Problem: 2

a) $E[z^T A z] = \mu^T A \mu + Tr(A\Sigma)$

$$E[z^T A z] = E[\Sigma_{ij} A_{ij} z_i z_j]$$

$$= \Sigma_{ij} A_{ij} \underbrace{E[z_i z_j]}_{\Sigma_{ij} + \mu_i \mu_j}$$

(Covariance & product of means)

$$= \Sigma_{ij} A_{ij} (\Sigma_{ij} + \mu_i \mu_j)$$

$$= \Sigma_{ij} A_{ij} \Sigma_{ij} + \Sigma_{ij} A_{ij} \mu_i \mu_j$$

$$= Tr(A\Sigma) + \mu^T A \mu$$

b) $Cov(Az) = A \Sigma A^T$

$$Cov(Az) = E[(Az - E[Az])(Az - E[Az])^T]$$

$$= E[A(z-\mu)(z-\mu)^T A^T]$$

$$= A E[(z-\mu)(z-\mu)^T] A^T$$

$$= A \Sigma A^T$$

c) $Cov(\hat{\beta}) = \sigma^2 (X^T X)^{-1}$

$$Cov(\hat{\beta}) = (X^T X)^{-1} X^T Cov(y) X (X^T X)^{-1}$$

$$= \sigma^2 (X^T X)^{-1} (X^T X)(X^T X)^{-1}$$

$$= \sigma^2 (X^T X)^{-1}$$

Problem: 3

$\Longrightarrow$

$$L(p: x, y, z) = (1-p)^{2x} \cdot 2p(1-p)^{y} \cdot p^{2z}$$

$\rightarrow$ log likelihood function:

$$\log(L(p: x, y, z)) = 2x\log(1-p) + y\log(2p(1-p)) + 2z(\log p)$$

maximum likelihood estimator

$\downarrow$

$$\frac{d \log(L(p: x, y, z))}{dp}$$

$$= \frac{-2x}{1-p} + \frac{2y}{p} + \frac{2z}{1-p} = 0$$

$$-2x(p) + 2y(1-p) + 2z(p) = 0$$