

$$Q.2 \quad \hat{\Sigma}^2 = (y - X\hat{\beta})^T (y - X\hat{\beta}) = y^T y - y^T X \hat{\beta} - \hat{\beta}^T X^T y + \hat{\beta}^T X^T X \hat{\beta}$$

$$= y^T y - 2\hat{\beta}^T X^T y + \hat{\beta}^T X^T X \hat{\beta}$$

$$\sum_{i=1}^n (y_i - \bar{y})^2 = \sum_{i=1}^n (y_i^2 - 2y_i \bar{y} + \bar{y}^2)$$

$$= y^T y - 2n\bar{y}\bar{y} + n\bar{y}^2$$

$$= y^T y - n\bar{y}^2$$

$$\bar{y} = \frac{\sum y_i}{n}$$

$$1 - \frac{\hat{\Sigma}^2}{\sum (y_i - \bar{y})^2} = 1 - \frac{y^T y - 2\hat{\beta}^T X^T y + \hat{\beta}^T X^T X \hat{\beta}}{y^T y - n\bar{y}^2}$$

$$= \frac{2\hat{\beta}^T X^T y - \hat{\beta}^T X^T X \hat{\beta} - n\bar{y}^2}{y^T y - n\bar{y}^2}$$

$$= \frac{2\hat{\beta}^T X^T y - \hat{\beta}^T X^T (y - \varepsilon) - n\bar{y}^2}{y^T y - n\bar{y}^2}$$

$$= \frac{\hat{\beta}^T X^T y + \hat{\beta}^T X^T \varepsilon - n\bar{y}^2}{y^T y - n\bar{y}^2}$$

$$\text{but } X^T \varepsilon = 0$$

$$= \frac{\hat{\beta}^T X^T y - n\bar{y}^2}{y^T y - n\bar{y}^2}$$