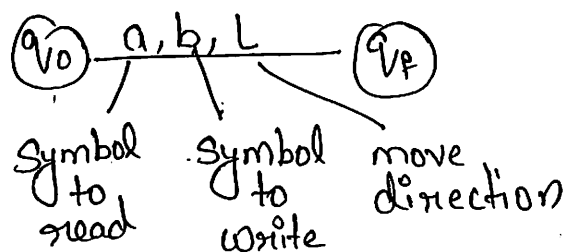


Rules of operation

1) Read the current symbol

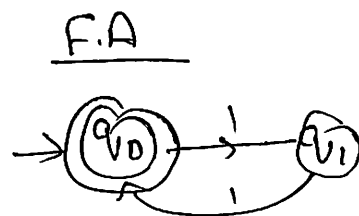
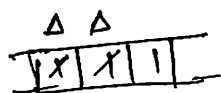
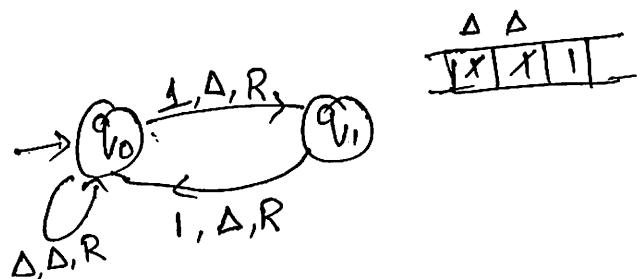
↓
update (write) the same cell

↓
move exactly one cell (left or right)



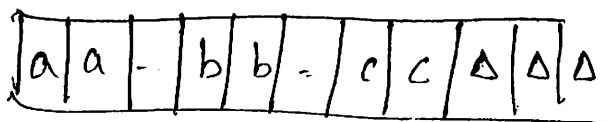
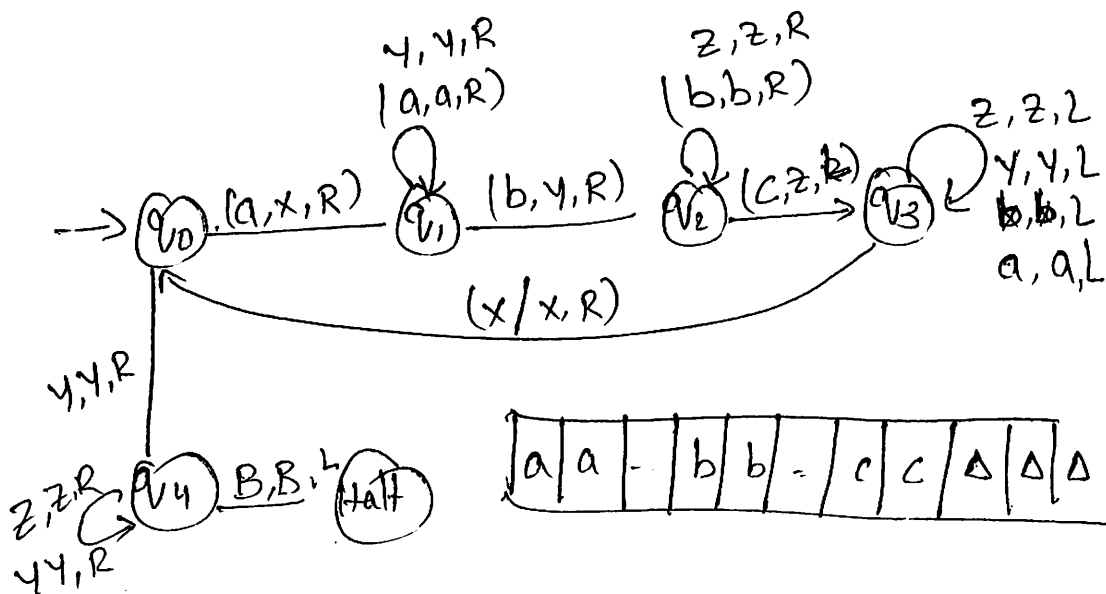
→ Construct T.M for string having even no. of 1's
over $\Sigma = \{1\}$

$L = \{11, 1111, \dots\}$



→ Construct T.M for string having $a^n b^n c^n, n \geq 1$

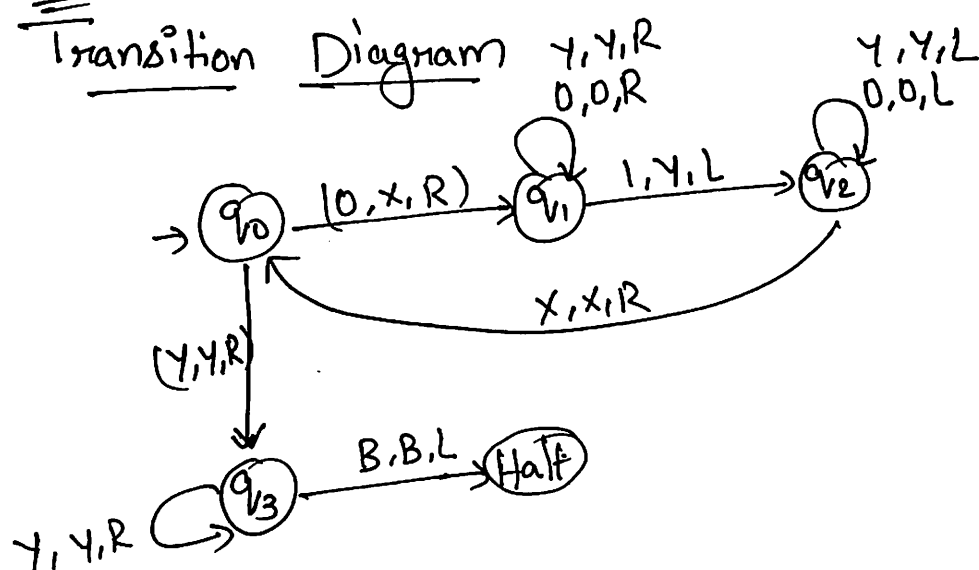
$L = \{abc, aabbcc, aaabbbccc, \dots\}$



→ Construct a Turing machine for $L = \{0^n 1^n \mid n \geq 1\}$
 Transition function is given as:

States	0	1	x	y	B
q_0	(q_1, x, R)			(q_3, y, R)	
q_1	$(q_1, 0, R)$	(q_2, y, L)		(q_1, y, R)	
q_2	$(q_2, 0, L)$		(q_0, x, R)	(q_2, y, L)	
q_3				(q_3, y, R)	Halt

Sol



Consider 0011

B	0	0	1	1	B
---	---	---	---	---	---

 -

$$\delta(q_0, 0) = (q_1, x, R)$$

B	x	0	1	1	B
---	---	---	---	---	---

$$\delta(q_1, 0) = (q_1, 0, R)$$

B	x	0	1	1	B
---	---	---	---	---	---

$$\delta(q_1, 1) = (q_2, y, L)$$

B	x	0	y	1	B
---	---	---	---	---	---

$$\delta(q_2, 0) = (q_2, 0, L)$$

B	x	<u>0</u>	y	1	B
---	--------------	----------	---	---	---

$$\delta(q_2, x) = (q_2, x, R)$$

B	x	<u>0</u>	y	1	B
---	---	----------	---	---	---

$$\delta(q_0, 0) = (q_1, x, R)$$

B	x	x	<u>y</u>	1	B
---	---	---	----------	---	---

$$\delta(q_0, y) = (q_1, y, R)$$

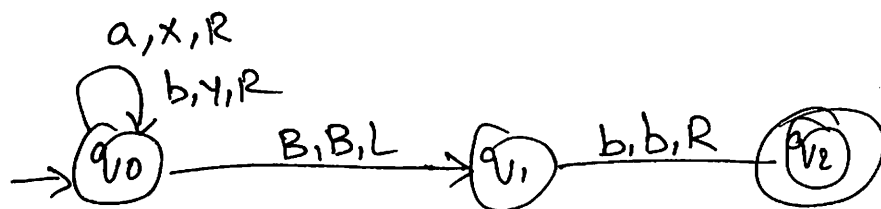
B	x	x	y	<u>1</u>	B
---	---	---	---	----------	---

$$\delta(q_1, 1) = (q_2, y, L)$$

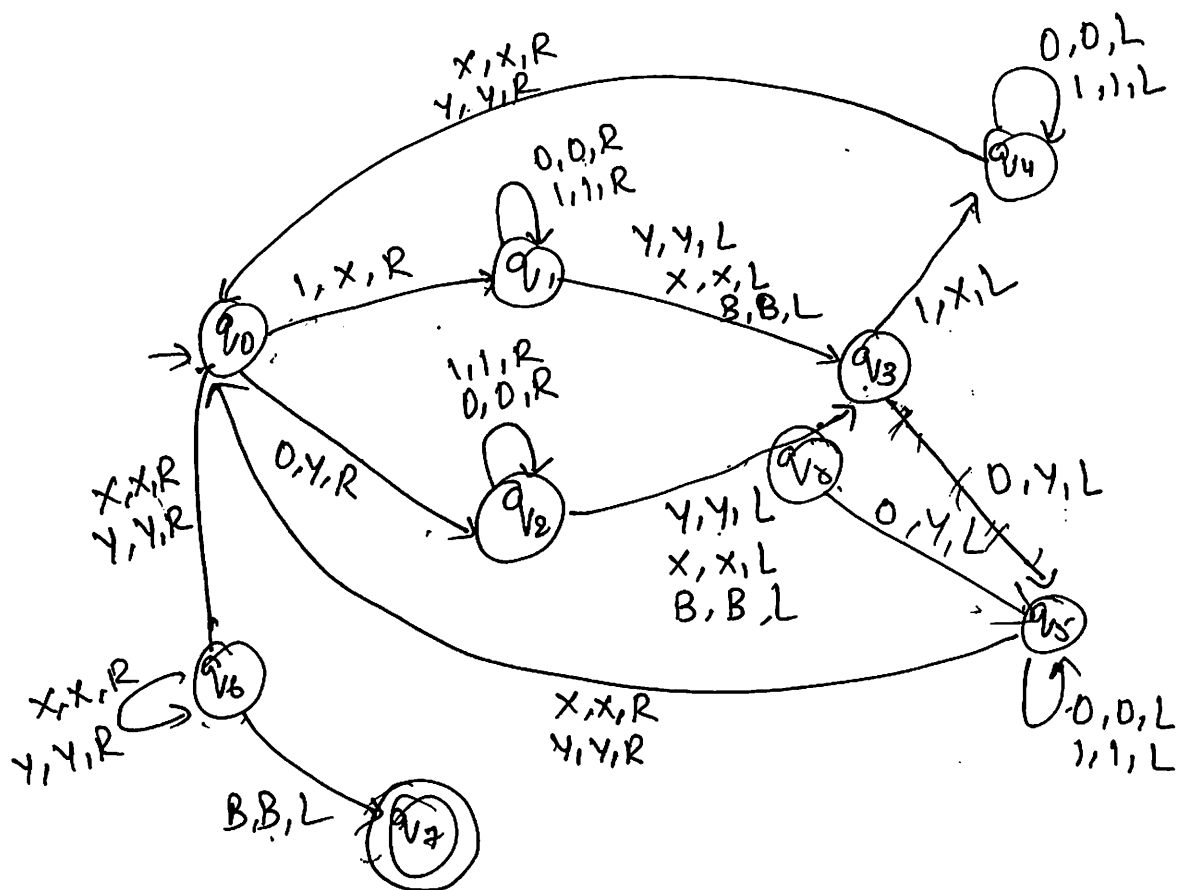
B	x	x	y	y	B
---	---	---	---	---	---

Continue the process until it halts

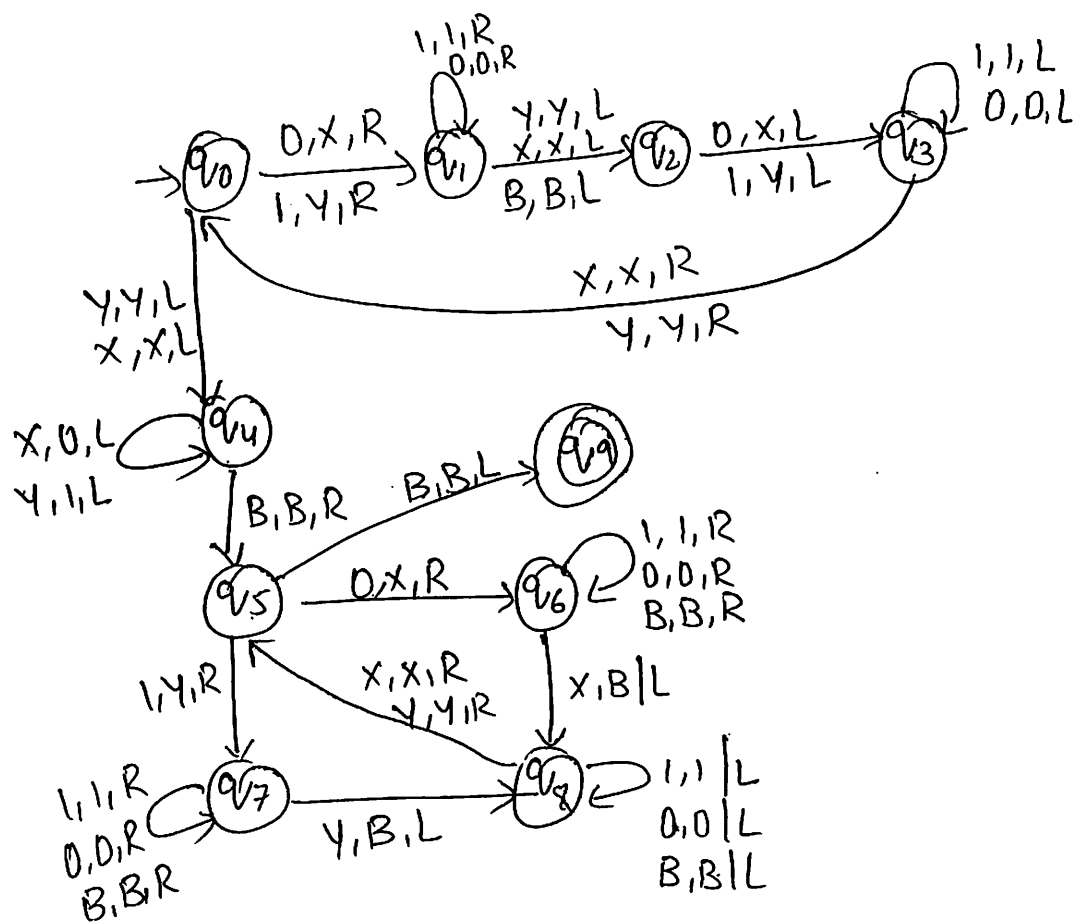
→ $\Sigma = \{a, b\}$ & String ends with $\#b'$



$$\rightarrow L = \{ ww^R \mid w \in \{0,1\}^* \}$$



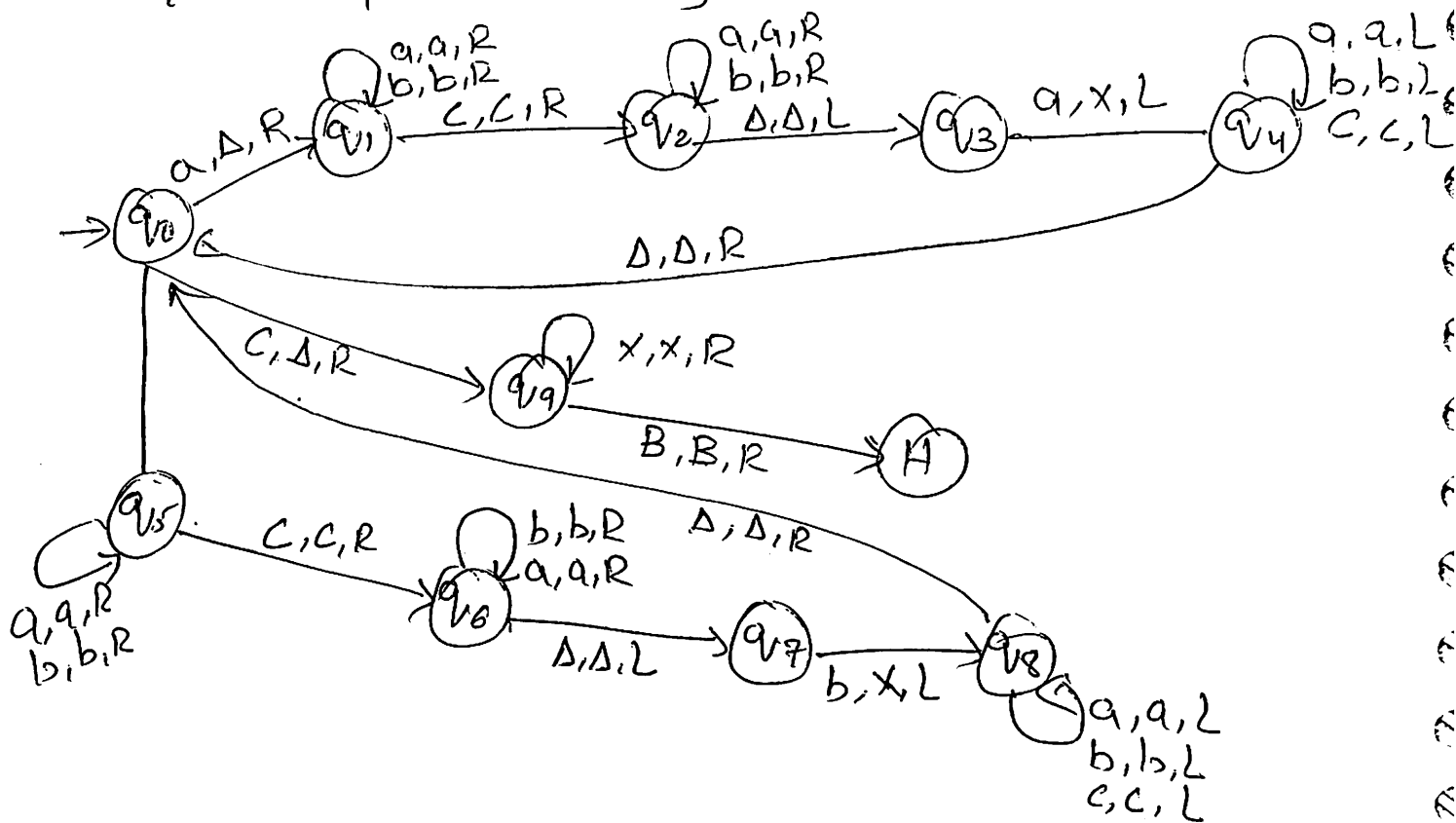
→ T.M for $L = \{ww \mid w \in \{0,1\}^*\}$



i/p - aba.aba / 010.010

010 010	⇒ XYX.XYX
X10010	⇒ X40.XYX
X1001X	= X10.XYX
XY001X	B010.XYX
X400YX	X10.BYX
X4X0YX	X40BBX
X4X8YX	X4XBBB

$$L = \{wcw^R \mid w \in (a,b)^*\}$$



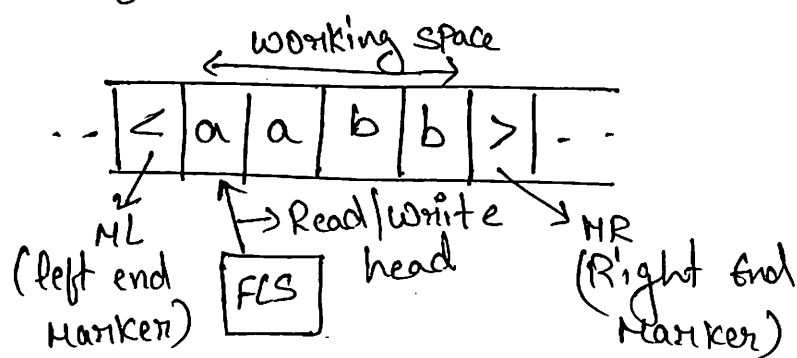
Church's thesis (or) Church Turing thesis:

Linear Bounded Automata [LBA]

A Turing machine that uses only the tape space occupied by the input string is called as LBA. A LBA is described by 8 tuple notation

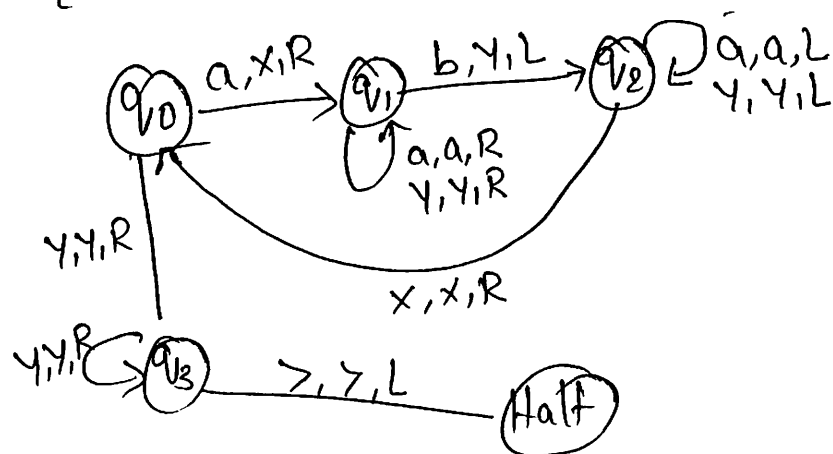
i.e. $M = (Q, \Sigma, q_0, \delta, F, \Gamma, \langle, \rangle)$

$$\delta: Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R\}$$



1) $L = \{a^n b^n \mid n \geq 0\}$

$L = \{ab, aabb, aaabbb, \dots\}$



$aabb$
 $xa\gamma b$
 $xx\gamma b$
 $xx\gamma\gamma$

It is similar to multi stack Turing Machine but the only difference between this is in place of each stack, there is a counter, which contains non -ve integers. Each move of counter machine depends on its state, current i/p symbol.

In one move of counter machine, CM can

- a) change state
- b) add or subtract one from any one of its counters.

-ve counters are not allowed at all

→ Counter Machine is similar to restricted multi stack machine with following restrictions.

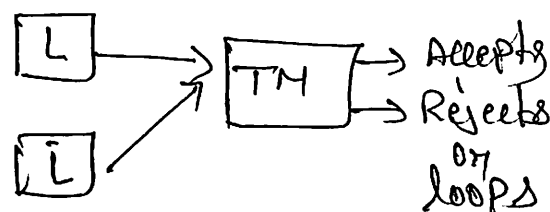
- i) There are only two stack symbols z_0, x
- ii) z_0 is bottom of stack marker. It is initially on each stack.
- iii) Replace z_0 only by string of the form $x^i z_0$; $i \geq 0$
- iv) Replace x only by x^i , $i \geq 0$ i.e. z_0 appears on the bottom of each stack & all other stack symbols are x
- v) Every language accepted by CM is recursively enumerable.

Recursively enumerable language:

A language L is ^{rec_{en}} enumerable then there is a Turing machine that accepts every word in L & either rejects or loops for every string in $\text{Lang } \bar{L}$ (i.e. complement of L)

$$\text{Accept}(M) = L$$

$$\text{Rejects}(M) \text{ or } \text{loops}(M) = \bar{L}$$



eg:- $L = aa^*$

The language accepted by this TM is aa^*

1) For the i/p aaa , it accepts

2) For the i/p aba , it loops

Recursive languages:

let L be a language L is said to be recursive language if there exists Turing machine 'T' that accepts every string or word in L & rejects every word or string in L' .

$$\text{Accept}(M) = L$$

$$\text{Reject}(M) = L'$$

Church's Hypothesis:

The assumption that the intuitive notion of Computable functions can be identified with Partial recursive functions

→ However this hypothesis cannot be proved

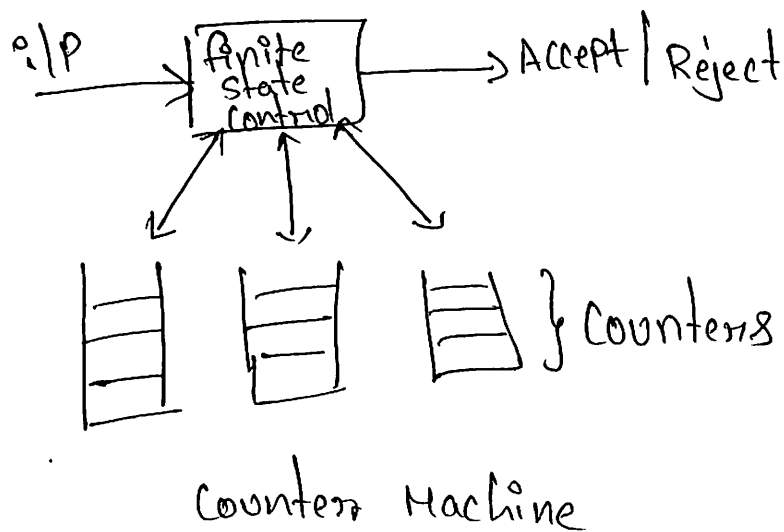
→ Computability of recursive functions is based on the following assumption

i) Each elementary function is Computable

ii) Let f be Computable function & g be another function which can be obtained by applying an elementary operations to f , then g becomes Computable function.

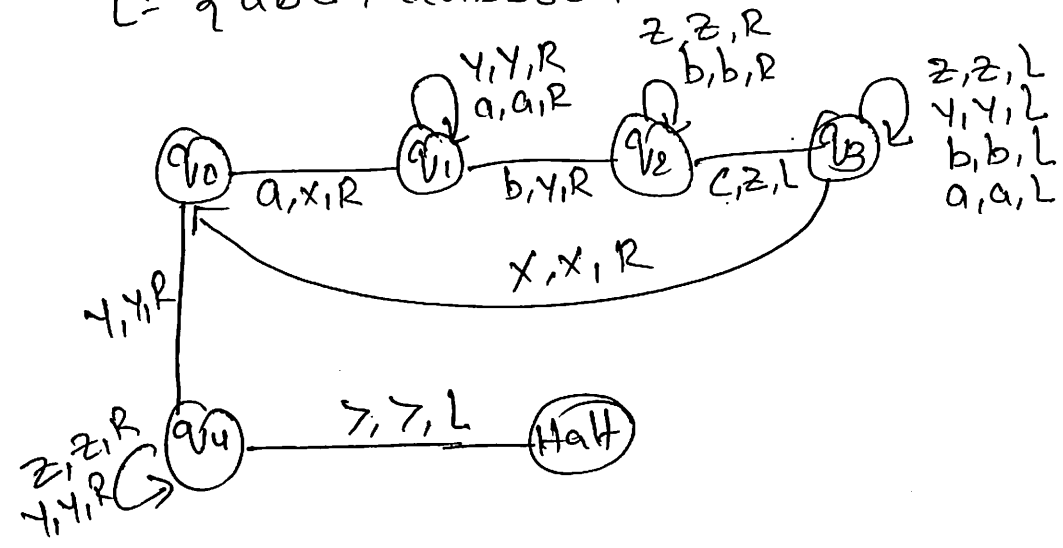
iii) Any function becomes Computable if it is obtained by rule i & ii

Counter Machine:



a) $L = \{a^n b^n c^n \mid n > 0\}$

$L = \{abc, aabbcc, aaabbbccc, \dots\}$



Context Sensitive Languages

Context sensitive grammar [CSG]

$$G = (V, T, P, S)$$

The production rules are in the following form

$\alpha \rightarrow \beta$ where α, β are set of terminals & non terminals. $|\beta| \geq |\alpha|$

$\rightarrow \in$ Productions are not allowed in CSG

① Find out the lang for the foll grammar

$$S \rightarrow aSBC \mid aBC$$

$$CB \rightarrow BC$$

$$aB \rightarrow ab$$

$$bB \rightarrow bb$$

$$bC \rightarrow bc$$

$$cC \rightarrow cc$$

$S \rightarrow abc$ [$ab \rightarrow ab$]
 abc [$bc \rightarrow bc$]
 abc

$S \rightarrow aSBC$ [$S \rightarrow aBC$]
 $\rightarrow aaBCBC$ [$ab \rightarrow ab$]
 $\rightarrow aabCBC$ [$CB \rightarrow BC$]
 $\rightarrow aabBCC$ [$bB \rightarrow bb$]
 $\rightarrow aabbCC$ [$bc \rightarrow bc$]
 $aabbcc$ [$cC \rightarrow cc$]
 $aabbcc$

$\therefore L = \{abc, aabbcc, \dots\}$

② Find the lang for the foll grammars.

$S \rightarrow ABC \mid ABCS$

$\therefore L = \{abc, bac, acb, \dots\}$

$AB \rightarrow BA$

$BC \rightarrow CB$

$BA \rightarrow AB$

$CB \rightarrow BC$

$CA \rightarrow AC$

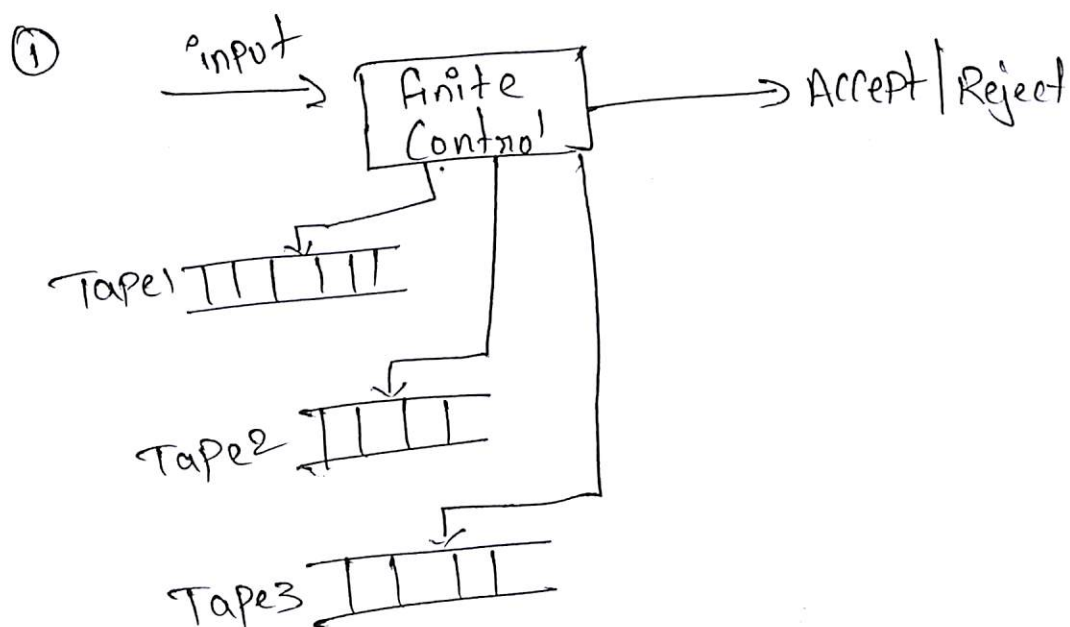
$A \rightarrow a$

$B \rightarrow b$

$C \rightarrow c$

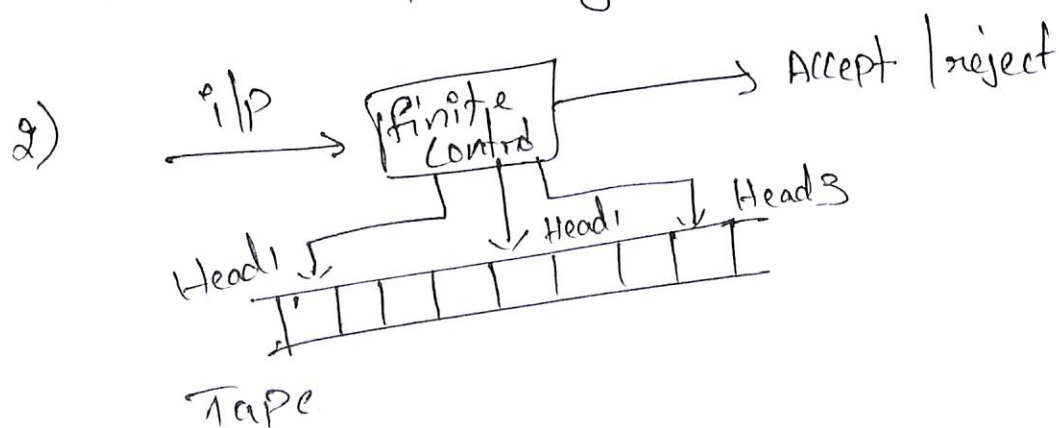
Types of Turing machine:

- 1) Turing machine with multiple tapes
- 2) Turing machine with one tape multiple heads
- 3) Turing machine with two dimensional tape
- 4) Turing machine with infinite tape.
- 5) non-deterministic Turing machine

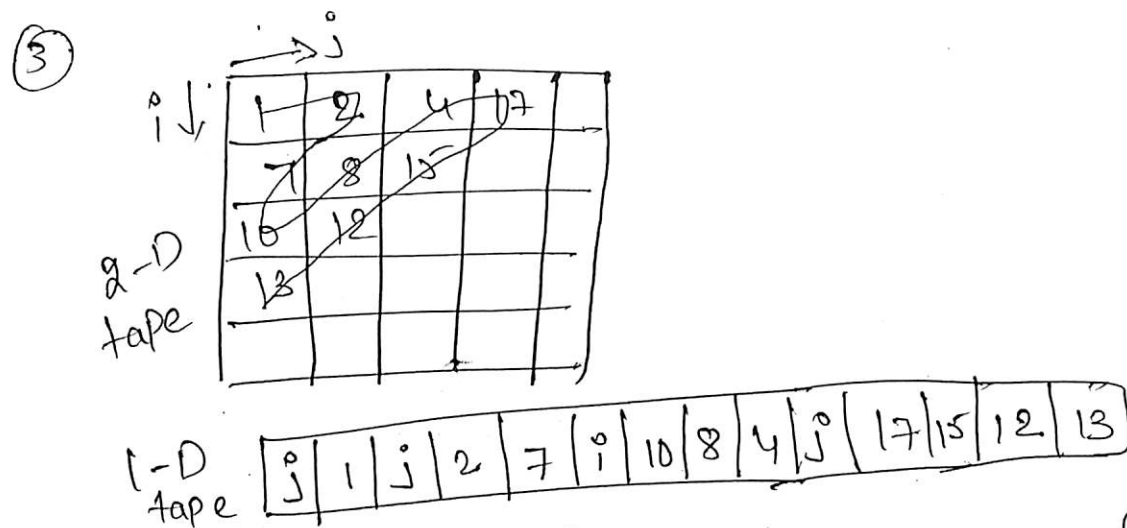


It has finite control, more than one tapes having its own read write head.

The language accepted by n-tape Turing machine can be accepted by 1-tape Turing machine



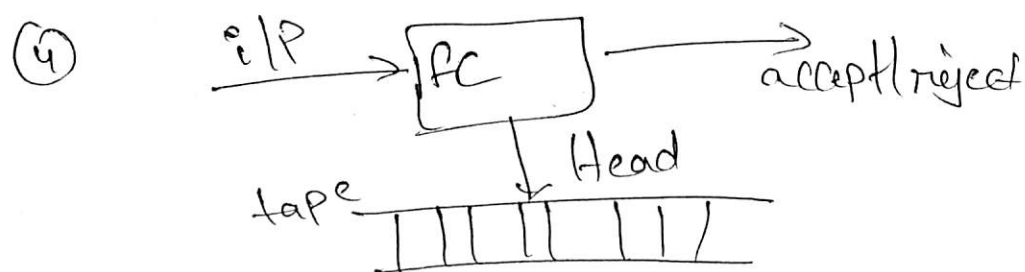
There are n -heads but in state only one head can move. This type of turing machine are powerful as one tape turing machine



It is divided into small squares formed due to corresponding rows & columns

→ Turing machine with 1-D tape is equally Powerful to that of 2-D tape

→ The head of 2-D tape moves one square up, down, left, right

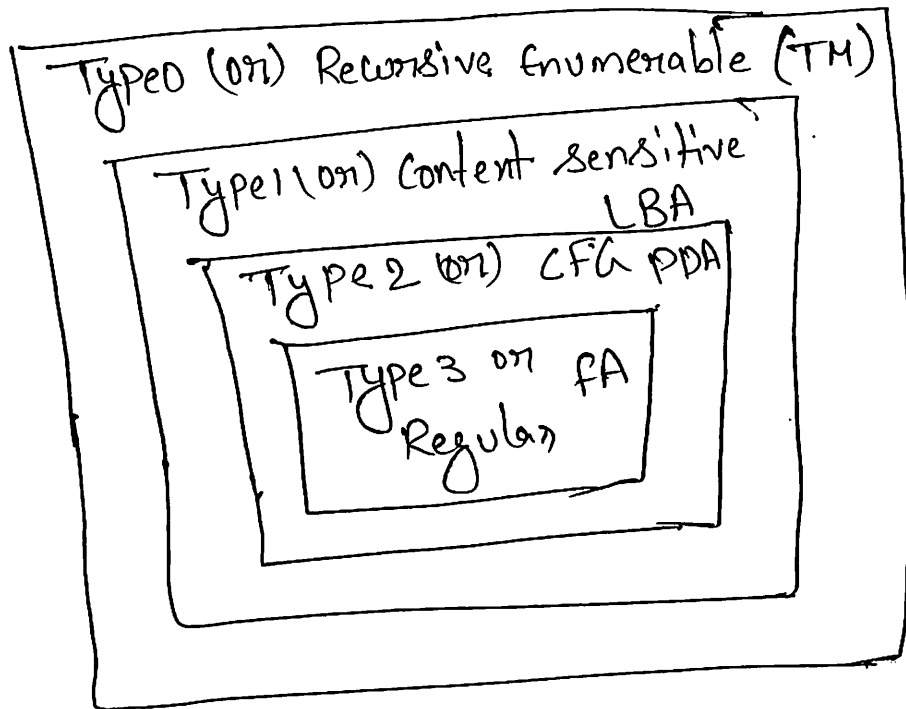


Turing machine that have one finite control & one tape which extends infinitely both directions. This type of turing machine is Powerful as one-tape TM whose tape has left end.

⑥ Similar to NFA, for any state & any i/p symbol. It can take any action from a set rather than a definite pre-determined action even in some situations it may take different actions at different times.

eg: Turing machine which accepts language L
 $L = \{ww \mid w \in (a+b)^*\}$ is non-deterministic TM

Chomsky Hierarchy of Languages:



One way of exhibiting relationship between languages in chomsky hierarchy: initial classification into 4 categories of language

- Type 0
- Type 1
- Type 2
- Type 3.