5}{5!}$

$= 0.2706 + 0.1799 + 0.089 + 0.030$

$= 0.5755.$

13) $\lambda = 1.8$

i) Atleast one

$P(x \geq 1) = 1 - (P(x=0) + P(x=1))$

$= 1 - (e^{-1.8} + e^{-1.8}(1.8))$

$= 1 - [e^{-1.8} + 1.8e^{-1.8})$

$= 1 - 0.462$

$= 0537$

iii) At most one

$P(x \le 1) = P(x=0) + P(x=1)$

$= 0.462.$

14) $P(x < 35) = 0.07$ & $P(x < 63) = 0.89$

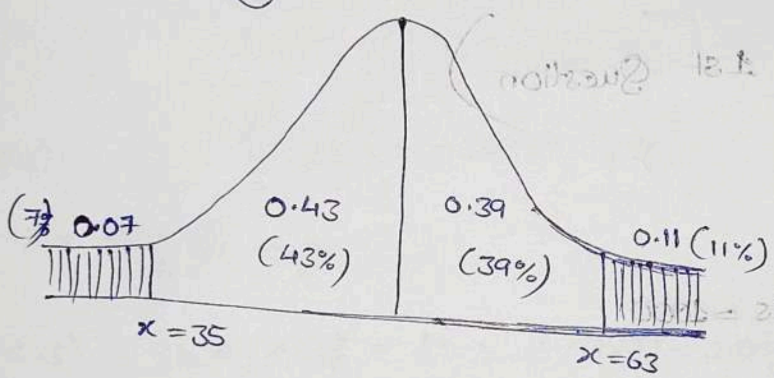
$x_1 = 35$; $x_2 = 63$

$P(x \ge 63) = 1 - P(x < 63)$

$= 1 - 0.89$

$= 0.11$

$Z_1 = \frac{x_1 - \mu}{\sigma} = \frac{35-\mu}{\sigma} \rightarrow ①$; $Z_2 = \frac{x_2 - \mu}{\sigma} = \frac{63-\mu}{\sigma} \rightarrow ②$



$P(0 < Z < Z_1) = 0.43$

$Z_1 = 1.48$ (Using Z-table) $\rightarrow ③$

$P(0 < Z < Z_2) = 0.39$

$Z_2 = 1.23 \rightarrow ④$

equate ① & ② with ③ & ④

$\frac{35-\mu}{\sigma} = 1.48$ | $\frac{63-\mu}{\sigma} = 1.23$

$35 - \mu = 1.48\sigma \rightarrow ⑤$ | $63 - \mu = 1.23\sigma \rightarrow ⑥$

Solving (4) & (5)

$35 - \mu = -1.48\,\sigma$

$63 - \mu = 1.23\,\sigma$

$(-)\quad(+)\quad(-)$

$-28 = -2.71\,\sigma$

$\sigma = 10.33$

$35 - \mu = -1.48 \times 10.33$

$\mu = 35 + 15.2884$

$\mu = 50.2884.$

Variance $= \sigma^2 = 106.7089$

$\therefore\ \sigma = 10.33$

$\sigma^2 = 106.7089$

$\mu = 50.2884.$

15) (Same as 1st Question)

16) No. of Students $= 1000$

$\mu = 35\ ;\ \sigma = 5$

i) $P(25 \le x \le 40)$

$z_1 = \frac{x_1 - \mu}{\sigma} = \frac{25-35}{5} = -2.\ \ ;\ z_2 = \frac{x_2 - \mu}{\sigma} = \frac{40-35}{5} = 1.$

$P(-2 \le z \le 1) = |A(z_2) + A(z_1)|$

$= |0.3413 + 0.4772|$

$= 0.8185$

No. of Students $= 1000 \times 0.8185 = 818.5 = 819$

ii) $P(x > 40)$

$$z = \frac{40-35}{5} = 1$$

$$P(z>1) = |0.5 - A(z_1)| = |0.5 - 0.3413| = 0.1587$$

$$\text{No. of Students} = 0.1587 \times 1000 = 158.7$$
$$= 159$$

iii) $P(x < 20)$

$$z = \frac{20-35}{5} = -3.$$

$$P(z < -3) = |(0.5) - A(z)| = |0.5 - 0.4987| = 0.0013.$$

$$\text{No. of Students} = 0.0013 \times 1000 = 1.3$$
$$= 1$$

iv) $P(x > 50)$

$$z = \frac{50-35}{5} = 3$$

$$P(z < 3) = |0.5 - A(z_1)| = |0.5 - 0.4987| = 0.0013$$

$$\text{No. of studets} = 1.3 = 1$$

17)

x	0	1	2	3	4	5
f	2	14	20	34	22	8

$$n = 5 \quad ; \quad N = \sum f_i = 100$$

$$\mu = \frac{0 + 14(1) + 2(20) + 3(34) + (4 \times 22) + 5(8)}{100} = \frac{284}{100}$$

$$\mu = 2.84$$

$$np = 2.84 \quad ; \quad 5p = 2.84 \quad ; \quad p = 0.57$$

$$q = 1 - 0.57 = 043$$

$P(X=0) = {}^5C_0 (0.57)^0 (0.43)^5$

$= 0.015$

$F = N \cdot P_i(x)$

$= 100 \times 0.015 = 1.5$

$P(x=1) = {}^5C_1 (0.57)^1 (0.43)^4$

$= 0.098$

$F = 100 \times 0.098 = 9.8$

$P(X=2) = {}^5C_2 (0.57)^2 (0.43)^3$

$= 0.260$

$F = 100 \times 0.260 = 26.$

$P(X=3) = {}^5C_3 \cdot (0.57)^3 (0.43)^2$

$= 0.341$

$F = 100 \times 0.341 = 34.1$

$P(X=4) = {}^5C_4 (0.57)^4 (0.43)^1$

$= 0.22$

$F = 100 \times 0.22 = 22$

$P(X=5) = {}^5C_5 (0.57)^5 (0.43)^0$

$= 0.059.$

$F = 100 \times 0.059 = 5.9$

$\Sigma F = 99.3 \simeq 100.$

18) Let λ be the mean.

$$P(X=x) = \frac{e^{-\lambda}\cdot\lambda^x}{x!} \rightarrow ①$$

$$P(X=x-1) = \frac{e^{-\lambda}\cdot(\lambda)^{x-1}}{(x-1)!} \rightarrow ②$$

$$① \div ② \Rightarrow \frac{P(x)}{P(x-1)} = \frac{\cancel{e^{-\lambda}}\cdot\lambda^x}{x!} \Big/ \frac{\cancel{e^{-\lambda}}\cdot\lambda^{x-1}}{(x-1)!}$$

$$\frac{P(x)}{P(x-1)} = \frac{\lambda}{x}$$

$$\therefore P(x) = \frac{\lambda}{x}(P(x-1)).$$

Recurrence relation for poisson distribution.

19) $\mu = 155$; $\sigma = 19$

i) $P(136 \le x \le 174)$

$$z_1 = \frac{136-\mu}{\sigma} = \frac{136-155}{19} = \frac{-19}{19} = -1$$

$$z_2 = \frac{174-155}{\sigma} = \frac{19}{19} = 1.$$

$$P(-1 \le z \le 1) = |A(z_2)+A(z_1)|$$

$$= |0.3413 + 0.3413|$$

$$= 0.6826.$$

ii) $P(x < 117)$

$$z = \frac{117-155}{19} = \frac{-38}{19} = -2.$$

$$P(z < -2) = |0.5 - A(z_1)| = |0.5 - (0.4722)| = 0.0278$$

iii) $P(x > 195)$

$$z = \frac{195-155}{19} = \frac{40}{19} = 2.105$$

$$P(z > 2.10) = |0.5 - A(z_1)|$$

$$= |0.5 - (0.4821)| = 0.0179.$$

20) $p = 1/3$; $q = 1 - 1/3 = 2/3.$; $n = 5$

i) $P(x \leq 3) = P(x=0) + P(x=1) + P(x=2) + P(x=3)$

$$= {}^5C_0 \left(\tfrac{1}{3}\right)^0 \left(\tfrac{2}{3}\right)^5 + {}^5C_1 \left(\tfrac{1}{3}\right)^1 \left(\tfrac{2}{3}\right)^4 + {}^5C_2 \left(\tfrac{1}{3}\right)^2 \left(\tfrac{2}{3}\right)^3 + {}^5C_3 \left(\tfrac{1}{3}\right)^3 \left(\tfrac{2}{3}\right)^2$$

$$= 0.131 + 0.328 + 0.329 + 0.164.$$

$$= 0.952$$

ii) $P(x \geq 2) = 1 - (P(x=0) + P(X=1) + P(X=2))$

$$= 1 - (0.131 + 0.328 + 0.329)$$

$$= 0.212.$$

Part-C

6) Given, mean $= np = 4$

Variance $= npq = 3.$

$q = \frac{3}{np} = \frac{3}{4}$

$P = 1 - q = 1/4$

$n(1/4) = 4$

$n = 16$

i) $P(x=1) = 16C_1 (1/4)^1 (3/4)^{15}$

$= 16(0.25)(0.75)^{15}$

$= 0.0534$

ii) $P(x \geq 1) = 1 - P(x \leq 1)$

$= 1 - (P(x=0) + P(x=1))$

$= 1 - (0.01 + 0.0534)$

$= 1 - 0.0634.$

$= 0.9366.$

iii) $P(0 < x < 3) = P(x=1) + P(x=2)$

$= 0.0534 + 16C_2 (0.25)^2 (0.75)^{14}.$

$= 0.0534 + 0.1336.$

$= 0.187.$

9) Similar to part-B (14)

7)

x	0	1	2	3	4	5	6
f	13	25	52	58	32	16	4.

$n = 6$; $N = \sum f_i = 200$

$\mu = \frac{\sum f_i x_i}{N} = \frac{0 + 25 + 2(52) + 3(58) + 4(32) + 5(16) + 6(4)}{200}$

$= \frac{535}{200} = 0.013$

$np = 0.013$

$P = \frac{535}{200} \times \frac{1}{6}$

$P = 0.44$

$q = 0.56$

$P(x=0) = {}^6C_0 \cdot (0.44)^0 \cdot (0.56)^6$

$= 0.0308$

$P(x=1) = {}^6C_1 (0.44)^1 (0.56)^5$

$= 0.1454$

$P(x=2) = {}^6C_2 (0.44)^2 (0.56)^4$

$= 0.2856$

$P(x=3) = {}^6C_3 (0.44)^3 (0.56)^3$

$= 0.2992$

$P(x=4) = {}^6C_4 (0.44)^4 (0.56)^2$

$= 0.1763$

$P(x=5) = {}^6C_5 (0.44)^5 (0.56)^1$

$= 0.0554$

$P(x=6) = {}^6C_6 (0.44)^6 (0.56)^0$

$= 0.0072.$

$F = N \cdot \sum P(x) = 200(0.0308 + 0.1454 + 0.2856 + 0.2992 + 0.1763 + 0.0554 + 0.0072)$

$F = 200 \times 0.9999$

$F = 199.98 \approx 200$