⇒ Method of Successive Approximation :-

This method is also known as direct substitution method or method of iterations or method of fixed iterations, is applicable if the equation

$$f(x) = 0$$

can be expressed as

$$x_1 = g(x)$$

If x_1 is the initial approximation to the root then the next approximation to the root is given by

$$x_2 = g(x_1)$$

and the next approximation will be

$$x_3 = g(x_2)$$

In general

$$x_i = g(x_{i-1})$$

$$\boxed{x_{i+1} = g(x_i)}$$

The iterative cycle will terminate when the relative error in the new approximation is within the prescribed tolerance.

NOTE :-

This method is converse if and only if

$$\boxed{|g'(x)| < 1}$$

Ex 2.11 Given that one root of non-liner equation $2^x - x - 3 = 0$ lies in the interval $(-3, -2)$. Find the root correct to three decimal places

Solve

Since we want the solution correct to three significant digits, the iterative process will be terminated as soon as the successive iterations produces no change at first four significant positions.

$$2^x - x - 3 = 0$$

$$x = 2^x - 3$$

$$\therefore x = g(x)$$

$$g(x) = 2^x - 3$$

Iteration 1 :- Let us take intial approximation

$$x_1 = -3$$

$$x_2 = g(x_1)$$

$$x_2 = 2^{-3} - 3 = -2.875$$

Iteration 2 :-

Now we take $x_2 = -2.875$ as the current appro. to obtain the next appro.

$$x_3 = g(x_2)$$

$$x_3 = 2^{-2.875} - 3 = -2.8637$$

Iteration 3:- Now we take $x_3 = -2.8637$ as the current appro. to obtain the next app. as

$$x_4 = g(x_3)$$

$$x_4 = 2^{-2.8637} - 3 = -2.8626$$

Iteration 4:-

Now we take $x_4 = -2.8626$ —//—

$$x_5 = g(x_4)$$

$$x_5 = 2^{-2.8626} - 3 = -2.8625$$

Interation 5:-

Now we take $x_5 = -2.8625$ —//—

$$x_6 = g(x_5)$$

$$= 2^{-2.8625} - 3$$

$$x_6 = -2.8625$$

We see that after iteration three number. there is no change at the first four significant digits. Therefore, we take $x = -2.862$ as the desired solution correct to four significant digits or 3 decimal plance.

Pg 88
Q.20

Find the root of equition $2x = \cos x + 3$ correct to three decimal places by using Iteration method.

Solve

$$f(x) = 2x - \cos x - 3 = 0$$

$$f(0) = -4$$

$$f(1) = -1.54$$

$$f(2) = +1.41$$

so, rigion (1,2)

then quadrat root lies between (1,2)

$$x = \frac{1}{2}(\cos x + 3)$$

$$g(x) = \frac{1}{2}(\cos x + 3)$$

$$g'(x) = \frac{1}{2}(-\sin x)$$

$$g'(x) = \left|\frac{1}{2}(-\sin x)\right|$$

$$\frac{1}{2} < 1 \qquad \because -1 < \sin x < 1$$

So, this method is applicable for this eq^n

$$x = g(x)$$

$$x = \frac{1}{2}(\cos x + 3)$$

Interation 1 :-

Let us take intial approx. as $x_1 = 1.5$

$$x_2 = g(x_1)$$

$$x_2 = \frac{1}{2}(\cos(1.5) + 3) = 1.5353$$

Scanned by CamScanner

Iteration-2:- Now we take $x_2 = 1.5353$ — // —

$x_3 = g(x_2)$

$x_3 = \frac{1}{2}[\cos(1.5353)+3] = 1.5177$

Iteration-3:- Now we take $x_3 = 1.5177$ — // —

$x_4 = g(x_3)$

$x_4 = \frac{1}{2}[\cos(1.5177)+3] = 1.5265$

Iteration-4:- Now we take $x_4 = 1.5265$ — // —

$x_5 = g(x_4)$

$x_5 = \frac{1}{2}[\cos(1.5265)+3] = 1.5221$

Iteration-5:- Now we take $x_5 = 1.5221$ — // —

$x_6 = g(x_5)$

$x_6 = \frac{1}{2}[\cos(1.5221)+3] = 1.5243$

Iteration-6:- Now we take $x_6 = 1.5243$

$x_7 = g(x_6)$

$x_7 = \frac{1}{2}[\cos(1.5243)+3] = 1.5232$

Iteration-7 Now — // — $x_7 = 1.5232$ — // —

$x_8 = \frac{1}{2}[\cos(1.5232)+3] = 1.5237$

Iteration 8 Now — // — $x_8 = 1.5237$ — // —

$x_9 = \frac{1}{2}[\cos(1.5237)+3]$

$x_9 = 1.5235$

we see that after Iteration

Q. 1

Find the root of the eq^n $2x - \log_{10} x = 7$
~~by using method of f~~
correct to the 3 decimal places using iteration method.

Solve

$$f(x) = 2x - \log_{10} x - 7 = 0$$

$$f(0) = -7$$

$$f(1) = 2 - 0 - 7 = -5$$

$$f(2) = 4 - 0.3010 - 7 = -3.301$$

$$f(3) = 6 - 0.4771 - 7 = -1.4771$$

$$f(4) = 8 - 0.6020 - 7 = 0.398$$

then quadrate root liest B/W (3, 4)

$$2x - \log_{10} x - 7 = 0$$

So,

$$x = \left(\frac{\log_{10} x + 7}{2}\right)$$

$$g(x) = \frac{\log_{10} x + 7}{2}$$

$$g'(x) = \frac{1}{x}\log_{10} e = \frac{0.493}{x}$$

So, this method satisfies the $cond^n$ $g'(x) < 1.50$ its applicable for this eq^n.

Ite

It

Iteration 1: let take appro. x

$$x_1 = 3.5$$

so,

$$x_2 = g(x_1)$$

$$x_2 = \frac{1}{2}[\log_{10}(3.5)+7]$$

$$x_2 = \frac{1}{2}[0.5440+7]$$

$$x_2 = 3.7720$$

Iteration 2:

$$x_2 = 3.7720$$

$$x_3 = g(x_2)$$

$$x_3 = \frac{1}{2}[\log_{10}(3.7720)+7]$$

$$x_3 = 3.7882$$

Iteration 3:

$$x_3 = 3.7882$$

$$x_4 = g(x_3)$$

$$= \frac{1}{2}[\log_{10}(3.7882)+7]$$

$$x_4 = 3.7892$$

Iteration 4:-

$$x_4 = 3.7892$$

$$x_5 = g(x_4)$$

$$= \frac{1}{2}[\log_{10}(3.7892)+7]$$

$$x_5 = 3.7892$$

Iteration 5:-

$$x_5 = 3.7892$$

$$x_6 = \frac{1}{2}[\log_{10}(3.7892)+7] = 3.7892$$

Here 3.7892 is desired soln

Q.2 Find the one root of the eqn $3x - 2e^{x} = 0$ lies b/w (0,1). Find that root with tolrence 0.001

Method of Synthetic division :-

The Division of Polynomial f(x) by the factor (x−r) can be carried rapidly by synthetic division the remender of the above division will give the value of polynomial for x=r ∴ How every if r happen to the root of the polynomial f(x) then the remender resulting from the division will be zero.

Similarly the value of derivative f′(x) of the polynomial f(x) for x=r is equal to the remender obtained by second synthetic division of the result obtain from the first synthetic division which will be a polynomial of degree 1 less to the origional polynomial.

Q. Divide the polynomial $x^5-3x^4-10x^3+5x^2+22x+6$ by $(x-4)$.

Solve

4	1	-3	-10	5	22	6	
		4	4	-24	-76	-216	
	1	1	-6	-19	-54	$\boxed{-210}$	$f(x)$
		4	20	56	148		
	1	5	14	37	$\boxed{94}$		$f'(x)$
	1						

Q. $f(x) = x^6 - x^4 - x^3 - 1$, find the value of $f'(2)$ using synthetic division also find $f'''(2)$.

Soln

$x - 2 = 0$

$x = 2$

2	1	0	-1	-1	0	0	-1	
		2	4	6	10	20	40	
	1	2	3	5	10	20	$\boxed{39}$	$f(2)$
		2	8	22	54	128		
	1	4	11	27	64	$\boxed{148}$		$f'(2)$
		2	12	46	146			
	1	6	23	73	$\boxed{210}$			$f''(2)$
		2	16	78				
	1	8	39	$\boxed{151}$				$f'''(2)$
	1							

Scanned by CamScanner

Q Suppose the polynomial is $x^5-3x^4-10x^3+10x^2+44x+48$ is to be divided by the quadratic factor x^2+2x+1

Solve

The division of the Polynomial $f(x)$ by the quadratic $x^2-\alpha$, $x-\beta$

$x^2+2x+1=0$

$x(x+1)+1(x+1)=0$

$(x+1)(x+1)=0$

$x=-1,-1$

1	1	-3	-10	+10	44	48
-1						
		-	-	1		$\boxed{}$

$f(x)$

$x^3-4x^2-7x+21$

Remender 30 (x.

Q Suppose the polynomial is $x^5-3x^4-10x^3+10x^2+44x+48$ is to be divided by the quadratic factor x^2+2x+1.

Solve

The division of the Polynomial $f(x)$ by the quadratic $x^2-\alpha$, $x-\beta$

$\alpha=-2$, $\beta=-1$

	1	-3	-10	10	44	48
-2		-2	10	2	-34	-22
-1			-1	5	1	-17
	1	-5	-1	17	11	$\boxed{9}$ → f(x)

$x^3 - 5x^2 - x + 17$

Remender $11(x+2) + 9 = 11x + 31$

Q divide $2x^5 - 3x^4 + 5x^2 + 6x - 9$ By factor $x^2 - x + 2$ By Synthetic division

Solve

$\alpha = +1, \quad \beta = -2$

	2	-3	0	5	+6	-9
1		2	-1	-5	2	18
-2			-4	2	10	-4
	2	-1	-5	2	18	5

$2x^3 - x^2 - 5x + 2$

Remender $18(x-\alpha) + 5 = 18(x-1) + 5$
$\Rightarrow 18x - 13$

Bairstrow Method :-

This method is used to find two real number p and q.
इस विधि का उपयोग दो वास्तविक संख्याएँ p व q ज्ञात करने के लिए किया जाता है जो इस प्रकार हो की $x^2 + px + q$, $P_n(x)$ का एक factor हो, p_0 व q_0 से प्रारंभ करके हम Iterates $\{(p_k, q_k)\}$ निम्नलिखित सूत्रो से प्राप्त करते है :-

Scanned by CamScanner

$$p_{k+1} = p_k + \Delta p_k$$
$$q_{k+1} = q_k + \Delta q_k$$

जहाँ

$$\Delta p_k = -\frac{b_n c_{n-3} - b_{n-1} c_{n-2}}{c_{n-2}^2 - c_{n-3}(c_{n-1} - b_{n-1})}$$

$$\Delta q_k = \frac{b_{n-1}(c_{n-1} - b_{n-1}) - b_n c_{n-2}}{c_{n-2}^2 - c_{n-3}(c_{n-1} - b_{n-1})}$$

The value of b_i's and c_i's are obtain from recorance Relation

$$b_i = a_i - p_k b_{i-1} - q_k b_{i-2}$$
$$c_i = b_i - p_k c_{i-1} - q_k c_{i-2}$$ जहाँ $i = 1, 2, ---$

with

$$c_0 = b_0 = a_0 \quad , \quad c_{-1} = b_{-1} = 0 .$$

$\rightarrow$ $a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} + ----+ a_{n-1} x + a_n$

	a_0	a_1	a_2	-----	a_{n-1}	a_n
$-p_k$		$-p_k b_0$	$-p_k b_1$	----	$-p_k b_{n-2}$	
$-q_k$			$-q_k b_0$	----	$-q_k b_{n-3}$	
	$a_0 = b_0$	$a_1 - p_k b_0 = b_1$	$a_2 - p_k b_1 - q_k b_0 = b_2$		$a_{n-1} - p_k b_{n-2} - q_k b.$	

यह विधि दो कोटि की है। जब p व q इच्छित शुद्धता तक ज्ञात कर चुके हो। तो de·

$$Q^{n}_{(n-2)} = b_0 x^{n-2} + b_1 x^{n-3} + \cdots + b_{n-3} x + b_{n-1}$$

Q Using Bairstow method obtain in quadratic factor :-

$$x^4 - 3x^3 + 20x^2 + 44x + 54 = 0 \quad \text{with } (P, q) = (2, 2)$$

Sol^n We use the synthetic division method to determine b_i's and c_i's

Iteration 1: $P_0 = 2$, $q_0 = 2$

	a_0	a_1	a_2	a_3	a_4
	1	-3	20	44	54
-2		-2	10	-56	4
-2			-2	10	-56
	1 (b_0)	-5 (b_1)	28 (b_2)	-2 (b_3)	2 (b_4)
		-2	+14	-80	
			-2	+14	
	1 (c_0)	-7 (c_1)	40 (c_2)	-68 (c_3)	

$$\Delta P = -\frac{b_4 c_1 - b_3 c_2}{c_2^2 - c_1(c_3 - b_3)}$$

$$= -\frac{(2)(-7) - (-2)(40)}{(40)^2 - (-7)(-68) + (-7)(-2)}$$

$$\Delta P = -0.0580$$

$$\boxed{P_1 = 2 + (-0.0580) = 1.9420}$$

$$\Delta q = \frac{b_3(c_3 - b_3) - b_4 c_2}{c_2^2 - c_1(c_3 - b_3)}$$

$$= \frac{b_3 c_3 - b_3^2 - b_4 c_2}{c_2^2 - c_1 c_3 + c_1 b_3}$$

$$= \frac{(-2)(-68) - (-2)^2 - (2)(40)}{(40)^2 - (-7)(-68) + (-7)(-2)}$$

$$= \frac{136 - 4 - 80}{1600 - 476 + 14}$$