Task 3.

For $A \in R^{m \times n}$ & $u, v \in R^m$, proove Sherman Morrison Formula.

We can proove by considering LHS $= X^{-1}$ & RHS $= Y$

so $X^{-1} = Y$
if & only if $XY = YX = I$.

Starting with $XY = I$.

$$XY = (A + uv^T)\left(A^{-1} - \frac{A^{-1}uv^TA^{-1}}{1 + v^TA^{-1}u}\right)$$

$$= AA^{-1} + uv^TA^{-1} - \frac{AA^{-1}uv^TA^{-1} + uv^TA^{-1}uv^TA^{-1}}{1 + v^TA^{-1}u}$$

$$= I + uv^TA^{-1} - \frac{uv^TA^{-1} + uv^TA^{-1}uv^TA^{-1}}{1 + v^TA^{-1}u}$$

$$= I + uv^TA^{-1} - \frac{u(1 + v^TA^{-1}u)v^TA^{-1}}{1 + v^TA^{-1}u}$$

$$= I + uv^TA^{-1} - uv^TA^{-1}$$

$$= I$$

for $YX = I$

$$= \left(A^{-1} - \frac{A^{-1}uv^TA^{-1}}{1+v^TA^{-1}u}\right)(A + uv^T)$$

$$= A^{-1}A + A^{-1}uv^T - \frac{A^{-1}uv^TA^{-1}A}{1+v^TA^{-1}u} - \frac{A^{-1}uv^TA^{-1}uv^T}{1+v^TA^{-1}u}$$

$$= A^{-1}A + A^{-1}uv^T - \frac{(A^{-1}uv^T + A^{-1}uv^TA^{-1}uv^T)}{1+v^TA^{-1}u}$$

$$= I + A^{-1}uv^T - \frac{A^{-1}u(v^T + v^TA^{-1}uv^T)}{1+v^TA^{-1}u}$$

$$= I + A^{-1}uv^T - \frac{A^{-1}u(1 + v^TA^{-1}u)v^T}{1+v^TA^{-1}u}$$

$$= I + A^{-1}uv^T - A^{-1}uv^T$$

$$= I$$

Thus we have prooved $XY = YX = I$

Hence y is inverse of X

$$\therefore X^{-1} = (A + uv^T)^{-1} = Y = \left(A^{-1} - \frac{A^{-1}uv^TA^{-1}}{1+v^TA^{-1}u}\right)$$