Q1) Normal distribution

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{(x-\mu)^2}{2\sigma^2}\right)}$$

mean (μ) $= \theta_1$ variance $= \theta_2$

size $= n$

$$L(\mu) = \prod_{i=1}^{n} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x_i-\mu)^2}{2\sigma^2}}$$

$$\log[L(\mu)] = -n\log(\sigma\sqrt{2\pi}) - \sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\sigma^2}$$

$$\frac{\partial \log L(\mu)}{\partial \theta_1} = \frac{1}{\sigma^2}\sum_{i=1}^{n}(x_i-\mu) = \frac{n\bar{x} - n\theta_1}{\sigma^2} = 0$$

or $\theta = \bar{x}$

$$\frac{\partial \log L(\theta_1)}{\partial \theta_1} = \frac{n}{\sigma^2}(\bar{x}-\mu)$$

$$\frac{\partial^2 \log L(\theta_1)}{\partial \theta_1^2} = -\frac{n}{\sigma^2}$$

$$\theta_2 = \frac{\sigma^2}{n}$$ $\rightarrow$ varian of sampl mean

Q1) $B(m, \theta)$ $\theta \in [0, 1)$

pmf : $P(X = k) = {}^{m}C_{k}\, \theta^{k} (1-\theta)^{m-k}$

$k \rightarrow$ no. of success in m trials

$\theta \rightarrow$ probability of success

$1-\theta \rightarrow$ " " failure

likelihood function $\rightarrow$ $L(\theta) = \prod_{i=1}^{n} {}^{m}C_{x_i}\, \theta^{x_i} (1-\theta)^{m-x_i}$

$$\ln(L(\theta)) = \sum_{i=1}^{n} \ln({}^{m}C_{x_i}) + \sum_{i=1}^{n} x_i \ln\theta + \sum_{i=1}^{n} (m-x_i) \ln(1-\theta)$$

$$\frac{\partial}{\partial\theta} \ln(L(\theta)) = \sum_{i=1}^{n} \frac{x_i}{\theta} - \frac{m-x_i}{1-\theta} = 0$$

$$\sum_{i=1}^{n} \frac{x_i}{\theta} = \sum_{i=1}^{n} \frac{m-x_i}{1-\theta}$$

$$\frac{\sum_{i=1}^{n} x_i}{\theta} = \frac{\sum^{n} (m-x_i)}{1-\theta}$$

$$\frac{\sum_{i=1}^{n} x_i}{\theta} = \frac{m.n - \sum_{i=1}^{n} x_i}{1-\theta}$$

$$\theta.(nm - \sum x_i) = (1-\theta) \sum x_i$$

$$\theta.n.m = \sum x_i$$

$$\boxed{\theta = \frac{\sum x_i}{nm} = \frac{\bar{X}}{m}}$$