---1120201058

T. Yuvitha

Tutorial - 06

01 1. $\int \sin(x) + 10\cos^2(x)\, dx$

$\int \sin(x)\,dx + 10\int \cos^2(x)\,dx$

$-\cos(x) - 10\cot(x) + C$

2. $\int 4e^x + 15 - \frac{1}{6z}\, dz$

$4\int e^x\, dz + 15\int dz - \frac{1}{6}\int \frac{1}{z}\, dz$

$4e^z + 15z - \frac{1}{6}\ln|z| + C$

3. $\int_2^4 \frac{3z^2+1}{(z+1)(z-5)^2}\, dz$

$$\frac{3z^2+1}{(z+1)(z-5)^2} = \frac{A}{z+1} + \frac{B}{z-5} + \frac{C}{(z-5)^2}$$

$3z^2+1 = A(z-5)^2 + B(z+1)(z-5) + C(z+1)$

$3z^2+1 = (A+B)z^2 + (-10A - 4B + c)z + (25A - 5B + c)$

z^2	z	z
$A+B=3$	$-10A - 4B + C = 0$	$25A - 5B + 10A + 4B = 1$
$36A = 4$	$C = 10A + 4B$	$35A - B = 1$
$A = 1/9$		

$$\frac{3z^2+1}{(z+1)(z-5)^2}\,dz = \frac{1}{9(z+1)} + \frac{26}{9(z-5)} + \frac{38}{3(z-5)^2}$$

$$\int_2^4 \frac{3z^2+1}{(z+1)(z-5)^2} = \frac{1}{9}\int_2^4 \frac{1}{(z+1)}\,dz + \frac{26}{9}\int_2^4 \frac{dz}{(z-5)} + \frac{38}{3}\int_2^4 \frac{dz}{(z-5)^2}$$

$$= \frac{1}{9}\Big[\ln|z+1|\Big]_2^4 + \frac{26}{9}\Big[\ln|z-5|\Big]_2^4 + \frac{38}{3}\left[\frac{-1}{z-5}\right]_2^4$$

$$= \frac{1}{9}\left(\ln|5| - \ln|3|\right) + \frac{26}{9}\left(\ln|1| - \ln|3|\right) + \frac{152}{9}$$

$$= \frac{1}{9}\ln|5| - 3\ln|3| + \frac{152}{9}$$

(02) 1. $\int 2x\,(x^2+3)^4\,dx$

$u = x^3+3$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

$= \int (x^2+3)^4 \cdot 2x\,dx$

$= \int u^4\,du$

$= \frac{u^5}{5} + c$

$= \frac{(x^2+3)^5}{5} + c$

2. $x\sqrt{x+3}\,dx$

$u = x+3 \qquad x = u-3$

$\frac{du}{dx} = 1$

$du = dx$

$$\int x\sqrt{x+3}\,dx = \int (u-3)\sqrt{u}\,du$$
$$= \int (u\sqrt{u} - 3\sqrt{u})\,du$$
$$= \int (u^{3/2} - 3u^{1/2})\,du$$
$$= \int u^{3/2}\,du - 3\int u^{1/2}\,du$$
$$= \frac{2}{5}u^{5/2} - \frac{6}{3}u^{3/2}$$
$$= \frac{2}{5}u^{5/2} - 2u^{3/2} + c$$
$$= \frac{2}{5}(x+3)^{5/2} - 2(x+3)^{3/2} + c$$

3. $\int \frac{4-x}{\sqrt{16-x^2}}\,dx$

$$= \int \frac{4}{\sqrt{16-x^2}}\,dx - \int \frac{x}{\sqrt{16-x^2}}\,dx$$

$u = 16 - x^2$

$\frac{du}{dx} = -2x$

$du = -2x\,dx$

For $\int \frac{4}{\sqrt{16-x^2}}\,dx$, use the substitution $x = 4\sin\theta$

$dx = 4\cos\theta \cdot d\theta$

$16-x^2 = 16-16\sin^2\theta$

$= 16\cos^2\theta$

$\sin x = 4\sin\theta$

$\therefore \theta = 4\sin^{-1}(x/4)$

$\frac{4}{\sqrt{16-x^2}}\,dx = \frac{4}{4\cos\theta} \cdot 4\cos\theta\,d\theta$

$\int 4\,d\theta = 4\theta + C_1$

$4\sin^{-1}\left(\frac{x}{4}\right) + C_1$

For $\int \frac{x}{\sqrt{16-x^2}}\,dx$

$= \int \frac{x}{\sqrt{u}} \cdot -\frac{1}{2x}\,du$

$= -\frac{1}{2}\int \frac{1}{\sqrt{u}}\,du$

$= -\frac{1}{2} \cdot 2u^{1/2} + C_2$

$= -\sqrt{u} + C_2$

$= -\sqrt{16-x^2} + C_2$

$\therefore \int \frac{4-x}{\sqrt{16-x^2}}\,dx = 4\sin^{-1}\left(\frac{x}{4}\right) - \sqrt{16-x^2} + C$

Q3) 1. $\int 4x\cos(2-3x)\,dx$

$u = 4x$ $\qquad$ $dv = \cos(2-3x)$

$du = 4\,dx$ $\qquad$ $v = \int \cos(2-3x)\,dx$

$w = 2-3x$

$dw = -3\,dx$

$dx = -\frac{1}{3}\,dw$

$V = \int \cos(2-3x)\,dx = \int \cos(w)\left(-\frac{1}{3}\,dw\right) = -\frac{1}{3}\sin(w) + c$

$v = -\frac{1}{3}\sin(2-3x) + c$

$\int u.dv = uv - \int dv.du$

$\int 4x\cos(2-3x)\,dx = 4x\left[-\frac{1}{3}\sin(2-5x)\right] - \int -\frac{1}{3}\sin(2-3x)\,4$

$= -\frac{4x}{3}\sin(2-3x) + \frac{4}{3}\int \sin(2-3x)\,dx$

Again, $w = 2-3x$

$dw = -3\,dx$

$dz = -\frac{1}{3}\,dw$

$\int \sin(2-3x)\,dx = \int \sin(w)\left(-\frac{1}{3}\,dw\right)$

$= -\frac{1}{3}\int \sin(w)\,dw$

$= \frac{1}{3}\cos w + c$

$= \frac{1}{3}\cos(2-3x) + c$

$= \frac{4x}{3}\sin(2-3x) + \frac{4}{9}\cos(2-3x) + C$

2. $\int (3t + t^2)\sin(2t)\,dt$

$= \int 3t\sin(2t) + t^2\sin(2t)\,dt$

$= 3\int t\sin(2t)\,dt + \int t^2\sin(2t)\,dt$

$= \int 3t\sin(2t)\,dt$

$u = 3t$ $\qquad\qquad$ $dv = \sin(2t)\,dt$

$du = 3\cdot dt$ $\qquad\qquad$ $v = \int \sin(2t)\,dt$

$w = 2t$

$dt = \frac{1}{2}\,dw$

$v = \int \sin(2t)\cdot dt$

$= \int \sin(w)\cdot \frac{1}{2}\,dw$

$= -\frac{1}{2}\cos(2t)$

$\int \cos(2t)\cdot dt$

$w = 2t$

$dt = \frac{1}{2}\,dw$

$\int \cos(2t)\cdot dt = \frac{1}{2}\int \cos(w)\cdot dw$

$= \frac{1}{2}\sin(2t)$

$\int 3t\sin(2t)\cdot dt - \frac{3t}{2}\cos(2t) + \frac{3}{4}\sin(2t) + c$

$\int t^2\sin(2t)\,dt$ $\qquad\qquad$ $dv = \sin(2t)\,dt$

$u = t^2$

$du = 2t\cdot dt$ $\qquad\qquad$ $v = -\frac{1}{2}\cos(2t)$

$$\int u \cdot dv = u \cdot v - \int v \, du$$

$$\int t^2 \sin(2t)\,dt = \frac{t^2}{2}\cos(2t) - \int t \cdot \cos(2t) \cdot dt$$

$$= \int t \cdot \cos(2t)\,dt$$

$u = t$

$du = dt$

$dv = \cos(2t)\,dt$

$v = \frac{1}{2}\sin(2t)$

$$\int t \cdot \cos(2t)\,dt = \frac{t}{2}\sin(2t) - \frac{1}{2}\int \sin(2t)\,dt$$

$$= \frac{t}{2}\sin(2t) + \frac{1}{4}\cos(2t$$

$$\int t^2 \sin(2t)\,dt = -\frac{t^2}{2}\cos(2t) + \frac{t}{2}\sin(2t) + \frac{1}{4}\cos(2t) + C$$

$$\int (3t + t^2)\sin(2t)\,dt = \frac{-3t}{2}\cos(2t) + \frac{3}{4}\sin(2t)$$

$$- \frac{t^2}{2}\cos(2t) + \frac{t}{2}\sin(2t)$$

$$+ \frac{t}{2}\sin(2t) + \frac{1}{4}\cos(2t) + C$$

$$= 3\left[\frac{1}{4}\sin(2t) - \frac{t}{2}\cos(2t)\right] - \frac{t^2}{2}\cos(2t)$$

$$+ \frac{1}{4}\left[2t\sin(2t) + \cos(2t)\right] + C$$

3. $\int 6\tan^{-1}\left(\frac{8}{w}\right)dw$

$dv = dw$

$v = w$

$u = 6\tan^{-1}\left(\frac{8}{w}\right)$

$du = 6 \cdot \frac{d}{dw}\left[\tan^{-1}(8/w)\right]$

$= \frac{6}{1+(8/w)^2} \times \frac{d}{dw}(8/w)$

$= \frac{6}{1+\frac{64}{w^2}} \times \frac{-8}{w^2}$

$du = \frac{-48}{w^2+64}$

$\int u \cdot dv = u \cdot v - \int v \cdot du$

$\int 6\tan^{-1}(8/w)dw = 6\tan^{-1}(8/w) \cdot w + 48\int \frac{w}{w^2+64}dw$

$u = w^2+64$

$du = 2w \cdot dw$

$w \cdot dw = \frac{1}{2}du$

So $\int \frac{w}{w^2+64}dw = \frac{1}{2}\int \frac{1}{u}du$

$= \frac{1}{2}\ln|w| + c$

$= \frac{1}{2}\ln|w^2+64| + c$

$\int 6\tan^{-1}\left(\frac{8}{w}\right)dw = 6w\tan^{-1}\left(\frac{8}{w}\right) + 24\ln|w^2+64| + C$

4. $\int e^{2x} \cos\left(\frac{1}{4}z\right) dz$

$u \cdot \cos(z/4)$ $\qquad$ $dv = e^{2z} dz$

$du = \frac{-1}{4} \sin\left(\frac{z}{4}\right) dz$ $\qquad$ $v = \frac{1}{2} e^{2z}$

$\int u \cdot dv = u \cdot v - \int v \cdot du$

$\int e^{2z} \cos(z/4)\, dz = \cos(z/4) \frac{1}{2} e^{2z} + \int \frac{1}{2} e^{2z} \cdot \frac{1}{4} \sin(z/4) dz$

$= \frac{e^{2z}}{2} \cos\left(\frac{z}{4}\right) + \frac{1}{8} \int e^{2z} \sin\left(\frac{z}{4}\right) dz$

$\int e^{2z} \sin\left(\frac{z}{4}\right) dz$

$u = \sin \frac{z}{4}$ $\qquad$ $dv = e^{2z} dz$

$du = \frac{1}{4} \cos\left(\frac{z}{4}\right) dz$ $\qquad$ $v = \frac{1}{2} e^{2z}$

$\int e^{2z} \sin\left(\frac{z}{u}\right) dz = \frac{1}{2} e^{2z} \sin\left(\frac{z}{4}\right) - \frac{1}{8} \int e^{2z} \cos\left(\frac{z}{4}\right) dz$

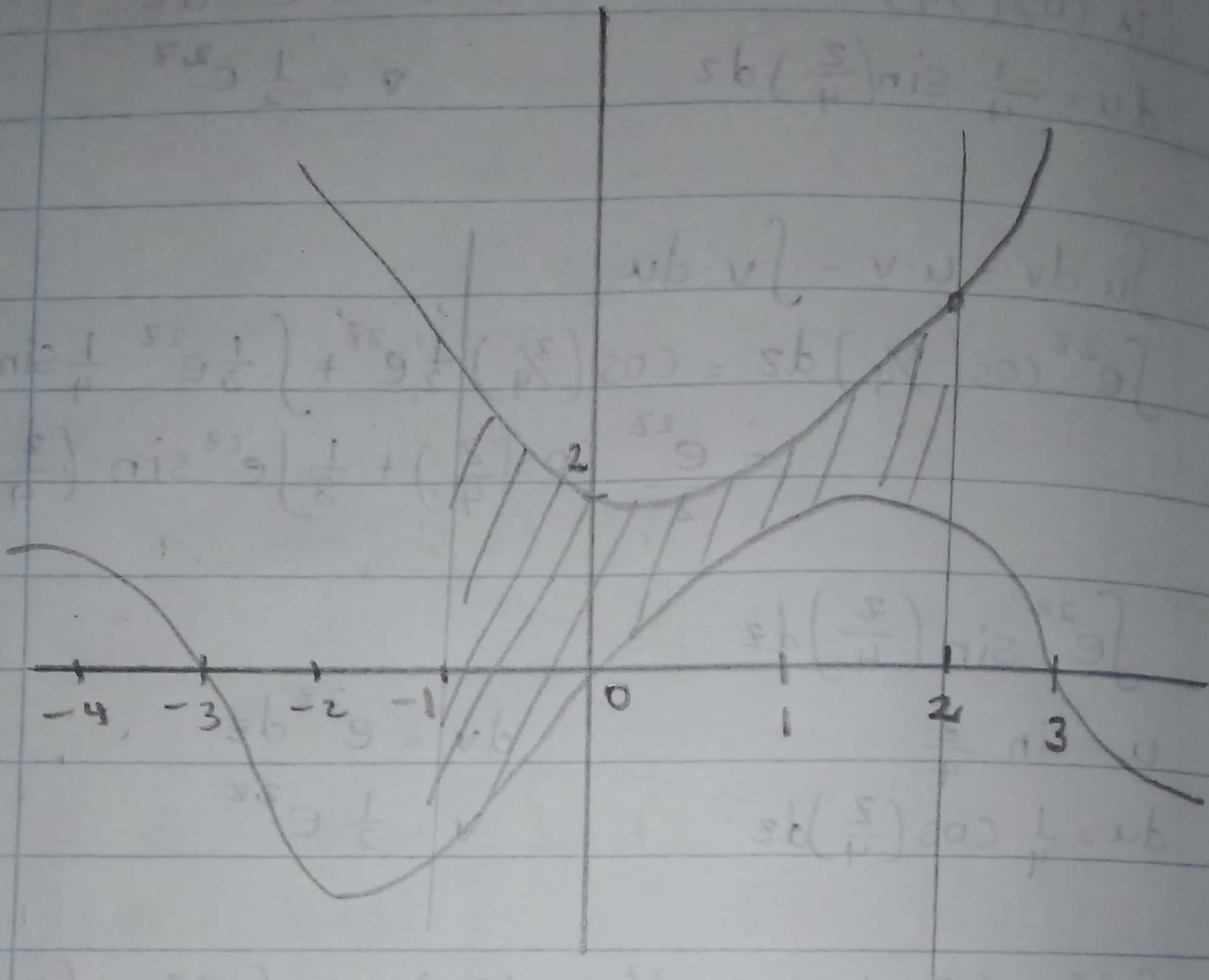
Let $I = \int e^{z} \cos\left(\frac{z}{4}\right) dz$

$I = \frac{1}{2} e^{2z} \cos\left(\frac{z}{4}\right) + \frac{1}{8}\left[\frac{1}{2} e^{2z} \sin\left(\frac{z}{4}\right) - \frac{1}{8} I\right]$

$I = \frac{32}{65} e^{2z} \cos\left(\frac{z}{4}\right) + \frac{4}{65} e^{2z} \sin\left(\frac{z}{4}\right) + C$

$\int e^{2z} \cos\left(\frac{z}{4}\right) = \frac{32}{65} e^{2z} \cos\left(\frac{z}{4}\right) + \frac{4}{65} e^{2z} \sin\left(\frac{z}{4}\right) +$

(04) i) $y = x^2 + 2$, $y = \sin(x)$, $x = -1$, $x = 2$



$$A = \int_{-1}^{2} (x^2 + 2 - \sin(x))\,dx$$

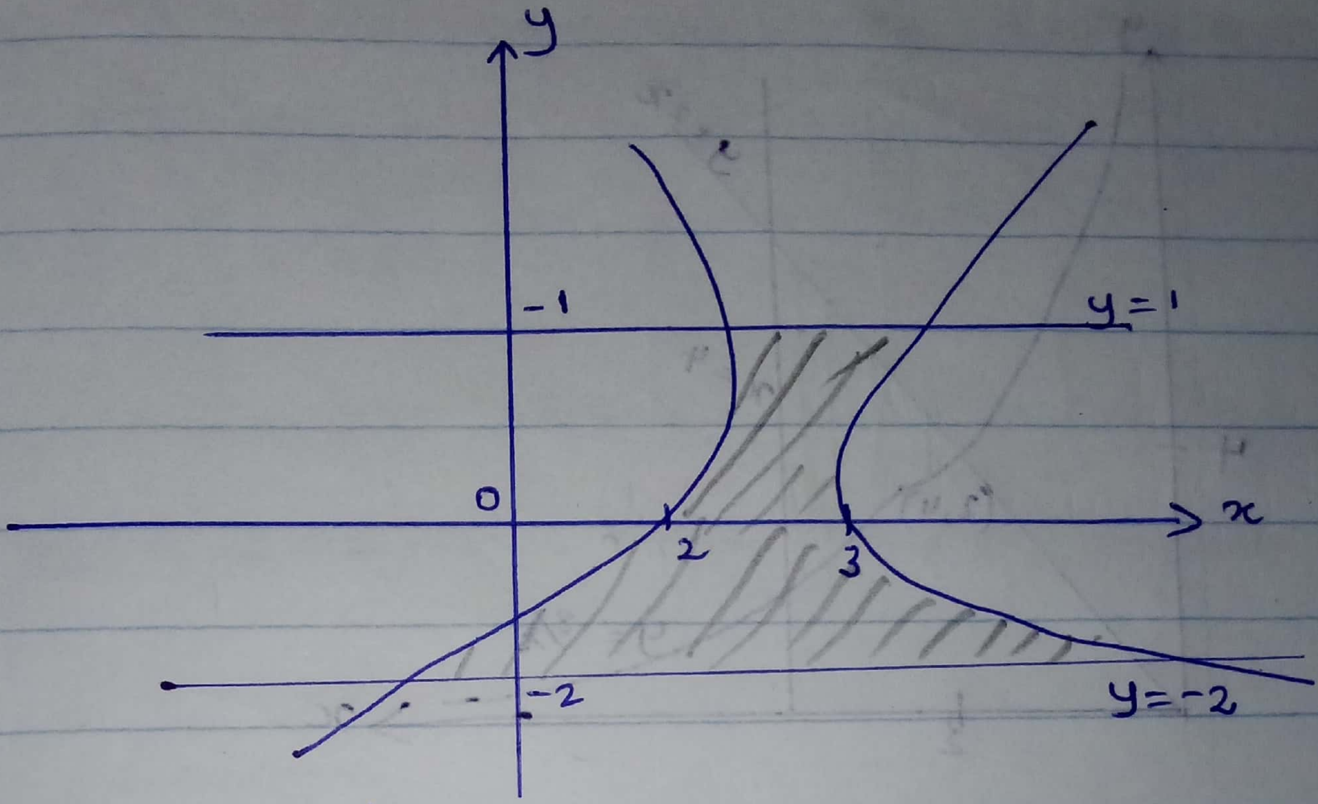
$$= \left[\frac{x^3}{3} + 2x + \cos(x) \right]_{-1}^{2}$$

$$= \frac{2^3}{3} + 2 \cdot 2 + \cos(2) - \frac{(-1)^3}{3} - 2(-1) - \cos(-1)$$

$$= 9 + \cos(2) - \cos(-1)$$

$A \simeq 8.044$

ii) $x = 3 + y^2$, $x = 2 - y^2$, $y = 1$, $y = 2$



$$\text{Area} = \int_{-2}^{1} \left((3+y^2) - (2-y^2) \right) dy$$

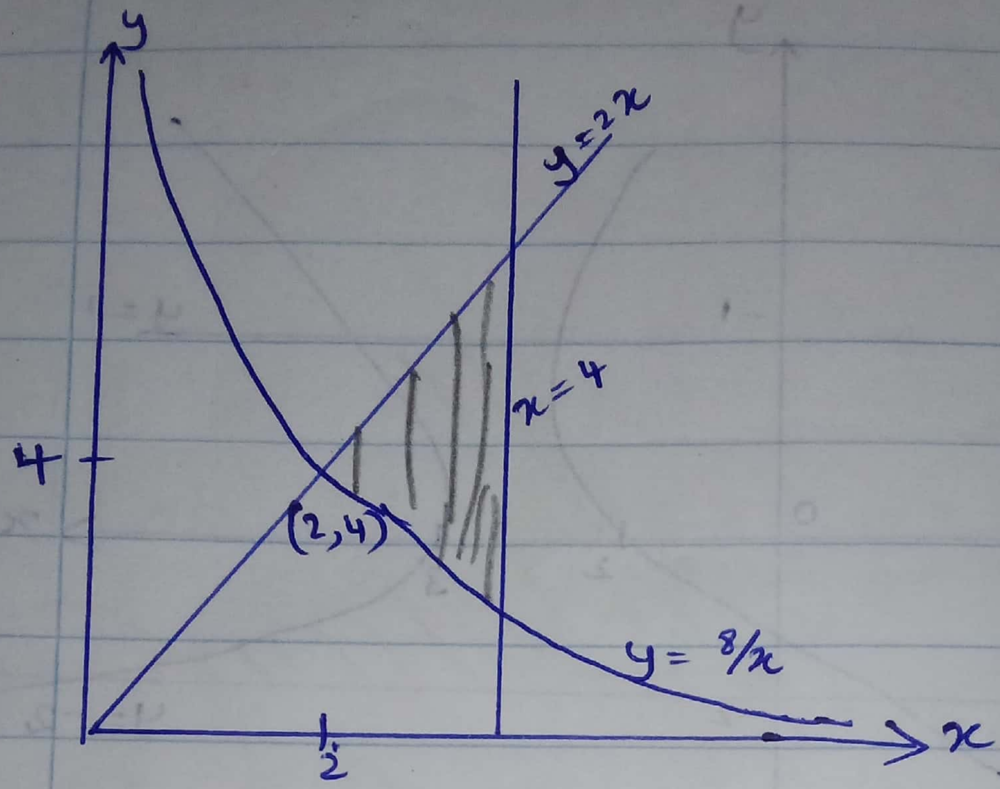
$$= \int_{-2}^{1} (1 + 2y^2)\, dy$$

$$= \left(y + \frac{2y^3}{3} \right) \Big|_{-2}^{1}$$

$$= \left(1 + \frac{2}{3} \right) - \left(-2 - \frac{16}{3} \right)$$

$$= 9$$

iii) $y = \frac{8}{x}$, $y = 2x$, $x = 4$



$$\text{Area} = \int_2^4 \left(2x - \frac{8}{x}\right) dx$$

$$= \left(x^2 - 8\ln x\right)\Big|_2^4$$

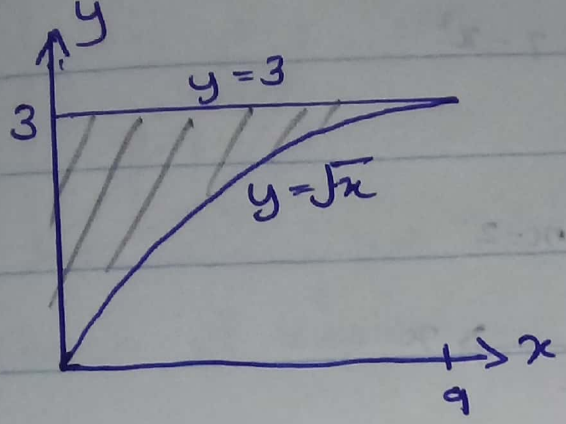
$$= \left(16 - 8\ln 4\right) - \left(4 - 8\ln 2\right)$$

$$= 12 - 8\ln 2$$

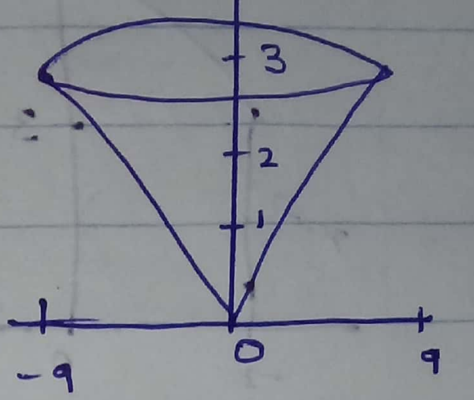
$$\simeq 6.4548$$

(5)

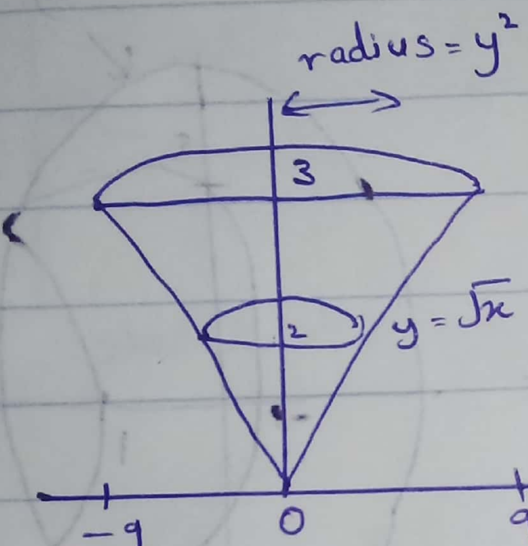
i) $y = \sqrt{x}$, $y = 3$ and y axis ; about the y axis



Sketch of region



sketch of Solid



sketch of disk

Radius $= y^2$

Area of $(y) = \pi \,(\text{radius})^2$

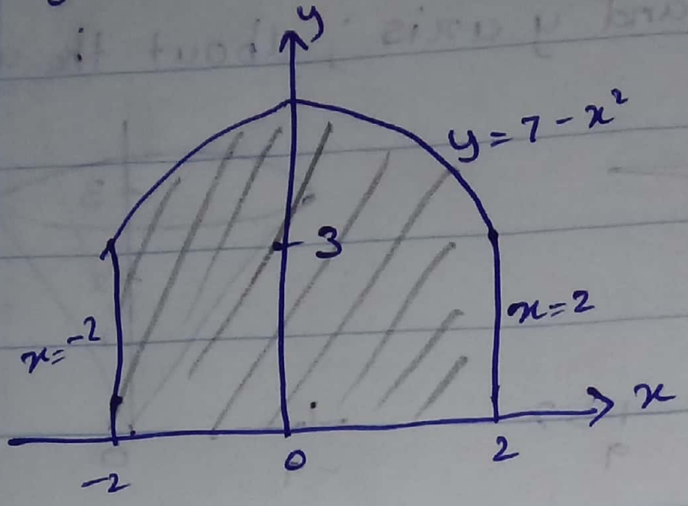
$= \pi (y^2)^2$

$= \pi y^4$

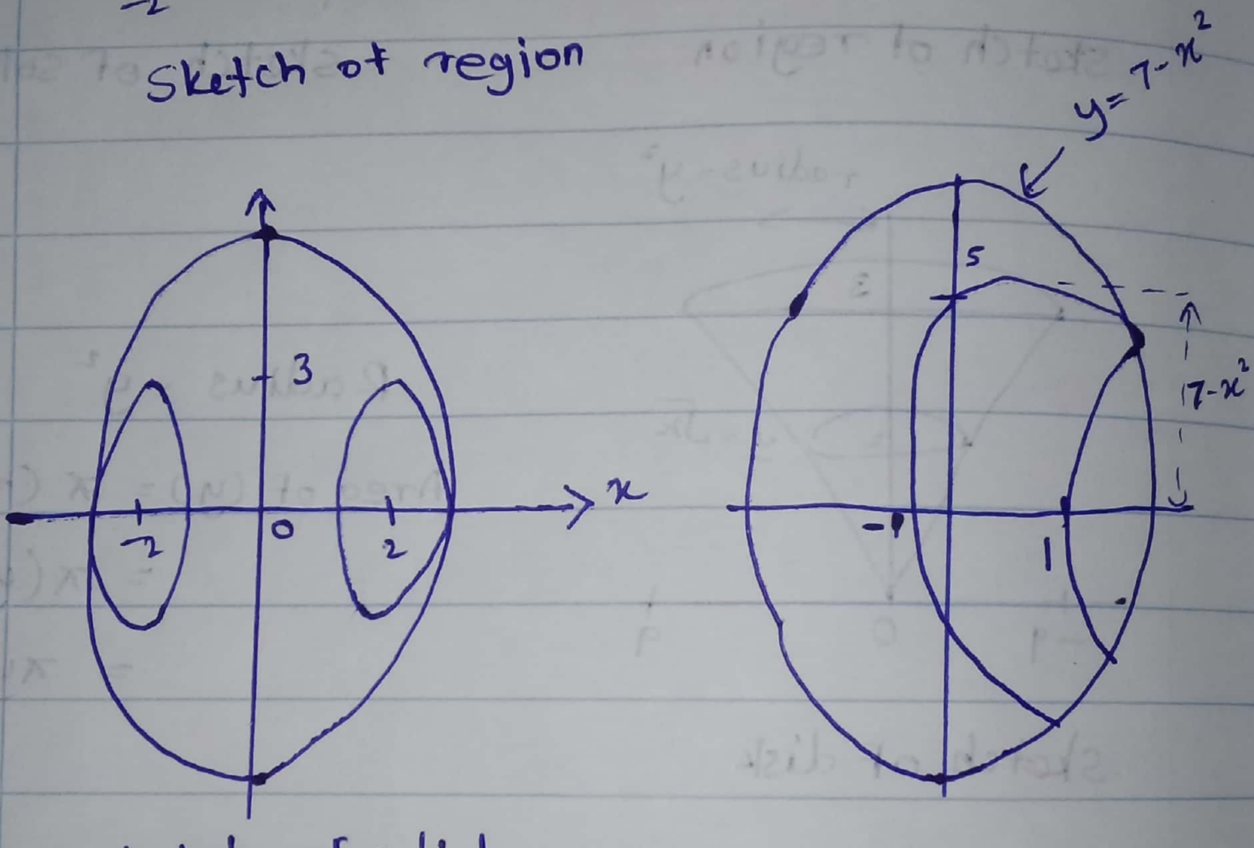
For the limits on the integral, first disk in solid occurs at $y = 0$ & last disk would occur at $y = 3$, limit are $0 \le y \le 3$

Volume is, $V = \int_0^3 \pi y^4 \, dy = \frac{1}{5} \pi y^5 \Big|_0^3 = \frac{243}{5} \pi$

iii) $y = 7 - x^2$, $x = -2$, $x = 2$, and the x axis; about the x axis:



Sketch of region



Sketch of solid

Radius $= 7 - x^2$

$A(x) = \pi(\text{Radius})^2 = \pi(7 - x^2)^2 = \pi(49 - 14x^2 + x^4)$

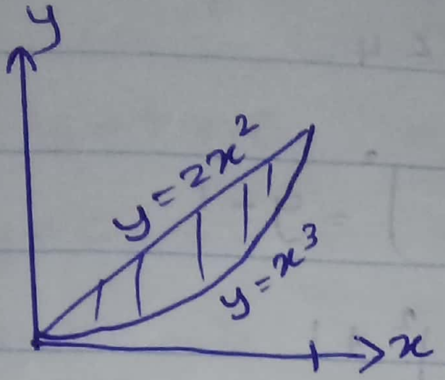
For the limits on the integral, 1st disk in the soilid occurs at $x = -2$, last disk occur at $x = 2$

limits $-2 \leq x \leq 2$

$$V = \int_{-2}^{2} \pi (49 - 14x^2 + x^4)\,dx = \pi\left(49x - \frac{14x^3}{3} + \frac{1}{5}x^5\right)\Bigg|_{-2}^{2}$$

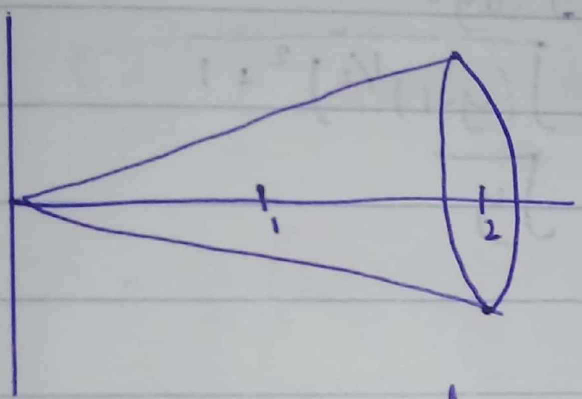
$= \frac{2012}{15}\pi$

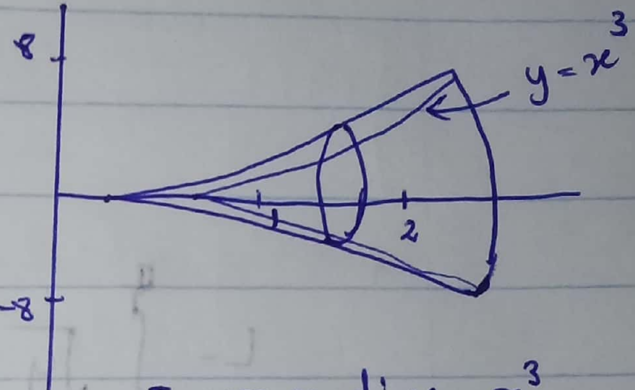
iii) $y=2x^2$, $y=x^3$; about the x axis



$$x^3 = 2x^2$$
$$x^3 - 2x^2 = 0$$
$$x^2(x-2) = 0 \Rightarrow x=0,\ x=2$$
$$(0,0)\ \&\ (2,8)$$

sketch of region





sketch of solid

Inner radius $= x^3$
outer radius $= 2x^2$

$$A(x) = \pi\left(\left((2x)^2\right)^2 - (x^3)^2\right)$$
$$= \pi(4x^4 - x^6)$$

First ring in solid occurs at $x=0$
Last ring occurs at $x=2$

limits $0 \le x \le 2$

Volume is $V = \int_0^2 \pi(4x^4 - x^6)\,dx$

$$= \pi\left(\frac{4}{5}x^5 - \frac{1}{7}x^7\right)\Big|_0^2$$

$$= \frac{256\pi}{35}$$

(06)

i) $x = \frac{2}{3}(y-1)^{3/2}$, $1 \leq y \leq 4$

$$\frac{dx}{dy} = (y-1)^{1/2} \Rightarrow \left(\frac{dx}{dy}\right)^2 = y-1$$

$$\left(\frac{dx}{dy}\right)^2 + 1$$

$$\sqrt{\left(\frac{dx}{dy}\right)^2 + 1}$$

$$= \sqrt{\left((y-1)^{1/2}\right)^2 + 1}$$

$$= \sqrt{y}$$

$$L = \int_1^4 \sqrt{y}\, dy$$

$$= \frac{2}{3} y^{3/2} \Big|_1^4$$

$$= \frac{14}{3}$$

ii) $x = \frac{1}{2}y^2$, $0 \leq x \leq \frac{1}{2}$. Assume that y is positive

$$\frac{dx}{dy} = y \Rightarrow \sqrt{1 + \left(\frac{dx}{dy}\right)^2}$$

$$= \sqrt{1+y^2}$$

$$L = \int_0^1 \sqrt{1+y^2}\, dy$$

$y = \tan\theta$ $\quad dy = \sec^2\theta\, d\theta$

$y = 0 \rightarrow 0 = \tan\theta \rightarrow \theta = 0$

$y = 1 \rightarrow 1 = \tan\theta \rightarrow \theta = \pi/4$

$\sqrt{1+y^2} = \sqrt{1+\tan^2\theta} = \sqrt{\sec^2\theta} = |\sec\theta| = \sec\theta$

$$L = \int_0^{\pi/4} \sec^3\theta\, d\theta$$

$$= \frac{1}{2}\left(\sec\theta \tan\theta + \ln|\sec\theta + \tan\theta\right)\Big|_0^{\pi/4}$$

$$= \frac{1}{2}\left(\sqrt{2} + \ln(1+\sqrt{2})\right)$$

iii) $x = \ln y - \frac{y^2}{8}$, $\quad 1 \leq y \leq 2$

$$L = \int_a^b \sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy$$

$x = \ln y - \frac{y^2}{8}$

$$\frac{dx}{dy} = \frac{d}{dy}\left(\ln y - \frac{y^2}{8}\right) = \frac{1}{y} - \frac{y}{4}$$

$$L = \int_1^2 \sqrt{1+\left(\frac{1}{y} - \frac{y}{4}\right)^2}\, dy$$

$$\left(\frac{1}{y}-\frac{y}{4}\right)^2 = \left(\frac{1-y^2/4}{y}\right)^2$$

$$= \frac{1-\frac{y^2}{2}+\frac{y^4}{16}}{y^2}$$

$$= \frac{(y^2+4)^2}{16y^2}$$

$$\sqrt{1+\left(\frac{1}{y}-\frac{y}{4}\right)^2} = \sqrt{\frac{(y^2+4)^2}{16y^2}} = \frac{y^2+4}{4y}$$

$$L = \int_1^2 \frac{y^2+4}{dy}\,dy = \int_1^2 \left(\frac{y}{4}+\frac{1}{y}\right) dy$$

$$L = \int_1^2 \frac{y}{4}\,dy + \int_1^2 \frac{1}{y}\,dy$$

$$L = \frac{3}{8} + \ln 2$$

(7)

i) $y = \sqrt{x}, \quad 2 \le x \le 6$

$$\text{Surface Area} = 2\pi \int_2^6 (\text{radius}) \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

$$= 2\pi \int_2^6 (x^{1/2}) \sqrt{1 + \left(\left(\tfrac{1}{2}\right) x^{-1/2}\right)^2} dx$$

$$= 2\pi \int_2^6 (x^{1/2}) \sqrt{1 + \frac{1}{4x}} dx$$

$$= 2\pi \int_2^6 (x^{1/2}) \frac{\sqrt{4x+1}}{2x^{1/2}} dx$$

$$= \pi \int_2^6 \sqrt{4x+1}\, dx$$

Substitution $\rightarrow u = 4x+1$

$\frac{du}{dx} = 4 \Rightarrow dx = \frac{du}{4}$

when $x=2$, $u = 4(2)+1 = 9$

when $x=6$, $u = 4(6)+1 = 25$

$$A = \pi \int_9^{25} \sqrt{u} \cdot \frac{du}{4} \qquad \sqrt{u} = u^{1/2}$$

$$= \frac{2}{3} u^{3/2} \qquad = \frac{49\pi}{3}$$

ii) $y = 7x, -1 \le x \le 1$

$$A = 2\pi \int_a^b y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

$y = 7x$

$\frac{dy}{dx} = 7$

$$A = 2\pi \int_{-1}^{1} 7x \sqrt{1+7^2}\, dx$$

$1 + 7^2 = 50 \quad = 5\sqrt{2}$

$7x \cdot 5\sqrt{2} = 35\sqrt{2} \cdot x$

$$A = 2\pi \int_{-1}^{1} 35\sqrt{2} \cdot x\, dx$$

$$= 70\pi\sqrt{2} \int_{-1}^{1} x\, dx$$

$$\int_{-1}^{1} x\, dx = \left[\frac{x^2}{2}\right]_{-1}^{1} = 0$$

iii) $y = x+1 \quad , \quad 0 \leq x \leq 3$

$$A = 2\pi \int_a^b y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx$$

$y = x+1$

$\frac{dy}{dx} = 1$

$$A = 2\pi \int_0^3 (x+1) \sqrt{1+(1)^2} \, dx$$

$= \sqrt{2}$

integran $= (x+1)\sqrt{2}$

Surface area $A = 2\pi \int_0^3 (x+1)\sqrt{2} \, dx$

$$A = 2\pi\sqrt{2} \int_0^3 (x+1) \, dx$$

$A = 15\pi\sqrt{2}$

(08) $y = 4 + 3x^2 \quad , \quad 1 \le x \le 2$

$$ds = \sqrt{1 + \left[\frac{dy}{dx}\right]^2}\, dx$$

$$\frac{dy}{dx} = 6x \implies ds = \sqrt{1+36x^2}\, dx$$

Surface Area,

$$SA = \int 2\pi x \, ds$$

$$= \int_1^2 2\pi \sqrt{1+36x^2}\, dx$$

$$= \frac{\pi}{54}\left(1+36x^2\right)^{3/2}\Big|_1^2$$

$$= \frac{\pi}{54}\left(145^{3/2} - 37^{3/2}\right)$$

$$= 88.4864$$