MOD - 4

Push down Automata

$a^n b^n \to$ B15 , B18 (1st part)
B8

er language $L = \{a^n b^n \mid n \geq 1\}$

Sol: $M = \{Q, \Sigma, \Gamma, \delta, q_0, Z_0, F\}$

Step 1: Initially push all a's onto the stack

Step 2: Whenever 'b' occurs, change the state and pop 'a' from the stack

Step 3: Repeat Step 2 until stack is Empty.

$L = \{ab, aabb, aaabbb, \ldots\ldots\}$

a	a	a	b	b	b	ϵ

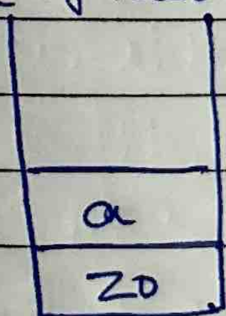
↑

First, the Input symbol reads the 'a' and according to Step 1, it pushes into the stack which already contains Z_0 as first element

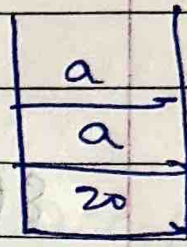
$\delta(q_0, a, Z_0) = (q_0, aZ_0)$

- q_0 → present State
- a → Input Symbol
- Z_0 → Top of Stack
- (q_0, aZ_0) → next State

→ Stack Symbols

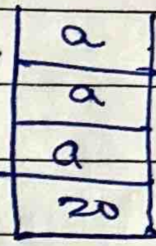


a	a	a	b	b	b	ε
	↑					



$\delta(q_0, a, a) = \delta(q_0, aaZ_0)$

a	a	a	b	b	b	ε
		↑				



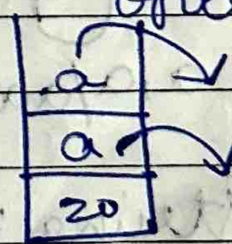
$\delta(q_0, a, a) = \delta(q_0, aaaZ_0)$ → No need to mention this step

a	a	a	b	b	b	ε
			↑			

Now, the Input symbol reads 'b' and according to step 2, it changes the state & a is popped from stack

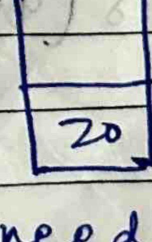
$\delta(q_0, b, a) = \delta(q_1, \epsilon)$ → indicates pop operation

a	a	a	b	b	b	ε
				↑		



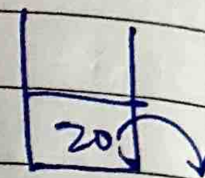
$\delta(q_1, b, a) = \delta(q_1, \epsilon)$

a	a	a	b	b	b	ε
					↑	



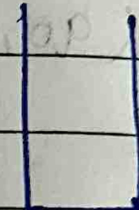
$\delta(q_1, b, a) = \delta(q_1, \epsilon)$ → no need to mention

a	a	a	b	b	b	ε



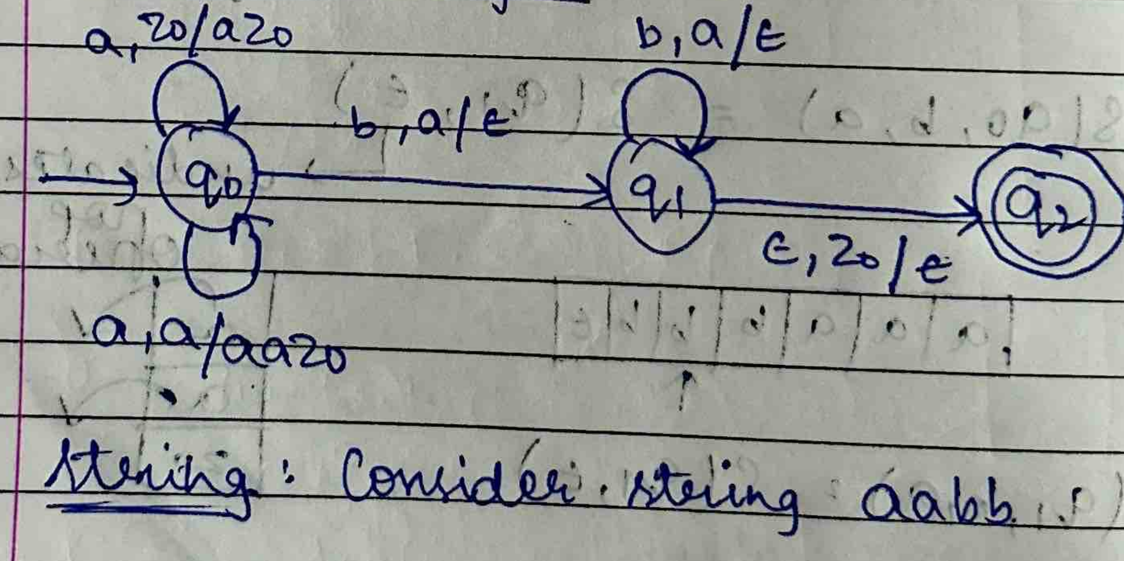
$\delta(q_1, \epsilon, Z_0) \vdash \delta(q_2, \epsilon)$

when ε is encountered, the δ-transition moves to next state.



Since the stack is empty, the string is accepted.

Transition diagram



String : Consider string aabb

$\delta(q_0, aabb, Z_0) \vdash \delta(q_0, abb, aZ_0)$

$\vdash \delta(q_0, bb, aaZ_0)$

$\vdash \delta(q_1, b, aZ_0)$

$\vdash \delta(q_1, \epsilon, Z_0)$

$\vdash \delta(q_2, \epsilon)$

We have reached the final state of the stack is empty, so string is accepted

$M = \{Q, \Sigma, \Gamma, \delta, z_0, F$

$Q = \{q_0, q_1, q_2\}$
$\Sigma = \{a, b\}$
$\Gamma = \{z_0, a\}$
$Q_0 = \{q_0\}$
$z_0 = \{z_0\}$
$F = \{q_2\}$

$xcx^r \to B9$

for $L = \{wcw^R \mid w \in (a+b)^*\}$

Sol: $L = \{abacaba, ababcbaba, \ldots\}$

We have 4 cases:

$\delta(q_0, a, z_0) = \delta(q_0, az_0)$
$\delta(q_0, b, z_0) = \delta(q_0, bz_0)$

Case 1: $\delta(q_0, a, a) = (q_0, aa)$
$\delta(q_0, a, b) = (q_0, ab)$
$\delta(q_0, b, a) = (q_0, ba)$
$\delta(q_0, b, b) = (q_0, bb)$

Push the symbol into the stack

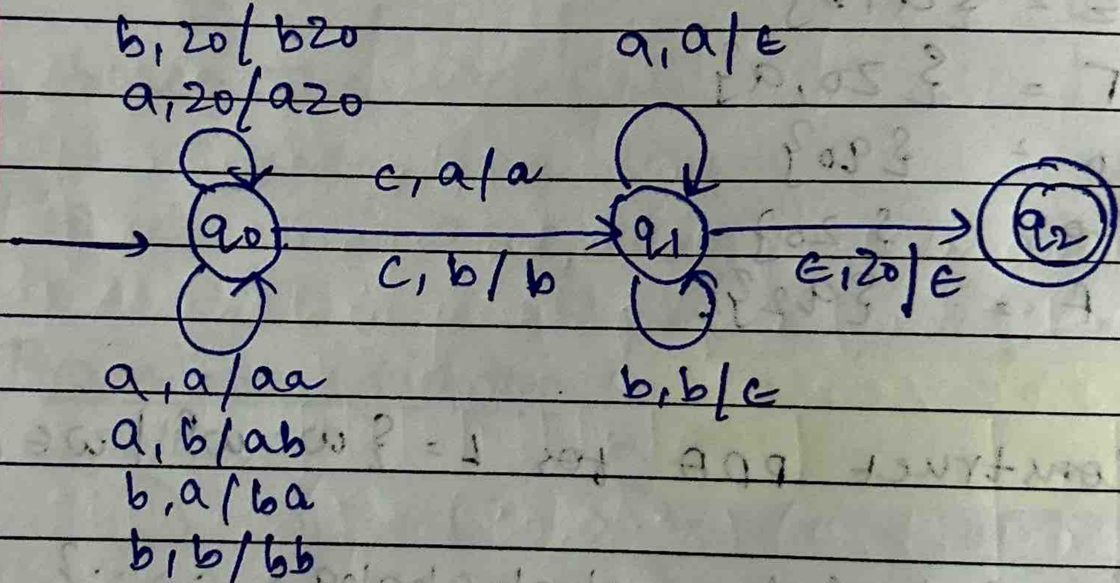
Case 2: $\delta(q_0, c, a) = (q_1, a)$
$\delta(q_0, c, b) = (q_1, b)$

keep the stack as it is

Case 3: $\delta(q_1, a, a) = \delta(q_1, \epsilon)$ } Pop operation
$\delta(q_1, b, b) = \delta(q_1, \epsilon)$

Case 4: $\delta(q_1, \epsilon, z_0) = \delta(q_2, \epsilon)$ } when we reach end of stack

Transition diagram



$M = \{Q, \Sigma, \Gamma, \delta, z_0, F\}$

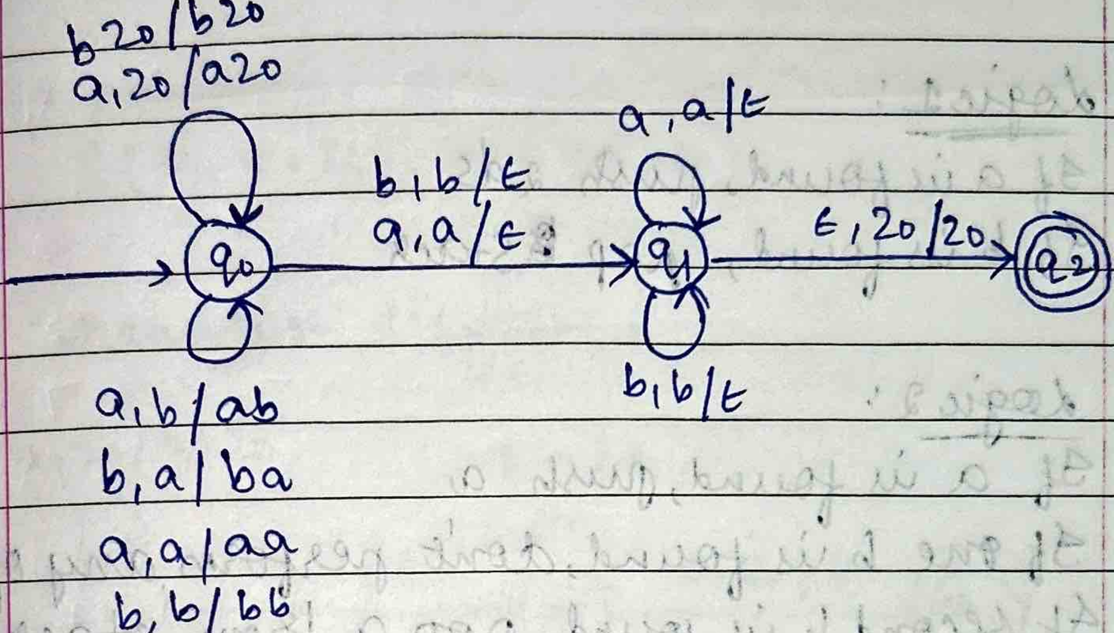
$Q = \{q_0, q_1, q_2\}$
$\Sigma = \{a, b\}$
$\Gamma = \{a, b\}$
$q_0 = \{q_0\}$
$z_0 = \{z_0\}$
$F = \{q_2\}$

$ww^R \to B16$

$x = x^R \to b20$

3. Design PDA for $L = \{ww^R \mid w \in (a+b)^*\}$.

$L = \{abba, abaaba, aaaaaa \ldots\}$



String: abba

$(q_0, abba, Z_0) \vdash (q_0, bba, aZ_0)$

$\cancel{\vdash (q_0, ba, baZ_0)}$

~~NC~~ $\cancel{\vdash (q_0, a, bbaZ_0)}$

~~C~~ $\cancel{\vdash (q_1, a, aZ_0)}$

$\vdash (q_0, ba, baZ_0)$

NC | C

$(q_0, a, bbaZ_0)$ | (q_1, a, aZ_0)

$(q_0, \epsilon, abbaZ_0)$ | $\vdash (q_1, \epsilon, Z_0)$

X transition doesnt exist | $\vdash (q_2, Z_0)$

final state is reached

$a^n b^{2n} \rightarrow$ B3
B18 (2nd part)

... for a language $\{a^n b^{2n} \mid n \geq 1\}$

Sol: $L = \{abb,\ aabbbb, \ldots\}$

Logic 1:
If a is found, push 2 a's
If b is found, pop stack

Logic 2:
If a is found, push a
If one b is found, don't perform any operation
If second b is found, pop a from stack.

a	a	b	b	b	b	ϵ

a
a
z_0

$\delta(q_0, a, z_0) = (q_0, aaz_0)$

a	a	b	b	b	b	ϵ

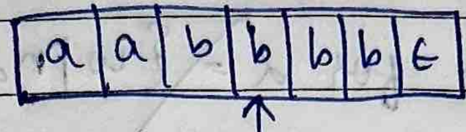
a
a
a
a
z_0

$\delta(q_0, a, a) = (q_0, aaaaz_0)$

a	a	b	b	b	b	ϵ

a
a
a
z_0

$\delta(q_0, b, a) = (q_1, \epsilon)$

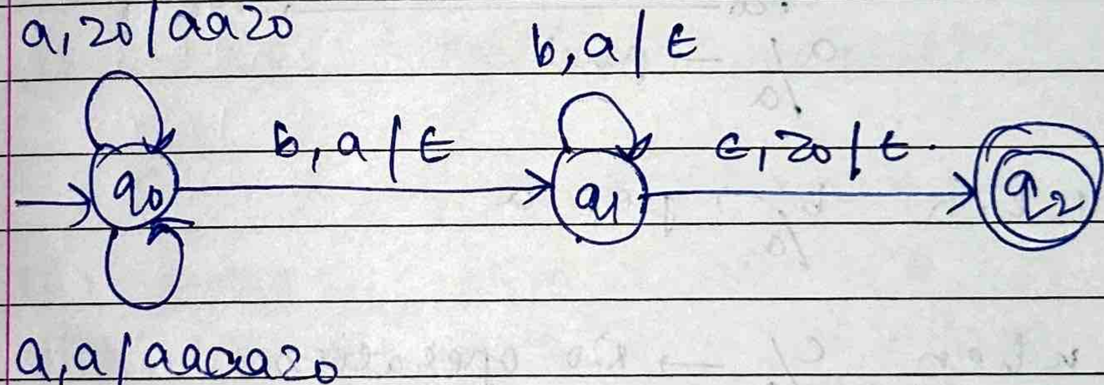


$\delta(q_1, b, a) = (q_1, \epsilon)$

$\vdots$

$\delta(q_1, \epsilon, z_0) = (q_2, \epsilon)$

Transition diagram :



$M = \{Q, \Sigma, \Gamma, \delta, q_0, z_0, F\}$

$Q = \{q_0, q_1, q_2\}$

$\Sigma = \{a, b\}$

$\Gamma = \{z_0, a\}$

$q_0 = \{q_0\}$

$z_0 = \{z_0\}$

$F = \{q_2\}$

Extra

Q) ~~Construct PDA for $L = \{w \mid n_a(w) = n_b(w)\}$~~

~~Sol: $L = \{\epsilon, aabb, abab, baab, \dots\}$~~

~~We have to follow the following steps:~~

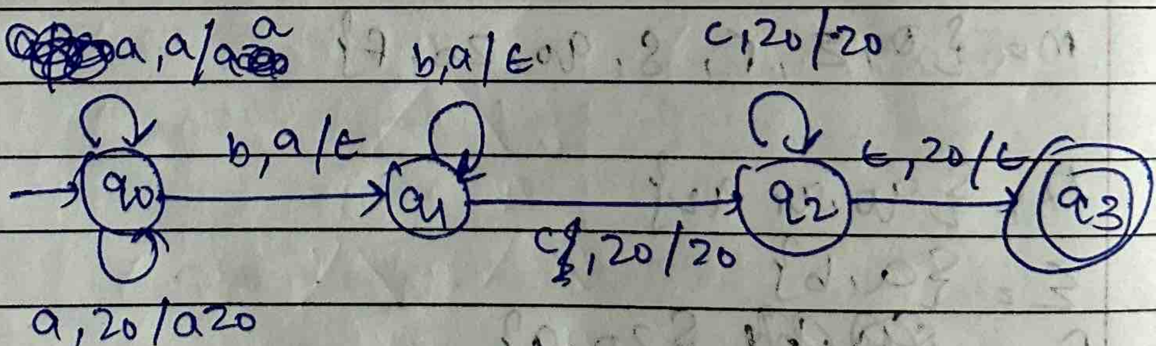
(a) $L = \{a^n b^n c^m \mid n, m \geq 1\}$

when $a/z_0 \rightarrow$ push ~~top of stack~~

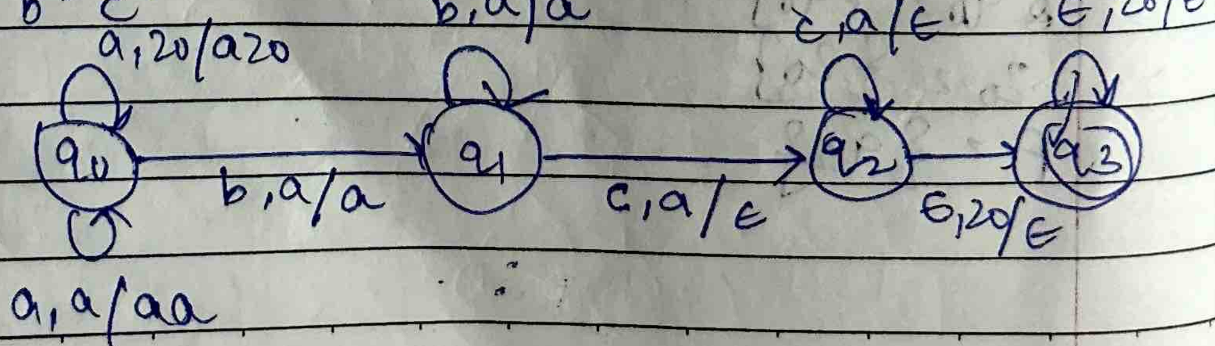
$a/a \rightarrow$ push

when $b/a \rightarrow$ pop a

when $c/z_0 \rightarrow$ No operation



(b) $a^n b^m c^n$



Conversion of CFG to PDA:

Rule 1 terminals: a, b, +, -, 0, 1 $\quad \delta(q, +, +) = (q, \epsilon)$

Rule 2. Non-terminal : $A \to \alpha$

$$\delta(q, \epsilon, A) = (q, \alpha)$$

1. Contruct PDA for CFG:

$S \to AB$

$A \to 0S \mid 0$

$B \to 1S \mid 1$

(i) $S \to AB$ (~~S~~ Non-terminal)

$S \to AB, \quad \delta(q, \epsilon, S) = (q, AB)$

(ii) $A \to 0S, \quad \delta(q, \epsilon, A) = (q, 0S)$

(iii) $A \to 0, \quad \delta(q, \epsilon, A) = (q, 0)$

(iv) $B \to 1S, \quad \delta(q, \epsilon, B) = (q, 1S)$

(v) $B \to 1, \quad \delta(q, \epsilon, B) = (q, 1)$

Here, the terminal symbols are 0, 1

so,

$\delta(q, 0, 0) = (q, \epsilon)$

$\delta(q, 1, 1) = (q, \epsilon)$

2. Construct PDA for the following CFG and test whether "abbabb" is in $N(P)$

$R = \{S \to AA \mid a$

$A \to SA \mid b\}$

Set of variables : $\{S, A\}$

terminals : $\{a, b\}$

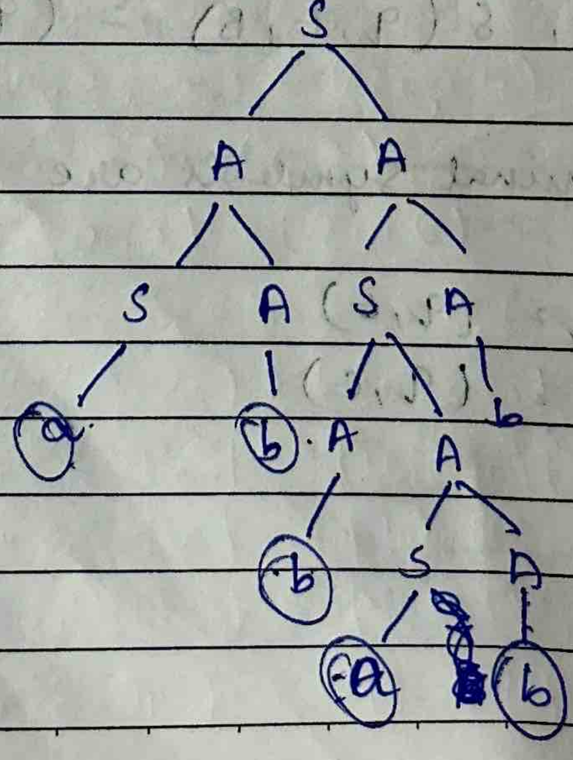
$\delta(q, \epsilon, S) = \{(q, AA), (q, a)\}$

$\delta(q, \epsilon, A) = \{(q, SA), (q, b)\}$

$\delta(q, a, a) = (\{q, \epsilon\})$

$\delta(q, b, b) = \{q, \epsilon\}$

abbabb



$\delta(q, abbabb, S)$

$\vdash \delta(q, abbabb, AA)$

$\vdash \delta(q, abbabb, SAA)$

$\vdash \delta(q, abbabb, aAA)$

$\vdash \delta(q, bbabb, AA)$

$\vdash \delta(q, \cancel{b}babb, \cancel{b}A)$

$\vdash \delta(q, babb, A)$

$\vdash \delta(q, babb, SA)$

$\vdash \delta(q, babb, AAA)$

$\vdash \delta(q, babb, bAA)$

$\vdash \delta(q, abb, AA)$

$\vdash \delta(q, abb, SAA)$

$\vdash \delta(q, abb, aAA)$

$\vdash \delta(q, bb, AA)$

$\vdash \delta(q, bb, bA)$

$\vdash \delta(q, b, A)$

$\vdash \delta(q, b, b)$

$\vdash \delta(q, \epsilon)$

Stack is Empty, string is accepted

PDA to CFG

CFG = (V, T, P, S)

Q (, ε ... Γ)

1. Eg: $M = (\{q_0, q_1\}, \{a, b\}, \{a, z_0\}, \delta, q_0, z_0, \phi\}$

$G = (V, T, P, S)$

→ Combine (Q, Γ, ...)

Variables ← $V = [S, [q_0 a q_0], [q_0 a q_1], [q_1 a q_0] [q_1 a q_1]$
$[q_0 z_0 q_0], [q_0 z_0 q_1], [q_1 z_0 q_0] [q_1 z_0 q_1]]$

Terminals ← $T = \{a, b\}$

Start ← $S = [q_0 z_0 q_0]$ } Combine Initial state,
$S = [q_0 z_0 q_1]$ } Total states, Top of stack.

Q condition:

$\delta(q_0, a, z_0) = (q_0, a z_0)$ $\delta(q_0, a, a) = (q_0, aa)$

$\delta(q_0, b, a) = (q_1, a)$ $\delta(q_1, b, a) = (q_1, a)$

$\delta(q_1, a, a) = (q_1, \epsilon)$ $\delta(q_1, \epsilon, z_0) = (q_1, \epsilon)$

Sol: (i) $\delta(q_0, a, z_0) = (q_0, a z_0)$ → if there are 2 symbols, 4 → P

$[q_0 z_0 q_0] \rightarrow a[q_0 a q_0] [q_0 z_0 q_0]$

$[q_0 z_0 q_0] \rightarrow a[q_0 a q_1] [q_1 z_0 q_0]$

$[q_0 z_0 q_1] \rightarrow a[q_0 a q_0] [q_0 z_0 q_1]$

$[q_0 z_0 q_1] \rightarrow a[q_0 a q_1] [q_1 z_0 q_1]$

$\delta(q_0, a, a) = (q_0, aa)$

$[q_0\ a\ q_0] \quad a\,[q_0\ a\ q_0]\,[q_0\ a\ q_0]$

$[q_0\ a\ q_0] \quad a\,[q_0\ a\ q_1]\,[q_1\ a\ q_1]$

$[q_0\ a\ q_1] \quad a\,[q_0\ a\ q_0]\,[q_0\ a\ q_0]$

$[q_0\ a\ q_1] \quad a\,[q_0\ a\ q_1]\,[q_1\ a\ q_1]$

$\delta(q_0, b, a) = (q_1, a)$

$[q_0\ a\ q_0] \rightarrow b\,[q_1\ a\ q_0]$

$[q_0\ a\ q_1] \rightarrow b\,[q_1\ a\ q_1]$

$\delta(q_1, b, a) = (q_1, a)$

$[q_1\ a\ q_0] \rightarrow b\,[q_1\ a\ q_0]$

$[q_1\ a\ q_1] \rightarrow b\,[q_1\ a\ q_1]$

$\delta(q_1, a, a) = (q_1, \epsilon)$

$[q_1\ a\ q_1] \rightarrow \cancel{q_1}\ a$

$\delta(q_1, \epsilon, z_0) = (q_1, \epsilon)$

$[q_1, z_0, q_1] \rightarrow [\epsilon]$

Q/B

Part - A

$n_a(x) = n_b(x) \rightarrow$ B17

... equal no. of a's and equal no. of b's.

(Or)

$L = \{ w \mid n_a(w) = n_b(w) \}$

Sol: $L = \{ \epsilon, ab, aabb, aaabbb \ldots\ldots \}$

Step 1: If stack is Empty & input symbol is a or b → Push

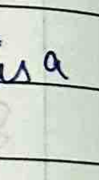
Step 2: If top of stack is a, Input is a
↳ Push

If top of stack is a, Input is b
↳ Pop

Step 3: If top of stack is b, Input is b
↳ Push

If top of stack is b, Input is a
↳ Pop.

$\delta(q_0, a, z_0) = \delta(q_0, az_0)$



$\delta(q_0, b, z_0) = \delta(q_0, bz_0)$

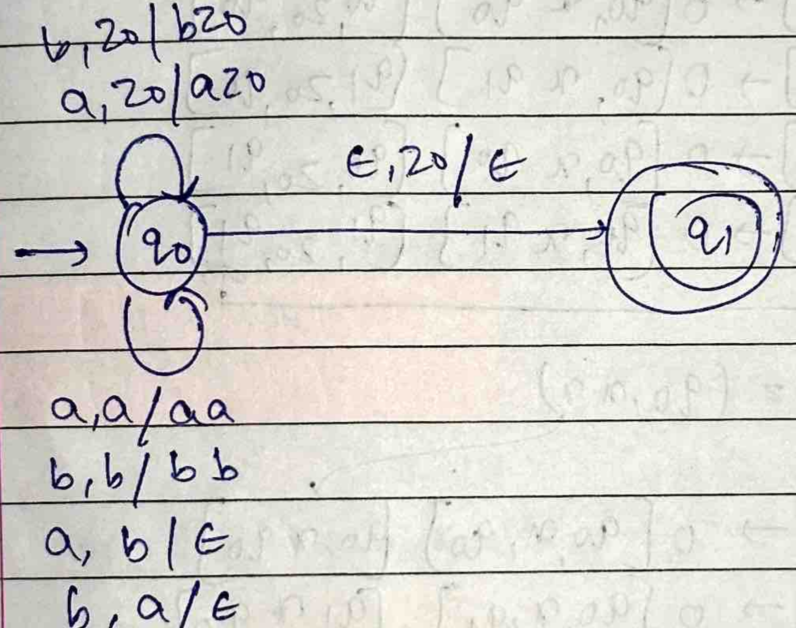
$\delta(q_0, a, a) = \delta(q_0, aa)$

$\delta(q_0, b, b) = \delta(q_0, bb)$

$\delta(q_0, a, b) = \delta(q_0, \epsilon)$

$\delta(q_0, b, a) = \delta(q_0, \epsilon)$

$\delta(q_0, \epsilon, Z_0) = \delta(q_1, \epsilon)$



$PDA = \{(q_0, q_1), \{a, b\}, \{Z_0, a, b\}, \delta, q_0, q_1, Z_0\}$

PDA TO CFG → B2.

(3) ... o CFG

$\delta(q_0, 0, Z_0) = (q_0, xZ_0)$

$\delta(q_0, 0, x) = (q_0, xx)$

$\delta(q_0, 1, x) = (q_1, \epsilon)$

$\delta(q_1, 1, x) = (q_1, \epsilon)$

$\delta(q_1, \epsilon, x) = (q_1, \epsilon)$

$\delta(q_1, \epsilon, Z_0) = (q_1, \epsilon)$

(i) $S \to (q_0, Z_0, q_0)$

$S \to (q_0, Z_0, q_1)$

(ii) $\delta(q_0, 0, Z_0) = (q_0, x Z_0)$

$[q_0, Z_0, q_0] \to 0\,[q_0, x\, q_0]\,[q_0, Z_0, q_0]$

$[q_0, Z_0, q_0] \to 0\,[q_0, x\, q_1]\,[q_1, Z_0, q_0]$

$[q_0, Z_0, q_1] \to 0\,[q_0, x\, q_0]\,[q_0, Z_0, q_1]$

$[q_0, Z_0, q_0] \to 0\,[q_0, x\, q_1]\,[q_1, Z_0, q_1]$

(iii) $\delta(q_0, 0, x) = (q_0, x x)$

$[q_0, x, q_0] \to 0\,[q_0, x, q_0]\,[q_0\, x\, q_0]$

$[q_0, x, q_0] \to 0\,[q_0, x, q_1]\,[q_1\, x\, q_0]$

$[q_0, x, q_1] \to 0\,[q_0, x, q_0]\,[q_0\, x\, q_1]$

$[q_0, x, q_1] \to 0\,[q_0, x, q_1]\,[q_1\, x\, q_1]$

(iv) $\delta(q_0, 1, x) = (q_1, \epsilon)$

opp

$[q_0\; x\; q_0] \to \{1\}$

(v) $\delta(q_1, 1, x) = (q_1, \epsilon)$

$[q_1, x, q_0] \to 1$

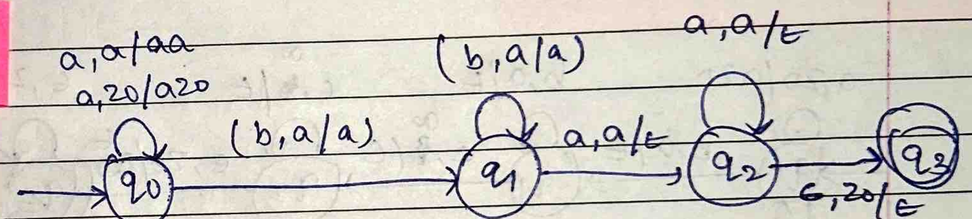
(vi) $\delta(q_1, \epsilon, x) = (q_1, \epsilon)$

$[q_1, x, q_0] \to \epsilon$

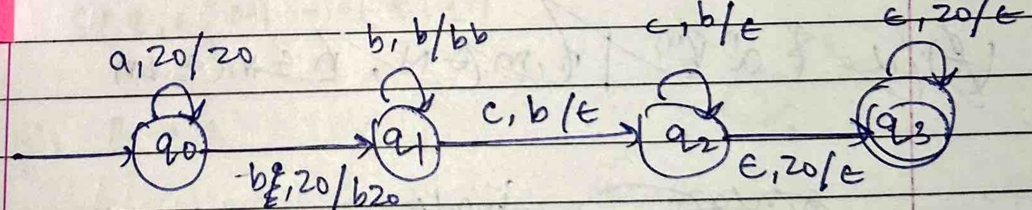
(vii) $\delta(q_1, \epsilon, z_0) = (q_1, \epsilon)$

$[q_1, z_0, q_0] \to \epsilon$.

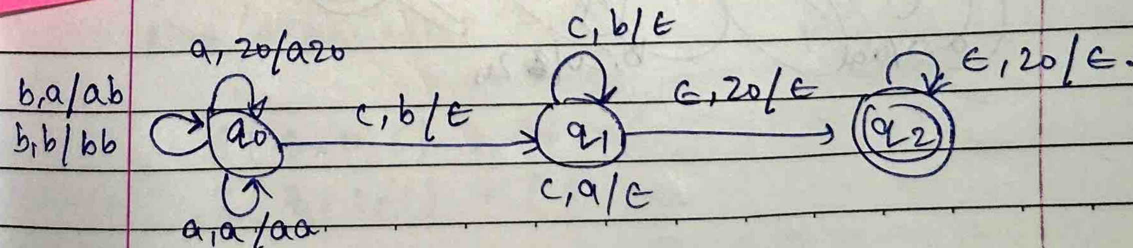
(5) a) $L = \{a^n b^m a^n \mid m, n \in N\}$



b) $L = \{a^n b^m c^m \mid m, n \in N\}$



c) $L = \{a^i b^j c^k \mid i, j, k \in N, \; i+j=k\}$



(vi) $\delta(q_1, \epsilon, x) = (q_1, \epsilon)$

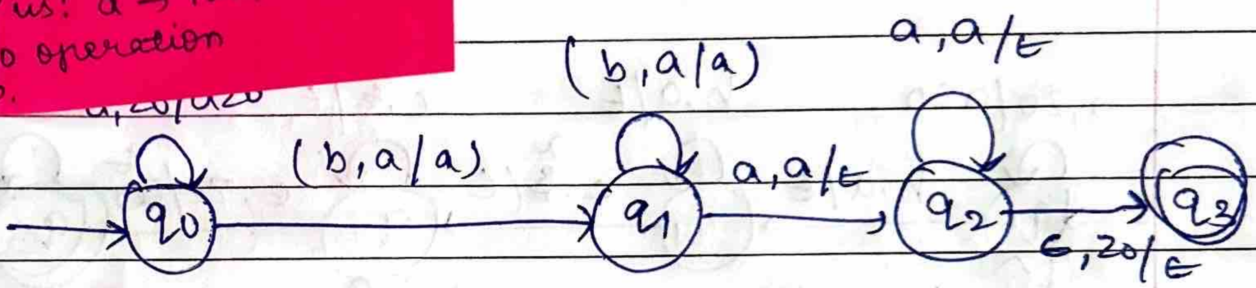
$[q_1, x, q_0] \rightarrow \epsilon$

(vii) $\delta(q_1, \epsilon, z_0) = (q_1, \epsilon)$

$[q_1, z_0, q_0] \rightarrow \epsilon$.

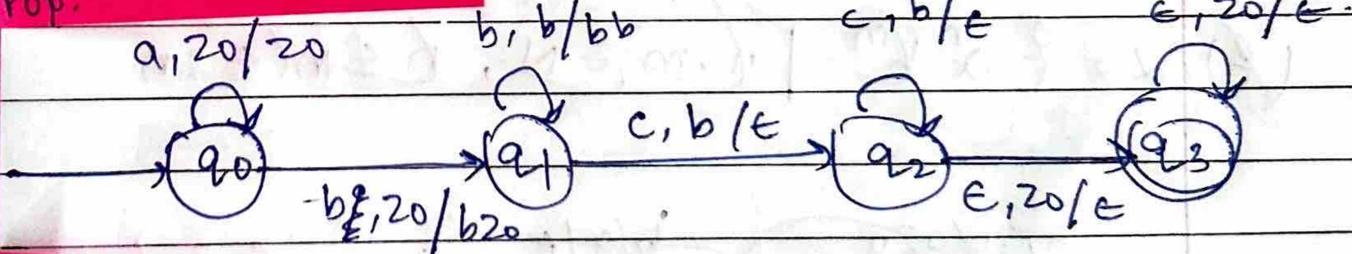
5 a) $\alpha = \{a^n b^m a^n$ /m, n ∈ N}

when I/P is: a → push
b → No operation
a → Pop.



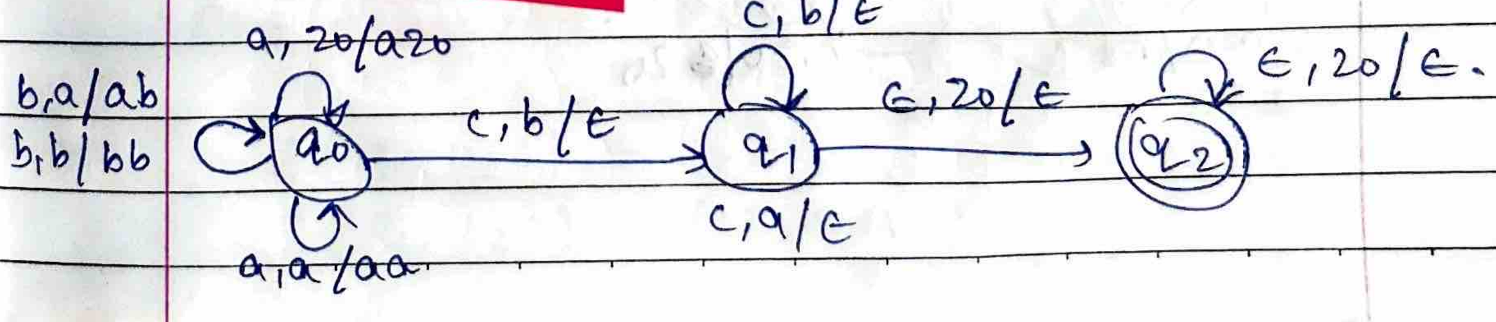
when I/P is a : No operation
b → push
c → pop.

m, n ∈ N}



when I/P is a : push
b : push
c : pop

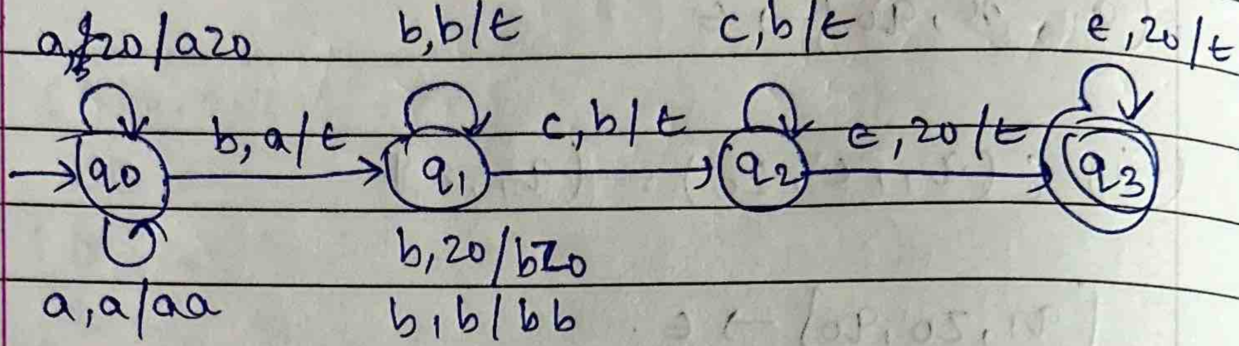
| i, j, k ∈ N, i+j = k}



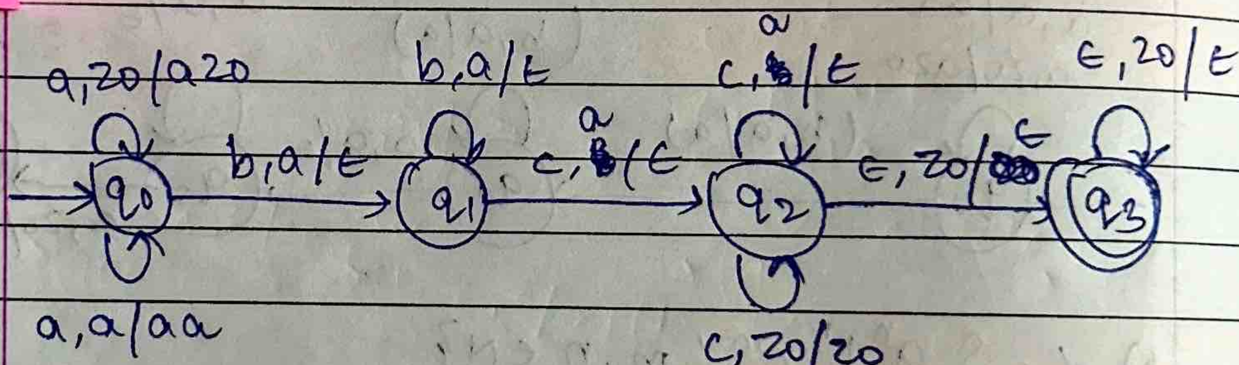
TOS : Top of Stack

when I/P is a : Push, z0, b
I/P = b & top of stack = a : Pop
I/P = c & top of Stack is b : pop.

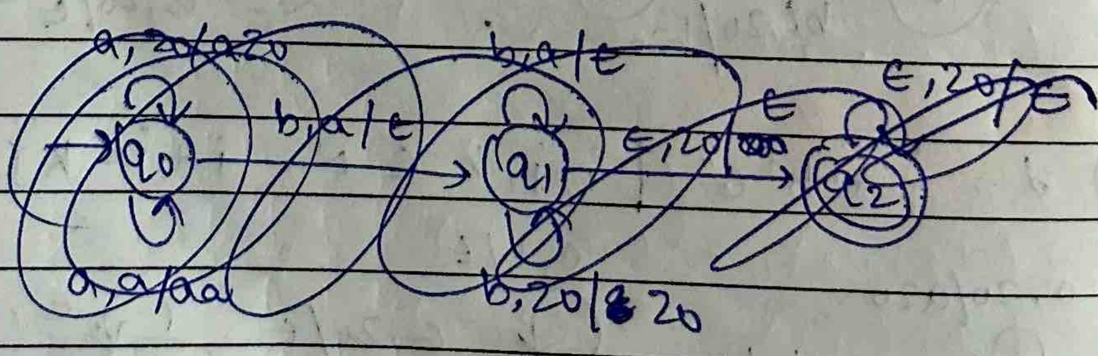
(d) $L = \{a^i b^j c^k \mid i, j, k \in N, \; j = i + k\}$



) $L = \{a^i b^j c^k \mid i, j, k \in N, \; i > j\}$



~~(f) $L = \{a^n b^m \mid n, m \in N, \; n \le m \le 2n\}$~~

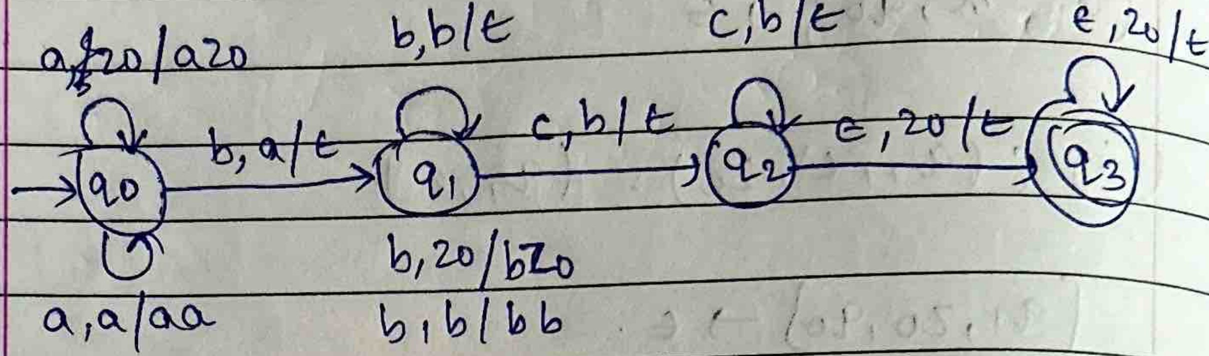


TOS : Top of Stack

Page No.:
Date:

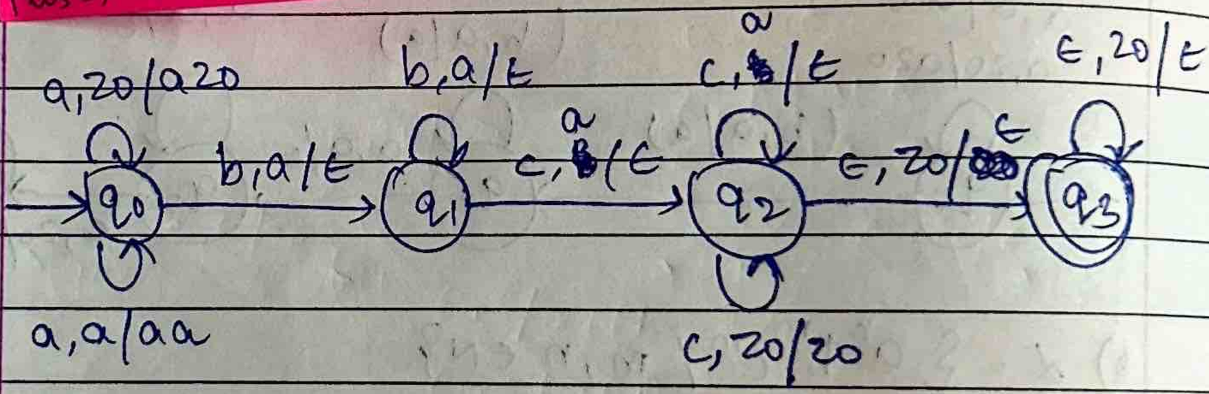
when I/P is a : Push z0, b
I/P = b & top of stack = a : Pop
I/P = c & top of Stack is b : pop.

(d) $L = \{a^i b^j c^k \mid i, j, k \in N,\ j = i + k\}$

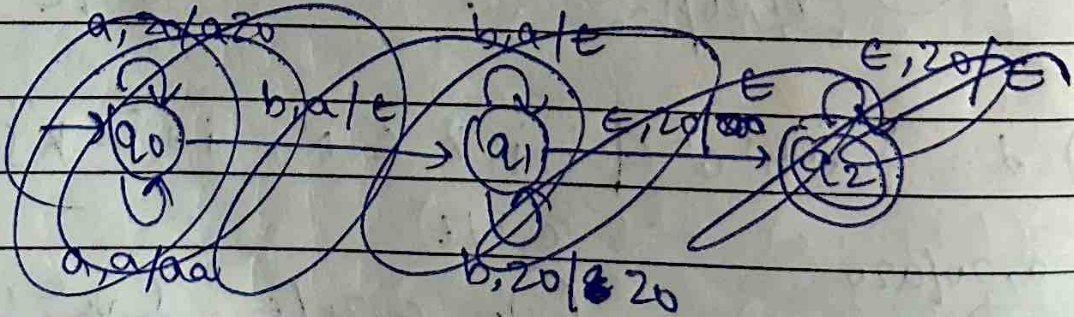


when a is I/P → push
when I/P is b, TOS is a → Pop
when I/P is c, TOS is a, z0 → Pop.

$i, j, k \in N,\ i > j\}$



~~(f) $L = \{a^n b^m \mid n, m \in N,\ n \leq m \leq 2n$~~



Part - B

S
0A
0AB
011

} Left most deriva-tion

(6) Given CFG: $S \to 0A$,
$A \to 0AB / 1$,
$B \to 1$.

Convert to PDA.

Sol: set of Variables: $\{S, A, B\}$
set of non-terminals: $\{0, 1\}$

(i) $S \to 0A$

$\delta(q, \epsilon, S) = (q, 0A)$

(ii) $A \to 0AB \mid 1$

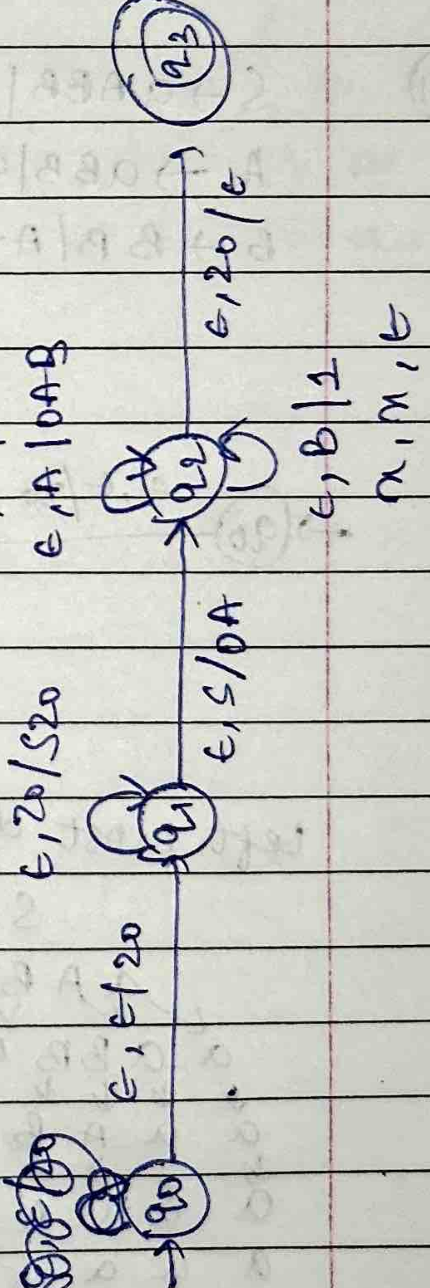
$\delta(q, \epsilon, A) = (q, 0AB)$
$\delta(q, \epsilon, A) = (q, 1)$

(iii) $B \to 1$

$\delta(q, \epsilon, B) = (q, 1)$

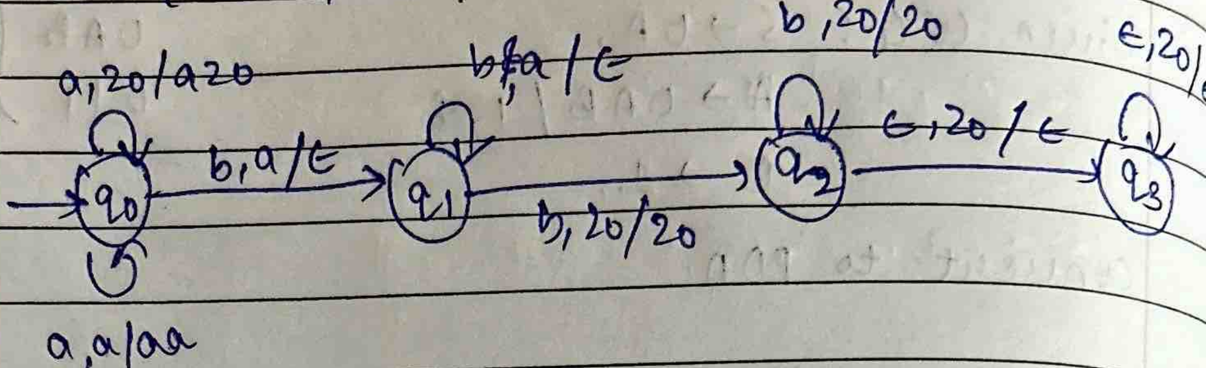
Non terminals: $\{0, 1\}$

$\delta(q, 0, 0) = (q, \epsilon)$
$\delta(q, 1, 1) = (q, \epsilon)$

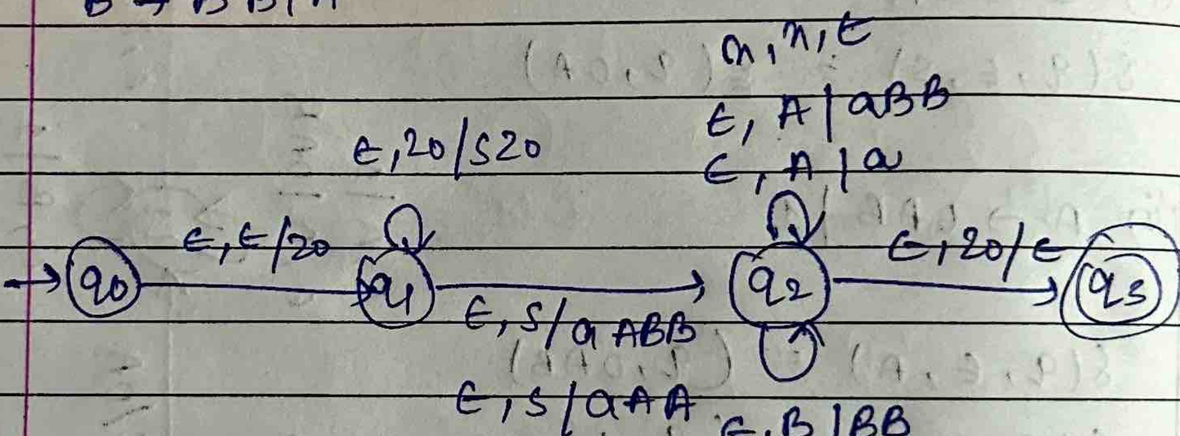


I/P = a, TOS = z_0/a → push
I/P = b, TOS = a → pop
I/P = b, TOS = z_0 → No operation

B(5): $L = \{a^m b^n / n > m\}$



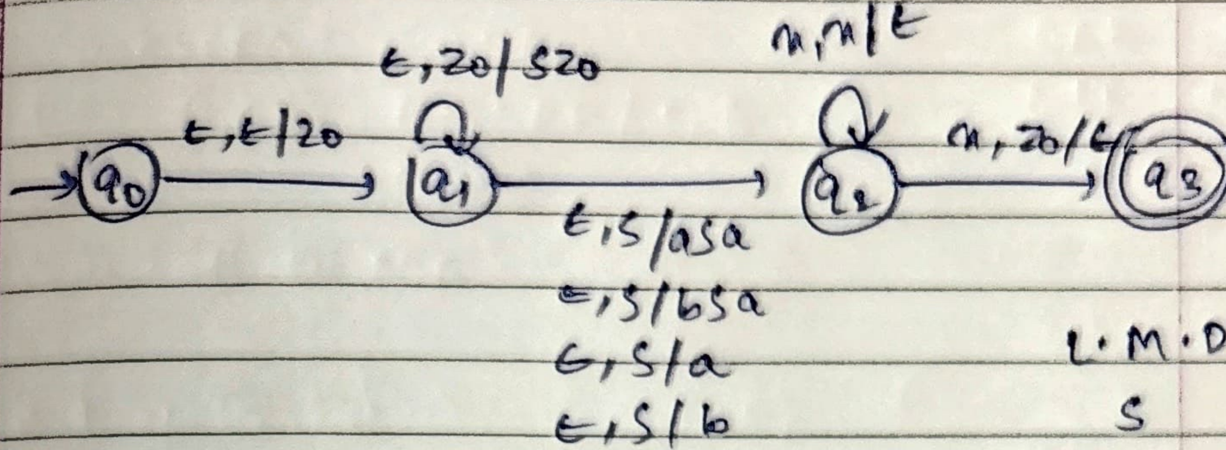
B(11) S → aABB | aAA
A → aBB | a
B → BB | A.



Left most derivation

S
a A B B
a a B B B B
a a A B B B
a a a A B B
a a a a B B
a a a a A B
a a a a a A
a a a a a a

B (12) S → aSa | bSb | a | b



L.M.D

S

a S a

a b S b a

a b a b a.