

# TMMV01 Aerodynamics - Assignment 2018

## Rotor Aerodynamics and Blade Element Momentum Theory

The aim of the assignment is to:

- Apply theoretical knowledge to an engineering problem.
- Use a calculation (simulation) model to investigate the performance of a wind turbine.
- Show skills to interpret and analyze results.
- Communicate (present) aerodynamic methodology, results, and analysis in a clear and convincing way.

The grade is pass/fail.

### Introduction

Energy can be extracted from the wind and converted to electric power as the wind blows over a wind turbine, which then, due to the resulting aerodynamic forces, starts to rotate. A theoretical upper limit for how much of the kinetic energy of the wind that can be converted to mechanical energy is given by the Betz limit, Eq. (3.2.15) in [1], to 59%. From now on all equations, figures, and chapters which are referred to are found in [1], and the reference will not be written.

The blades of a wind turbine work exactly as wings of an aircraft; due to the motion of the fluid (air) aerodynamic forces act on the blades. For the aircraft, the lift (perpendicular to the freestream) keeps it in the air, whereas the drag resists the motion. For the wind turbine blade one force component contributes to the power generating torque on the rotor, whereas the other component creates a bending moment on the blade. The major difference between a wing and a rotor blade is the velocity component due to the rotation of the rotor. See Fig. 3.21 for an illustration of forces and velocity component on the rotor blade.

The generated power depends on wind speed  $U$ , angular velocity of the rotor  $\Omega$ , number of blades  $B$ , angle of attack  $\alpha(r)$ , pitch angle along the blade  $\theta_p(r)$ , chord length  $c(r)$ , shape of the profile (airfoil) etc, where  $r$  is a coordinate along the blade;  $r = 0$  at the hub.

### Blade Element Momentum Theory

Blade element momentum theory (BEM) is a calculation method originally developed for propellers, but just as applicable for other rotors, e.g. wind turbines, to compute forces, torques, and subsequently the produced power. For the analysis the rotor blade is divided into a number of elements (hence the part "blade element" of the name) in the radial direction, see Fig. 3.19. Each of these is treated as an airfoil (2D), and the resulting forces and moments on the blade are obtained as the sum of the contributions from all elements. Read section 3.5.3 for a detailed description of the BEM and turbine blade aerodynamics. Eq. (3.5.12) and Eq. (3.5.14) are fundamental for the work.

Somehow, the extracted energy from the wind must also enter the problem and this is accomplished by introducing momentum theory, which relates forces on the rotor to the changes of momentum of the wind as it passes the rotor. This is discussed in Sec. 3.2-3.3 and summarized in Sec. 3.5.2, with Eq. (3.5.1) and (3.5.2) being the fundamental results.

By combining the expressions from momentum theory with those from blade element theory the result can be used to determine the forces and torques on the blade. Depending on the problem, different solution methods can be used. Here, we will follow Method 2 in Sec. 3.7.1.4. This assumes that everything is specified except  $C_l$ ,  $C_d$ , and  $\alpha$ , which then can be computed. In short, this is an iterative procedure where  $a$ ,  $a'$ , and  $\alpha$  are calculated while  $c_l$  and  $c_d$  are taken from a diagram or table<sup>1</sup>. Subsequently, when  $\alpha$ ,  $c_l$ , and  $c_d$  are known for each element, the forces and power can be computed.

## Problem Formulation

You are working with aerodynamic analyses at a consultant company, and a customer wants to investigate the possibilities to build new wind power plants at a site where the average wind speed is  $X$  m/s. The customer desires a power production of about  $Y$  MW from a single wind turbine.  $X$  and  $Y$  are group specific parameters, a list for selection of  $X$  and  $Y$  will be available.

### To do

Propose design parameters for a wind power plant with three blades that delivers the desired power  $Y$  at the average wind speed  $X$ . I.e. determine blade length, angular velocity of the rotor, chord length, and pitch angle along the blade, i.e. as functions of the radius. Also, select at least two different airfoils, in order to meet the varying spanwise requirements. I.e. there will be different airfoils at the root and tip due to different needs. A link to a data base with wind turbine airfoils and aerodynamic characteristics are found on the course web page. Divide the blade into at least 15 equally long elements. Assume that the hub covers the first 5 % of the span. Start to read section 3.7.3 and modify the equations in the lecture notes to take tip losses into account. The angle of attack must be close to the one for maximum  $C_l/C_d$  along most of the span. Follow Method 2, p. 116 in [1], and see the lecture slides for detailed instructions about the methodology. Required parameters which are not specified must be chosen with reasonable motivation. Show the obtained power and the corresponding power coefficient.

In addition, use your program to investigate the performance when the wind speed varies from 4 m/s to 26 m/s for your recommended configuration and comment on the major reasons for reduced performance at off-design conditions. If the solver does not work for some conditions, explain why. Based on your results and findings, suggest how to improve the design for better performance for a larger interval of wind speeds (the suggested findings do not have to be implemented).

### Figures to present:

- Reynolds number, rotational velocity, and wind speed (for comparison) along the blade.
- Angle of attack, pitch angle and  $\phi$  along the blade. Show the AoA for max  $C_l/C_d$ .
- $a$  and  $a'$  along the blade.
- Forces and force coefficients along the blade.
- Tip loss correction  $F$  factor along the blade.
- Power coefficient for varying wind speed.

Analyze and discuss the results and how they were obtained. Convince the customer (=examiner) that you understand what you have done by relating the results to theory discussed in the course, and comment on strengths and weaknesses of the methodology. Discuss all figures in the result section thoroughly. The customer would appreciate information about which parameters that have the largest impact on the results, to use as a decision support for further studies.

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<sup>1</sup>Tip: store the tabulated AoA,  $c_l$ , and  $c_d$  in three vectors in Matlab and use `interp1` to interpolate the values for the calculated AoA.

## Presentation

Present your work in a technical report containing a short Introduction, Method, Results, Discussion, and Conclusion. The approximate word count is 1500 words, i.e. the assignment can be presented within this number of words. Irrelevant content can be a reason for correction, even with less than 1500 words. Write the number of words on the front page! The number of diagrams/plots is limited to ten. Start with a very short introduction to the work, and describe the method so that the report with appropriate references contains sufficient information to redo the work. Equations are allowed but not a requirement and should only be included if they are discussed explicitly, e.g. in the analysis, or illustrative/important for the understanding of the method. Focus of the report should be on results and discussion, i.e. these parts must constitute the largest part of the report. *The report must be prepared by the authors on the front page. To submit copies of others' works, parts or whole, is not acceptable. All reports will be checked for plagiarism.*

See the general guide lines concerning the report format and layout.

Good luck!

## Errata

Eq. (3.2.7):

$$U_2 = \frac{U_1 + U_4}{2}$$

Eq. (3.2.11):  $V_2 \rightarrow U_2$

Eq. (3.3.6):

$$\frac{a(1-a)}{a'(1+a')}$$

Eq. (3.3.20)

$$(1-a_2)(4a_2-1)^2/(1-3a_2)$$

Eq. (3.7.4):

$$\frac{a'}{1-a} = \frac{\sigma' C_l}{4\Omega \sin\varphi}$$

P. 116 (and on other pages)  $\varphi = \alpha + \theta_p$

## References

[1] Manwell, J.F., McGowan J.G., and Rogers, A.L., *Wind Energy Explained - Theory, Design and Application*, John Wiley & Sons Ltd, 2002, Electronic.