P&S Module 1 Part A Solutions

@ Vishnu Nadella and Ujjwal Acharya

1. State the classical definition of probability?

> The probability of an event is defined as the ratio of the number of cases favorable to it, to the number of all possible outcomes.

$$\text{Probability} = \frac{\text{No. of favorable comes}}{\text{No. of all possible outcomes}}$$

2. If $E(X) = 6$ and $E\left(X^2\right) = 100$ find the variance.

$E(x) = 6$

$E(x^2) = 100$

Variance $V(x) = E(x^2) - [E(x)]^2$

$\Rightarrow V(x) = 100 - (6)^2$

$V(x) = 100 - 36 = \underline{\underline{64}}$

3. If three coins are thrown at a time and X denotes the random variable which is defined as X(x) = no of heads, write its probability distribution table.

$S = \{HHT, HTH, HHH, THH, HTT, TTH, THT, TTT\}$

X = no. of heads

X	0	1	2	3
$P(x)$	$1/8$	$3/8$	$3/8$	$1/8$

4. If $E(X) = 7,\ E\left(X^2\right) = 40$ find the value of $E\left(5X^2 - 11X + 8\right)$.

$E(x) = 7 \quad E(x^2) = 40$

$5(E(x^2)) - 11\, E(x) + 8$

$5(40) - 11(7) + 8$

$200 - 77 + 8 = 131$

5. State the definitions of discrete and continuous random variables with a suitable example.

> **Discrete**
> A discrete random variable has a countable number of possible values. The probability of each value of a discrete random variable is between 0 and 1, and the sum of all the probabilities is equal to 1.
> Ex:- The number of eggs that a hen lays in a given day (it can't be 2.3)
> **Continuous**
> A random variable is said to be continuous if it take all possible values between certain limits.
> Ex:-continuous random variables are height, weight and age

6. List out the important Properties of probability density function.

> Let x be the continuous random variable with density function f(x), the probability distribution function should satisfy the following conditions:
>
> . $\int_a^b \ f\,(x)\,dx$
>
> . The probability density function is non-negative for all the possible values, i.e. $f(x) \geq 0, \forall x$
>
> . The area between the density curve and horizontal X-axis is equal to 1, i.e. $\int_{-\infty}^{\infty} \ f\,(x)\,dx = 1$
>
> . Due to the property of continuous random variable, the density function curve is continuous for all over the given range

7. Find the probability distribution of getting number tails if we toss three coins calculate mean.

$S = \{HHT, HTH, HHH, THH, HTT, TTH, THT, TTT\}$

X = no. of tails

X	0	1	2	3
$P(x)$	$1/8$	$3/8$	$3/8$	$1/8$

$$\bar{X} = \sum x . P(x) = 0 \times \frac{1}{8} + 1 \times \frac{3}{8} + 2 \times \frac{3}{8} + 3 \times \frac{1}{8}$$

$$= \frac{3+6+3}{8} = \frac{3}{2} = 1.5$$

8. State the definition of mathematical expectation of a probability distribution function.

> Mathematical expectation, also known as the expected value, which is the summation of all possible values from a random variable. It is denoted by P(x), and the value corresponding with the actually observed occurrence of the event.
> For a random variable expected value is a useful property. E(X) is the expected value and can be computed by the summation of the overall distinct values that is the random variable. The mathematical expectation is denoted by the formula:
> $E\left(X\right) = \ \Sigma\ \left(x1p1,\ x2p2,\ \ldots,\ xnpn\right),$
> where, x is a random variable with the probability function, f(x),
> p is the probability of the occurrence,
> and n is the number of all possible values.

9. State the definition of the Mean and Variance of a probability mass function.

> **Mean:**
> In probability theory, the expected value of a random variable X denoted E(X) or E[X] is the arithmetic mean of a large number of independent realizations of X. the expected value is also known as the expectation, mathematical expectation, mean, average, or the first moment.
> $E\left(X\right) = \sum Pi.\left(Xi\right)$
>
> **Variance:**
> In probability theory and statistics, variance is the expectation of the squared deviation of a random variable from its mean. In other words, it measures how far a set of numbers is spread out from their average value.
> $\sigma^2 = E\left(X^2\right) - \left(E\left(X\right)\right)^2$

10. State the definition of the Mean and Variance of a probability density function.

> **Mean:**
> If a random variable X has a density function f(x), then we define the mean value (also known as the average value or the expectation) of X as
> $\mu = \int_{-\infty}^{\infty} x.f\,(x)\,dx$
>
> **Variance:**
> The variance of a continuous random variable is defined by the integral:
> $\sigma^{\,2} = \int_{-\infty}^{\infty}\ (x - \mu\,X)^2\ fX\,(x)\,dx$, where μ is the mean of the random variable x.

11. Find the probability distribution for sum of scores on dice if we throw two dice.

x	2	3	4	5	6	7	8	9	10	11	12
$P(x)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

Expectation $= \sum P_i x_i$

$= 7 =$ Mean

Variance $= E(x)^2 - (\text{Mean})^2$

$= 54.833 - 49$

$= 5.833$

12. Out of 24 mangoes, 6 mangoes are rotten. If we draw two mangoes. Obtain probability distribution of a number of rotten mangoes that can be drawn. Also, find the expectation.

Let x denote the no. of defective items among 2 mangoes drawn from 24 mangoes

x can take 0, 1, 2

$P(x=0) = \frac{6C_2}{24C_2} = \frac{15}{276} = 0.054$

$P(x=1) = \frac{6C_1 \cdot 18C_1}{24C_2} = \frac{108}{276} = 0.39$

$P(x=2) = \frac{18C_2}{24C_2} = \frac{153}{276} = 0.55$

x	0	1	2
$P(x)$	0.05	0.39	0.55

$E(x) = \sum P_i x_i$

$\Rightarrow 0.39 + 1.10 = 1.49$

13. If X is a random variable then show that $E\left[X+K\right]=E\left[X\right]\ +\ K$ where 'K' is constant.

Proof: By the definition of expectation $E(X)=\sum\limits_{i=1} P_i x_i$ — ①

Now consider LHS

$$E(X+k) = \sum_{i=1}^{n} P_i(x_i+k)$$

$$= \sum_{i=1}^{n} P_i x_i + k\sum_{i=1}^{n} P_i \qquad \left[\sum_{i=1}^{n} P_i = 1\right]$$

$$= \sum_{i=1}^{n} P_i x_i + k \qquad [\text{from } ①]$$

$$= E(X)+k$$

14. Show that $\sigma^2\ =\ E\left(X^2\right)\ -\ \mu^2.$

Proof:- $V(X) = \sigma^2 = E(X-\mu)^2$

$$= \sum P_i(x_i-\mu)^2 \qquad [\because \text{expectation def}]$$

$$= \sum P_i\,(x_i^2+\mu^2-2x_i\mu)$$

$$= \sum P_i x_i^2 + \sum P_i \mu^2 - 2\underbrace{\sum P_i x_i \mu} \qquad [\because \sum P_i = 1]$$

$$= E(X^2)+\mu^2-2\mu^2 \qquad \hookrightarrow \left[\sum P_i x_i \cdot \mu = E(X)\mu = \mu \cdot \mu = \mu^2\right]$$

$$= E(X^2) - \mu^2$$

$$= E(X^2) - [E(X)]^2$$

15. State the definitions of the probability mass function and probability density of random variables.

> **Probability Mass Function:**
> If an experiment has k possible distinct outcomes, then we can describe those outcomes using the discrete random variable X, consisting of the values x0,x1,x2,...,xk.
> The corresponding probabilities that the outcomes occur would be given by p(x0),p(x1),p(x2),...,p(xk).
> The function p(x) is a valid probability mass function if the following two constraints are satisfied:
> 0<p(x)≤1 for any x∈{x1,x2,...,xk}
> and
> $\sum Px\left(x\right)=1$
>
> **Probability Density of Random Variables:**

> The Probability Density Function (PDF) is used to specify the probability of the random variable falling within a particular range of values, as opposed to taking on anyone value. This probability is given by the integral of this variable's PDF over that range—that is, it is given by the area under the density function but above the horizontal axis and between
> the lowest and greatest values of the range. The probability density function is nonnegative everywhere, and its integral over the entire space is equal to 1

16. If X is Discrete Random variable then show that $V\,[aX\ +\ b\,] =\ a^2V\,(X)$.

$$\begin{aligned}
Var(aX+b) &= E((aX+b)^2) - (E(aX+b))^2 \\
&= E(a^2X^2 + 2abX + b^2) - (aE(X)+b)(aE(X)+b) \\
&= a^2E(X^2) + 2abE(X) + b^2 - a^2(E(X))^2 - 2abE(X) - b^2 \\
&= a^2E(X^2) - a^2(E(X))^2 \\
&= a^2(E(X^2) - (E(X))^2) \\
&= a^2Var(X)
\end{aligned}$$

17. State the classical definition of probability. If a fair coin is tossed six times. calculate the probability of getting four heads.

> The probability of an event is defined as the ratio of the number of cases favorable to it, to the number of all possible outcomes.

$$P(A) = \frac{\text{Successful Events}}{\text{Total Events of Sample Space}}$$

$$= \frac{15}{64}$$

Exactly 4 Heads

= 0.23

P(A) = 0.23

$$P(A) = \frac{\text{Successful Events}}{\text{Total Events of Sample Space}}$$

$$= \frac{22}{64}$$

Atleast 4 Heads

= 0.34

P(A) = 0.34

18. State the definition of different types of random variables with example.

> Random Variables are of two types:-
> **(i)Discrete Random Variable:**
> A discrete random variable is one which may take on only accountable number of distinct values such as 0,1,2,3,4,........
> Discrete random variables are usually (but not necessarily) counts.
> If a random variable can take only a finite number of distinct values, then it must be discrete. Examples of discrete random variables include, the number of children in a family, the Friday night attendance at a cinema, the number of patients in a doctor's surgery, the number of defective light bulbs in a box of ten.
> **(ii)Continuous Random Variable:**

> A continuous random variable is one which takes an infinite number of possible values. Continuous random variables are usually measurements.
> Examples include height, weight, the amount of sugar in an orange, the time required to run a mile.

19. Outline the classical definition of probability. A coin is tossed 9 times. calculate the probability of getting 5 heads.

> The probability of an event is defined as the ratio of the number of cases favorable to it, to the number of all possible outcomes.

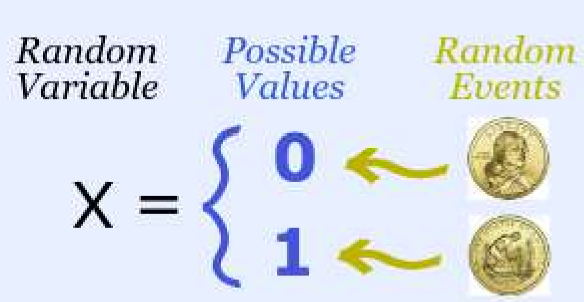
for 5 Heads in 9 Coin Flips		
	Atleast 5 Heads	**Exactly 5 Heads**
Total Events n(S)	512	512
Success Events n(A)	256	126
Probability P(A)	0.5	0.25

20. State the definition of random variable with an example.

> A random variable is a variable whose value is unknown or a function that assigns values to each of an experiment's outcomes. Random variables are often designated by letters and can be classified as discrete, which are variables that have specific values, or continuous within a specific range.

Example: Tossing a coin: we could get Heads or Tails.

Let's give them the values **Heads=0** and **Tails=1** and we have a Random Variable "X":



In short:

X = {0, 1}

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1. Let x denotes the maximum of the 2 no.'s that appear when a pair of fair dice is thrown once. Calculate the
(i) Discrete Probability distribution
(ii) Expectation (iii) Variance.

Sol: When 2 dice are thrown, total no. of outcomes is
$6 \times 6 = 36$.

$$\begin{bmatrix} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\ (4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6) \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6) \end{bmatrix}$$

If R.V 'x' assigns for max then

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 2 & 3 & 4 & 5 & 6 \\ 3 & 3 & 3 & 4 & 5 & 6 \\ 4 & 4 & 4 & 4 & 5 & 6 \\ 5 & 5 & 5 & 5 & 5 & 6 \\ 6 & 6 & 6 & 6 & 6 & 6 \end{bmatrix}$$

Max of 2 no.'s when 2 dice are thrown.

$P(x=1) = \frac{1}{36}$ $\quad P(x=4) = \frac{7}{36}$

$P(x=2) = \frac{3}{36}$ $\quad P(x=5) = 9/36$

$P(x=3) = \frac{5}{36}$ $\quad P(x=6) = \frac{11}{36}$.

∴ The probability distribution is

x	1	2	3	4	5	6
P(x)	$\frac{1}{36}$	$\frac{3}{36}$	$\frac{5}{36}$	$\frac{7}{36}$	$9/36$	$11/36$.

(ii) Expectation = Mean = $\sum_i p_i x_i$

i.e., $E(x) =$
$$1\left(\frac{1}{36}\right) + 2\left(\frac{3}{36}\right) + 3\left(\frac{5}{36}\right) + 4\left(\frac{7}{36}\right) + 5\left(\frac{9}{36}\right) + 6\left(\frac{11}{36}\right)$$

$$= \frac{1}{36}(1+6+15+28+45+66)$$

$$\mu = \frac{1}{36}(161) = \frac{161}{36} = 4.47$$

(iii) Variance $= \sum_i p_i x_i^2 - \mu^2$

$$= \frac{1}{36}(1) + 4\left(\frac{3}{36}\right) + 9\left(\frac{5}{36}\right) + 16\left(\frac{7}{36}\right) +$$
$$25\left(\frac{9}{36}\right) + 36\left(\frac{11}{36}\right) - \left(\frac{161}{36}\right)^2$$

$$= \frac{1}{36}(1+12+45+112+225+396) - \left(\frac{161}{36}\right)^2$$

$$= \frac{1}{36}(791) - (4.47)^2$$

$$= 21.97 - 19.98$$

$$\sigma^2 = 1.99.$$

S.D $\sigma = \sqrt{1.99} = 1.41$

(2) Let x denotes the no. of heads in a single toss of 4 fair coins. Determine

$P(x<2)$ (ii) $P(1<x\leq 3)$

Sol: The req P.D is

x	0	1	2	3	4
$P(x)$	$1/16$	$4/16$	$\frac{6}{16}$	$\frac{4}{16}$	$\frac{1}{16}$

TTTT, TTTH, TTHT, THTT, HTTT, TTHH, THTH, HTHT, HHTT, THHT, HTTH, THHH, HTHH, HHTH, HHHT, HHHH.

(i)

$$P(x<2) = P(x=0)+P(x=1)$$

$$= \frac{1}{16}+\frac{4}{16} = \frac{5}{16}$$

(ii)

$$P(1<x\leq 3) = P(x=2)+P(x=3)$$

$$= \frac{6}{16}+\frac{4}{16} = \frac{10}{16} = \frac{5}{8}$$

(3) A R.V x has the following Probability funⁿ

x	-1	0	1	2	3
$P(x)$	0.3	0.1	0.1	0.3	0.2

Calculate (i) Expectation (ii) Variance (iii) S.D

Sol: Expectation = Mean $= \sum_1 f_i x_i$

i.e, $E(x) = (-1)(0.3)+0(0.1)+1(0.1)+2(0.3)+3(0.2)$

$$\mu = -0.3+0.1+0.6+0.6$$

$$= 1$$

(ii) Variance $= E(x^2) - \mu^2$

$$= \sum_1 x_i^2 f_i - [E(x)]^2$$

$$= (-1)^2(0.3)+0(0.1)+1(0.1)+2^2(0.3)+3^2(0.2)-1$$

$$= 0.3+0.1+1.2+1.8-1$$

$$\sigma^2 = 2.4$$

(iii) S.D $= \sigma = \sqrt{2.4}$

$$= 1.54$$

(4) Find mean & Variance of the uniform P.D given by $P(x) = 1/n$ for $x = 1,2,3,\dots n$.

⑤ A. R.V x has the following Probability function.

x	8	12	16	20	24
$p(x)$	$1/8$	$1/6$	$3/8$	$1/4$	$1/12$

Calculate (i) Expectation (ii) variance (iii) Standard

Sol: Expectation = Mean = $\sum_i f_i x_i$

i.e., $\mu = 8\left(\frac{1}{8}\right)+12\left(\frac{1}{6}\right)+16\left(\frac{3}{8}\right)+20\left(\frac{1}{4}\right)+24\left(\frac{1}{12}\right)$

$\mu = 1+2+6+5+2$

$= 16$

(ii) Variance $= \sum_i f_i x_i^2 - \mu^2$

$\sigma^2 = 64\left(\frac{1}{8}\right)+144\left(\frac{1}{6}\right)+256\left(\frac{3}{8}\right)+400\left(\frac{1}{4}\right)+576\left(\frac{1}{12}\right)-(16)^2$

$= 8+24+96+100+48-256$

$= 276-256$

$= 20$

$\sigma = 2\sqrt{5}$

⑥ The length of time (in minutes) that a certain lady speaks on the telephone is found to be random phenomenon, with a Probability fun specified by the function

$$f(x) = \begin{cases} Ae^{-x/5}, & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

(i) Calculate the value of A so that makes f(x) a probability density fun. (ii) Calculate the prob that she will take over the phone is more than 20 minutes?

Sol: $f(x) \geq 0 \ \forall x \in [0, \infty)$

(i) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$= \int_0^{\infty} Ae^{-x/5}\,dx = 1$

$= A\left[\frac{e^{-x/5}}{-1/5}\right]_0^{\infty} = 1$

$= \left[-Ae^{-x/5} \cdot 5\right]_0^{\infty} = 1$

$A = 1/5$

$= -5A\left[e^{-x/5}\right]_0^{\infty} = 1$

$= -5A\left[-e^0\right] = 1$

$5A = 1$

$\boxed{A = 1/5}$

(ii) $P(x>20) = 1-P(0<x\leq 20)$

$= 1-\int_0^{20} f(x)\,dx$

$= 1-\int_0^{20} Ae^{-x/5}\,dx$

$= 1-\int_0^{20}\frac{1}{5}e^{-x/5}\,dx$

$= 1-\frac{1}{5}\int_0^{20} e^{-x/5}\,dx.$

$$= 1 - \frac{1}{5}\left[\frac{e^{-x/5}}{-1/5}\right]_0^{20}$$

$$= 1 - \left[e^{-x/5}\right]_0^{20}$$

$$= 1 - \left[e^{-20/5} - e^{-0/5}\right]$$

$$= 1 - \left[e^{-4} - e^{0}\right]$$

$$= 1 - \left[\frac{1}{e^4} - 1\right]$$

$$= 1 - \frac{1}{e^4} + 1 \qquad \frac{e^4 - 1 + e^4}{e^4}$$

$P(x > 20)$ $\qquad 2 - \frac{1}{e^4} \,/\, \frac{2e^4 - 1}{e^4}$

$$= 2 - \frac{1}{e^4}$$

$$= \frac{2e^4 - 1}{e^4}$$

(7) If x denote the sum of the numbers that appear when a pair of fair dice is tossed. Estimate the (i) Distribution function (ii) Mean and (iii) Variance

Sol:
(i)

(1,1) (1,2) (1,3) (1,4) (1,5) (1,6)

(2,1) (2,2) (2,3) (2,4) (2,5) (2,6)

(3,1) (3,2) (3,3) (3,4) (3,5) (3,6)

(4,1) (4,2) (4,3) (4,4) (4,5) (4,6)

(5,1) (5,2) (5,3) (5,4) (5,5) (5,6)

(6,1) (6,2) (6,3) (6,4) (6,5) (6,6)

$X = x_i$	2	3	4	5	6	7	8
$P_i = P(x_i)$	1/36	2/36	3/36	4/36	5/36	6/36	5/36

$X = x_i$	9	10	11	12
$P_i = P(x_i)$	4/36	3/36	2/36	1/36

(ii) Mean $\sum_i f_i x_i$

$$2\left(\frac{1}{36}\right) + 3\left(\frac{2}{36}\right) + 4\left(\frac{3}{36}\right) + 5\left(\frac{4}{36}\right) + 6\left(\frac{5}{36}\right) +$$

$$7\left(\frac{6}{36}\right) + 8\left(\frac{5}{36}\right) + 9\left(\frac{4}{36}\right) + 10\left(\frac{3}{36}\right) +$$

$$11\left(\frac{2}{36}\right) + 12\left(\frac{1}{36}\right)$$

$$= \frac{1}{36}\left(2+6+12+20+30+42+40+36+30+22+12\right)$$

$$= \frac{1}{36}(252) = \frac{252}{36} = 7$$

(iii) Variance $= \sum_1^1 f_i x_i^2 - \mu^2$

$\sigma^2 = 4\left(\frac{1}{36}\right) + 9\left(\frac{2}{36}\right) + 16\left(\frac{3}{36}\right) + 25\left(\frac{4}{36}\right) +$
$36\left(\frac{5}{36}\right) + 49\left(\frac{6}{36}\right) + 64\left(\frac{5}{36}\right) + 81\left(\frac{4}{36}\right)$
$+ 100\left(\frac{3}{36}\right) + 121\left(\frac{2}{36}\right) + 144\left(\frac{1}{36}\right) - (7)^2$

$= \frac{1}{36}\left(4+18+48+100+180+294+294\right.$
$\left.320+324+300+242+144\right) - 49$

$\frac{1}{36}(1974) - 49$

$= 54.83 - 49$

$= 5.83.$

⑧ Is the function defined as follows a density fun

$f(x) = \begin{cases} e^{-x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$. If so, estimate

the probability that the variate having. This density will fall in the interval (1,2)? Calculate the Cumulative probability F(2)?

Sol: $f(x) \begin{cases} e^{-x} & x \geq 0 \\ 0 & x < 0 \end{cases}$

$P(x) = \int_1^2 f(x)\,dx$

$= \int_1^2 e^{-x}\,dx$

$= \left[-e^{-x}\right]_1^2$

$= -e^{-2} + e^{-1}$

$= -0.135 + \cancel{0.367}$ 0.367

$= \cancel{0.23287}$ $\cancel{0.502}$

$= 0.23287.$

Cumulative Probability

$= \int_{-\infty}^{x} f(x)\,dx$

$= \int_{-\infty}^{2} e^{-x}\,dx = \int_{-\infty}^{0} 0\,dx + \int_0^2 e^{-x}\,dx$

$= \left[-e^{-x}\right]_0^2$

$= -e^{-2} - (-e^{0})$

$= -e^{-2} + 1$

$= -0.135 + 1$

$= 0.865.$

(9) If Probability density funⁿ

$$f(x) = \begin{cases} Kx^3, & 0 \le x \le 3 \\ 0, & \text{else where} \end{cases}$$

Calculate the value of K and calculate the probability btw $x = 1/2$ & $x = 3/2$

Sol: The value of Probability density funⁿ over the whole range is equal to '1'

(i) $\int_0^3 f(x)\,dx$

$= \int_0^3 Kx^3\,dx = 1$

$K\left[\frac{x^4}{4}\right]_0^3 = 1$

$= K\left[\frac{81}{4} - 0\right] = 1$

$= K \cdot \frac{81}{4} = 1$

$= 81K = 4$

$K = 4/81$

$P\left(\frac{1}{2} < x < 3/2\right)$

$\int_{1/2}^{3/2} Kx^3\,dx$

$= \frac{4}{81}\left[\frac{x^4}{4}\right]_{1/2}^{3/2}$

$= \frac{4}{81}\left(\frac{\left(\frac{3}{2}\right)^4}{4} - \frac{\left(\frac{1}{2}\right)^4}{4}\right)$

$= \frac{4}{81} \cdot \frac{1}{4}\left(\frac{81}{16} - \frac{1}{16}\right)$

$\frac{1}{81}\left(\frac{80}{16}\right)$

$= \frac{80}{81 \times 16}$

$= \frac{80}{1296}$

$= 5/81$

(10) A random variable X has the following probability funⁿ:

X	0	1	2	3	4	5	6
p(x)	0	K	2k	2k	3k	k^2	$2k^2$

Calculate (i) k (ii) $P(X<6)$ (iii) $P(X \ge 6)$

Sol: (i) Since $\sum_{x=0}^{6} p(x) = 1$

$0 + K + 2K + 2k + 3k + k^2 + 2k^2 = 1$

$3k^2 + 8K - 1 = 0$

$K = \frac{-4 \pm \sqrt{19}}{3}$

$K = 0.11$

(ii) $P(X<6) = P(x=0) + P(x=1) + P(x=2) + P(x=3) + P(x=4) + P(x=5)$

$= 0 + K + 2k + 2k + 3k + k^2$

$= 8k + k^2$

$= 8(0.11) + (0.11)^2$

$= 0.88 + 0.0121$

$= 0.8921$

(iii) $P(X \ge 6) = \cancel{1 - P(X<6)}$

$= \cancel{1 - 0.8921}$

$= \cancel{0.1079}$

$P(x=6) = 2k^2$

$= 2(0.11)^2$

$= 2 \times 0.0121$

(11) let x denotes the minimum of the two no.'s that appear when a pair of fair dice is thrown once. Calculate the

(i) Discrete probability distribution

(ii) Expectation (iii) Variance

Sol: when two dice are thrown, total no. of outcomes is $6 \times 6 = 36$

(1,1) (1,2) (1,3) (1,4) (1,5) (1,6)

(2,1) (2,2) (2,3) (2,4) (2,5) (2,6)

(3,1) (3,2) (3,3) (3,4) (3,5) (3,6)

(4,1) (4,2) (4,3) (4,4) (4,5) (4,6)

(5,1) (5,2) (5,3) (5,4) (5,5) (5,6)

(6,1) (6,2) (6,3) (6,4) (6,5) (6,6)

If R.V 'x' assigns for min then

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 & 2 & 2 \\ 1 & 2 & 3 & 3 & 3 & 3 \\ 1 & 2 & 3 & 4 & 4 & 4 \\ 1 & 2 & 3 & 4 & 5 & 5 \\ 1 & 2 & 3 & 4 & 5 & 6 \end{bmatrix}$$

The Probability dist is

x	1	2	3	4	5	6
$P(x)$	$\frac{4}{36}$	$\frac{9}{36}$	$\frac{7}{36}$	$\frac{5}{36}$	$\frac{3}{36}$	$\frac{1}{36}$.

(ii) Expectation = Mean $= \sum_i P_i x_i$

i.e, $E(x) = 1\left(\frac{11}{36}\right) + 2\left(\frac{9}{36}\right) + 3\left(\frac{7}{36}\right) + 4\left(\frac{5}{36}\right) + 5\left(\frac{3}{36}\right) + 6\left(\frac{1}{36}\right)$

$\mu = \frac{1}{36}(11+18+21+20+15+6) = \frac{91}{36} = 2.53$

(iii) Variance $= \sum_1 P_i x_i^2 - \mu^2$

$= \frac{11}{36}(1) + \frac{9}{36}(4) + \frac{7}{36}(9) + \frac{5}{36}(16) + \frac{3}{36}(25) + \frac{1}{36}(36) - \left(\frac{91}{36}\right)^2$

$= \frac{1}{36}(11+36+63+80+75+36) - \left(\frac{91}{36}\right)^2$

$\sigma^2 = 8.3611 - 6.3896$

$= 1.97145$.

$\sigma = \sqrt{1.97145} = 1.4041$

(12) A R.V x has the following Prob fon:

x	-3	-2	-1	0	1	2	3
$P(x)$	k	0.1	k	0.2	2k	0.4	2k

Calculate (i) k (ii) mean (iii) Variance

Sol: Since $\sum_{-3}^{3} P(x) = 1$

$k + 0.1 + k + 0.2 + 2k + 0.4 + 2k = 1$

$6k + 0.7 - 1 = 0$

$6k - 0.3 = 0$

$6k = 0.3$

$k = \frac{0.3}{6}$

$k = \frac{3}{60} = \frac{1}{20} = 0.05$

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(i) Mean $= \sum_1^4 P_i x_i$

$(-3)(K)+(-2)(0.1)+(-1)(K)+0(0.2)$

$+1(2K)+2(0.4)+3(2K)$

$[\because K=0.05]$

$(-3)(0.05)+(-2)(0.1)+(-1)(0.05)+0(0.2)$

$+1(2\times0.05)+2(0.4)+3\underbrace{(0.05\times2)}_{0.1}$

$-0.15-0.2-0.05+0+0.1+0.8+0.3$

$=-0.4+1.2$

$=0.8$

(iii) Variance $= \sum_1^4 P_i x_i^2 - \mu^2$

$= \cancel{9K+4(0.1)}$

$=9(0.05)+4(0.1)+1(0.05)+0(0.2)+1(0.1)+4(0.4)+9(0.1)-(0.8)^2$

$=0.45+0.4+0.05+0.1+1.6+0.9-0.64$

$=3.5-0.64$

$=2.86$

(13) A continuous random variable has the probability density function

$$f(x)=\begin{cases}Kxe^{-\lambda x}, & \text{for } x\geq 0, \lambda>0\\ 0 & \text{otherwise}\end{cases}$$

Evaluate (i) Mean (ii) Variance by finding k.

Sol: (i) Since the probability is unity, we have

$$\int_{-\infty}^{\infty} f(x)\,dx=1$$

i.e., $\int_{-\infty}^{0} 0\,dx+\int_0^{\infty} Kxe^{-\lambda x}\,dx=1$

i.e., $K\int_0^{\infty} xe^{-\lambda x}\,dx=1$

i.e., $K\left[x\left(\frac{e^{-\lambda x}}{-\lambda}\right)-1\cdot\left(\frac{e^{-\lambda x}}{\lambda^2}\right)\right]_0^{\infty}=1$

i.e., $K\left[(0-0)-\left(0-\frac{1}{\lambda^2}\right)\right]=1$

or $k=\lambda^2$

$f(x)$ becomes

$$f(x)=\begin{cases}\lambda^2 x e^{-\lambda x} & \text{for } x\geq 0, \lambda>0\\ 0 & \text{otherwise}\end{cases}$$

(i) Mean of the distribution

$$\mu=\int_{-\infty}^{\infty} x f(x)\,dx$$

$$\mu=\int_{-\infty}^{0} 0\,dx+\int_0^{\infty} x\lambda^2 x e^{-\lambda x}\,dx$$

$$=\lambda^2\int_0^{\infty} x^2 e^{-\lambda x}\,dx$$

$$=\lambda^2\left[x^2\left(\frac{e^{-\lambda x}}{-\lambda}\right)-2x\left(\frac{e^{-\lambda x}}{\lambda^2}\right)+2\left(\frac{e^{-\lambda x}}{-\lambda^3}\right)\right]$$

$$=\lambda^2\left[(0-0+0)-\left(0-0-\frac{2}{\lambda^3}\right)\right]=\frac{2}{\lambda}$$

(iii) Variance of the distribution

$$\sigma^2 = \int_{-\infty}^{\infty} x^2 f(x)\,dx - \mu^2$$

$$\sigma^2 = \int_0^{\infty} x^2 f(x)\,dx - \left(\frac{2}{\lambda}\right)^2$$

$$= \lambda^2 \int_0^{\infty} x^3 e^{-\lambda x}\,dx - \frac{4}{\lambda^2}$$

$$= \lambda^2 \left[x^3\left(\frac{e^{-\lambda x}}{-\lambda x}\right) - 3x^2\left(\frac{e^{-\lambda x}}{\lambda^2}\right) + 6x\left(\frac{e^{-\lambda x}}{-\lambda^3}\right) - 6\left(\frac{e^{-\lambda x}}{\lambda^4}\right)\right]_0^{\infty} - \frac{4}{\lambda^2}$$

$$= \lambda^2\left[(0-0+0-0) - \left(0-0+0-\frac{6}{\lambda^4}\right)\right] - \frac{4}{\lambda^2}$$

$$= \frac{6}{\lambda^2} - \frac{4}{\lambda^2} = \frac{2}{\lambda^2}.$$

(14) If the Probability density fun of random variable is

$f(x) = K(1-x^2)$, $0 < x < 1$, then calculate

(i) K (ii) $p(0.1 < x < 0.2)$ (iii) $P(x > 0.5)$.

Sol: Given $f(x) = \begin{cases} K(1-x^2), & 0<x<1 \\ 0, & \text{otherwise} \end{cases}$

(i) W.K.T $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\int_{-\infty}^{0} f(x)dx + \int_0^1 f(x)dx + \int_1^{\infty} f(x)\,dx$$

i.e, $0 + \int_0^1 K(1-x^2)dx + 0 = 1$

$$K\left(x - \frac{x^3}{3}\right)_0^1 = 1 \text{ or } K\left(1-\frac{1}{3}\right) = 1$$

$$K = \frac{3}{2}.$$

(ii) $P(0.1 < x < 0.2) =$

$$\int_{0.1}^{0.2} f(x)\,dx = \int_{0.1}^{0.2} K(1-x^2)\,dx$$

$$= \frac{3}{2}\left(x - \frac{x^3}{3}\right)_{0.1}^{0.2} \quad (\because K = 3/2)$$

$$= \frac{3}{2}\left[\left(0.2 - \frac{0.008}{3}\right) - \left(0.1 - \frac{0.001}{3}\right)\right]$$

$$= \frac{3}{2}\left[0.1 - \frac{0.007}{3}\right] = 0.2965.$$

(iii) $P(x > 0.5) = \int_{0.5}^{\infty} f(x)\,dx = \int_{0.5}^{1} f(x)dx + \int_1^{\infty} f(x)dx$

$$= \frac{3}{2}\int_{0.5}^{1}(1-x^2)dx + 0 = \frac{3}{2}\left(x - \frac{x^3}{3}\right)_{0.5}^{1}$$

$$= \frac{3}{2}\left[\left(1-\frac{1}{3}\right) - \left(0.5 - \frac{0.125}{3}\right)\right]$$

$$= \frac{3}{2}\left(\frac{2}{3} - 0.4583\right) = 0.3125.$$

(15) A R.V x has the following Probability function.

x	4	5	6	8
$p(x)$	0.1	0.3	0.4	0.2

Calculate (i) Expectation (ii) Variance (iii) Standard deviation

Sol: Expectation = Mean = $\sum_i P_i x_i$

$$\mu = 4(0.1) + 5(0.3) + 6(0.4) + 8(0.2)$$

$$= 0.4 + 1.5 + 2.4 + 1.6$$

$$= 5.9$$

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(ii) Variance $= \sum_1^4 P_i x_i^2 - \mu^2$

$= 16(0.1) + 25(0.3) + 36(0.4) + 64(0.2) - (5.9)^2$

$= 1.6 + 7.5 + 14.4 + 12.8 - 34.81$

$= 36.3 - 34.81$

$= 1.49$

$\therefore \sigma^2 = 1.49$

(iii) $S.D = \sigma = \sqrt{1.49} = 1.22$

(16) If X is a continuous random variable whose density fun^n is

$$f(x) = \begin{cases} x & \text{if } 0 < x < 1 \\ 2-x & \text{if } 1 \le x < 2 \\ 0 & \text{elsewhere} \end{cases}$$

Q. Evaluate $E(25X^2 + 30X - 5)$.

Sol:

(20) The Probability density fun^n of R.V X is

$f(x) = \frac{k}{x^2+1}$, $-\infty < x < \infty$. Calculate k & the distribution fun^n $F(x)$.

Sol: $\int_{-\infty}^{0} \frac{k}{x^2+1} + \int_{0}^{\infty} \frac{k}{x^2+1} = 1$

$k\left[\tan^{-1}x\right]_{-\infty}^{0} + k\left[\tan^{-1}x\right]_{0}^{\infty} = 1$

$k\left[\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right] = 1$

$k = 1/\pi$

$f(x) = \frac{d}{dx}\left[F(x)\right]$

$\int_{-\infty}^{x} f(x)\,dx = \frac{1}{\pi}\left[\tan^{-1}x\right]_{-\infty}^{x}$

$= \frac{1}{\pi}\left[\tan^{-1}x + \frac{\pi}{2}\right]$

(17)

(18) Two Coins are tossed simultaneously. let x denotes the number of heads then Calculate $E[x]$, $E(x^2)$, $E(x^3)$, $v(x)$.

Sol: $n=2$

Possibilities HH 2

HT 1

TH 1

TT 0

x	2	1	0
$P(x)$	$1/2$	$2/4$	$1/4$

$E[x] = \frac{2}{4} + \frac{2}{4} + 0$

$= \frac{1}{2} + \frac{1}{2} = 1$

$E(x^2) = \frac{4}{4} + \frac{2}{4} = 1 + \frac{1}{2} = \frac{3}{2}$

$= 1.5$

$E(x^3) = \frac{8}{4} + \frac{2}{4} = 2 + 1/2 = 5/2$

$= 2.5$

$v(x) = 1.5 - (1)^2$

$= 0.5$

(19) If the function defined by

$$f(x) = \begin{cases} 0 & , x < 2 \\ \frac{1}{18}(2x+3) & , 2 \le x \le 4 \\ 0 & , x > 4 \end{cases}$$

a Probability density function? find the Probability that a variate having $f(x)$ as density fun will fall in the interval $2 \le x \le 3$.

Sol: (i) For all points x in $-\infty \le x \le \infty$, $f(x) \ge 0$ and

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{2} 0 \cdot dx + \int_{2}^{4} \frac{1}{18}(2x+3)\,dx + \int_{4}^{\infty} 0 \cdot dx$$

$$= \frac{1}{18}\int_{2}^{4}(2x+3)\,dx = \frac{1}{18}\left[\frac{(2x+3)^2}{4}\right]_2^4$$

$$= \frac{1}{72}(121-49) = 1$$

Here $f(x)$ is a probability density fun.

(ii) The Probability that the density will fall in the interval $2 \le x \le 3$ is

$$P(2 \le x \le 3) = \int_{2}^{3} f(x)\,dx = \frac{1}{18}\int_{3}^{2}(2x+3)\,dx$$

$$= \frac{1}{18}\left(x^2+3x\right)_2^3 = \frac{1}{18}(18-10)$$

$$= \frac{8}{18} = \frac{4}{9}.$$

Part-C

1. The Probability density fun of a random variable X is

$$f(x) = \begin{cases} 3x^2 & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

Calculate value of a, if $P(a \le x \le 1) = \frac{19}{81}$

Sol:

$$\int_a^1 3x^2 = \frac{19}{81}$$

$$\left[x^3\right]_a^1 = \frac{19}{81}$$

$$1 - a^3 = \frac{19}{81}$$

$$a^3 = \frac{62}{81}$$

$$a = \sqrt[3]{\frac{62}{81}}$$

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(2) The daily Consumption of electric Power is a random Variable having the probability density fun?

$$f(x) = \begin{cases} \frac{1}{9} x e^{-x/3}, & x > 0 \\ 0, & x \le 0 \end{cases}$$

If the total production is 12 million KW-hours, determine the probability that there is power cut (shortage) on any given day.

Sol: Probability that the power consumed is btw 0 to 12 is

$P(0 \le x \le 12 \text{ million KW-hours}) = \int_0^{12} f(x)\, dx$

$= \frac{1}{9}\int_0^{12} x e^{-x/3}\, dx$

$= \frac{1}{9}\left[x \frac{e^{-x/3}}{(-1/3)} - 1 \cdot \frac{e^{-x/3}}{1/9} \right]$

$= \frac{1}{9}\left[-36 e^{-4} - 9 e^{-4} + 9\right]$

$= \frac{1}{9}\left(9 - 45 e^{-4}\right)$

$= 1 - 5e^{-4}.$

Power supply is inadequate if daily consumption exceeds 12 million KW, i.e.,

$P(x > 12) = 1 - P(0 \le x \le 12)$

$= 1 - (1 - 5e^{-4})$

$= 5e^{-4} = 0.091578,$

(3) A fair coin is tossed until a head or five tails ~~tails~~ occurs. Find the expected no. E of tosses of the coin. $= 20.$

Sol: The tossing of Given x denotes the no. of Coins. Coins is tossed until a head or 5 tails occur i.e., it is clear if on x = 1 head comes then the process will be stopped & if tail comes then coin will be tossed second time. It will be repeated again & again till 5 tails come maximum. Then the value of X will be 1, 2, 3, 4, 5

$S = \{H, TH, TTH, TTTH \text{ or } TTTTH, TTTTT\}$

∴ Probability that head comes in 1st throw

$P(x=1) = 1/2$

$\text{lly } P(x=2) = P(TH) = P(T)P(H) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

$P(x=3) = P(TTH) = P(T)\,P(T)\,P(H)$

$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = 1/8$

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$P(x=4) = P(TTTH) = P(T)P(T)P(T)P(H)$

$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{16}$

$P(x=5) = P(TTTTH \cup TTTTT)$

$= P(T)P(T)P(T)P(T)P(H) + P(T)P(T)P(T)P(T)P(T)$

$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

$= \frac{1}{32} + \frac{1}{32} = \frac{1}{16}$

P.D will be

x	1	2	3	4	5
$P(x)$	$1/2$	$1/4$	$1/8$	$1/16$	$1/16$

$\therefore$ Mean $= \sum_{i=1}^{5} p_i x_i$

$= 1\left(\frac{1}{2}\right) + 2\left(\frac{1}{4}\right) + 3\left(\frac{1}{8}\right) + 4\left(\frac{1}{16}\right) + 5\left(\frac{1}{16}\right)$

$= \frac{1}{2} + \frac{1}{2} + \frac{3}{8} + \frac{1}{4} + \frac{1}{16}$

$= \frac{16+16+12+8+5}{32} = \frac{31}{16} = 1.9$

(4) A fair die is tossed. let the R.V X denote the twice the no. appearing on the die (i) Construct the P.d of X hence find mean and variance

Let x denote the event of getting twice the no. Then X can take the values 1, 2, 3, 4, 5 & 6.

Thus the Probability dist of X is

x	1	2	3	4	5	6
$P(x)$	$11/36$	$9/36$	$7/36$	$5/36$	$3/36$	

Mean $= \sum_1 p_i x_i$

$\mu = 1\left(\frac{11}{36}\right) + 2\left(\frac{9}{36}\right) + 3\left(\frac{7}{36}\right) + 4\left(\frac{5}{36}\right) + 5\left(\frac{3}{36}\right) + \dots$

$= \frac{1}{36}(11 + 18 + 21 + 20 + 15 + 6)$

$= \frac{91}{36} = 2.52$

Variance $= \sum_1 p_i x_i^2 - \mu^2$

$\sigma^2 = 1\left(\frac{11}{36}\right) + 4\left(\frac{9}{36}\right) + 9\left(\frac{7}{36}\right) + 16\left(\frac{5}{36}\right) + 25\left(\frac{3}{36}\right) + 36\left(\dots\right) - (2.5)^2$

$= \frac{1}{36}(11 + 36 + 63 + 80 + 75 + 36) - 6.25$

$= \frac{1}{36}(301) - 6.25$

$= 8.36 - 6.25$

$= 8.36 - 6.25$

$= 2.11$

(5) If $f(x) = ke^{-|x|}$ is Probability density fun in the interval is real, then evaluate: (ii) mean (iii) variance (iv) $P(0 < x < 4)$ By finding k.

Sol: $\int_{-\infty}^{\infty} f(x) = 1$

$= \int_{-\infty}^{\infty} ke^{-|x|} = 1$

By solving:

$$2\int_0^{\infty} Ke^{-|x|} = 1$$

$$K = 1/2$$

Mean $= \mu =$

$$\mu = \int_{-\infty}^{\infty} x\, f(x)\,dx$$

odd funⁿ

Ans is 0.

Variance

$$\sigma^2 = \int_{-\infty}^{\infty} x^2 f(x) - \mu^2$$

$$= 2\int_0^{\infty} x^2 \times \frac{1}{2} e^{-|x|} - 0$$

$$= \int_0^{\infty} x^2 e^{-x} =$$

$$= \left[x^2(-e^{-x}) - 2x(e^{-x}) + 2(-e^{-x})\right]_0^{\infty}$$

$$= [0-(-2)] = 2$$

$$P(0 \le x \le 4) = \int_0^4 f(x)\,dx$$

$$= \frac{1}{2}\int_0^4 e^{-|x|}$$

$$= \frac{1}{2}\left(-e^{-x}\right)_0^4 = \frac{1}{2}\left[-e^{-4}+1\right]$$

$$= \frac{1}{2}\left[1-0.018\right]$$

$$= \frac{0.982}{2} = 0.491.$$

(6) The function $f(x) = Ax^2$, in $0 < x < 1$ is valid probability density function then calculate the value of A.

Sol:

$$\int_0^1 Ax^2 = 1$$

$$\left[\frac{Ax^3}{3}\right]_0^1 = 1$$

$$\frac{A}{3} = 1$$

$$A = 3$$

(7) The density function of a random variable X is

$$f(x) = \begin{cases} e^{-x}, & x \ge 0 \\ 0, & \text{otherwise} \end{cases}$$

evaluate $E[x]$, $E(x^2)$, $V(x)$.

Sol:

$$\int_0^{\infty} xe^{-x} = E[x]$$

$$= \left[x[-e^{-x}] - 1[e^{-x}]\right]_0^{\infty}$$

$$= -[-1[1]]$$

$$= 1$$

$$E[x^2] = \int_0^{\infty} x^2 e^{-x}$$

$$\left[x^2[-e^{-x}] - 2x[e^{-x}] + 2[-e^{-x}]\right]_0^{\infty}$$

2.

2

$E[x] = 1$; $E[x^2] = 2$

$V(x) = 2 - 1 = 1$

(8) If $E[x] = 10$, $V(x) = 1$, then Calculate $E(2x(x+10))$

Sol: $E[2x[x+10]]$

$= E[2x^2 + 20x]$

$= 2E[x^2] + 20E[x]$

$= 2(101) + 20(10)$

$= 202 + 200$

$= 402.$

$E[x] = 10$

$V(x) = 1$

$1 = x - 100$

$x = 101$

$E[x^2] = 101.$

(4) Sol: X = Denotes twice the no.

$x = 2, 6, 4, 8, 10, 12$

Mean $= \sum_{i}^{} x \, p(x)$

$= \frac{2}{6} + \frac{4}{6} + \frac{6}{6} + \frac{8}{6} + \frac{10}{6} + \frac{12}{6}$

$= \frac{42}{6} = 7$

Variance $= \sum_{i}^{} x^2 p(x) = \frac{4}{6} + \frac{16}{6} + \frac{36}{6} + \frac{64}{6} + \frac{100}{6} + \frac{144}{6} = 60.67$

$V(x) = 60.67 - (7)^2 = 60.67 - 49$

$= 11.67$

⑨ A discrete R.V x has the following P.d. Calculate (i) k (ii) $P(x<3)$ (iii) $P(x>5)$.

x	1	2	3	4	5	6	7	8
$p(x)$	2k	4k	6k	8k	10k	12k	14k	4k

(i) $\sum_{1}^{8} P(x) = 1$

$= 2k+4k+6k+8k+10k+12k+14k+4k = 1$

$= 60K = 1$

$\boxed{K = \frac{1}{60}}$

(ii) $P(x<3) = P(x=1)+P(x=2)+\cancel{P(x=3)}$

$= 2k+4k+6k$

$= 12k = 12\left(\frac{1}{60}\right) =$

$= \frac{1}{5}$

$= 6k = 6\left(\frac{1}{60}\right)$

$= \frac{1}{10} = 0.1$

$P(x>5) = 1-[P(x=1)+P(x=2)+P(x=3)+P(x=4)+P(x=5)]$

$= 1-[2k+4k+6k+8k+10k]$

$= 1-[30k]$

$= \cancel{1-20\left(\frac{1}{60}\right)}$

$= \cancel{1-\frac{1}{3} = \frac{2}{3}}.$

$1-30\left(\frac{1}{60}\right)$

$= 1-\frac{3}{6} = \frac{6-3}{6} = \frac{3}{6} = 1/2.$

$\frac{30}{60} = 1/2$

⑩ For the continuous R.V X whose P.D.f is given by

$$f(x) = \begin{cases} cx(2-x), & 0 \le x \le 2 \\ 0, & \text{otherwise} \end{cases}$$

Calculate c, mean & variance of X.

Sol: (i) Since the total probability is unity, we have

$$\int_{-\infty}^{\infty} f(x)\,dx = 1 \quad \text{So.} \quad \int_{0}^{2} f(x)\,dx = 1$$

i.e., $\int_0^2 cx(2-x)\,dx = 1$ i.e., $c\left(x^2 - \frac{x^3}{3}\right)_0^2 = 1$

i.e., $c\left(4 - \frac{8}{3}\right) = 1$ i.e., $\frac{4c}{3} = 1$ or $c = 3/4$

$\therefore f(x) = \begin{cases} \frac{3x}{4}(2-x), & \text{if } 0 \le x \le 2 \\ 0, & \text{otherwise} \end{cases}$

(ii) Mean of $X = E(X) = \int_{-\infty}^{\infty} x f(x)\,dx$

$\mu = \int_0^2 x \cdot \frac{3x}{4}(2-x)\,dx = \frac{3}{4}\int_0^2 (2x^2 - x^3)\,dx$

$= \frac{3}{4}\left(\frac{2x^3}{3} - \frac{x^4}{4}\right)_0^2 = \frac{3}{4}\left(\frac{2^4}{3} - \frac{2^4}{4}\right) = \frac{3}{2}(2^4)\left(\frac{1}{3} - \frac{1}{4}\right) = \frac{12}{12} = 1$

(iii) Variance of $X = V(X) = \int_{-\infty}^{\infty} x^2 f(x)\,dx - \mu^2 = \int_0^2 x^2 \cdot \frac{3x}{4}(2-x)\,dx - 1$

$= \frac{3}{4}\int_0^2 (2x^3 - x^4)\,dx - 1 = \frac{3}{4}\left(2 \cdot \frac{x^4}{4} - \frac{x^5}{5}\right)_0^2 - 1$

$= \frac{3}{4}\left[\frac{32}{4} - \frac{32}{5}\right] - 1 = 24\left(\frac{1}{4} - \frac{1}{5}\right) - 1 = \frac{6}{5} - 1 = \frac{1}{5}$