UNIT IV

Test of Hypothesis - I

PART - B

1. A Population consists of five numbers 2, 3, 6, 8 and 11. Consider all possible samples of size two which can be drawn with replacement from this population. Find
   i) The mean of the population
   ii) The standard deviation of the population.
   iii) The mean of the sampling distribution of means.
   iv) The standard deviation of the sampling distribution of means.

Sol:- i) Mean of the population :- $\mu = \frac{1}{N} \Sigma x_i$

$$= \frac{2+3+6+8+11}{5}$$

$$= 6$$

ii) Variance of the population :-

$$\sigma^2 = \Sigma \frac{(x_i - \mu)^2}{N}$$

$$= \frac{(2-6)^2 + (3-6)^2 + (6-6)^2 + (8-6)^2 + (11-6)^2}{5}$$

$$\sigma^2 = 10.8$$

$$\therefore \sigma = 3.29$$

Sampling with replacement (Infinite Population)

$$N^n = 5^2 = 25$$

(2,2)	(2,3)	(2,6)	(2,8)	(2,11)
(3,2)	(3,3)	(3,6)	(3,8)	(3,11)
(6,2)	(6,3)	(6,6)	(6,8)	(6,11)
(8,2)	(8,3)	(8,6)	(8,8)	(8,11)
(11,2)	(11,3)	(11,6)	(11,8)	(11,11)

The sample means are ($\bar{x}_i$)

2	2.5	4	5	6.5
2.5	3	4.5	5.5	7
4	4.5	6	7	8.5
5	5.5	7	8	9.5
6.5	7	8.5	9.5	11

(iii) Mean of the sampling distribution

$$\mu_{\bar{x}} = \frac{1}{N^n} \sum \bar{x}_i$$

$$= \frac{1}{25} \{2 + 2.5 + 4 + 5 + \cdots + 11\}$$

$$= 6$$

(iv) Variance of the sampling distribution

$$\sigma_{\bar{x}}^2 = \frac{1}{N^n} \sum (\bar{x}_i - \mu_{\bar{x}})^2$$

$$= \frac{1}{25} \{(2-6)^2 + (2.5-6)^2 + \cdots + (11-6)^2\}$$

$$= 5.40$$

∴ Standard deviation of the sampling distribution

$$\sigma_{\bar{x}} = 2.32$$

2. A population consists of 5, 10, 14, 18, 13, 24. Consider all possible samples of size two which can be drawn without replacement from this population.

Find (i) The mean of the population.

(ii) The standard deviation of the population

(iii) the mean of the sampling distribution of means.

(iv) the standard deviation of the sampling distribution of means.

Sol: (i) Mean of the population:-

$$\mu = \frac{\sum x_i}{N} = \frac{5+10+14+18+13+24}{6} = 14$$

ii) Variance of the population $\sigma^2 = \frac{\Sigma(X_i - \mu)^2}{N}$

$$= \frac{1}{6}\{(5-14)^2 + (10-14)^2 + (14-14)^2 + (18-14)^2 + (13-14)^2 + (24-14)^2\}$$

$= 35.67$

$\therefore$ Standard deviation of the population $= 5.97$

Sampling without replacement :-

$N_{C_n} = 6_{C_2} = 15$

(5,10) (5,14) (5,18) (5,13) (5,24)

(10,14) (10,18) (10,13) (10,24)

(14,18) (14,13) (14,24)

(18,13) (18,24)

(13,24)

The sample means are $(\bar{x}_i)$

7.5 9.5 11.5 9 14.5

12 14 11.5 17

16 13.5 19

15.5 21

18.5

iii) Mean of the sampling distribution of means :-

$$\mu_{\bar{x}} = \frac{1}{N_{C_n}} \Sigma \bar{x}_i$$

$$= \frac{1}{15}\{7.5 + 9.5 + \cdots 18.5\}$$

$= 14$

iv) Variance of the sampling distribution of means :-

$$\sigma^2_{\bar{x}} = \frac{1}{N_{C_n}} \Sigma(\bar{x}_i - \mu_{\bar{x}})^2$$

$$= \frac{1}{15}\{(7.5-14)^2 + (9.5-14)^2 + \cdots + (18.5-14)^2\}$$

$= 14.26$

$\therefore$ standard deviation of the sampling distribution of means $= 3.78$

3. A Population consists of five numbers 4, 8, 12, 16, 20, 24. Consider all possible samples of size two which can be drawn without replacement from this population. Find

i) The mean of the Population

ii) The standard deviation of the population

iii) The mean of the sampling distribution of means

iv) The standard deviation of the sampling distribution of means.

Sol:- i) Mean of the population:- $\mu = \frac{1}{N}\Sigma x_i$

$$= \frac{4+8+12+16+20+24}{6}$$

$$=14$$

ii) Variance of the population:-

$$\sigma^2 = \frac{\Sigma(x_i-\mu)^2}{N}$$

$$= \frac{1}{6}\{(4-14)^2 + (8-14)^2 + \cdots + (24-14)^2\}$$

$$= 46.67$$

∴ Standard deviation of the population = 6.83

Sampling without replacement

$$^{N}C_{n} = {}^{6}C_{2} = 15$$

(4, 8)	(4, 12)	(4, 16)	(4, 20)	(4, 24)
	(8, 12)	(8, 16)	(8, 20)	(8, 24)
		(12, 16)	(12, 20)	(12, 24)
			(16, 20)	(16, 24)
				(20, 24)

Sample means are $(\bar{x}_i)$

6	8	10	12	14
	10	12	14	16
		14	16	18
			18	20
				22

iii, Sol Mean of the sampling distribution of means

$$\mu_{\bar{x}} = \frac{1}{N_{C_n}} \Sigma \bar{x_i}$$

$$= \frac{1}{15}\{6+8+10+ \cdots +22\}$$

$$= 14$$

iv, Variance of the sampling distribution of means :-

$$\sigma_{\bar{x}}^2 = \frac{1}{N_{C_n}} \Sigma (\bar{x_i} - \mu_{\bar{x}})^2$$

$$= \frac{1}{15}\{(6-14)^2 + (8-14)^2 + \cdots + (22-14)^2\}$$

$$= 18.67$$

∴ Standard deviation of the sampling distribution of means $= 4.32$

4. Samples of size 2 are taken from the population 1, 2, 3, 4, 5, 6. which can be drawn with replacement? Find.

i, Mean of the population

ii Standard deviation of the population

iii Mean of the ~~st~~ sampling distribution of means.

iv The standard deviation of the sampling distribution of means.

Sol i, Mean of the Population $(\mu) = \frac{1}{N} \Sigma X_i$

$$= \frac{1}{6}\{1+2+3+4+5+6\} = 3.5$$

ii, ~~Mean~~/ Variance of the population $(\sigma^2) = \frac{1}{N} \Sigma (X_i - \mu)^2$

$$= \frac{1}{6}\{(1-3.5)^2 + (2-3.5)^2 + \cdots + (6-3.5)^2\}$$

$$= 2.9$$

∴ the standard deviation of the population $(\sigma) = 1.71$

iii, Sampling with replacement :-

$$N^n = 6^2 = 36$$

Samples are

(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

Sample means ($\bar{x}_i$) are

1	1.5	2	2.5	3	3.5
1.5	2	2.5	3	3.5	4
2	2.5	3	3.5	4	4.5
2.5	3	3.5	4	4.5	5
3	3.5	4	4.5	5	5.5
3.5	4	4.5	5	5.5	6

iii Mean of the sampling distribution of means

$$\mu_{\bar{x}} = \frac{1}{N^n} \sum \bar{x}_i$$

$$= \frac{1}{36} \{1 + 1.5 + 2 + \cdots + 5.5 + 6\}$$

$$= 3.5$$

iv, Variance of the sampling distribution of means

$$\sigma^2_{\bar{x}} = \frac{1}{N^n} \sum (\bar{x}_i - \mu_{\bar{x}})^2$$

$$= \frac{1}{36} \{ (1-3.5)^2 + (1.5-3.5)^2 + (2-3.5)^2 + \cdots + (6-3.5)^2 \}$$

$$= 1.46$$

$\therefore$ Standard deviation of the sampling distribution of means $= 1.2$

5. Samples of size 2 are taken from the population 3, 6, 9, 15, 27. which can be drawn with replacement? Find

i, The mean of the population

ii, the standard deviation of the population iii, Mean of the sampling distribution of means

iv Standard deviation of the sampling distribution of means

Sol: i, Mean of the population $(\mu) = \frac{1}{N} \Sigma x_i$

$$= \frac{1}{5} \{3+6+9+15+27\}$$

$$= 12$$

ii, Variance of the population $(\sigma^2) = \frac{1}{N} \Sigma (x_i - \mu)^2$

$$= \frac{1}{5} \{(3-12)^2 + (6-12)^2 + (9-12)^2 + (15-12)^2 + (27-12)^2\}$$

$$= 72$$

∴ standard deviation of the population = 8.48

Sampling distribution with replacement :-

$$N^n = 5^2 = 25$$

The samples

(3,3)	(3,6)	(3,9)	(3,15)	(3,27)
(6,3)	(6,6)	(6,9)	(6,15)	(6,27)
(9,3)	(9,6)	(9,9)	(9,15)	(9,27)
(15,3)	(15,6)	(15,9)	(15,15)	(15,27)
(27,3)	(27,6)	(27,9)	(27,15)	(27,27)

The Sample Means $(\bar{x}_i)$

3	4.5	6	9	15
4.5	6	7.5	10.5	16.5
6	7.5	9	12	18
9	10.5	12	15	21
15	16.5	18	21	27

iii Mean of the sampling distribution of means

$$\mu_{\bar{x}} = \frac{1}{N^n} \Sigma \bar{x}_i$$

$$= \frac{1}{25} \{3 + 4.5 + \cdots + 27\}$$

$$= 12$$

iv, Variance of the sampling distribution of means

$$\sigma_{\bar{x}}^2 = \frac{1}{N^n} \Sigma (\bar{x}_i - \mu_{\bar{x}})^2$$

$$= \frac{1}{25}\{(3-12)^2+(4.5-12)^2+ \cdots +(27-12)^2\}$$

$$\sigma_{\bar{x}}^2 = 36.36$$

$\therefore$ Standard deviation of sampling distribution of means $(\sigma_{\bar{x}}) = 6.03$

6. If the population is 3, 6, 9, 15, 27

i) List all possible samples of size 3 that can be taken without replacement

ii) Calculate the mean of each of the sampling distribution of means.

iii) Find the standard deviation of sampling distribution of means.

Sol: ii) Means

i) Sampling without replacement:-

$$^N C_n = {^5C_3} = 10$$

Samples are

(3,6,9) (3,6,15) (3,6,27)

~~(3,9,~~ (3,9,15) (3,9,27) (3,15,27)

(6,9,15) (6,9,27) (6,15,27)

(9,15,27)

Sample Means $(\bar{x}_i)$ are

6 8 12

9 13 15

10 14 16

17

ii) Mean of the sampling distribution of means

$$\mu_{\bar{x}} = \frac{1}{^NC_n}\sum(\bar{x}_i)$$

$$= \frac{1}{10}\{6+8+12+ \cdots +17\}$$

$$=12$$

iii) Variance of the sampling distribution of means

$$\sigma_{\bar{x}}^2 = \frac{1}{^NC_n}\sum(\bar{x}_i - \mu_{\bar{x}})^2$$

$$= \frac{1}{10}\{(6-12)^2+(8-12)^2+(12-12)^2+ \cdots +(17-12)^2\}$$

$$\sigma^2_{\bar{x}} = 12$$

$\therefore$ standard deviation of the sampling distribution of means

$$= \sigma_{\bar{x}} = \sqrt{12} = 3.464$$

7. The mean height of students in a college is 155 Cms and standard deviation is 15. What is the probability that the mean height of 36 students is less than 157 Cms.

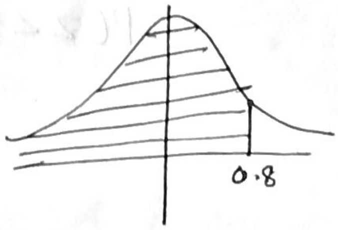
Sol: Mean of the population $\mu = 155$

$$\sigma = 15, \; n = 36, \; \bar{x} = 157$$

$$P(\bar{x} \le 157) = P(\bar{x} \le x_1)$$

$$Z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \frac{157-155}{15/\sqrt{36}} = 0.8$$

$$P(Z \le z_1) = P(Z \le 0.8)$$



$$= 0.5 + P(0 \le Z \le 0.8)$$

$$= 0.5 + 0.2881$$

$$= 0.7881$$

8. A random sample of size 100 is taken from an infinite Population having the mean 76 and the variance 256. What is the probability $\bar{x}$ will be between 75 and 78.

Sol: Given $n = 100, \; \mu = 75, \; \sigma^2 = 256 \Rightarrow \sigma = 16$

Let $P(\bar{x}_1 \le \bar{x} \le \bar{x}_2) = P(75 \le x \le 78)$

$$Z_1 = \frac{\bar{x}_1 - \mu}{\sigma/\sqrt{n}} = \frac{75-76}{16/\sqrt{100}} = -0.625$$

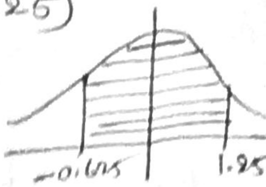
$$Z_2 = \frac{\bar{x}_2 - \mu}{\sigma/\sqrt{n}} = \frac{78-76}{16/\sqrt{100}} = 1.25$$

$$\therefore P(z_1 \le Z \le z_2) = P(-0.625 \le Z \le 1.25)$$

$= P(-0.625 \le z \le 0) + P(0 \le z \le 1.25)$

$= 0.2334 + 0.3944$

$= 0.628$

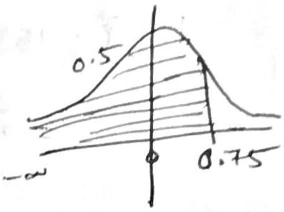


9. The mean of certain normal population is equal to the standard error of the mean of the samples of 64 from that distribution. Find the probability that the mean of the sample size 36 will be negative.

Sol: Given $n=64$, Standard Error $= \frac{\sigma}{\sqrt{n}} = \mu = \frac{\sigma}{\sqrt{64}} = \frac{\sigma}{8}$

$$z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \frac{\bar{x}-(\sigma/8)}{\sigma/6} \quad (\because n=36)$$

$$= \frac{6\bar{x}}{\sigma} - \frac{3}{4}$$



$\therefore P(z < 0.75) = P(-\infty < z < 0.75)$

$= 0.5 + 0.2734$

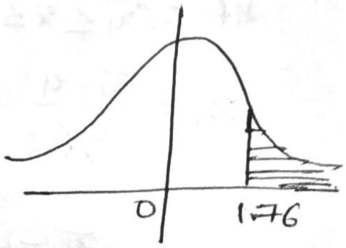
$= 0.7734$

10. A random sample of size 64 is taken from a normal population with $\mu = 51.4$, & $\sigma = 68$. What is the probability that the mean of the sample will i) exceed 52.9 ii) fall b/w 50.5 and 52.3 iii) be less than 50.6

Sol: Given $n=64$, $\mu = 51.4$, $\sigma = 68$

i) $P(\bar{x} > 52.9) = P(\bar{x} > \bar{x}_1)$

$$\text{PL } z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \frac{52.9-51.4}{6.8/\sqrt{64}} = 1.76$$



$P(z > 1.76) = 0.5 - P(0 < z < 1.76)$

$= 0.5 - 0.4608$

$= 0.0392$

ii. $P(50.5 < \bar{x} < 52.3) = P(\bar{x}_1 < x < \bar{x}_2)$

$$Z_1 = \frac{\bar{x}_1 - \mu}{\sigma/\sqrt{n}} = \frac{50.5 - 51.4}{6.8/\sqrt{64}} = -1.06$$

$$Z_2 = \frac{\bar{x}_2 - \mu}{\sigma/\sqrt{n}} = \frac{52.3 - 51.4}{6.8/\sqrt{64}} = 1.06$$

~~P(56.74~~

$P(-1.06 < Z < 1.06)$

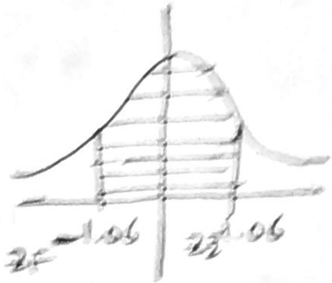
$= P(-1.06 < Z < 0) + P(0 < Z < 1.06)$

$= P(0 < Z < 1.06) + P(0 < Z < 1.06)$

$= 2P(0 < Z < 1.06)$

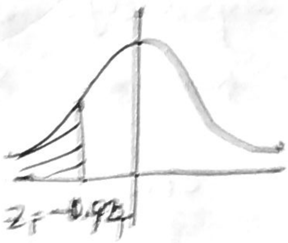
$= 2(0.3554)$

$= 0.7108$



iii. $P(\bar{x} < 50.6) = P(\bar{x} < \bar{x}_1)$

$$Z_1 = \frac{\bar{x}_1 - \mu}{\sigma/\sqrt{n}} = \frac{50.6 - 51.4}{6.8/\sqrt{64}} = -0.94$$



$P(Z < -0.94)$

$= 0.5 - P(-0.94 < Z < 0)$

$= 0.5 - P(0 < Z < 0.94)$

$= 0.5 - 0.3264$

$= 0.1736$

11. A sample of 400 items is taken from a population whose standard deviation is 10. The mean of sample is 40. Test whether the sample has come from a population with mean 38 also calculate 95% confidence interval for the population.

Sol: Given $\bar{x} = 40$, $\mu = 38$, $\alpha = 95\%$, $n = 400$, $\sigma = 10$

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

1) Null Hypothesis (H_0):- $\mu = 38$

2) Alternative Hypothesis (H_1):- $\mu \neq 38$

3) Level of significance :-

$$Z_{\alpha/2} = 1.96 \text{ at } \alpha = 0.05 \ (95\%)$$

4) Test of statistic:-

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{40-38}{10/\sqrt{400}} = 4 \ \& \ |z| = 4$$

5) conclusion:- $|z| > Z_\alpha$, we reject Null hypothesis.

12. The means of two large samples of sizes 1000 and 2000 members are 67.5 inches and 68.0 inches respectively. Can the samples be regarded as drawn from the same population of S.D 2.5 inches.

Sol: Given $n_1 = 1000$, $n_2 = 2000$

$\bar{x}_1 = 67.5$, $\bar{x}_2 = 68$

$\sigma = 2.5$, $\alpha = 95\% = 0.05$

i, Null hypothesis (H_0):- $\bar{x}_1 = \bar{x}_2$

ii, Alternative hypothesis (H_1):- $\bar{x}_1 \neq \bar{x}_2$

iii, Level of significance:- $Z_\alpha = 1.96$ at $\alpha = 0.05$

iv, Test Statistic:- $Z = \dfrac{\bar{x}_1 - \bar{x}_2}{\sigma\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \dfrac{67.5 - 68.0}{2.5\sqrt{\frac{1}{1000} + \frac{1}{2000}}}$

$$Z = -5.20$$

$$\& \ |z| = 5.20$$

v, Conclusion:- $|z| > Z_\alpha$

$\therefore$ we reject null hypothesis

13. An ambulance service claims that it takes on the average 8.9 minutes to reach its destination, in emergency calls. To check on this claim the agency which issues license to ambulance service has then timed on fifty emergency calls getting a mean of 9.2 minutes with 1.6 minutes. What can they conclude at 5% level of significance?

Sol: let $n=50$, $\bar{x}=9.2$, $\sigma=1.6$, $\mu=8.9$

i, Null hypothesis :- $\mu=8.9$

ii, Alternative hypothesis :- $\mu \neq 8.9$

iii, Level of significance :-

$Z_{\alpha/2}=1.96$, at $\alpha=0.05$

iv, Test statistic :- $Z=\dfrac{\bar{x}-\mu}{\sigma/\sqrt{n}}=\dfrac{9.2-8.9}{1.6/\sqrt{50}}=1.326$

& $|Z|=1.326$

v, Conclusion :- $|Z|<Z_\alpha$

$\therefore$ we accept null hypothesis.

14. According to norms established for a mechanical aptitude test persons who are 18 years have an average weight of 73.2 with S.D 8.6. If 40 randomly selected persons have average 76.7 test the hypothesis $H_0: \mu=73.2$ against alternative hypothesis : $\mu>73.2$

Sol: Given $\mu=73.2$, $\bar{x}=76.7$, $\sigma=8.6$, $n=40$, $\alpha=0.05$

i, Null Hypothesis :- $\mu=73.2$

ii, Alternative Hypothesis :- $\mu>73.2$

iii, Level of Significance :- $Z_\alpha=1.645$ at $\alpha=0.05$

iv) Test Statistic :- $z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \frac{76.7-73.2}{8.6/\sqrt{140}} = 2.57$

& $|z| = 2.57$

v) Conclusion :- $|z| > z_\alpha$,

we reject null hypothesis

15. A Sample of 100 electric bulbs produced by manufacturer "A" showed a mean life time of 1190 hrs and S.D of 90 hrs. A Sample of 75 bulbs produced by manufacturer "B" showed a mean life time of 1230 hrs with Standard deviation of 120 hrs. Is there difference b/w the mean life times of the two brands at a significance level of 0.05.

Sol:- Given $n_1 = 100$, $n_2 = 75$

$\bar{x}_1 = 1170$, $\bar{x}_2 = 1230$

$S_1 = 90$, $S_2 = 120$

i) Null hypothesis :- $\bar{x}_1 = \bar{x}_2$

ii) Alternative hypothesis :- $\bar{x}_1 \neq \bar{x}_2$

iii) Level of significance :-

$\alpha = 0.05$ at $z_\alpha = 1.96$

iv) Test statistic :- $\cancel{z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}}}$

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{1190-1230}{\sqrt{\frac{90^2}{100} + \frac{120^2}{75}}} = -2.42$$

$|z| = 2.42$

v) conclusion :- $|z| > z_\alpha$,

we reject null hypothesis

16. On the basis of their total scores, 200 candidates of a civil service examination are divided into two groups; the first group is 30% and the remaining 70%. Consider the first question of the examination among the first group, 40 had the correct answer, whereas among the second group, 80 had the correct answer. On the basis of these results, can one conclude that the first question is not good at discriminating ability of the type being examined here.

$N = 200$

Sol: Given $n_1 = 30\% = \frac{30}{100} \times 200 = 60$, $x_1 = 40$, $x_2 = 80$

$$n_2 = 70\% = 200 - 60 = 140$$

$$P_1 = \frac{x_1}{n_1} = 0.67, \quad P_2 = \frac{x_2}{n_2} = 0.57$$

$$P = \frac{n_1 P_1 + n_2 P_2}{n_1 + n_2} = \frac{60 \times 0.67 + 140 \times 0.57}{60 + 140} = 0.67$$

$$q = 1 - P = 0.33$$

i) Null hypothesis :- $P_1 = P_2$

ii) Alternative hypothesis :- $P_1 \neq P_2$

iii) Level of significance :- $Z_\alpha = 1.96$ at $\alpha = 0.05$

iv) Test statistic :-

$$Z = \frac{P_1 - P_2}{\sqrt{pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{0.67 - 0.57}{\sqrt{(0.67)(0.33)\left(\frac{1}{60} + \frac{1}{140}\right)}}$$

$$Z = 1.37$$

$$\& \ |Z| = 1.37$$

v) Conclusion :- $|Z| < Z_\alpha$

$\therefore$ we accept null hypothesis

17. A cigarette manufacturing firm claims that brand A line of Cigarettes out sells its brand B by 8%. If it is found that 42 out of a sample of 200 Smokers prefer brand A and 18 out of another sample of 100 Smokers prefer brand B. Test whether 8% difference is a valid claim.

Sol: Given $n_1 = 200$, $n_2 = 100$

$x_1 = 42$, $x_2 = 18$

$p_1 = \frac{x_1}{n_1} = 0.21$, $p_2 = x_2/n_2 = 0.18$

$P = 8\%$, $q = 1 - 0.08 = 0.92$

i, Null hypothesis:- $P_1 = P_2$

ii, Alternative hypothesis:- $P_1 \neq P_2$

iii, Level of significance:- $Z_\alpha = 1.96$ at $\alpha = 0.05$

iv, Test statistic:-

$$Z = \frac{p_1 - p_2}{\sqrt{pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{0.21 - 0.18}{\sqrt{(0.08)(0.92)\left(\frac{1}{200} + \frac{1}{100}\right)}}$$

$= 0.90$

$|Z| = 0.90$

v, Conclusion:- $|Z| < Z_\alpha$

we accept null hypothesis

18. If 48 out of 400 persons in rural area possessed "cell" phones while 120 out of 500 in urban area. Can it be accepted that the proportion of "cell" phones in the rural area and urban area is same or not. Use 5% of level of significance.

Sol:- Given $\bar{x}_1 = 48$, $\bar{x}_2 = 120$ $\qquad p = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2} = 0.18$

$n_1 = 400$, $n_2 = 500$ $\qquad q = 1 - p = 0.82$

$$p_1 = \frac{x_1}{n_1} = \frac{48}{400} = 0.12, \quad p_2 = \frac{x_2}{n_2} = \frac{120}{500} = 0.24$$

i, Null hypothesis :- $P_1 = P_2$

ii, Alternative hypothesis:- $P_1 \neq P_2$

iii, Level of significance :- $Z_\alpha = 1.96$ at $\alpha = 0.05$

iv, Test statistic :-

$$Z = \frac{P_1 - P_2}{\sqrt{pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{0.12 - 0.24}{\sqrt{(0.18)(0.82)\left(\frac{1}{400} + \frac{1}{100}\right)}}$$

$= -4.8$

$|Z| = 4.8$

v, Conclusion :- $|Z| > Z_\alpha$

$\therefore$ we reject null hypothesis

19. Samples of students were drawn from two universities and from their weights in kilograms mean and S.D are calulated as shown below make a large sample test to the significance of difference b/w means.

	Mean	S.D	Sample size
University - A	55	10	400
University - B	57	15	100

Sol:- Given $\bar{x}_1 = 55$, $\bar{x}_2 = 57$, $n_1 = 400$, $n_2 = 100$

$s_1 = 10$, $s_2 = 15$

i, Null hypothesis:- $\bar{x}_1 = \bar{x}_2$

ii, Alternative hypothesis:- $\bar{x}_1 \neq \bar{x}_2$

iii, Level of significance :- $Z = \cancel{\frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}}$

$Z_\alpha = 1.96$ at $\alpha = 0.05$

iv, Test of statistic:-

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{55 - 57}{\sqrt{\frac{10^2}{400} + \frac{15^2}{100}}} = -1.26$$

& $|Z| = 1.26$

v, conclusion:- $|Z| < |Z_\alpha|$

$\therefore$ we accept null hypothesis

20. In a big city 325 men out of 600 men were found to be smokers. Does this information support the conclusion that the majority of men in this city are smokers?

Sol:- Given $n = 600$, $x = 325$, $p = \frac{x}{n} = 0.54$

$P = 0.5$, $Q = 0.5$

i, Null hypothesis:- $P = 0.5$

ii, Alternative hypothesis:- $P \neq 0.5$

iii, Level of significance:- $Z_\alpha = 1.96$ at $\alpha = 0.05$

iv, Test statistic:- $Z = \frac{p - P}{\sqrt{\frac{PQ}{n}}} = \frac{0.54 - 0.5}{\sqrt{\frac{0.5 \times 0.5}{100}}} = 1.95$

& $|Z| = 1.95$

v, Conclusion:- $|Z| < Z_\alpha$, we accept null hypothesis

PART-C

1. Let $S=\{1,5,6,8\}$. find the probability distribution of the sample mean for random sample of size 2 drawn without replacement. Find
i. The mean of the population
ii. The standard deviation of the population
iii. The mean of the sampling distribution of means
iv. The standard deviation of the sampling distribution of means.

Sol:
i. Mean of the population $(\mu) = \frac{1}{N}\sum x_i$

$= \frac{1}{4}\{1+5+6+8\}$

$= 5$

ii. Standard deviation of the population $(\sigma) = \sqrt{\frac{1}{N}\sum (x_i-\mu)^2}$

$= \sqrt{\frac{1}{4}\{(1-5)^2+(5-5)^2+(6-5)^2+(8-5)^2\}}$

$= 2.54$

Sampling without replacement:- $N_{C_n} = {}^4C_2 = 6$

Samples are

(1,5) (1,6) (1,8)

(5,6) (5,8)

(6,8)

Sample Means $(\bar{x}_i)$ are 3, 3.5, 4.5, 5.5, 6.5, 7

iii. Mean of the sampling distribution of means

$\mu_{\bar{x}} = \frac{1}{N_{C_n}}\sum(\bar{x}_i)$

$= \frac{1}{6}\{3+3.5+4.5+5.5+6.5+7\}$

$= 5$

iv, S.D of the sampling distribution of means

$$\sigma_{\bar{x}} = \sqrt{\frac{1}{N_{C_n}} \Sigma (\bar{x}_i - \mu_{\bar{x}})^2}$$

$$= \sqrt{\frac{1}{6} \{ (3-5)^2 + (3.5-5)^2 + \cdots + (7-5)^2}$$

$$= 1.612$$

2. Samples of size 2 are taken from the population 1, 2, 3, 4, 5, 6. Which can be drawn without replacement! Find
   i, The mean of the population
   ii, The Standard deviation of the population
   iii, The mean of the sampling distribution of means
   iv, The S.D of the sampling distribution of means.

Sol:- i, Mean of the Population $(\mu) = \frac{1}{N} \Sigma X_i$

$$= \frac{1}{6} \{1+2+3+4+5+6\}$$

$$= 3.5$$

ii, Variance of the population $(\sigma^2) = \frac{1}{N} \Sigma (X_i - \mu)^2$

$$= \frac{1}{6} \{ (1-3.5)^2 + (2-3.5)^2 + (3-3.5)^2 + (4-3.5)^2 + (5-3.5)^2 + (6-3.5)^2 \}$$

$$= 2.917$$

$\therefore$ S.D of the population $\sigma = 1.71$

Sampling without replacement :- $N_{C_n} = 6_{C_2} = 15$

Samples are

(1,2) (1,3) (1,4) (1,5) (1,6)
(2,3) (2,4) (2,5) (2,6)
(3,4) (3,5) (3,6)
(4,5) (4,6)
(5,6)

Sample means ($\bar{x}_i$) are

1.5	2	2.5	3	3.5
	2.5	3	3.5	4
		3.5	4	4.5
			4.5	5
				5.5

iii, Mean of the sampling distribution of means

$$\mu_{\bar{x}} = \frac{1}{N_{C_n}} \Sigma (\bar{x}_i)$$

$$= \frac{1}{15} \{ 1.5 + 2 + 2.5 + \cdots + 5.5 \}$$

$$= 3.5$$

iv, Variance of the sampling distribution of means

$$\sigma_{\bar{x}}^2 = \frac{1}{N_{C_n}} \Sigma (\bar{x}_i - \mu_{\bar{x}})^2$$

$$= \frac{1}{15} \{ (1.5 - 3.5)^2 + (2 - 3.5)^2 + (2.5 - 3.5)^2 + \cdots + (5.5 - 3.5)^2 \}$$

$$= 1.16$$

$\therefore$ S.D of the sampling distribution of means $= 1.07$

3. A normal Population has a mean of 0.1 and S.D of 2.1. Find the probability that mean of a sample of size 900 will be negative.

Sol: Given $\mu = 0.1$, $\sigma = 2.1$, $n = 900$

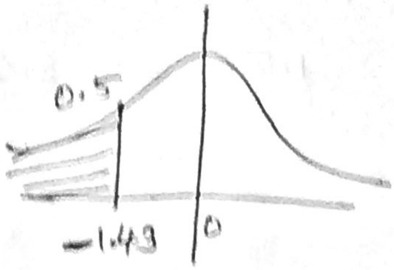
$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{\bar{x} - 0.1}{2.1/\sqrt{900}} = \frac{\bar{x} - 0.1}{0.07}$$

$$\Rightarrow \bar{x} = 0.1 + 0.007 Z$$

$$\therefore P(\bar{x} < 0) = P(0.1 + 0.07 Z < 0)$$

$= P\left(z < \frac{-0.1}{0.07}\right)$

$= P(z < -1.43)$



$= 0.5 - P(-1.43 \le z \le 0)$

$= 0.5 - P(0 \le z \le 1.43)$

$= 0.5 - 0.4236$ [$\because$ By tables]

$= 0.0764$

4. A random sample of size 64 is taken from an infinite population having the mean 45 and the standard deviation 8. What is the probability that $\bar{x}$ will be between 46 and 47.5.

Soln: Given $n = 64$, $\mu = 45$, $\sigma = 8$,

$P(46 \le \bar{x} \le 47.5)$

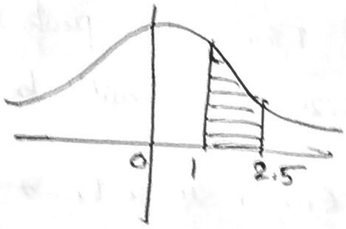
$= P(\bar{x}_1 \le x < \bar{x}_2)$

$z_1 = \dfrac{\bar{x}_1 - \mu}{\sigma/\sqrt{n}} = \dfrac{46 - 45}{8/\sqrt{64}} = 1$

$z_2 = \dfrac{\bar{x}_2 - \mu}{\sigma/\sqrt{n}} = \dfrac{47.5 - 45}{8/\sqrt{64}} = 2.5$

$\therefore P(1 < z < 2.5)$

$= P(0 < z < 2.5) - P(0 < z < 1)$



$= 0.4938 - 0.3413$

$= 0.1525$

5. If a 1-gallon can of paint covers on an average 513 square feet with a S.D of 31.5 sq. ft. what is the probability that the mean area covered by a sample of 40 of these 1-gallon cans will be anywhere from 510 to 520 sq.ft.

Sol Given $n=40$, $\mu=513$, $\sigma=31.5$

$$P(510<\bar{x}<520)$$

$$=P(\bar{x}_1<\bar{x}<\bar{x}_2)$$

$$z_1=\frac{\bar{x}_1-\mu}{\sigma/\sqrt{n}}=\frac{510-513}{31.5/\sqrt{40}}=-0.6$$

$$z_2=\frac{\bar{x}_2-\mu}{\sigma/\sqrt{n}}=\frac{520-513}{31.5/\sqrt{40}}=1.4$$

$$\therefore P(z_1<z<z_2)$$

$$=P(-0.6<z<1.4)$$

$$=P(-0.6<z<0)+P(0<z<1.4)$$

$$=P(0<z<0.6)+P(0<z<1.4)$$

$$=0.2258+0.4192$$

$$=0.645$$

6. A sample of 900 members has mean of 3.4 & S.D of 2.61 is this sample has been taken from a large Population mean 3.25 and S.D 2.61. Also Calculate 95% confidence interval.

Sol Given $n=900$, $\bar{x}=3.4$, $\mu=3.25$, $S=2.61$

i, Null hypothesis (H_0):– $\mu=3.25$

ii, Alternative hypothesis (H_1):– $\mu\neq3.25$

iii, Level of Significance (α):– $Z_\alpha=1.96$ at $\alpha=0.05$

iv, Test statistic:– $z=\frac{\bar{x}-\mu}{\sigma/\sqrt{n}}=\frac{3.4-3.25}{\frac{2.61}{\sqrt{900}}}=1.72$

& $|z|=1.72$

v, Conclusion:– $|z|<Z_\alpha$

$\therefore$ we accept null hypothesis

7. It is claimed that a random sample of 49 tyres has a mean life of 15200 kms. This sample was taken from population whose mean is 15150 kms and S.D is 1200 km test 0.05 level of significant.

Sol:-

Given $n=49$, $\bar{x}=15200$

$\mu=15150$, $\sigma=1200$, $\alpha=0.05$

i) Null hypothesis (H_0):- $\mu=15150$

ii, Alternative hypothesis (H_1):- $\mu\neq15150$

iii, Level of significance (α):- $Z_\alpha=1.96$ at $\alpha=0.05$

iv Test statistic:- $Z = \dfrac{\bar{x}-\mu}{\sigma/\sqrt{n}}$

$= \dfrac{15200-15150}{\frac{1200}{\sqrt{49}}}$

$=-79.5$

$\& \ |Z|=79.5$

v, Conclusion:- $|Z|>Z_\alpha$,

$\therefore$ we reject null hypothesis

8. A manufactures claims that at least 95% of the equipment which he supplied to a factory conformed to specifications. An examination of sample of 200 pieces of equipments received 18 were faulty test the claim at 0.05 level.

Sol: Given $P = 95\% = 0.95$, $Q=1-P=0.05$, $\alpha=0.05$

$n=200$, $x=18$, $p=\frac{x}{n}=\frac{18}{200}=0.09$

i. Null hypothesis (H_0):- $P = 0.95$

ii. Alternative hypothesis (H_1):- $P \neq 0.95$

iii. Level of significance (α):- $Z_\alpha = 1.96$ at $\alpha = 0.05$

iv. Test statistic:- $$Z = \frac{p - P}{\sqrt{\frac{PQ}{n}}}$$

$$Z = \frac{0.09 - 0.95}{\sqrt{\frac{0.95 \times 0.05}{200}}} = -55.8$$

$$Z = -55.8 \quad \& \quad |Z| = 55.8$$

v. conclusion:- $|Z| > Z_\alpha$

$\therefore$ we reject null hypothesis

9. Among the items produced by a factory out of 500, 15 were defective. In another sample of 400, 20 were defective, test the significant difference between two proportions at 5% level.

Sol: Given $n_1 = 500$, $n_2 = 400$

$x_1 = 15$, $x_2 = 20$, $\alpha = 0.05$

$$p_1 = \frac{x_1}{n_1} = 0.03, \quad p_2 = \frac{x_2}{n_2} = 0.05$$

$$p = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2} = 0.03, \quad q = 1 - p = 0.97$$

i) Null hypothesis (H_0):- $P_1 = P_2$

ii. Alternative hypothesis (H_1):- $P_1 \neq P_2$

iii. Level of significance (α):- $Z_\alpha = 1.96$ at $\alpha = 0.05$

iv, Test statistic:-

$$Z = \frac{p_1 - p_2}{\sqrt{pq\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} = \frac{0.03-0.05}{\sqrt{(0.03)(0.97)\left(\frac{1}{500}+\frac{1}{400}\right)}} = -1.81$$

$$|Z| = 1.81$$

v, Conclusion:- $|Z| < Z_\alpha$, we accept null hypothesis.

10. A manufacturer produced 20 defective articles in a batch of 400. After over hauled it produced 10 defective in a batch of 300 has a machine being improved after over hauling.

Sol:- Given $n_1 = 400$, $n_2 = 300$

$x_1 = 20$, $x_2 = 10$ $\alpha = 0.05$

$$p_1 = \frac{x_1}{n_1} = 0.05, \quad p_2 = \frac{x_2}{n_2} = 0.03$$

$$p = \frac{x_1 + x_2}{n_1 + n_2} = 0.04, \quad q = 1 - p = 0.96$$

i, Null hypothesis (H0) :- $P_1 = P_2$

ii, Alternative hypothesis (H1) :- $P_1 \neq P_2$

iii, Level of significance (α) :- $Z_\alpha = 1.96$ at $\alpha = 0.05$

iv, Test statistic :- $$Z = \frac{P_1 - P_2}{\sqrt{pq\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} = \frac{0.05-0.03}{\sqrt{(0.04)(0.96)\left(\frac{1}{400}+\frac{1}{300}\right)}}$$

$$Z = 1.42$$

$$|Z| = 1.42$$

v, Conclusion :- $|Z| < Z_\alpha$, we accept null hypothesis