

Module - II
Regular Languages

Topic 1:

Regular Set: A special set of words over input alphabet Σ is called a regular set, if it follows the following properties recursively.

- (ii) If R_1 and R_2 are regular sets over Σ , then $R_1 \cup R_2$, $R_1 R_2$ & also R_1^* are Regular sets

Regular Set can be applied obtained by finite application of 1 and 2

Example: Let $\Sigma = \{1\}$ then the set of strings $\{1, 11, 111, \dots\}$ is a regular set

Example: Let $\Sigma = \{0, 1\}$

Example: Let $\Sigma = \{a, b\}$ then the set of strings $\{ab, ba\}$ is a regular set.

So, In short we can say Regular sets are the sets which are accepted by finite automata

Example ③: Let $\Sigma = \{0, 1, 2\}$ then the set of strings that contain even no. of 0's or even no. of 1's or even no. of 2's is a regular set.

Yes, it is a regular set. Even no. of 0's or even no. of 1's or even no. of 2's can be obtained by repeating the operations like union, concatenation and

Klein closure. ✓

Topic 2:

Regular Expression : Let Σ be the alphabet. The RE over Σ and the sets that they denote are defined recursively as follows.

- (i) Any symbol $a \in \Sigma$ is a RE and it denotes the set $\{a\}$.

- (i) Any symbols from Σ , $\{\emptyset\}$ and ϵ are RE '0' length

(ii) If R_1 & R_2 are RE, then $R_1 + R_2$, $R_1 R_2$ is RE that denote R_1 or $R_1 R_2$ and R_1^* respectively

(iii) A string is a RE if it can be derived from Primitive RE by finite application of 2.

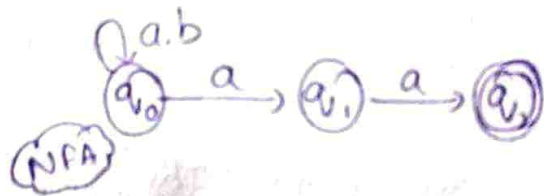
→ RE ^{are} means to represent certain sets of strings in some algebraic manner.

→ RE described in the language accepted by FA.

Example: {Set of strings of a's and b's ending with aa}

$L = \{aa, aaa, baa, bbaa, babaa, \dots\}$

→ Regular Set.



$(a+b)^*aa$

↓ Algebraic Expression.

Regular Expression.

Examples: Design a RE over $\{1\}$ that accepts all string of any length

The string of the language are

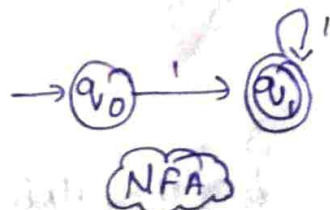
Sol: ϵ
1
11
111
1111
...



$RE = (1)^*$

② Design a RE over $\{1\}$ that accepts all the string of length atleast 1

Sol: 1
11
111
...

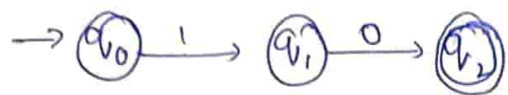


Positive Kleen closure

$RE = (1)^+$

③ Design a RE over $\{0,1\}$ that accepts string 10

Sol: $\{10, 010, 110, 0100, 1101, \dots\}$



$10 \Rightarrow$ concatenation operation

(Q) Design RE over $\{0,1\}$ that accepts $\{010, 111, 100\}$

Sol: RE = $010 + 111 + 100$

any one.

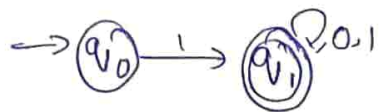
'+' Union operations

$010 \text{ OR } 111 \text{ OR } 100$

In RE we use '+' symbol

(Q) Design a RE over $\{1\}$ that starts with 1

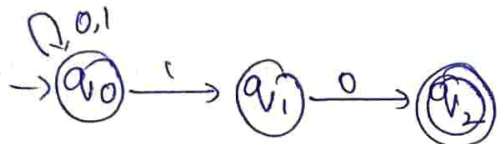
Sol: $1, 10, 11, 101, \dots, 110, \dots$



$1(0+1)^* = \text{R.E}$

(Q) Design a R.E that ends with 10

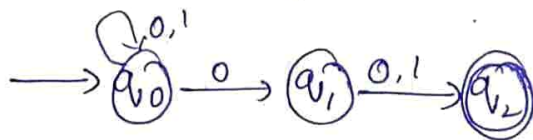
Sol: $10, 010, 110, \dots, 10$



$(0+1)^* 10 \Rightarrow \text{RE}$

(Q) Design a R.E over Σ such that automata accept strings in which second character from right is 0

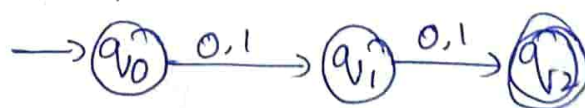
Sol: $01, 00, \dots$
 $00(0+1)^*$
 $10(0+1)^*$



$(0+1)^* 0 (0+1)^* \Rightarrow \text{RE}$

(Q) Design a RE over Σ such that automata accepts string length exactly 2

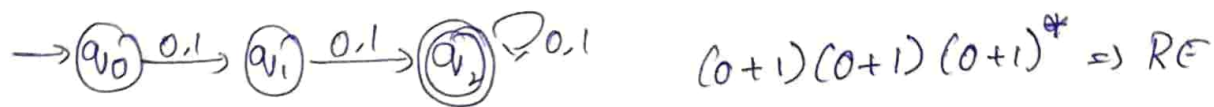
Sol: $00, 11, 01, 10,$



$(0+1) \cdot (0+1) \Rightarrow \text{RE}$

(Q) Design a RE over $\{0,1\}$ such that the automata accepts string length atleast 2

sol: 01, 10, 00, 01, 000, 111, ...



(Q) Design a RE over $\{0,1\}$ such that the automata accept string length atmost 2

sol: $\epsilon, 0, 1, 00, 01, 10, 11$



Topic 3 : Identity rules.

To Simplify Regular Expression.

$\emptyset \rightarrow$ null set $\epsilon \rightarrow$ null string.

If P, Q, R are RE then.

Rule 1 : $\emptyset + R = R$ Rule 2 : $\emptyset \cdot R = R \cdot \emptyset = \emptyset$

Rule 3 : $\epsilon \cdot R = R \cdot \epsilon = R$ Rule 4 : $R + R = R$

Rule 5 : $R^* \cdot R^* = R^*$

Rule 6 : $R^* R = R \cdot R^* = R^*$

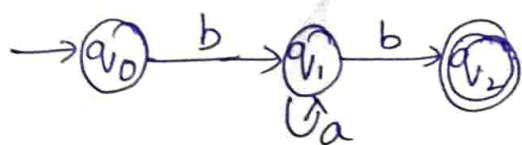
Rule 7 : $(R^*)^* = R^*$ Rule 8 : $\epsilon + R R^* = \epsilon + R^* R = R^*$

Rule 9 : $(P \cdot Q)^* P = P (Q P)^*$

Topic 4 : Construct a finite automata for a given Regular Expression

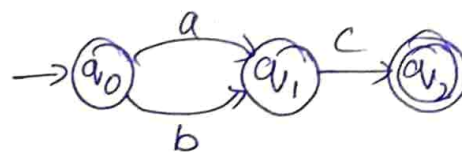
(1) ba^*b

sol: $L = \{bb, bab, baab, \dots\}$



(2) $(a+b)^+c$ $\xrightarrow{+}$ (OR)

sol:

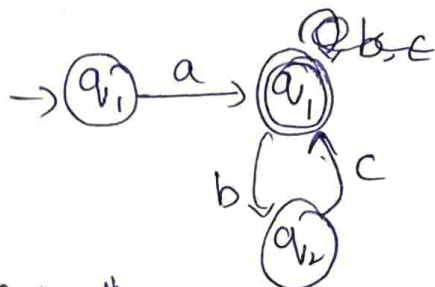


$L = \{ac, bc\}$

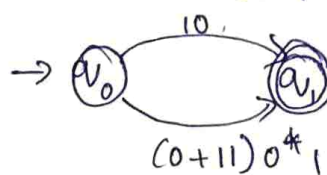
ing

③ $a(bc)^*$

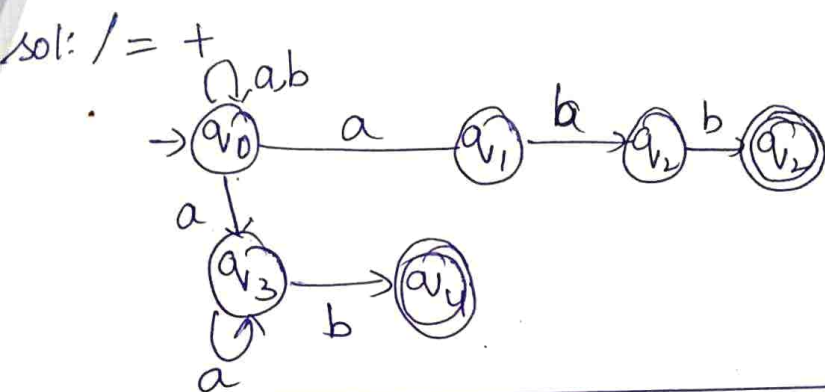
sol: $L = \{abc, abcb, abcbcb, \dots\}$



⑤ $10 + (0+11)0^*$



④ $(a|b)^+ (abb|a^+b)$



$a^+ = \{a, aa, aaa, \dots\}$

$a^* = \{\epsilon, a, aa, aaa, \dots\}$

1st May

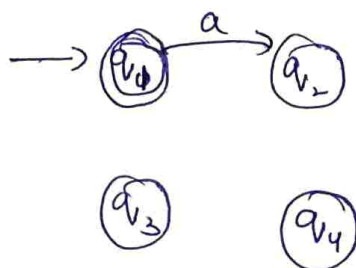
$G_0, G_2, G_3, G_4, G_7, G_8, G_9, H_1, H_2, H_3, H_8, H_7, J_0, J_2, J_3, K_2, K_6, K_7, K_8$

$L_1, L_3, L_4, L_5, L_6, L_7, M_1, M_2, M_3, M_4, M_6, M_8, N_0, N_4, N_8, N_9, P_1, P_3, P_5, P_6$

$L_e, 14, 16, 17, 18, 19$

Rough

Topic 5: Conversion of finite automata to Regular Expression



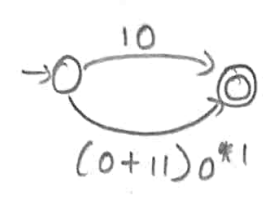
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10 Conversion of Regular Expression to finite automata

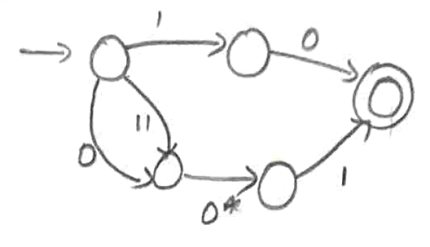
① Convert the following RE to its equivalent finite automata

Sol: $10 + (0+11)0^*1$

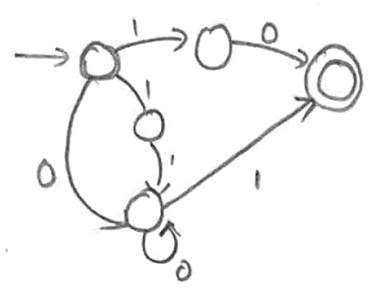
Step 1:



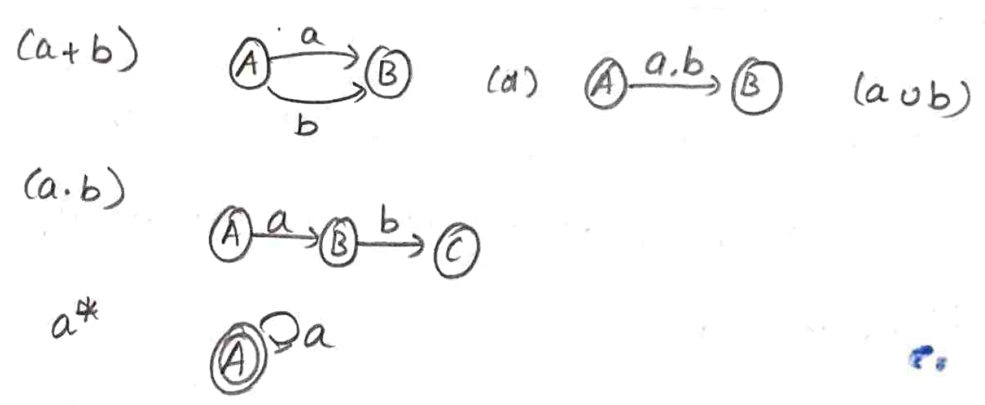
Step 2:



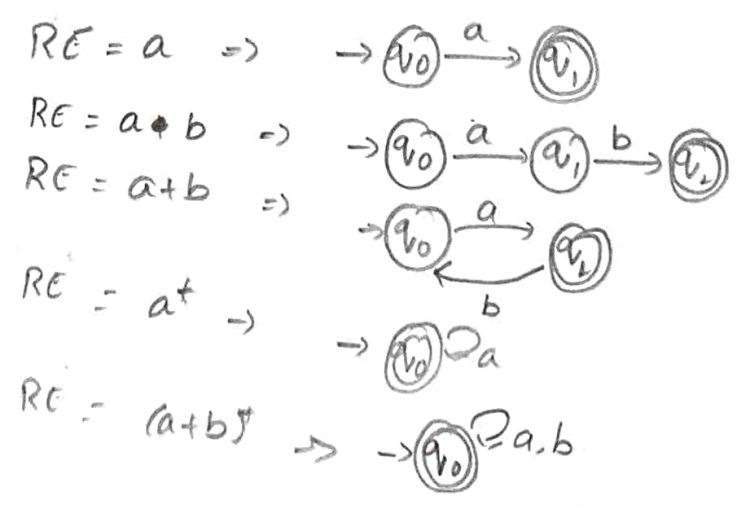
Step 3:



② Rules

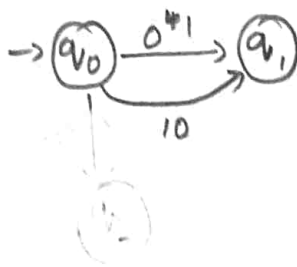


Direct Method



① $0^*1 + 10$

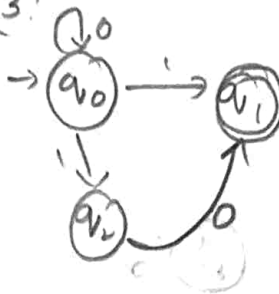
Step 1:



Step 2:

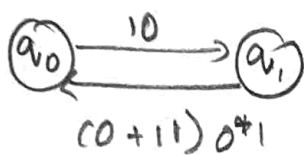


Step 3:

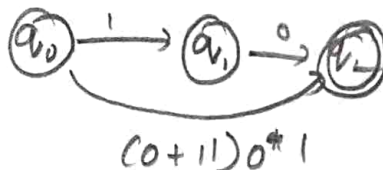


② $10 + (0 + 11)0^*1$

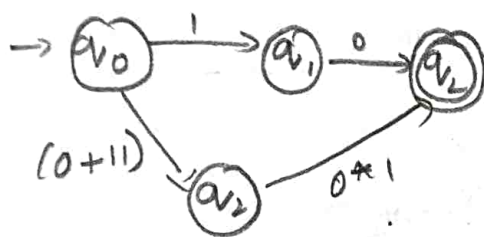
Step 1:



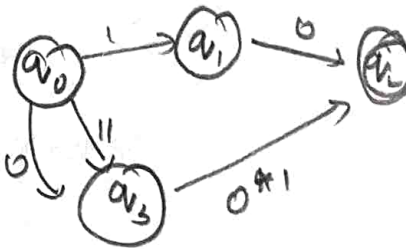
Step 2:



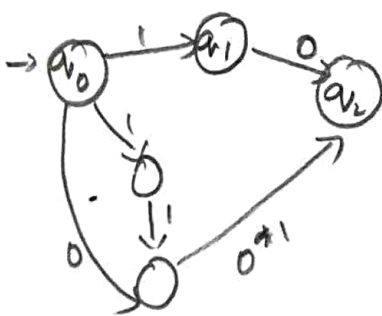
Step 3:



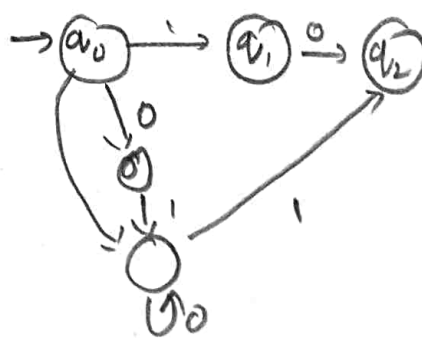
Step 4:



Step 5:

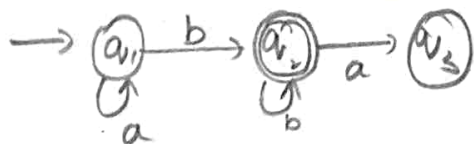


Step 6:



Using Arden's Theorem

① Construct RE from given DFA



Sol: Let's write down the Equations for each state

Now: State = input coming to it * Source state of i/p.

$$q_1 = q_1 a + \epsilon$$

Since q_1 is starting state ϵ will be added.

$$q_2 = q_1 b + q_2 b$$

Let us simplify q_1

$q_1 = q_1 a + \epsilon$ it can also be written as.

$$q_1 = \epsilon + q_1 a$$

it is in the form of $R = Q + RP$ which can be further reduced to $R = QP^*$

Assuming $R = q_1$, $Q = \epsilon$, $P = a$
we get,

$$q_1 = \epsilon \cdot a^* \quad q_1 = a^* \quad (\epsilon \cdot R = R)$$

Sub value of q_1 in q_2 we get

$$q_2 = q_1 b + q_2 b$$

$$q_2 = a^* b + q_2 b$$

We can compare this Equation with $R = Q + RP$ assuming $R = q_2$, $Q = a^* b$, $P = b$. which gets reduced to $R = QP^*$

$$q_2 = a^* b \cdot b^* \quad (R \cdot R^* = R^+)$$

$$q_2 = a^* b^+$$

$$a_3 = a_2 \cdot 0 + a_3(0+1)$$

final states are a_1, a_2 we concentrate on those states.

Similarly to $a_1 = e + a_1 \cdot 0$
 $R = 0 + R \cdot p$ $R = a_1$

$$a_1 = e \cdot (0)^* \quad a_1 = 0^* \quad (e \cdot R^* = R)$$

sub a_1 in a_2 we get

$$a_1 = 0^{\text{st}} \quad a_2 = 0^{\text{st}} + a_2 \cdot 1$$

$$a_2 = 0^* (1)^*$$

$$R = Q + RP \Rightarrow OP^t$$

Regular Expression is given by

$$r = q_1 + q_2.$$

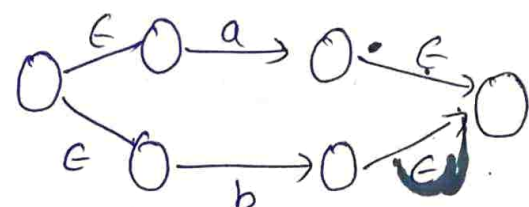
$$= O^q + O^k | \cdot |^k$$

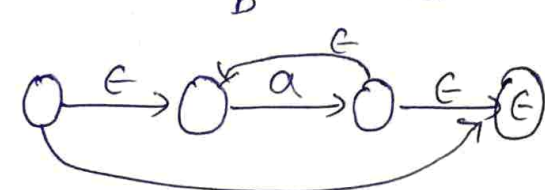
$$\gamma = 0^+ + 0^+ / + \quad \text{o. i.} \quad 1 \cdot 1^+ = 1^+$$

Conversion of Regular Expression to DFA ①

→ RE to E-NFA ~~to DFA~~

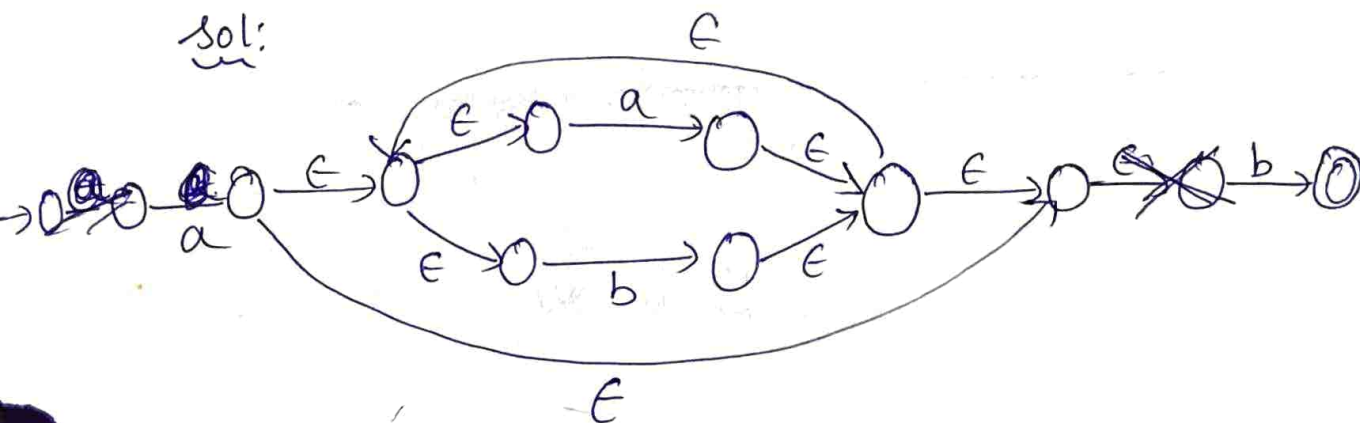
$ab \Rightarrow$  (d) 

$a+b \Rightarrow$ 

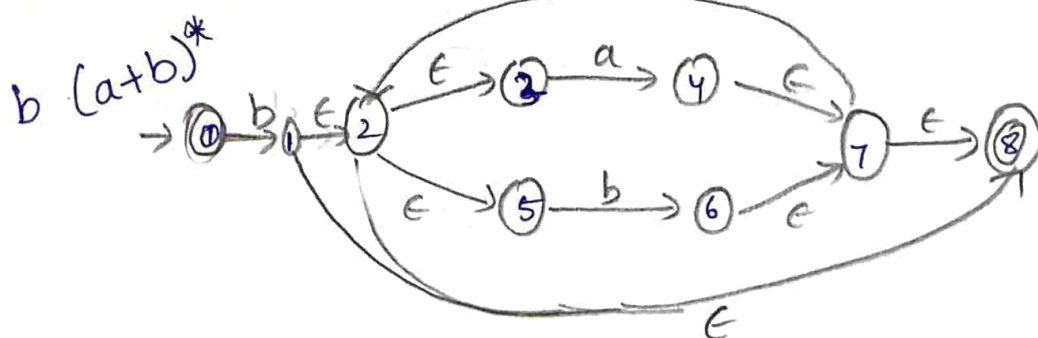
$a^* \Rightarrow$ 

Ex: $a(a+b)^*b$

Sol:



RE to E-NFA to DFA



(2)

Assume A as starting state of DFA.

$$\text{Evaluate } A = \epsilon\text{-closure}(0) = \{0\}$$

$$\{a, b\} = \epsilon$$

New state is A in DFA

So,

$$\delta(A, a) = \epsilon\text{-closure}(\delta(0, a)) = \emptyset \Rightarrow \text{Dead state}$$

$$\delta(D, a) = \delta(D, b) = \emptyset \Rightarrow \emptyset$$

$$\delta(A, b) = \epsilon\text{-closure}(\delta(0, b)) = 1$$

$$= \epsilon\text{-closure}(1) = \{1, 2, 5, 3, 8\} \Rightarrow B$$

$$\delta(B, a) = \epsilon\text{-closure}(\delta(1, a) \cup \delta(2, a) \cup \delta(5, a) \cup \delta(3, a) \cup \delta(8, a))$$

$$= \epsilon\text{-closure}(\emptyset \cup \emptyset \cup \emptyset \cup 4 \cup \emptyset)$$

$$= \epsilon\text{-closure}(4) = \{4, 7, 8, 2, 3, 5\} \Rightarrow C$$

$$\delta(B, b) = \epsilon\text{-closure}(\delta(1, b) \cup \delta(2, b) \cup \delta(5, b) \cup \delta(3, b) \cup \delta(8, b))$$

$$= \epsilon\text{-closure}(\emptyset \cup \emptyset \cup 6 \cup \emptyset \cup \emptyset)$$

$$= \epsilon\text{-closure}(6) = \{6, 7, 8\} \Rightarrow \emptyset$$

$$\delta(C, a) \Rightarrow \epsilon\text{-closure}(\delta(4, a) \cup \delta(7, a) \cup \delta(2, a) \cup \delta(3, a) \cup \delta(5, a))$$

$$= \epsilon\text{-closure}(\emptyset \cup \emptyset \cup \emptyset \cup 4 \cup \emptyset)$$

$$= \epsilon\text{-closure}(4) = \{4, 7, 8, 2, 3, 5\}$$

$$\delta(C, b) \Rightarrow \epsilon\text{-closure}(\delta(4, b) \cup \delta(7, b) \cup \delta(2, b) \cup \delta(3, b) \cup \delta(5, b))$$

$$= \epsilon\text{-closure}(\emptyset \cup \emptyset \cup \emptyset \cup \emptyset \cup 6)$$

$$= \epsilon\text{-closure}(6)$$

$$\delta(E, a) = E\text{-closure}(\delta(6, a) \cup \delta(7, a) \cup \delta(8, a) \cup \delta(2, a) \cup \delta(3, a) \cup \delta(5, a)) \quad (3)$$

$$= E\text{-closure}(\emptyset \cup \emptyset \cup \emptyset \cup \emptyset \cup \emptyset \cup 4)$$

$$= E\text{-closure}(4)$$

$$\delta(E, b) = E\text{-closure}(\delta(6, b) \cup \delta(7, b) \cup \delta(8, b) \cup \delta(2, b) \cup \delta(3, b) \cup \delta(5, b))$$

$$= E\text{-closure}(\emptyset \cup \emptyset \cup \emptyset \cup \emptyset \cup \emptyset \cup 6)$$

$$= E\text{-closure}(6)$$

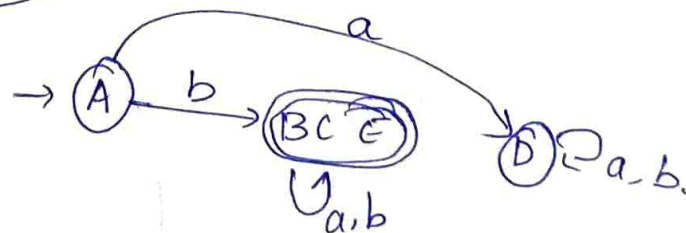
~~8, 6, 4~~, * final state \rightarrow which ever state contains final state & then that state/set becomes final set

Transition table.

	a	b
$\rightarrow A$	D	B
B *	C	E
C *	C	E
D	D	D
E *	C	E

\Rightarrow

	a	b
A	D	{B, C, E}
D	D	D
B, C, E	BCE	BCE



DFA to Regular Expression:

① $a \rightarrow b \rightarrow c$ (or) $ab \rightarrow c$.

Ex: $\rightarrow \textcircled{1} \xrightarrow{a} \textcircled{2} \xrightarrow{b} \textcircled{3} \xrightarrow{a,b}$

Step 1:

$\rightarrow \textcircled{1} \xrightarrow{a} \textcircled{2} \xrightarrow{a^*b} \textcircled{3} \xrightarrow{a,b}$

Step 2:

$\rightarrow \textcircled{1} \xrightarrow{ba^*b} \textcircled{3} \xrightarrow{a,b}$

Step 3:

$\rightarrow \textcircled{1} \xrightarrow{a^*ba^*b} \textcircled{3} \xrightarrow{a+b}$

$\rightarrow \textcircled{1} \xrightarrow{a^*ba^*b(a+b)} \textcircled{3}$

Re: $a^*ba^*b(a+b)^*$

Two final state

② $\rightarrow \textcircled{1} \xrightarrow{a} \textcircled{2} \xrightarrow{b} \textcircled{3} \xrightarrow{a+b}$

$\rightarrow \textcircled{1} \xrightarrow{a} \textcircled{2} \xrightarrow{b} \textcircled{3} \xrightarrow{a^*ba^*}$

$\rightarrow \textcircled{1} \xrightarrow{a} \textcircled{2} \xrightarrow{b} \textcircled{3} \xrightarrow{a+b}$

$a^*ba^*b(a+b)^*$

$a^*ba^* + a^*ba^*b(a+b)^*$

Pumping Lemma: is used to check language is Regular or Not
 Language $\begin{cases} \text{finite always Regular.} \\ \text{infinite may be Regular or Not.} \end{cases}$

↓ P.L.T.
 ②
 Pass / fail
 undecidable / Not Regular Language

Pumping Lemma:

If L is an infinite language then there exists some positive integer 'n' (pumping lemma) such that any string $w \in L$ has length greater than equal to 'n' i.e., $|w| \geq n$ then w can be divided into three parts, $w = xyz$ satisfy following conditions ① $i \geq 0$, $xy^iz \in L$.

② $|y| > 0$ ③ $|xy| \leq n$

Ex: $a^n b^{2n}$ where $n \geq 0$

suppose if $n=2$
then

$$xy^i z \Rightarrow \text{Pump } xy^{i+1} z$$

$$\begin{array}{ccc} aa & bbbb & \\ \downarrow & \downarrow & \downarrow \\ x & y & z \end{array} \quad \text{Assume & divide. } \in L$$
$$w = xy^2 z$$

now just pump y

$$\begin{array}{ccc} aa & bbbb & bb \\ \downarrow & \downarrow & \downarrow \\ x & y & z \end{array} \notin L$$

if it is not belonging to language then we can say
that PLT is failed and language is not Regular.

Ex2

$$\begin{array}{ccc} aa & bbbb & \\ \downarrow & \downarrow & \downarrow \\ x & y & z \end{array} \in L$$

$$\begin{array}{ccc} a & abab & bbb \\ \downarrow & \downarrow & \downarrow \\ x & y & z \end{array} \notin L \quad \text{Not a regular language.}$$

Pumping Lemma for Regular sets

Ex3: Using pumping lemma Prove that language $A = \{a^n b^n / n \geq 0\}$ is not regular

Sol: Assume that A is regular

Pumping length = p

$$S = a^p b^p$$

$$\begin{array}{ccc} & | & \\ \hline x & y & z \end{array}$$

Assume $p=7$

$$= aaaaaaa bbbbbbb$$

Case(i), The y is in the 'a' part.

$$\begin{array}{ccc} aaaaaa & a & bbbbbbb \\ \downarrow & \downarrow & \downarrow \\ x & y & z \end{array}$$

case (ii) The y is in the 'b' part

aaaaaaabbbbbb
 \downarrow \downarrow \downarrow
 x y z

case (iii) The y is the 'a' and 'b' part

aaaaaaabbbbbb
 \downarrow \downarrow \downarrow
 x y z

xy^1z

① xy^2z aa aaaa aaaa a bbbbbb

$11 \neq 7$

② $xy^1z \Rightarrow xy^2z$

aaaaaaabbb bbbb bbbb

$7 \neq 11$

③ $xy^1z \Rightarrow xy^2z$

aaaaa aabbaabb bbbb

~~etc~~ pattern is not followed.

case ①

$|xy| \leq p$ 657 ✓

case ②

$|xy| \leq p$ 1357 ✗

case ③

$|xy| \leq p$ 957 ✗

Pumping lemma for regular sets

Theorem: Let $M = (Q, \Sigma, \delta, q_0, f)$ be a finite automata with n states
 let L be the regular set accepted by M . let $w \in L$ and $|w| \geq m$. If $m \geq n$,
 then there exists x, y, z such that $w = xyz$, $y \neq \epsilon$ and $xy^i z \in L$ for each
 $i \geq 0$

Step 1: Assume L is regular. Let n be the number of states in the corresponding FA.

Step 2: Choose a string w such that $|w| \geq n$, use pumping lemma to write $w = xyz$, with $|xy| \leq n$ and $|y| > 0$

Step 3: Find a suitable integer i such that $xy^i z \notin L$. This contradicts our assumption. Hence L is not regular.



String Accepted by M .

Problem Show that the set $L = \{b^{i^2} \mid i > 1\}$ is not regular.

Step 1: Suppose L is regular and let there be n states in FA accepting L .

Step 1: Step 2: Let $w = b^{n^2}$ then $|w| = n^2 > n$. By step 2 we can write $w = xyz$ with $|xy| \leq n$ and $|y| > 0$.

Step 3: Consider xy^2z so $|xy^2z| = |x| + 2|y| + |z|$ and it should be greater than $|x| + |y| + |z|$ i.e. $|x| + 2|y| + |z| > |x| + |y| + |z|$ as $|y| > 0$

This means $n^2 = |xy^2z| = |x| + |y| + |z| < |xy^2z|$

As $|xy| \leq n$, $|y| \leq n$ therefore.

$$|xy^2z| = |x| + 2|y| + |z| \leq (n^2 + n), \text{ i.e.}$$

$$n^2 < |xy^2z| \leq n^2 + n < n^2 + n + n + 1$$

$$\text{So } n^2 < |xy^2z| \leq (n+1)^2$$

then $|xy^2z|$ is not a perfect square so $xy^2z \notin L$

But by pumping lemma $xy^2z \in L$

Thus this is a contradiction.

Properties of Regular Sets

① The union of two regular set is regular.

$$RE_1 = a(aa)^* \quad RE_2 = (aa)^*$$

$$L_1 = \{a, aaa, aaaa, \dots\}$$

$$L_2 = \{\epsilon, aa, aaaa, \dots\}$$

$$L_1 \cup L_2 = \{\epsilon, a, aaa, aaaa, aaaaa, \dots\}$$

(Strings of all possible length including null.)

$$RE(L_1 \cup L_2) = a^* \text{ (which is a regular Expression itself)}$$

② The intersection of two regular set is regular.

$$RE_1 = a(a^*) \text{ and } RE_2 = (aa)^*$$

$$L_1 = \{a, aa, aaa, aaaa, \dots\}$$

$$L_2 = \{\epsilon, aa, aaaa, aaaaaa, \dots\}$$

$$L_1 \cap L_2 = \{aa, aaaa, aaaaaa, \dots\} \text{ even length}$$

$$RE(L_1 \cap L_2) = aa(aa)^* \text{ which is a regular Expression itself}$$

③ The complement of regular set is regular.

$$RE_1 = (aa)^*$$

$$L = \{\epsilon, aa, aaaa, aaaaaa, \dots\}$$

complement of L is all strings not in L .

$$L' = \{a, aaa, aaaaa, \dots\}$$

$$RE(L') = a(aa)^*$$

Grammar

$\{V, T, P, S\} \rightarrow$ Quadruple.

$V \rightarrow$ Set of Non-terminals / Variables {upper case} similar to states.

$T \rightarrow$ Set of terminals (symbols) {Lower case}.

* $P \rightarrow$ Production Rules

$S \rightarrow$ start symbol

\rightarrow Represented as LHS \rightarrow RHS

\downarrow
Single Non
terminals

$\rightarrow \epsilon$ / String of terminals & non-terminals.

Grammar $\{ A \rightarrow aB \rightarrow$ Production Rule
 $B \rightarrow \epsilon / aB$

Grammar

Regular.

Grammar

LHS \rightarrow RHS

\downarrow Non terminal $\downarrow \epsilon$ terminal

terminal followed by NT

$\&$ \downarrow left linear Right linear.

Grammar Grammar

$D \rightarrow Ab.$

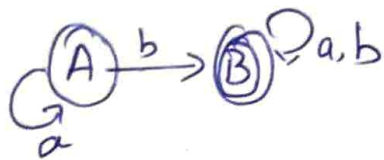
\downarrow
Non
Terminal
Should be on
left side

$C \rightarrow aA$

\downarrow
Right most.

Ex: $A \rightarrow \epsilon$
 $B \rightarrow a$
 $C \rightarrow aA$
 $D \rightarrow Ab$ } Regular Grammar

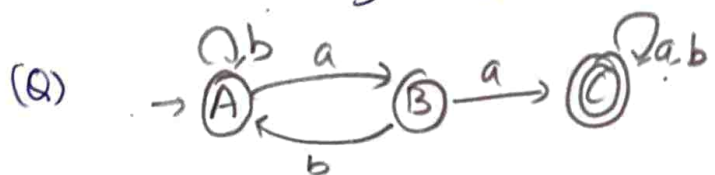
$A \rightarrow aBb$
 \rightarrow Not a
Regular
Grammar



Construct Regular Grammar.

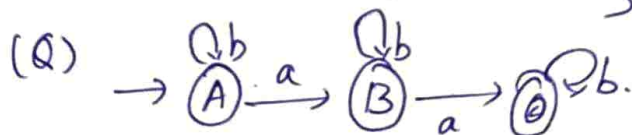
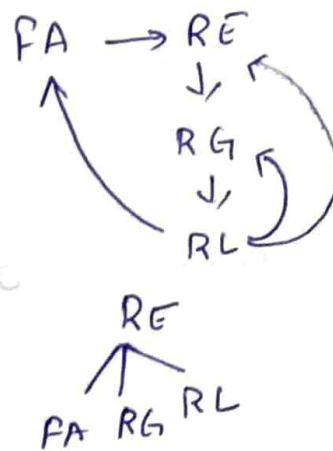
$$V = \{A, B\} \quad T = \{a, b\} \quad P = \{?\} \quad S = \{A\}$$

$$P = \left\{ \begin{array}{l} A \rightarrow aA \\ A \rightarrow bB / b \\ B \rightarrow aB / bB \\ B \rightarrow a \\ B \rightarrow b \end{array} \right. \begin{array}{l} \rightarrow \text{Right Linear Grammar.} \\ \rightarrow \text{Regular Grammar.} \\ \rightarrow \text{Terminal.} \end{array}$$



$$V = \{A, B, C\} \quad T = \{a, b\} \quad P = \{?\} \quad S = \{A\}$$

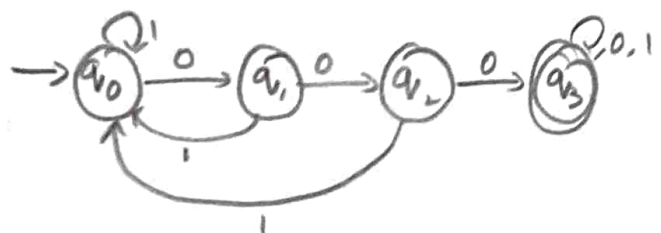
$$P = \left\{ \begin{array}{l} A \rightarrow aB, \\ A \rightarrow bA, \\ B \rightarrow aC, \\ B \rightarrow a, \\ C \rightarrow aC, \\ C \rightarrow a, \\ C \rightarrow bC, \\ C \rightarrow b. \end{array} \right\}$$



* Right Linear Grammar is Regular Grammar.

Sol:

$$\begin{array}{l} A \rightarrow aB \\ A \rightarrow bA \\ B \rightarrow aC \\ B \rightarrow a \\ B \rightarrow bB \\ C \rightarrow bC \\ C \rightarrow b \end{array}$$



$G_0, G_1, G_2, G_3, G_4, H_0, H_1$

$H \rightarrow 0, 1, 2, 6, 7, 9, 4$

$J \rightarrow 2, 4, 5, 6, 8, 9, 3$

$K \rightarrow 1, 2, 3, 4, 6, 7, 8, 9$

$L \rightarrow 0, 1, 3, 4, 6, 8$

$M \rightarrow 0, 1, 2, 6, 8, 9$

$N \rightarrow 0, 1, 6, 7, 8, 9$

$P \rightarrow 1, 2, 3, 6, 7, 8$

$Le - 13, 14, 15, 16, 17, 18,$

Conversion of right linear grammar to left linear grammar

$S \rightarrow bB$

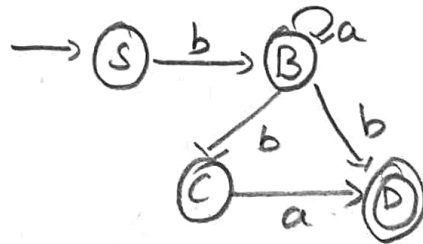
$B \rightarrow bC$

$B \rightarrow aB$

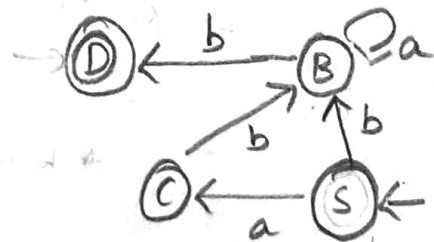
$C \rightarrow a$

$B \rightarrow b$

Step 1: Construct finite automata



Step 2: Reverse finite automata (interchange initial & final state and reverse edges)



Step 3: Write a Right linear grammar.

$S \rightarrow bB$

$S \rightarrow aC$

$C \rightarrow bB$

$B \rightarrow aB$

$B \rightarrow bD$

Step 4: Write left linear Grammar

$S \rightarrow Bb$

$S \rightarrow ca$

$C \rightarrow Bb$

$B \rightarrow Ba$

$B \rightarrow Db$

Construction of LLG to RLG.

$S \rightarrow Sa/Sb/Ab$

$A \rightarrow Aa/Ba$

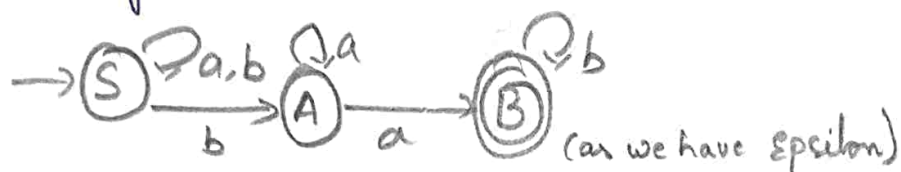
$B \rightarrow Bb/\epsilon$

Step 1: Design FA

Step 2: Interchange start state and final state

Step 3: Reverse the directions

Sol: Design finite Automata



Step 2:



Step 3:

$S \rightarrow bs/aA$

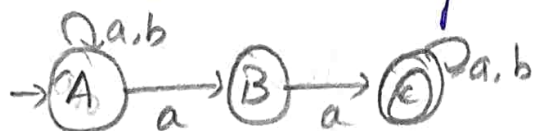
$A \rightarrow aA/bB$

$B \rightarrow aB/bB/\epsilon$

Equivalence of Regular Grammar and Finite Automata.

Yes, we have equivalence b/w RG & FA

ex: (Q) 'aa' as substring



$A \rightarrow aB$

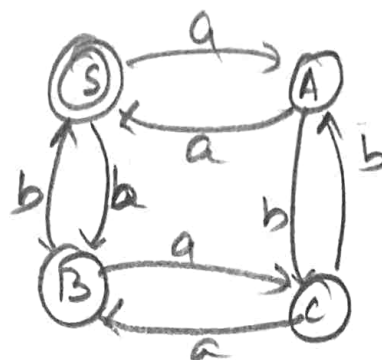
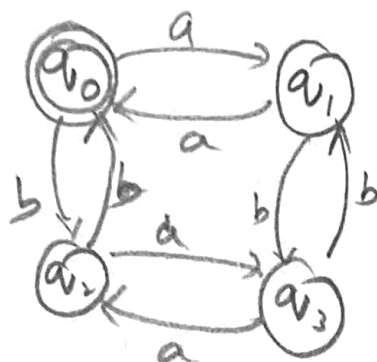
$A \rightarrow aA/bA$

$B \rightarrow aC$

$C \rightarrow aC/bC/\epsilon$

as it is final state

Even a's and Even b's.



$$S \rightarrow aA / bB / \epsilon$$

$$A \rightarrow bC / aS / \epsilon$$

$$B \rightarrow aC / bS$$

$$C \rightarrow bA / aB$$

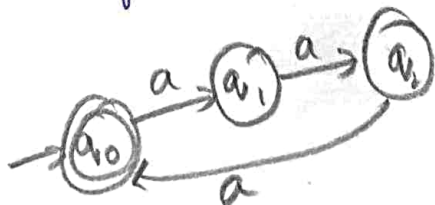
Language is said to be \rightarrow Regular Grammar iff \exists Regular Grammar generates it

Language is said to be Regular iff \exists FA accepting it

Regular Grammars

$$\begin{array}{ccc} \text{LHS} & \rightarrow & \text{RHS} \\ \downarrow & & \downarrow \\ \text{NT} & & T^+ / T^+ N \text{ (OR) } NT^+ / T^+ \end{array}$$

No. of a's divisible 3



$$S \rightarrow \underline{aaa} S / \epsilon$$

$$S \rightarrow aA / \epsilon$$

$$S \rightarrow aB$$

$$S \rightarrow aS$$

$G_0, G_2, G_5, G_7, G_8, H_0, H_1, H_4, H_8, H_9$

$J_4, J_6, J_7, J_8, K_1, K_2, K_3, K_4, K_6, K_7, K_8, K_9$

L_1, L_4, L_8, L_5, L_6

$M_1, M_2, M_3, M_6, M_8, M_9, N_0, N_4, N_6, N_7, N_8, N_9$

P_6, P_3, P_8

$Le_{13, 14, 18}$

Module - III

Context Free Grammar

Formal Definition : Rules to form a string, inturn forms a language

CFG : (V, T, P, S)

- ↳ finite of Non-Terminals (capital)
- ↳ finite set of Terminals (lower) ($V \cap T = \emptyset$)
- Production rules. $\alpha \rightarrow \beta$

$\begin{matrix} \text{L.H.S} & \text{R.H.S} \\ \text{Type 2} & \left\{ \begin{matrix} \text{only } \perp \\ \text{NT} \end{matrix} \right. & (V \cup T)^* \end{matrix}$

→ start Symbol. (in left hand side)

Ex: $S \rightarrow OS1$

$S \rightarrow \epsilon$

start symbol = $\{S\}$

$T = \{0, 1\}$ $P = 2$ (or) 1

Derivation Tree: $\{V, T, P, S\}$

$G: E \rightarrow E + E / E * E / E = E / id$

$w: id + id * id$

whether a word belong to Grammar or not