BRAC UNIVERSITY

FALL-2020

CSE 230

ASSINGMENT 1

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SECTION: 05

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Answer to the question no. 01

Finding Coefficient of x^{47} in the ~~expesion~~ expension of $\left(2x^3+\frac{1}{x}\right)^{29}$.

We know,

$$T_{(r+1)} = {}^{n}C_r\, a^{n-r}\, x^{r}$$

$$= {}^{29}C_r (2x^3)^{29-r} \left(\frac{1}{x}\right)^{r}$$

$$= {}^{29}C_r\, 2^{29-r}\, x^{87-3r-r}$$

$$= {}^{29}C_r\, 2^{29-r}\, x^{87-4r} \quad \text{----(i)}$$

According to the question:

$$87-4r = 47$$

$$\therefore r = 10.$$

Replacing $r=10$ in (i).

$$\cancel{{}^{29}C_r\, 2^{29-r}}$$

$${}^{29}C_{10}\, 2^{29-10}\, x^{87-(4\cdot 10)}$$

$$= 1.05 \times 10^{13}$$

According to the question:

$$^{21}C_2 (y^2)^{19} \left(\frac{2}{y}\right)^2 = {}^{37}C_2\, y^{35} \left(\frac{1}{3}\right)^2$$

$$\Rightarrow 840 \frac{y^{38}}{y^2} = 74\, y^{35}$$

$$\Rightarrow 840\, y^{36} = 74\, y^{35}$$

$$\Rightarrow \frac{y^{36}}{y^{35}} = \frac{74}{840}$$

$$\therefore y = \frac{37}{420} = 0.088 \text{ (Ans)}.$$

<u>Answer to the question no.03</u>

~~Finding the coefficient of z^4 in the expension of $(z^3+3z+1)^6$~~

$$~~\cdot (z^3+3z+1)~~$$

$$~~= (1+3z+z^3)\ (3z)^4~~$$

Answer to the question no. 03

Finding coefficient of z^4 in expression of $(z^3+3z+1)^6$

Let, $p+q+r=6$, $p=0$, $q=4$, $r=2$

$$\frac{6!}{0! \times 4! \times 2!} \cdot (z^3)^0 (3z)^4$$

$$= \frac{6 \times 5 \times 4!}{2! \times 4!} \, 81z^4$$

$$= 1215\, z^4 \quad \text{--- (1)}$$

Again; $\frac{6!}{1! \times 1! \times 4!} (z^3)^1 (3z)^1 (1)^4 \quad [p=1, q=1, r=4]$

$$= \frac{6 \times 5 \times 4!}{4!} \times 3(z^{3+1})$$

$$= 90z^4 \quad \text{--- (2)}$$

(1) + (2) $\Rightarrow$

$$1215\,z^4 + 90z^4 = 1305\,z^4.$$

So, z^4's coefficient $= 1305$ (Ans)

Answer to the question no.-04

Finding to coefficient of $a^5b^3c^2$ in the expansion of $(370a+285b+99c)^{11}$.

Let $p=5$, $q=\cancel{6}3$, $r=2$, $p+q+r=11$

Now,

$$\frac{11!}{p!\,q!\,r!}(370a)^5(285b)^q(99c)^r$$

$$=\frac{11!}{5!\,3!\,2!}(370)^5(285)^3(99)^2a^5b^3c^2$$

$$=4.36\times10^{28}a^5b^3c^2$$

So, The coefficient of $a^5b^3c^2$ is ~~4×36×10~~ 4.36×10^{28}

(Ans.)

~~Ans~~

Answer to the question no. 05

Number of Baltic countries, $n(B) = 3$.

Number of Scandinavian Countries $n(S) = 5$

Number of North American Countries, $n(N.A) = 3$.

Number of ways 3 countries can be travelled:

$n(S)$	$n(B)$	$n(N)$	~~Total~~ ways
$^5C_1$	$^3C_1$	$^3C_1$	$5 \times 3 \times 3 = 45$
$^5C_2$	$^3C_0$	$^3C_1$	$10 \times 1 \times 3 = 30$
$^5C_3$	$^3C_0$	$^3C_0$	10
$^5C_2$	$^3C_1$	$^3C_0$	30
$^5C_1$	$^3C_2$	$^3C_0$	15
$^5C_1$	$^3C_0$	$^3C_2$	15.

Total number of ways: $(45+30+10+30+15+15)$

$= 145$ (Ans).

Answer to the question no. 07

Given,

number of Digits = 5.

6's = 3.

9's = 2

number ways for 6's = $^5C_3$

number of ways for 9's = $^5C_2$

Total ways = $^5C_3 \times {}^5C_2$

= 100 ways

(Ans).

Answer to the question no. 08.

Given: $\left\{\frac{1}{4}, \frac{2}{10}, \frac{4}{28}, \frac{8}{82}, \frac{16}{244}, \frac{32}{730}\right\}$

Set of numerators of the functions:

$A = \{1, 2, 4, 8, 16, 32\}$.

$\Rightarrow A = \{2^0, 2^1, 2^2, 2^3, 2^4, 2^5\}$

Set of denominators of the function:

$B = \{4, 10, 28, 82, 244, 730\}$.

$= \{4, (4-0)\times 3, (10-4)\times 3, (28-10)\times 3, (82-28)\times 3, (244-84)\times 3\}$.

From the given explanation we can determine a set builder method to express the set:

$$\left\{ a,b \;\middle|\; \frac{a}{b} \in R,\ n \in N \quad \begin{array}{l} a \geq 0 \text{ and } a_n = 2^{n-1} \\ b > 3 \text{ and } b_n = (b_{(n-1)} - b_{(n-2)}) \times 3 \end{array} \right\}$$

(Ans.)

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$\underline{\underline{①}}$ 9

$$f(x) = \frac{x-2\sqrt{25-x^2}\ \ln(x+3)}{2x+5}$$

Non negative values for radical:

$25-x^2 \geq 0$

$\Rightarrow -x^2 \geq -25$

$\Rightarrow x^2 \leq 25$

$\Rightarrow x \leq 5.$

Again, Undefined points of : ~~25~~ 5

~~2x+5~~ .

$2x+5=0$

$\Rightarrow 2x = -5$

$\therefore x = -\frac{5}{2}$

Positive values for log; $\ln(x+3)$:

$\Rightarrow$ ~~lin~~ ~~ln (x+3)~~

$x+3>0$

$\therefore x>-3.$

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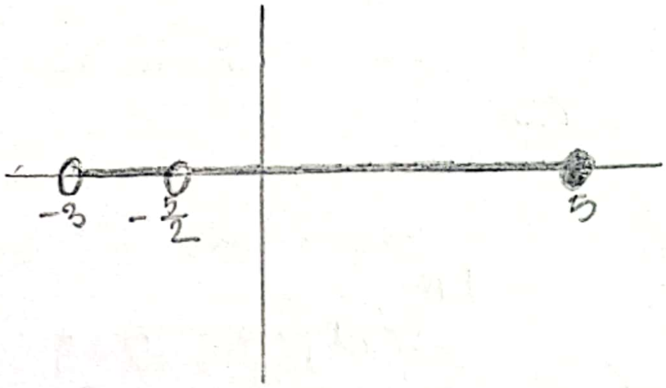
So, the domain of the function :

$\left(-3, -\frac{5}{2}\right) \cup \left(-\frac{5}{2}, 5\right]$



Answer to the question no. 10

Given, $f(x) = 4x - 5$, $g(x) = 7x^2 + 1$

let, $a, b \in R$, $a \neq b$

Then, $f(g(a)) = f(g(b))$

$\Rightarrow 4(g(a)) - 5 = 4(g(b)) - 5$

$\Rightarrow 4(7a^2 + 1) - 5 = 4(7b^2 + 1) - 5$

$\Rightarrow \cancel{28(}\ 28a^2 + 1 = 28b^2 + 1$

$\Rightarrow 28a^2 = 28b^2$

$a = b$

So, $f(g(x))$ is injective (Ans)

Answer to the question no. 11

Let the random number be x.

The number given from Alice to Bob $= x$

The number given from Bob to Carol: $9x$

The number given from Carol to David $= \frac{9x}{5+(3\times x)}$

According to the question:

$$\frac{9x}{5+3x} = 3$$

$\Rightarrow 9x = 15+9x$

$\Rightarrow 0 = 15$, Not an eligible value.

So, It is impossible for David to determine Alice's number if the number he gets from Carol is 3.

(Ans).