EE303

Probability and Random Processes

Assignment 2

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Objective

To verify the following approximations using MATLAB and report the limiting conditions for obtaining the same.

- Binomial Distribution tending to Normal Distribution
- Binomial Distribution tending to Poisson Distribution

- Poisson Distribution tending to Normal Distribution
- Hypergeometric Distribution tending to Binomial Distribution

Theory

Probability Distribution Function/Probability Mass Function for the distributions are as follows:-

- Binomial Distribution:-

  Binomial distribution is used to find the probability of ‘x’ successes in ‘n’ trials with probability of success ‘p’ on each trial. The Probability Mass Function of Binomial Distribution is given as

  $$P(x;\ n,\ p)\ =\ {}^{n}C_{x} \times p^{x} \times (1-p)^{(n-x)}$$

- Poisson Distribution: -

  Poisson Distribution is used to model the number of events occurring in a fixed interval of time. The Probability Mass Function of Poisson Distribution is as follows

  $$P(x;\ \lambda)\ =\ (e^{-\lambda} \times \lambda^{x})/x! \quad ,\ x\ =\ 0, 1, 2, 3, ...$$

  Where λ represents mean of Poisson Distribution

- Normal Distribution: -

  Normal Distribution is a continuous probability distribution which used to model real valued random variables. The Probability Distribution Function of Normal Distribution is given as

  $$f(x)\ =\ (e^{(-1/2)((x-\mu)/\sigma)^{2}})/(\sigma \times \sqrt{2\Pi})$$

  Where μ is the mean of distribution and σ is the standard deviation of the distribution.

- Hypergeometric Distribution: -

Hypergeometric Distribution is a discrete probability distribution which is used to get the probability of 'x' successes in 'n' trials without replacement for a fixed population size of 'N' that has exactly 'M' objects with that feature. The Probability Mass Function of Hypergeometric Distribution is given as follows

$$P(X = x) \ = \ ({}^{M}C_{x} \times {}^{(N-M)}C_{(n-x)}) / {}^{N}C_{n}$$

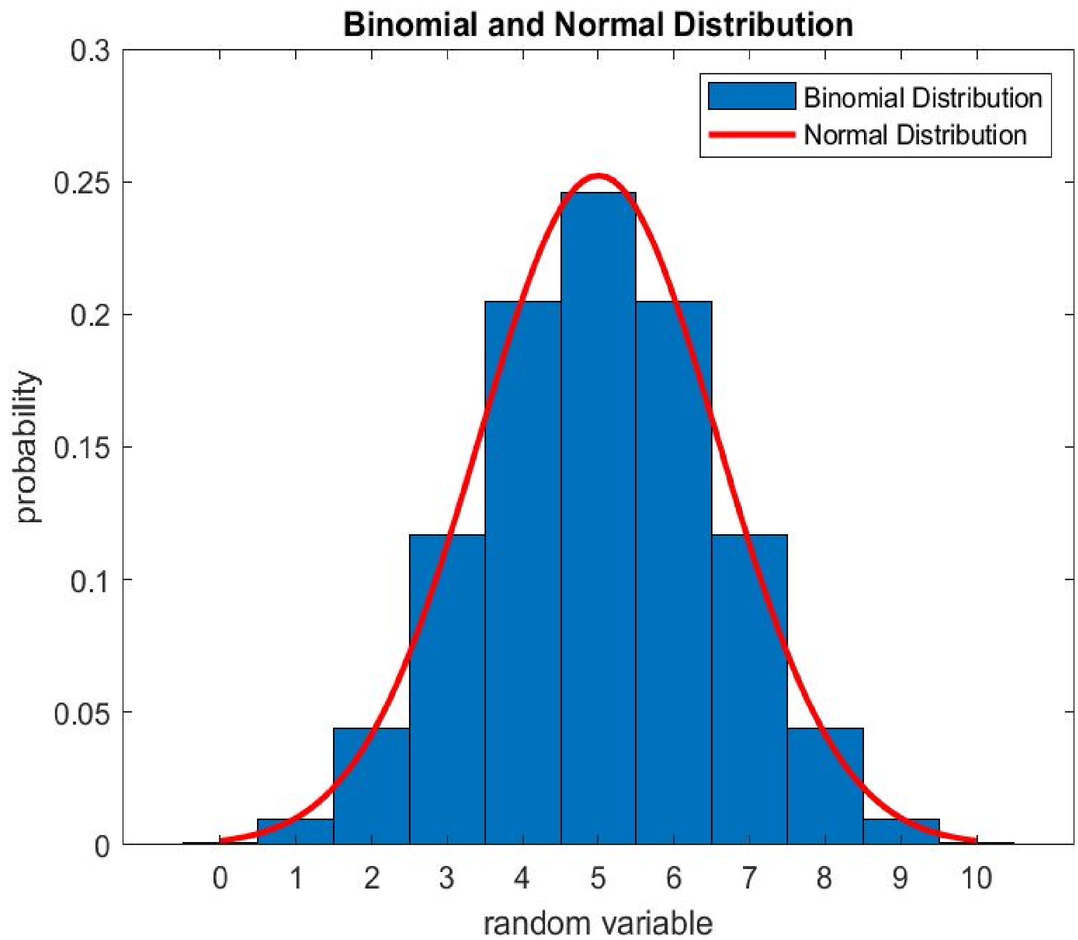
The limiting conditions for which the approximations mentioned in the objective holds are as follows:-

- For the Probability Mass Function of Binomial Distribution with parameters 'n' and 'p' to tend to Probability Distribution of Normal Distribution, 'n' should be large and 'p' should not be too small or too large or we can say that 'p' should be close to 0.5. If these conditions satisfy then Binomial Distribution with parameters 'n' and 'p' can be approximated by Normal Distribution with mean 'n*p' and variance 'n*p*(1-p)'.
- Binomial Distribution with parameters 'n' and 'p' can be approximated by Poisson Distribution if 'p' is small, say 0.05 provided 'n*p' is also small. In this situation, Probability Mass Function of Poisson Distribution with mean 'n*p' closely approximates the Probability Mass Function of Binomial Distribution with parameters 'n' and 'p'.
- Poisson Distribution with parameter 'lambda' can be approximated with Normal Distribution with mean 'lambda' and variance 'lambda' if 'lambda' is large, say 50.
- Hypergeometric Distribution with parameters 'N', 'M', 'n', 'x' can be approximated by Binomial Distribution if 'N'>>'n'. At Least 'N'>'20n'. If this condition satisfies the Binomial Distribution will have parameters 'n' and 'p=M/N'.

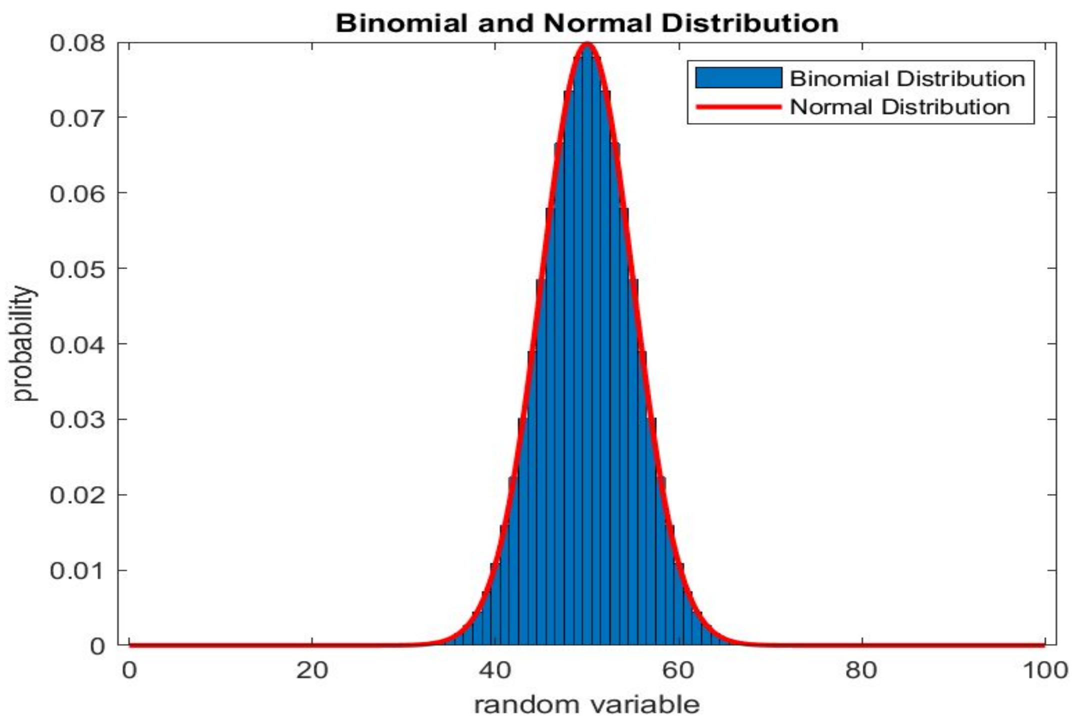
Results

The above limiting conditions or approximations were verified by plotting Probability Mass Functions and Probability Distribution Functions of the required distributions with different parameters. For plotting the functions MATLAB was used whose results and observations are as follows:-

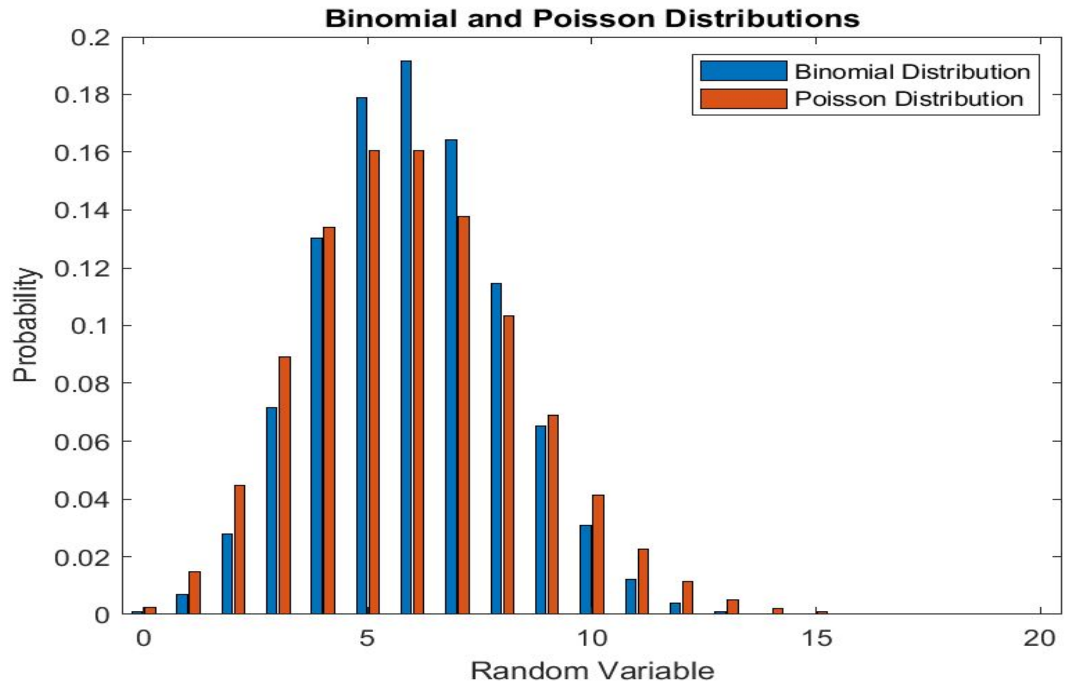
- binomial_and_normal.m is the MATLAB file used for verifying the approximations where Binomial Distribution tends to Normal Distribution.
    - Firstly parameters of binomial distributions were chosen as ‘n=10’ and ‘p=0.5’ which doesn’t satisfy the condition as ‘n’ is not large. Parameters of Normal Distribution were ‘mean= n*p’ and ‘variance=n*p*(1-p)’. The plot is given below for this case. As can be seen that Normal Distribution doesn’t approximate Binomial Distribution well.



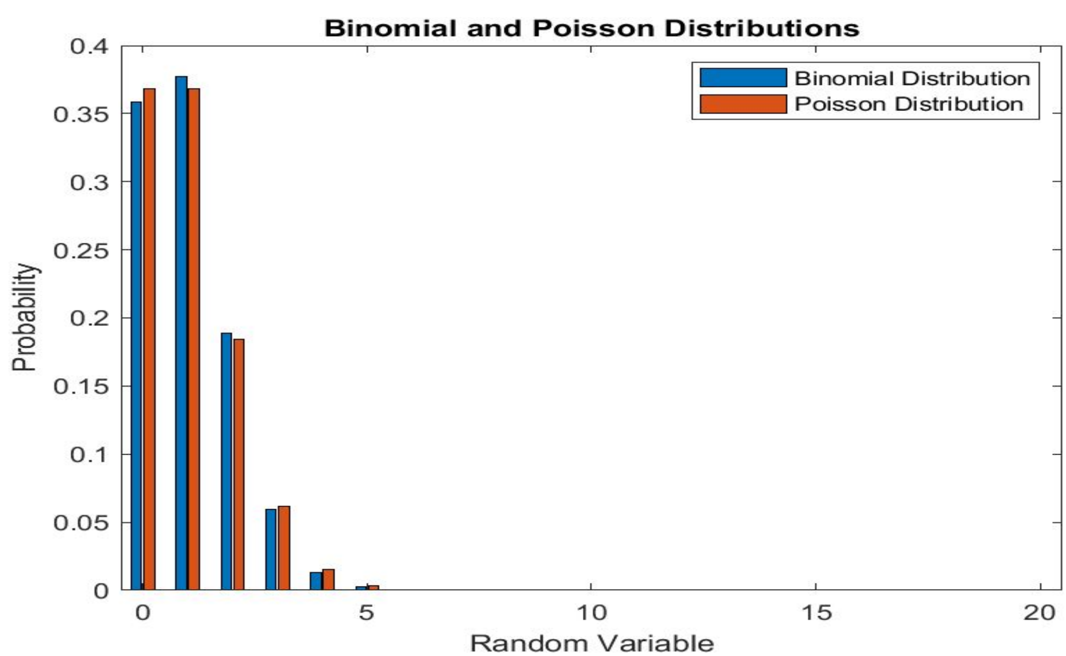
- Now we take ‘n=100’ and ‘p=0.5’. This case satisfies the approximation conditions as n is large and p is close to 0.5. This can be seen from the below plot



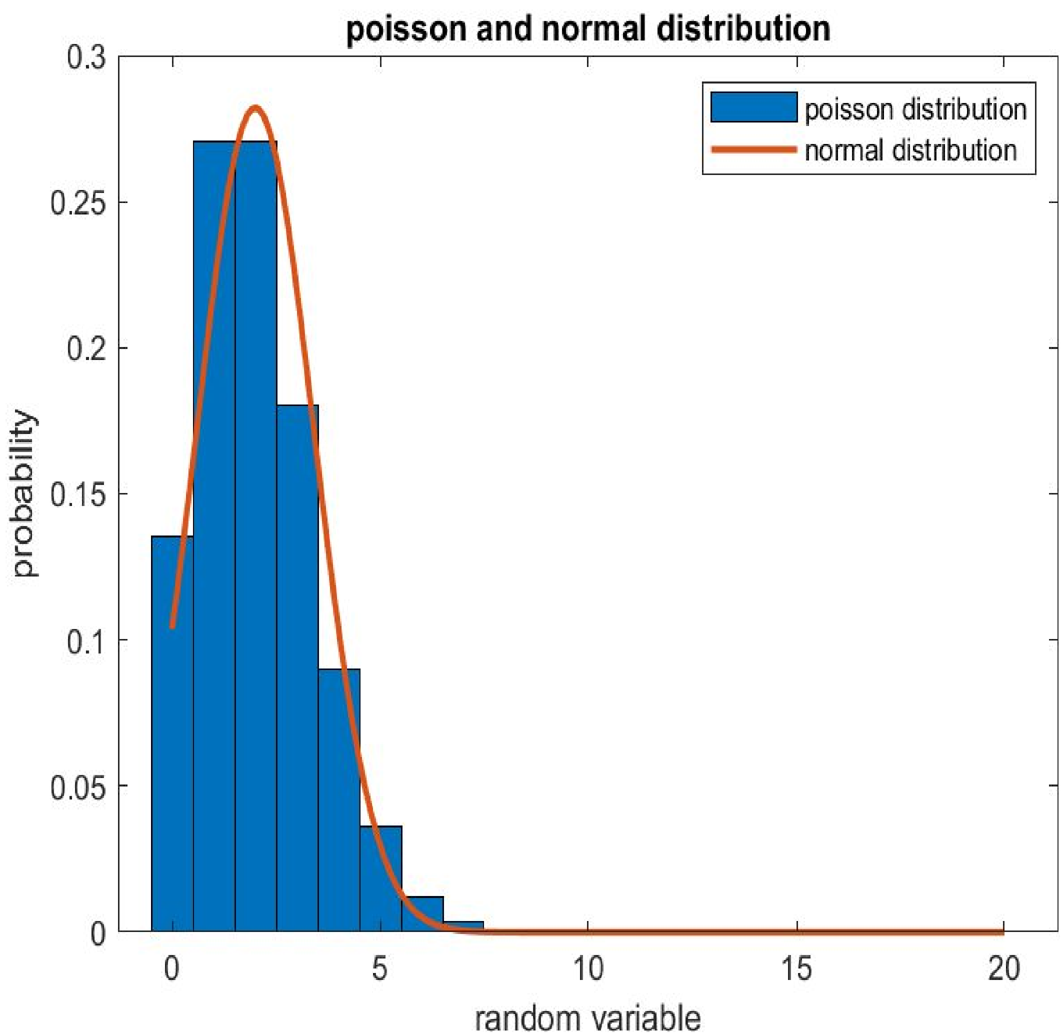
- binomial_and_poisson.m is the MATLAB file used to verify the approximation where Binomial Distribution tends to Poisson Distribution
  - In the first case parameters of Binomial Distribution were chosen as ‘n=20’ and ‘p=0.3’ which doesn’t satisfy the condition of approximation as ‘p’ is not very small. Mean of Poisson Distribution was chosen as ‘mean=n*p’ in this case. The plot of this case is as follows



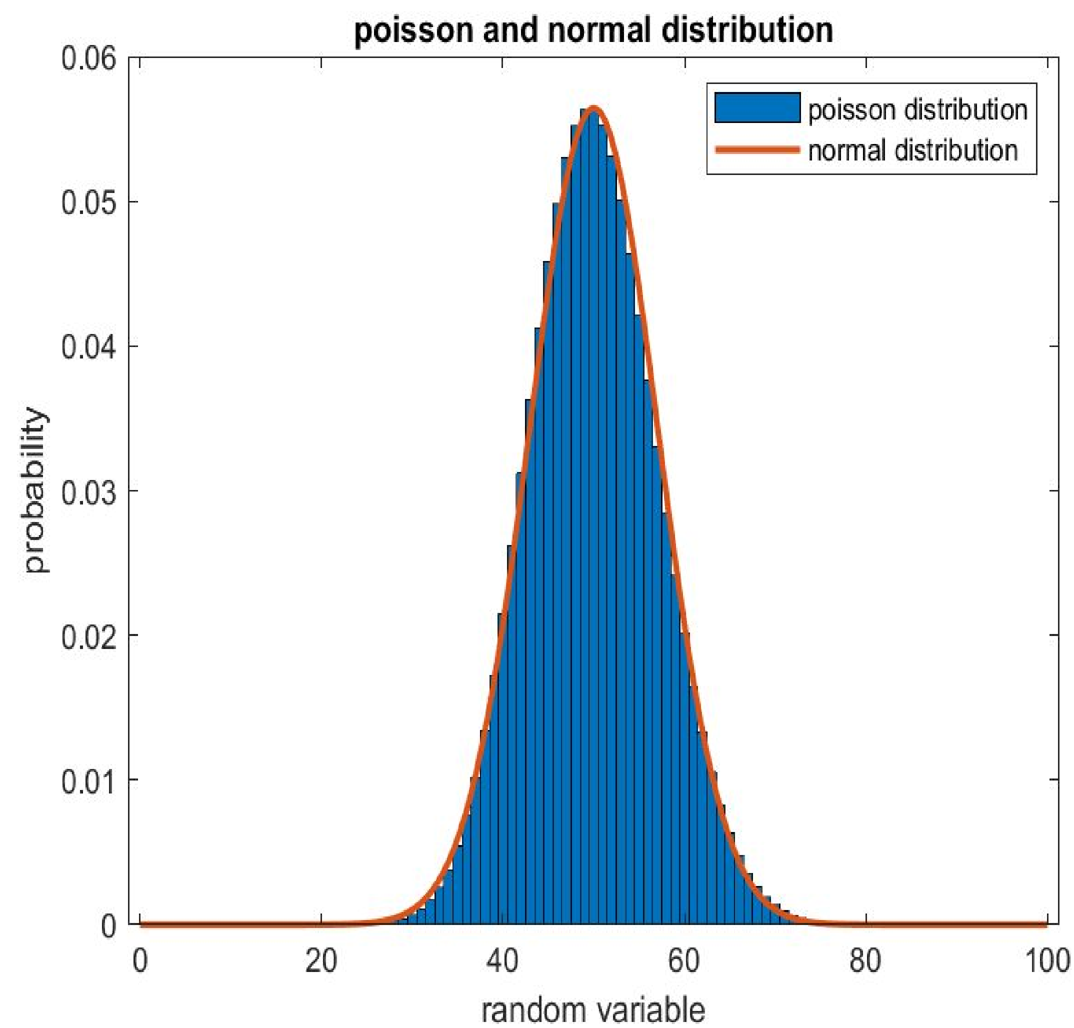
- Now the parameters of Binomial Distribution were chosen as ‘n=20’ and ‘p=0.05’ which satisfies the condition of approximation. Mean of Poisson Distribution was chosen as ‘mean=n*p’. Plot of this case is shown below and it can be clearly seen that approximation holds in this case.



- poisson_and_normal is the MATLAB file used to verify the approximations where Poisson Distribution tends to Normal Distribution.
    - In the first case, parameters of Poisson Distribution were chosen as ‘lambda=2’ and parameters of Normal Distribution were chosen as ‘mean=lambda’ and ‘variance=lambda’. It can be clearly stated that ‘lambda’ is not large which is the approximation condition. This can be clearly seen from the below plot that this case doesn’t satisfy the limiting condition of approximation where Poisson Distribution tends to Normal Distribution.

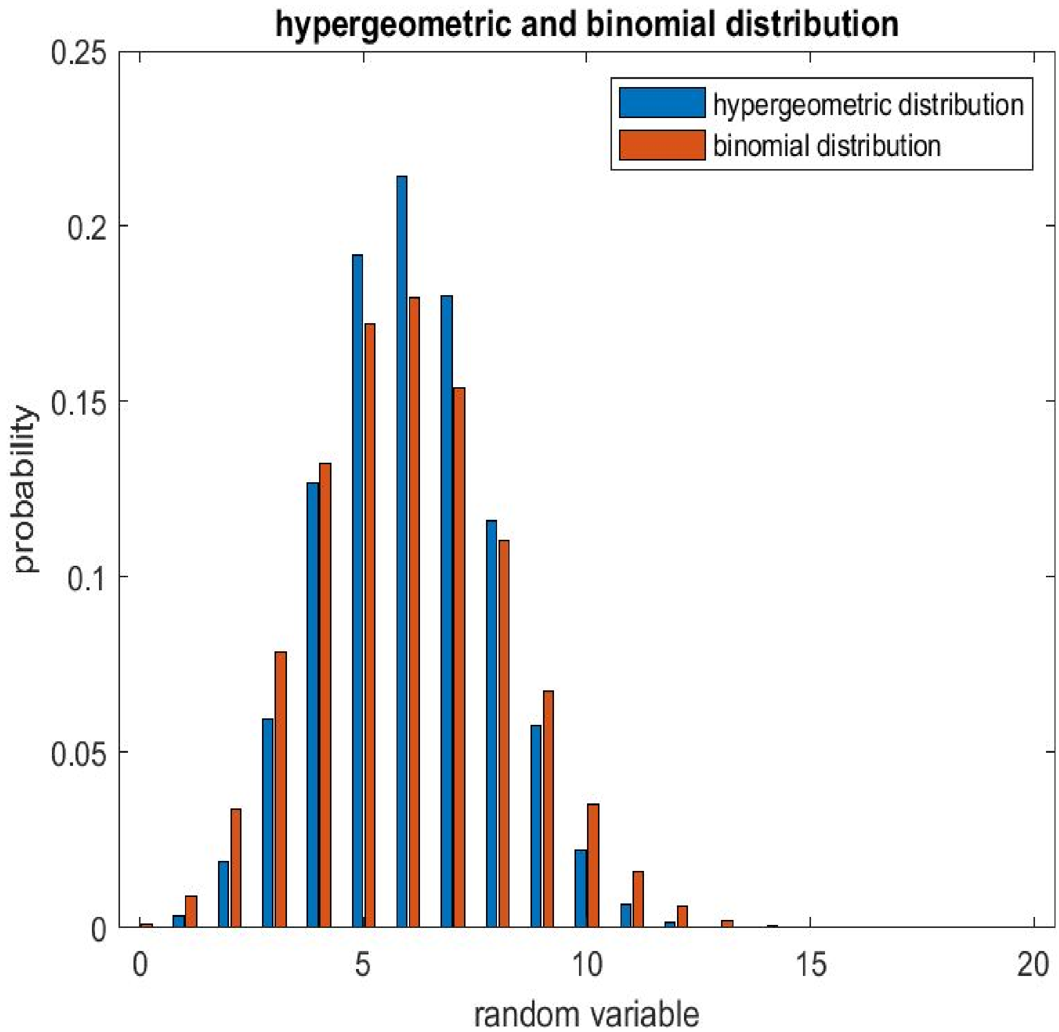


- Now the parameter of Poisson Distribution is chosen as 'lambda=50' and mean of Normal Distribution and variance of Normal Distribution are both equal to 'lambda'. This case satisfies the limiting condition as 'lambda' is large. Plot of this case is as shown below.

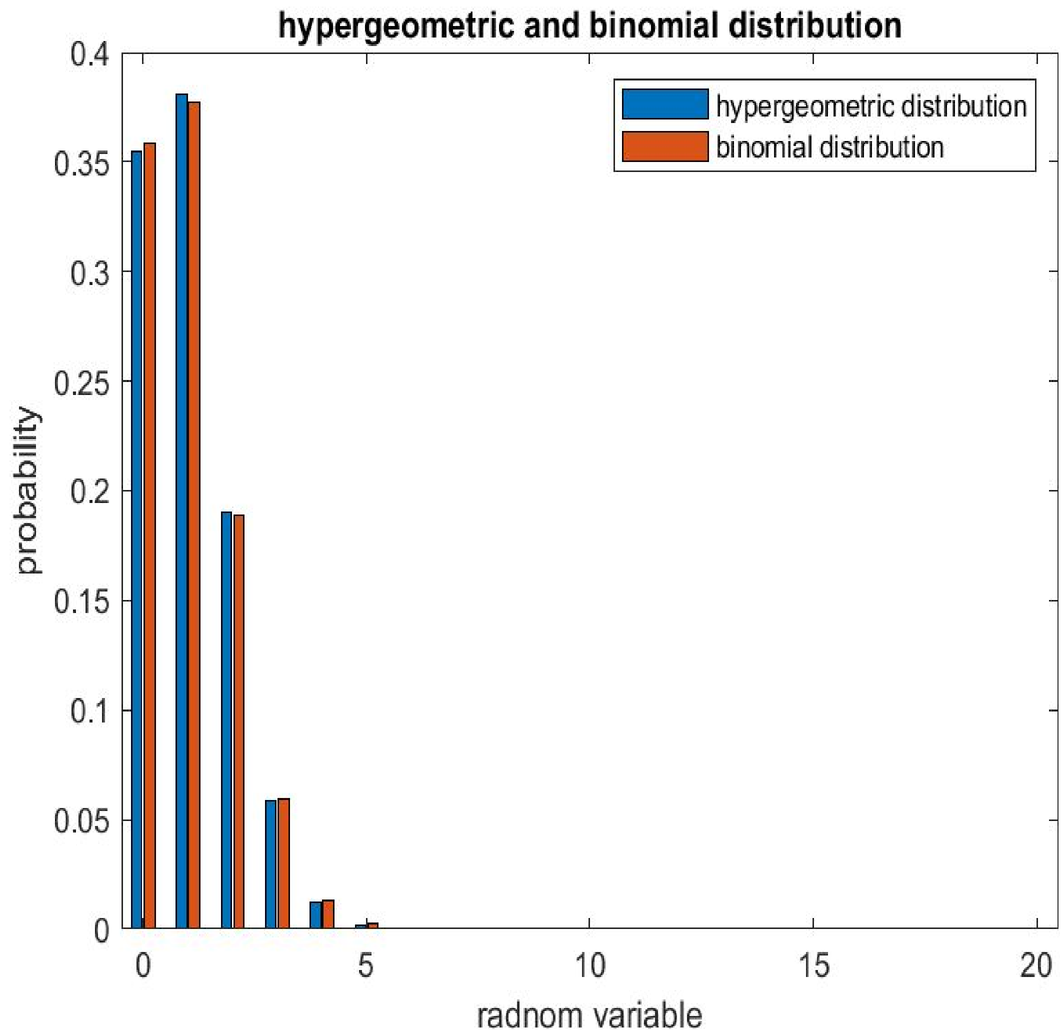


- hypergeometric_and_binomial is the MATLAB file used to verify the limiting condition were Hypergeometric Distribution tends to Binomial Distribution
    - Firstly the parameters of both distributions were chosen as:- For Hypergeometric Distribution: N=100, M=50, n=30 and For Binomial

Distribution: n=30, p=M/N. The plot of this case is shown below and it can be clearly seen that this case doesn't satisfy the limiting condition as N is not greater than 20*n.



- Now the parameters were changed to :- Hypergeometric Distribution: N=1000, M=50, n=20 and Binomial Distribution: n=20, p=M/N. The plot is shown below and it can be clearly seen that in this case Binomial Distribution closely approximates Hypergeometric Distribution



Conclusion

The limiting conditions for all the four required cases were verified using MATLAB. Also cases where the limiting conditions doesn't hold were also taken and plotted to show that if limiting conditions described above doesn't hold then the former distributions cannot be approximated by the later distributions.

References

- https://www.itl.nist.gov/div898/handbook/eda/section3/eda366i.htm
- https://www.itl.nist.gov/div898/handbook/eda/section3/eda3661.htm
- https://www.itl.nist.gov/div898/handbook/eda/section3/eda366j.htm
- https://in.mathworks.com/help/stats/binomial-distribution.html#:~:text=Normal%20Distribution%20%E2%80%94%20The%20normal%20distribution,Np(1%20%E2%80%93%20p).