unit - 1st :-

* METHODS of Successive Approximation :-

this method is also known as direct substitution method or method of iteration or method of fixed iteration is applicable if the eqn

$$F(x) = 0$$

can be expressed as

$$x = g(x)$$

eg. $e^x - x - 3 = 0$

$$x = e^x + 3$$

$$\boxed{x = g(x)}$$

If x_1 is the initial approximation to the route the Next approximation to the route is given by

$$\boxed{x_2 = g(x_1)}$$

and next approximation will be

$$x_3 = g(x_2)$$

$$\vdots$$

In GENERAL

$$\boxed{x_{i+1} = g(x_i)}$$

the iteration cycle will be terminate when the relative error is the Now approximation is within the presented tolerance

NOTE $\Rightarrow$ this procedure convey only if $\boxed{|g'(x)| < 1}$

Q. Given that one root of the non ... equation $2^x - x - 3 = 0$ lies in the interval $(-3, -2)$ than find the root correct up to three decimal places

Ans:- $\because$ we want the soln correct to three significant digits the initial process will be terminated as soon as the successive iteration produced no change at first four significant position

Further to apply the method of successive approximation the iteration

$$2^x - x - 3 = 0$$

$$x = 2^x - 3$$

$$x = g(x)$$

$$\boxed{g(x) = 2^x - 3}$$

Iteration 1st :-

let us start with initial appro.

$$x_1 = -3.$$

$$x_2 = 2^{(-3)} - 3$$

$$x_2 = -2.875$$

Iteration 2nd :-

Now we take $x_2 = -2.875$ as the equation to obtain the next approx

$x_3 = g(x_2)$

$x_3 = g(-2.875) - 3$

$= -2.863$

Similarly iteration 3rd :-

Now we take $x_3 = -2.863$ to obtain

Next approximation

$x_4 = g(x_3)$

$x_4 = 2^{-(2.863)} - 3$

$= -2.862$

Iteration No. 4 :- Now we take $x_4 = -2.8625$

as current approx. to obtain Next

approx.

$x_5 = g(x_4)$

$x_5 = 2^{-(2.862)} - 3$

$x_5 = -2.8625$

Iteration no. 5 →

Now we take x_5 to obtain to

Next appro.

$x_6 = g(x_5)$

$x_6 = 2^{-(2.8625)} - 3$

$(x_6 = -2.8625)$

Ans.

Q2.
Ans:-

$$2x - \cos x - 3 = 0$$

$$x = \frac{1}{2}(\cos x + 3)$$

find the root of above eqn correct up to three decimal places using iteration method

$$f(x) = 2x - \cos x - 3$$

$$f(0) = -4$$

$$f(1) = -1.54$$

$$f(2) = 1.42$$

Now we check convergence $g(x) < 1$

thus a root lies b/w 1 and 2

$$x = \frac{1}{2}(\cos x + 3)$$

$$x = g(x)$$

$$g(x) = \frac{(\cos x + 3)}{2}$$

$$g'(x) = \frac{-\sin x}{2}$$

$$|g'(x)| = \left|\frac{1}{2}(-\sin x)\right|$$

$$= \frac{1}{2}$$

Region
$(-1 < \sin x < 1)$

$(1/2 < 1)$

$\therefore$ this iteration method can be apply.

* we want soln correct up to three decimal palace the iteration process will terminate and obtain next approx.

Iteration 1st :-   (1, 2)   (1, 2)

Let us start with the initial process

$x_1 = 1.5$

$x_2 = 1/2 \, [\cos(1.5) + 3)]$

$= 1.5354$

Iteration 2nd :-

$x_2 = 1.5354$

$x_3 = 1/2 \, [\cos(1.5354) + 3]$

$= 1.5178$

Now we continue and find the ~~[illegible]~~n :-

Ans

Q Find the root of equation (not)

$$2x - \log_{10} x - 7 = 0$$

$$2x - 7 = \log_{10} x$$

$$x = \frac{2x-7}{\log_{10} e}$$

Q $\sin x - x - 2 = 0$ $x = -2.5$

Find the root with tolerence $\underline{0.001}$?

$$\sin x - x - 2 = 0$$

$$x = \sin x - 2$$

$$= g(x)$$

$$g'(x) = \cos x$$

$$|g'(x)| = |\cos x| < 1$$

$$|\cos(-2.5)| < 1$$

$$= 0.80 < 1$$

Iteration 1st :-

$$x_2 = \sin(-2.5) - 2$$

$$\boxed{x_2 = -2.5984}$$

Iteration 2nd

$$x_3 = -2.5984$$

$$x_4 = -2.5168$$

$$x_5 = -2.5849$$

Ans.

★ Concept of synthetic division:-

Division of polynomial $f(x)$ by the factor $(x-r)$ can be carried rapidly by synthetic division the remainder of above division will give the value of polynomial for $x=r$ how ever if r happens to the root of polynomial $f(x)$ than the remainder resulting from the division will be zero

Similarly the value of derivatives $f'(x)$ of the polynomial of $f(x)$. for $x=r$ is equal to the remainder obtain by second synthetic division of the result obtain from the first synthetic division which will be polynomial of degree less than to original polynomial

Q $x^5 - 3x^4 - 10x^3 + 5x^2 + 22x + 16$ by $(x-4)$

4	1	-3	-10	5	22	16	
	⊗	4	4	-24	-76	-216	
4	1	1	-6	-19	-54	$\boxed{-200}$	$= f(x)$
	X	4	20	56	148		
	1	5	14	37	$\boxed{94}$	$= f'(x)$	

$= f'(4)$

Ans.

$f'(4)$
94

Q. Find the multiplicity of the root $x = 2$ of the eq^n $x^4 - 5x^3 + 6x^2 + 4x - 8 = 0$ using synthetic division

2	1	-5	6	4	-8	
	X	2	-6	0	8	
2	1	-3	0	4	$\boxed{0}$	$= f(x)$
	X	2	-2	-4		
2	1	-1	-2	$\boxed{0}$	$= f'(x)$	
	X	2	2			
2	1	1	$\boxed{0}$	$= f''(x)$		
	X	2				
2	1	$\boxed{3}$	$= f'''(x)$			
	X					
	1	$= f^{IV}(x)$				

Ans.

Q. Suppose the polynomial $x^5 - 3x^4 - 10x^3 + 44x + 10x^2 + 48$ is divided by quadratic factor $x^2 + 2x + 1$

Ans Now above eq^n compare to $x^2 - \alpha x - \beta = 0$

$\alpha = -2 \quad \beta = -1$

-2, -1	1	-3	-10	10	44	48	
	X	-2	+10	+2	-34	-22	
			-1	5	1	-17	
	1	-5	-1	17	$\boxed{11}$	$\boxed{9}$	$= f(x)$ Rem

$(x - \alpha)$

$11(x+2) + 9$

$x^3 - 5x^2 - x + 17$

$\boxed{11(x+2) + 9 = 0}$

Ans. $\boxed{f(x) = 9}$

Q Divide $2x^5 - 3x^4 + 4x^3 - 5x^2 + 6x - 9$ by $x^2 - x + 2$

synthetically

compare it $(x^2 - \alpha x - \beta)$

$\alpha = 1$, $\beta = -2$

-1 2	2	-3	4	-5	6	-9
	X	2	-1	-1	-4	4
			-4	2	2	8
	2	-1	-1	-4	4	3

$$\boxed{a(x-\alpha) + b = 0}$$ (standard soln)

$= 4x - 1$

$= 2x^3 - x^2 - x - 4$

Ans.

Bairstow method :- this method is used to find two real no p and q such that $x^2 + px + q$ is a factor of $P_n(x)$

starting with p_0, q_0 we obtain us sequence of iterates $\{(p_k, q_k)\}$ from $p_{k+1} = p_k + \Delta p_k$, $q_{k+1} = q_k + \Delta q_k$

where
$$\Delta p_k = \frac{-b_n c_{n-3} - b_{n-1} c_{n-2}}{c_{n-2}^2 - c_{n-3}(c_{n-1} - b_{n-1})}$$

$$\Delta q_k = \frac{-b_{n-1}(c_{n-1} - b_{n-1}) - b_n c_{n-2}}{c_{n-2}^2 - c_{n-3}(c_{n-1} - b_{n-1})}$$

the value of b_i' & c_i' are obtained from ~~reccurance~~ recurrence relation.

Q using Bairstow method and ~~met~~ obtain the quadratic factor of the following eq.

a) $x^4 - 3x^3 + 20x^2 + 44x + 54 = 0$ with $(p, q) = 2, 2$

Initial $p_0 = 2$, $q_0 = 2$

	a_0	a_1	a_2	a_3	a_4
−2 / −2	1	−3	20	44	54
	✗	−2	10	−52	4
			−2	10	56
−2	1	−5	26	−2	2
	b_0	b_1	b_2	b_3	b_4
−2	✗	−2	14	−80	
			−2	14	
	1	−7	40	68	
	c_0	c_1	c_2	c_3	

$n = 4$

$$\Delta p_0 = \frac{-b_4 c_1 - b_3 c_2}{c_2^2 - c_1 c_3 + c_1 b_3} = -\frac{2(-7)-(-2)(40)}{(40)^2-(-7)(68)+(-7)(-2)}$$

$$= 0.057$$

$$\Delta q_0 = \frac{b_4 c_2 - b_3 c_3 + b_3^2}{c_2^2 - c_1 c_3 + c_1 b_3} = \frac{2(40)-(-2)\cdot 68+(-2)^2}{(40)^2-(-7)(68)+(-7)(-2)}$$

$$= -0.0457$$

$$p_1 = p_0 + \Delta p_0$$

$$= 2 + (-0.057) = \underline{1.9430}$$

$$q_1 = q_0 + \Delta q_0$$

$$= 2 + (-0.0457)$$

$$= 1.9543$$

Iteration IInd :-

-1.9430 -1.9543	1	-3	20	44	54
	X	-1.9430	9.6042	-53.7237	0.1236
			-1.9543	9.6601	-54.0361
-1.9430	1 (b_0)	-4.9430 (b_1)	27.6499 (b_2)	-0.0636 (b_3)	
-1.9543		-1.9430	13.3794	-75.9227	
			-1.9543	13.457	
	1 C_0	-6.886 C_1	39.875 C_2	-62.5293 C_3	

$$p_2 = p_1 + \Delta p_1 \qquad q_2 = q_1 + \Delta q_1$$

$$= -1.9430.$$

b self — Solve. $x^4 - x^3 + 6x^2 + 5x + 10 = 0$

with $(P, Q) = (1.14, 1.42)$

★ $a_{n-2}x^2 + a_{n-1}x + a_n = 0$

$$p_0 = \frac{a_{n-1}}{a_{n-2}} \quad \frac{x \text{ का गुणांक}}{x^2 \text{ का गुणांक}} \qquad q_0 = \frac{a_n}{a_{n-2}} = \frac{\text{no (last term)}}{x^2 \text{ का गुणांक}}$$

self Q ★ Performe the three iteration of Bairstow method to find a quadratic factor of a polynomial eqn using initial appro. $(-1.5 \;\&\; +1.5)$

$$x^4 - 4x^3 + 6x^2 - 8x + 4 = 0$$

self Q — Find the quadratic factor of polynomial eqn by these method.

$$x^4 + x^3 + 2x^2 + x + 1 = 0$$

$$p_0 = \frac{1}{2} = 0.5$$

$$q_0 = \frac{1}{2} = 0.5$$

then solve.

self Q — Find the root of eqn

$$(x^3 - 2x^2 + x - 2)$$

Shot on OnePlus

* Newton Raphson method :-

$$\boxed{x_{n+1} = x_n - \frac{f(x)}{f'(x_n)}}$$

Cond^n :- when initial cases fail

i) ranging position failed (when root are repeated)

ii) when root are parallel to initial case

* Newton Raphson method for Real multiplicity roots :

$$\boxed{x_{n+1} = x_n - m\frac{f(x)}{f'(x)}}$$

Q. find the root of the eqn near to $x=2$ by Newton Raphson method correct up to three decimal

$$f(x) = x^4 - x - 10.$$

$$f'(x) = 4x^3 - 1$$

$$x_{n+1} = x_n - \frac{[x_n^4 - x_n - 10]}{4x_n^3 - 1}$$

$$= \frac{4x_n^4 - \cancel{x_n} - x_n^4 + \cancel{x_n} + 10}{4x_n^3 - 1}$$

$$x_{n+1} = \frac{3x_n^4 + 10}{4x_n^3 - 1}$$

$n=0$

$$x_1 = \frac{3x_0^4 + 10}{4x_0^3 - 1}$$

$$58/31 = \frac{1.8709}{\cancel{1.8\ldots}}$$

(जब तक three digit same नहीं आ जाये be continued x_1, x_2 - - - Ans

Q Find the double root of the eq^n $\boxed{x_0 = 0.3}$

$$f(x) = x^4 - 6.75x^3 + 6.25x - 1.5 = 0$$

$$f'(x) = 4x^3 - 6.75 \times 3x^2 + 6.25$$

$$= x_n - \frac{\cancel{4x_n^3} - (\cancel{6.75 \times 3} x_n^2 + \cancel{6.25})2}{4x_n^3 - 6.\cancel{75 \times 3 x_n^2} + 6..}$$

Q

Q

Q

= ... the double root of

$$x^3 - x^2 - x + 1 = 0$$

$$\boxed{x_0 = 0.8}$$

* Newton Raphson method for finding the multiplicity root given (multiple root for)

$$\left[x_{n+1} = x_n - \frac{f(x_n)\, f'(x_n)}{\left[f'(x_n)^2 - f(x_n)\, f''(x_n) \right]} \right]$$

Q* Find the multiple root of the eqn

$$f(x) = 27x^5 + 27x^4 + 36x^3 + 28x^2 + 9x + 1 = 0$$

by using Newton method.

$$f'(x) = 135x^4 + 108x^3 + 108x^2 + 56x + 9$$

$$f''(x) = 540x^3 + 324x^2 + 216x + 56$$

multiplicity of the roots is known therefore multiple roots of there

$f(x) = 0$ can be considered as a simple root of a eqn

$$G(x) = 0$$

where $G(x) = \dfrac{f(x)}{f'(x)}$

Now

$$G(x) = \frac{27x^5 + 27x^4 + 36x^3 + 28x^2 + 9x + 1}{135x^4 + 108x^3 + 108x^2 + 56x + 9}$$

$x = 0 = 1/9$

$x = 1 = +ve$ region b/w $(-1$ to $0)$

$x = -1 = -0.5$

Now $x_0 = -0.5$

x_0 का मान अन्तराल के मध्य कुछ भी ले सकते हैं

$$f(x_n) = 27x_n^5 + 27x_n^4 + 36x_n^3 + 28x_n^2 + 9x_n + 1 = 0$$

$$f'(x_n) = 135x_n^4 + 108x_n^3 + 108x_n^2 + 56x_n + 9$$

$$f''(x_n) = 540x_n^3 + 324x_n^2 + 216x_n + 56$$

$n = 0$

$$x_1 = x_0 - \frac{f(x_0)\, f'(x_0)}{[f'(x_0)^2 - f(x_0) f''(x_0)]}$$

$$\boxed{x_0 = -0.5}$$

$$= -0.5 - \frac{(-0.156 \times 2.9375)}{(2.9375)^2 - (-0.156)(-38.5)}$$

Solve $= -0.5 - \frac{(\cdot 45825)}{}$

$= -0.5 + 0.17564$

$x_1 = -0.32436$

Ans

to be continue and find

x_2, x_3 ---

Q Find the multiple root of the eqn

$x^3 - 8.4x^2 + 23.14x - 20.48 = 0$

Ans

★ Newton Raphson method of symultanous transcendal eqn :-

$f(x, y)$

$g(x, y)$

$(x_0 y_0)$

$f(x_0 y_0)$

$g(x_0 y_0)$

$(f_x)(x_0 y_0)$

$f_y(x_0 y_0)$

$g_x(x_0 y_0)$

$g_y(x_0 y_0)$

$$\Delta = \begin{vmatrix} f(x)_{(x_0 y_0)} & f_y(x_0 y_0) \\ g(x)(x_0 y_0) & g_y(x_0 y_0) \end{vmatrix} \neq 0$$

$$\Delta x = \begin{vmatrix} (-f)(x_0 y_0) & f_y(x_0 y_0) \\ (-g)(x_0 y_0) & g_y(x_0 y_0) \end{vmatrix}$$

$$\Delta y = \begin{vmatrix} f_x(x_0 y_0) & (-f)_{x_0 y_0} \\ g_x(x_0 y_0) & -g(x_0 y_0) \end{vmatrix}$$

$$x_1 = x_0 + \frac{\Delta x}{\Delta}$$

$$y_1 = y_0 + \Delta y/y$$

Self Q

$f(x, y) = x^2 y + y^3 = 10$

$g(x, y) = x y^2 - x^2 = 8$

Given $(x_0 = 0.8, 2.2)$

$f(x_0 y_0) = (0.8)^2 \, 2.2 + (2.2)^3 - 10 = 2.056$

$g(x_0 y_0) = (0.8)(2.2)^2 - (0.8)^2 - 3 = 0.232$

$f_x(x_0 y_0) = 2xy = 2(0.8)(2.2) = 3.52$

$f_y(x_0 y_0) = x^2 + 3y^2 = (0.8)^2 + 3(2.2)^2 = 15.16$

$g_x(x_0 y_0) = y^2 - 2x = (2.2)^2 - 2(0.8) = 3.24$

$g_y(x_0 y_0) = 2xy = 2(0.8)(2.2) = 3.52$

First

$\Delta = -36.728$

$$\Delta x = \begin{vmatrix} -2.056 & 15.16 \\ -0.232 & 3.52 \end{vmatrix} = -3.72$$

$$\Delta y = \begin{vmatrix} 3.52 & -2.056 \\ 3.24 & -0.232 \end{vmatrix}$$

$\Delta y = 5.8448$

$x_1 = x_0 + \frac{\Delta x}{\Delta}$

$x_1 = 0.8 + \frac{(-3.72)}{-36.728}$

$= 0.9012$

$y_1 = y_0 + \Delta y / \Delta$

$y_1 = 2.0408$

Q. Find the real root of the eqn by Newton Raphson method correct two decimal

$$y\cos(xy)+1=0$$

$$\sin(xy)+x-y=0$$

$x_0=1 \quad y_0=2$ (value Rad में find करेंगे)

Q. Find the real root of the eqn

$$f(x,y) = x+3\log_{10}x-y^2=0$$

$$g(x,y) = 2x^2-xy-5x+1=0$$

$$x_0=3,\ y_0=1.5$$

*** Regula falsi method :-** this technique is similar to bisection method except the next estimate is taken at the intersection of ------ line b/w the pairs of x values and the x axis rather than the mid point & gives faster convergence by bisection

*** formula :-** If we can take two points x_n and x_{n-1} such that $\boxed{f(x_n)\, f(x_{n-1}) < 0}$

i.e $f(x_n)\, f(x_{n-1})$ are opposite sign

$f(x)$ is continuous over $[x_{n-1}, x_n]$ there exist at least one root of $f(x)$ in interval (x_{n-1}, x_n)

$$x_{n+1} = x_n - \frac{(x_n - x_{n-1})}{f(x_n) - f(x_{n-1})} f(x_n)$$

Q find the root of the eqn

i) $x^3 - 5x^2 - 17x + 20 = 0$

$$\left.\begin{array}{l} f(0) = 20 \\ f(1) = -1 \end{array}\right\} = 0, 1$$

$x_0 = 0 \quad x_1 = 1$

$f(x_0) = f(0) = 20$

$f(x_1) = f(1) = -1$

$\boxed{f(0) \cdot f(1) < 0}$

Iteration 1st :-

$n=1$

$$x_2 = x_1 - \frac{(x_1 - x_0)}{f(x_1) - f(x_0)} f(x_1)$$

$$x_2 = 1 - \frac{(1-0)}{-1-20}(-1) = 0.95238$$

$f(x_1) f(x_2) < 0$

To Be continue. ---- x_3 x_4 ----

Q Find the root of eqn by regula falsi.

$(4 \sin x - x^2 = 0)$

Q Find the root of eqn by regula falsi

$x^3 - 5x^2 - 17x + 20 = 0$

four decimal

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