# Ex 9 t2

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## Problem :

Consider the Markov chain that takes values on S = {1, 2, 3, 4}. The transition probability from state j to state i is given by

$$P[i,j] = \begin{pmatrix} 1/2 & 2/3 & 0 & 0 \\ 1/2 & 1/3 & 0 & 0 \\ 0 & 0 & 1/4 & 4/5 \\ 0 & 0 & 3/4 & 1/5 \end{pmatrix}$$

1. Suppose the Markov chain is initialized at X0 = 1. What is the distribution of Xt may tend towards?

With $X_0 = 1$:

$$P[i,j] = \begin{pmatrix} 1/2 & 2/3 & 0 & 0 \\ 1/2 & 1/3 & 0 & 0 \\ 0 & 0 & 1/4 & 4/5 \\ 0 & 0 & 3/4 & 1/5 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1/2 \\ 1/2 \\ \boxed{0} \\ \boxed{0} \end{pmatrix}$$

So for $p^n \cdot p = \begin{pmatrix} a \\ b \\ 0 \\ 0 \end{pmatrix}$ wher $a,b \geq 0$ and $a+b=1$

Starting with $X_0 = 1$, would result in a distribution of $\begin{pmatrix} a \\ b \\ 0 \\ 0 \end{pmatrix}$ for $a,b \geq 0$ and $a+b=1$ for $X_t$

2. Suppose the Markov chain is initialized at X0 = 3. What is the distribution of Xt may tend towards?

Starting with $X_0 = 3$

$$P[i,j] = \begin{pmatrix} 1/2 & 2/3 & 0 & 0 \\ 1/2 & 1/3 & 0 & 0 \\ 0 & 0 & 1/4 & 4/5 \\ 0 & 0 & 3/4 & 1/5 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \boxed{0} \\ \boxed{0} \\ 1/4 \\ 3/4 \end{pmatrix}$$

So for $p^n \cdot p = \begin{pmatrix} 0 \\ 0 \\ a \\ b \end{pmatrix}$ wher $a,b \geq 0$ and $a+b=1$

Starting with $X_0 = 3$ would result in a distribution of $\begin{pmatrix} 0 \\ 0 \\ a \\ b \end{pmatrix}$ with $a,b \geq 0$ and $a+b=1$ for $X_t$

3. Characterize all the invariant measure

$$P[i,j] = \begin{pmatrix} 1/2 & 2/3 & 0 & 0 \\ 1/2 & 1/3 & 0 & 0 \\ 0 & 0 & 1/4 & 4/5 \\ 0 & 0 & 3/4 & 1/5 \end{pmatrix}$$

$P[i,j]$ is Reducible to $P_{12}[i,j] \begin{pmatrix} 1/2 & 2/3 \\ 1/2 & 1/3 \end{pmatrix}$ and $P_{34}[i,j] \begin{pmatrix} 1/4 & 4/5 \\ 3/4 & 1/5 \end{pmatrix}$

we know that starting with $X_0 = 1$, we will stay in 1 and 2 since there is no connection between 1,2 and 3,4

Stationary of $P_{12}[i,j]$:

$$-\tfrac{1}{2} x_1 + \tfrac{2}{3} x_2 = 0$$

$$\tfrac{1}{2} x_1 - \tfrac{2}{3} x_2 = 0$$

$$= \begin{pmatrix} 2/3 \\ 1/2 \end{pmatrix} \cdot 6 = \begin{pmatrix} 4 \\ 3 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 4/7 \\ 3/7 \end{pmatrix}}}$$

Starting with $X_0 \in \{1,2\}$ would lead to a Stationary of $(4/7 \;\; 3/7)^T$ for $X_t$

$P_{34}[i,j] \begin{pmatrix} 1/4 & 4/5 \\ 3/4 & 1/5 \end{pmatrix}$

Stationary of $P_{34}[i,j]$:

$$-\tfrac{3}{4} x_1 + \tfrac{4}{5} x_2 = 0$$

$$\tfrac{3}{4} x_1 - \tfrac{4}{5} x_2 = 0$$

$$= \begin{pmatrix} 4/5 \\ 3/4 \end{pmatrix} = \begin{pmatrix} 16 \\ 15 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 16/31 \\ 15/31 \end{pmatrix}}}$$

Starting with $X_0 \in \{3,4\}$ would lead to a Stationary distribution of $(16/31 \;\; 15/31)^T$ for $X_t$