Indian Institute of Technology Bhilai

IC202: Calculus II

2023-24-M | Instructor: Dr. Raj Kumar Mistri

TUTORIAL SHEET-5

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1. Show that the following functions are differentiable everywhere.

   (a) $f(x, y, z) = x + 2y + 4z$. (b) $f(x, y, z) = xy + yz + zx$.

2. (a) Find the directional derivative of $f(x, y) = x^2 + xy$ at the point $(1, 2)$ in the direction of the unit vector which makes an angle of $\pi/4$ radian with the positive x-axis.

   (b) Find the directional derivative of $f(x, y, z) = x^3 - xy^2 - z$ at $(1, 1, 0)$ in the direction of the vector $u = 2i - 3j + 6k$.

3. Prove that the function
$$f(x, y) = \begin{cases} \frac{2xy^2}{x^2+y^4}, & \text{if } (x, y) \neq (0, 0); \\ 0, & \text{if } (x, y) = (0, 0). \end{cases}$$
has all the directional derivative at $(0, 0)$.

4. Find the direction in which the function f defined by
$$f(x, y) = \begin{cases} \frac{2xy}{x^2+y^2}, & \text{if } (x, y) \neq (0, 0); \\ 0, & \text{if } (x, y) = (0, 0). \end{cases}$$
has directional derivative at $(0, 0)$.

5. (a) Let $f(x, y) = x^2 - y^2$. Find the gradient of f at the point $(a, b) \in \mathbb{R}^2$. In what direction from $(0, 1)$ should one proceed to increase f the fastest.

   (b) Let $f(x, y, z) = \sin(xy)e^{-z^2}$. In what direction from $(1, \pi, 0)$ should one proceed to increase f most rapidly?

6. (a) Let $f(x, y) = x^2 + 3xy + y^3$, $x = u^2 - v^2$,$y = u^2 + v^2$. Find f_u and f_v.

   (b) Let $f(x, y, z) = x^2 + y - xz$, $x = u - v$,$y = uv$, $z = u + v$. Find f_u and f_v.

   (c) Let $f(x, y, z) = x^2 - y^2 + z^2$, $x = t$,$y = e^t$, $z = e^{-t}$. Find $\frac{df}{dt}$.

   (d) Let $f(x, y) = xy^2 + 3x + 2y$, $x = t^2$,$y = t + 1$. Find $\frac{df}{dt}$.

   (e) Using Chain Rule, find $\frac{df}{dt}$, where $f(t) = (\sin t)^{\tan t} + (\tan t)^{\sin t}$.

   (f) Let $f(x) = x^5 + x^3 - 1$, where $x = u^2 - v^2$. Find f_u and f_v.

   (g) Find $\frac{dy}{dx}$ if $y^2 - x^2 - \sin(xy) = 0$.

   (h) Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ at $(1, 2, 4)$ if $z^3 - xy + yz + y^3 - 2 = 0$.

7. Find the tangent plane and normal line of the level surface $f(x, y, z) = x^2 + y^2 + z - 9 = 0$ at the point $(1, 2, 4)$.