Lecture-4.

Three pictures of the Sun

(1) Image of the sun (through a filter).

(2) & (3) are taken during a solar eclipse

Corona

(1) (2) (3)

Both the photosphere & the chromosphere are eclipsed.

Corona is visible

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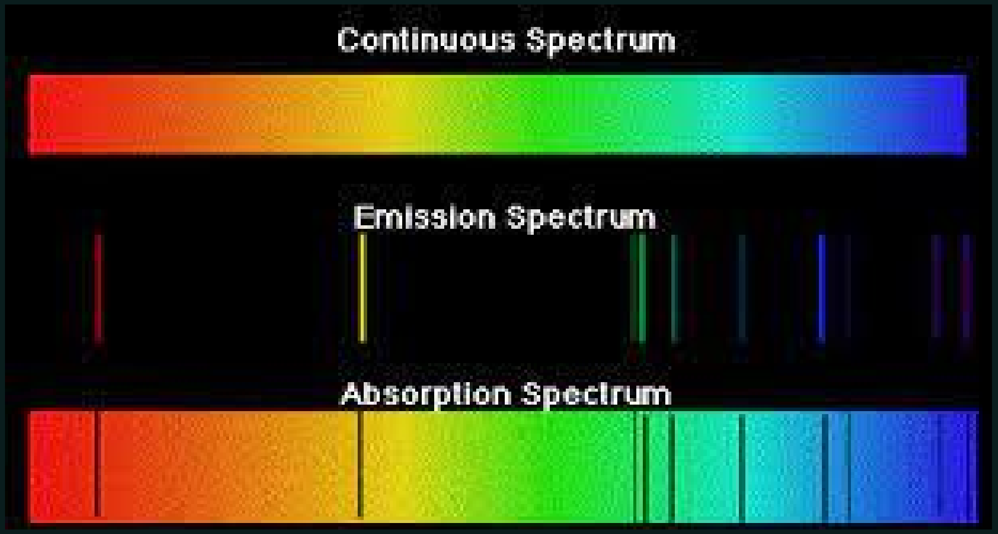
Photosphere

Chromosphere (Photosphere is blocked)

In the image (1) of photosphere, the central portion is bright while the outskirts are fainter / darker. This phenomenon is called Limb-darkening.

The Spectrum from the Sun

Something about the spectrum



(1)

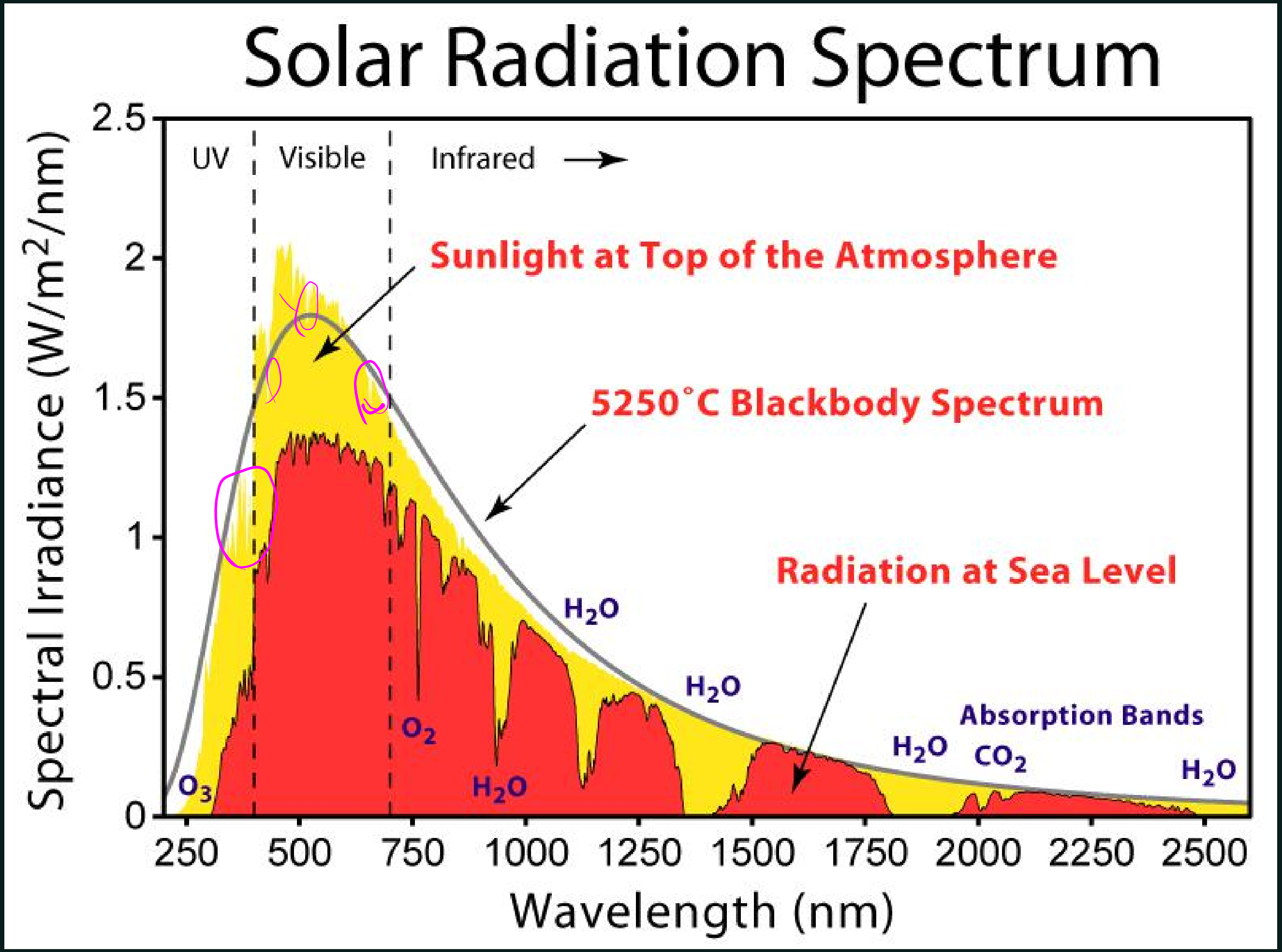
(2) outskirts of the solar disc

(3) → from the centre of the solar disc

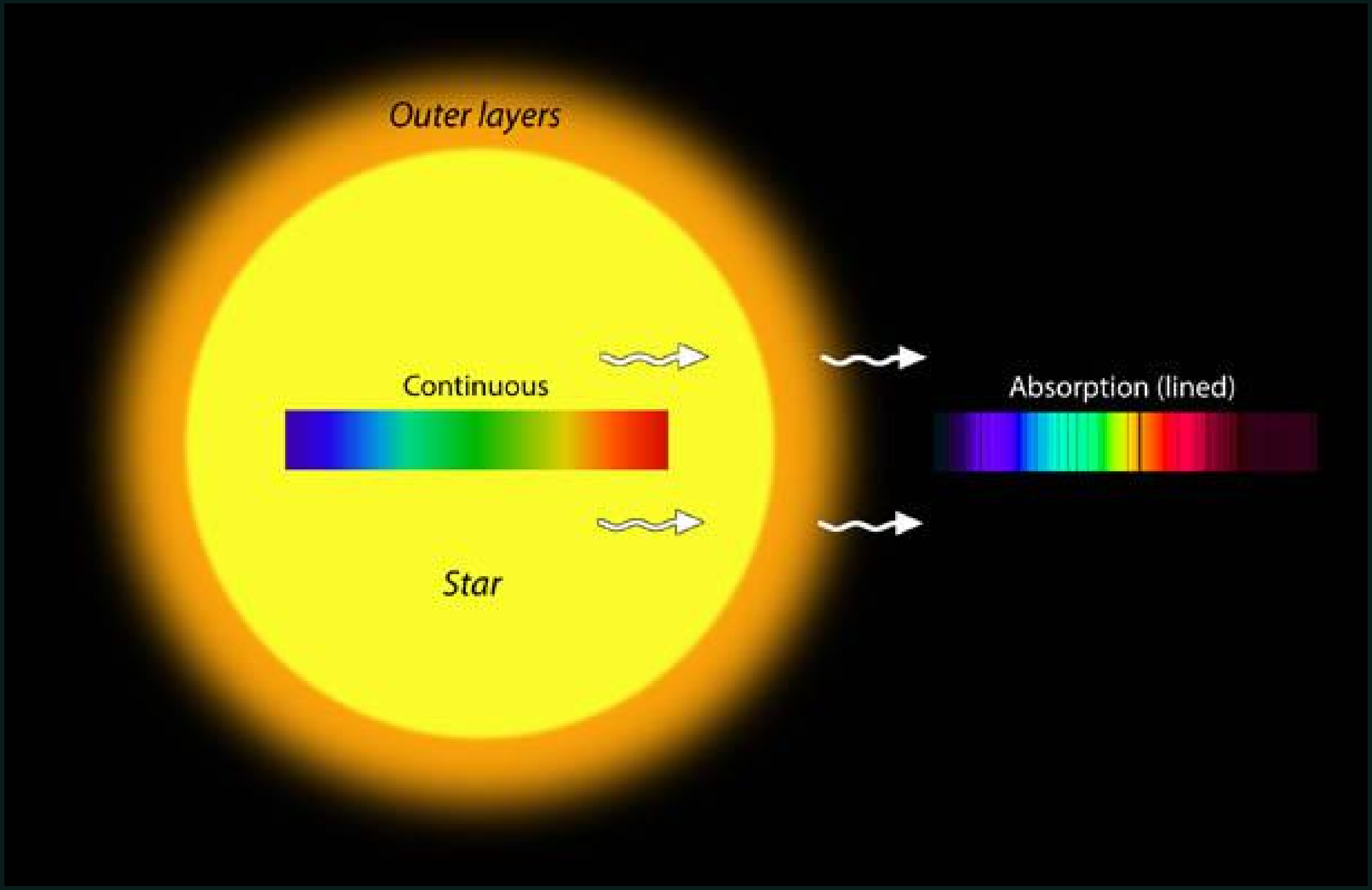
In (2) only a few frequencies/wavelengths are visible (which are emitted by the source)

In (3) most of the photons are there except a few that are missing / absorbed.

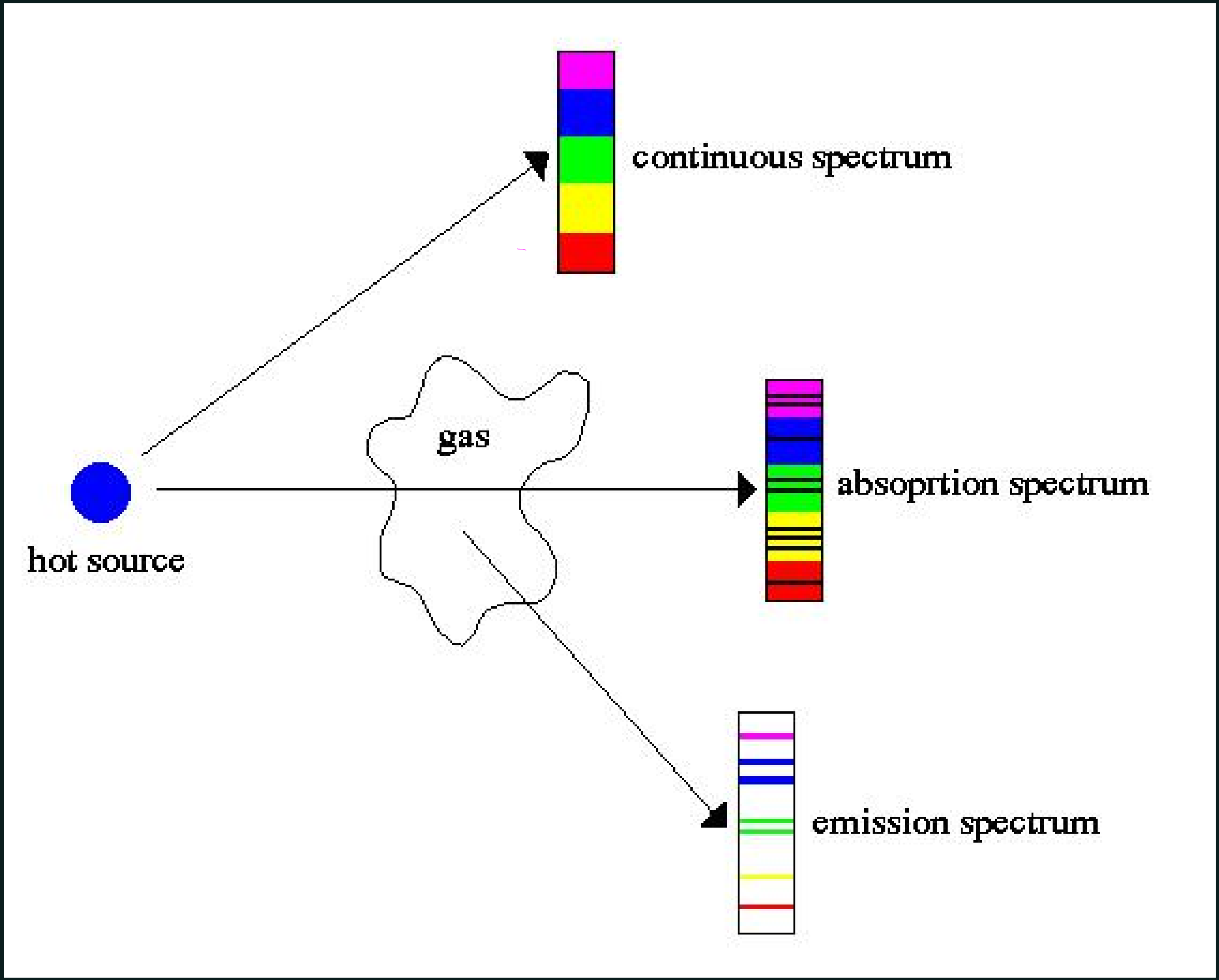
Sun's spectrum is like a Blackbody spectrum



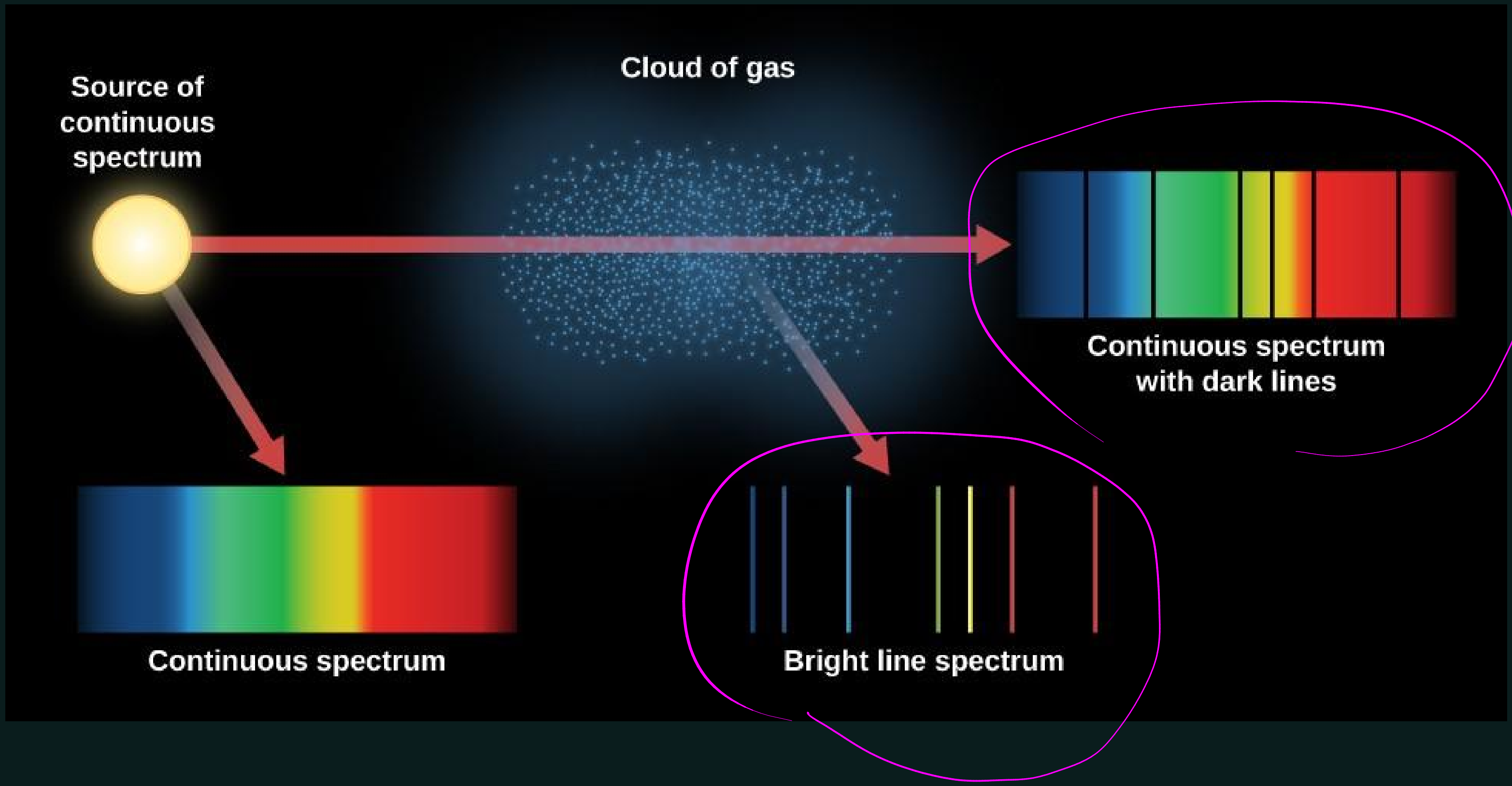
But there are differences



Kirchoff law



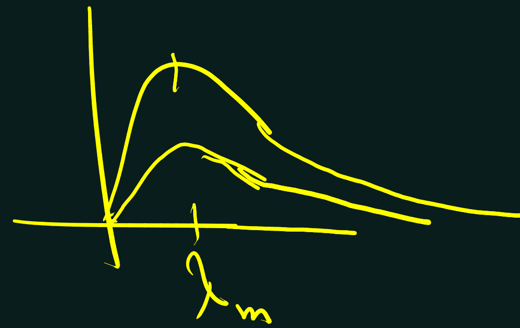
Kirchoff Law: spectrum from the Sun



Let's try to describe all this with equations

In the interior of the sun the photons of specific wavelengths are produced but they get scattered/absorbed remitted

Their energy redistribute; they come in equilibrium with the matter and all sort of wavelength are present



$$\lambda_m T = \text{constant}$$

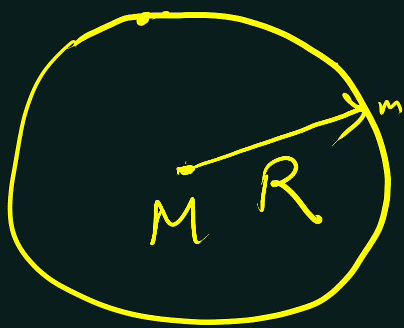
$$T = \frac{\text{constant}}{\lambda_m}$$

just by extracting the sample of radiation/photons that are present in a black body (in which the matter & radiation are in equilibrium) I can determine the temperature

Temperature thus measured for the sun is

$$\boxed{T \approx 5000 \text{ to } 6000\ ^\circ C}$$

Lets do consider the particles in the Sun.



Sun is a collection of particles under the influence of gravity. Radius $= R$

say total mass $= M$

$$\frac{1}{r} = \frac{GMm^2}{L^2}(1 + \epsilon\cos\theta)$$

Assume that the particle is doing a circular motion $\quad \epsilon = 0$

$$\frac{1}{r} = \frac{GMm^2}{L^2}$$

We want to find out the K.E., P.E., and the total energy

circular orbit $\quad \epsilon = \sqrt{1 + \dfrac{2EL^2}{G^2M^2m^3}} = 0$

$$\frac{2EL^2}{G^2M^2m^3} = -1$$

$$\Rightarrow E = -\frac{G^2M^2m^3}{2L^2}$$

$$= -\frac{1}{2}\frac{GMm}{r} \qquad \left(\frac{GMm^2}{L^2}\right) r$$

$$E = -\frac{1}{2}\frac{GMm}{r}\left(\frac{GMm^2}{L^2}\right)r = 1/r$$

But $\epsilon = 0 \Rightarrow$ $$\frac{1}{r} = \frac{GMm^2}{L^2}(1+\cancel{\epsilon\cos\theta}) = \frac{GMm^2}{L^2}$$

$$\Rightarrow \boxed{E = -\frac{1}{2}\frac{GMm}{r}}$$

$$E = KE + PE$$

$$PE = -\frac{GMm}{r}$$

$$E = -\frac{1}{2}\frac{GMm}{r} = \frac{1}{2}\left(-\frac{GMm}{r}\right)$$

$$E = \frac{1}{2}V = \frac{V}{2}$$

$$K.E. = E - V = \frac{V}{2} - V = -\frac{V}{2}$$

$$\boxed{\begin{aligned} K.E. &= -\frac{1}{2}\,P.E. \\ E &= \frac{1}{2}\,P.E. \end{aligned}}$$

Virial theorem

If you repeat for elliptical orbit then it is doesn't hold true to the exact energies. But it still holds true if we define averaged over time period

$$\left\{ \begin{aligned} \langle K.E \rangle_{\text{time period}} &= -\frac{1}{2} \langle P.E \rangle \\ E &= \frac{1}{2} \langle P.E \rangle \end{aligned} \right\}$$

Virial theorem

This virial theorem also holds true for a collection of particles say sun

$$\langle K.E. \rangle = -\frac{1}{2} \langle P.E. \rangle$$

of all the particles in the sun

of all the particles in the sun

But what is this for the gas in the sun.

P.E. $\sum \frac{G m_i m_j}{r_{ij}}$

P.E. $\approx -\frac{3}{5}\frac{G M M}{R}$

Total P.E. of sun $\approx -\left(\frac{3}{5}\right)\frac{G M^2}{R}$

Each particle is not moving freely; they are colliding

Net momentum change for a particle colliding of the walls of tiny cubical volume in the sun

$\Delta p = 2mv$

By N atoms colliding:

$\Delta p_{total} = 2mvN$

time taken $\Delta t = \dfrac{2L}{v}$

Rate of change of momentum

$\Downarrow$

Force

$$\frac{\Delta p_{total}}{\Delta t} = \frac{2mvN}{2L/v}$$

Force exerted on the wall $= \dfrac{2mv^2N}{2L}$

Pressure exerted on the wall $= \dfrac{F}{A} = \dfrac{2mv^2N}{2L \cdot L^2}$

$\dfrac{N}{L^3} = n$ = number density

$$P = mv^2 n$$

$$P = m\left(v_x^2 + v_y^2 + v_z^2\right) n \approx \frac{1}{2} m v^2 n$$

$$P = \underline{n k_B T} = \underbrace{\overbrace{\frac{1}{2} m v^2 n}}$$

K.E. of all the particles in the sun

$$k_B T \propto K.E.$$

Kinetic theory of Gases.

Total K.E. of the sun is roughly $k_B T N$

$R \approx 7 \times 10^{8}$ m

Virial Theorem.

$$K.E. = -\frac{1}{2} P.E.$$

$$Nk_B T = -\frac{1}{2}\left(-\frac{3}{5}\frac{GM^2}{R}\right)$$

$$\boxed{T = \frac{3}{10}\frac{GM^2}{RNk_B}}$$

$k_B \rightarrow 1.38 \times 10^{-23}$

$$\boxed{N = \frac{2M}{m_p}}$$

$M = 2 \times 10^{30}$ kg

$m_p = 1.67 \times 10^{-27}$ kg

$$T = \frac{3}{10}\frac{GM^2}{RNk_B} = \frac{3}{10}\frac{GM^{\cancel{2}}m_p}{R\,2\cancel{M}\,k_B}$$

$$T = \frac{3}{20}\frac{GMm_p}{k_B R}$$

$M = 2 \times 10^{30}$ kg $\qquad R = 7 \times 10^{8}$ m

$k_B = 1.38 \times 10^{-23} \qquad m_p = 1.67 \times 10^{-27}$ kg

$T \cong 3 \times 10^{6}$ (more than a million K)

So the actual temperature is more than a million kelvin.

{ But we found from the Black-body Spectrum calculation, that the temperature is about 5000 K. }

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This is the surface temperature.

Problems:

(1) Prove the virial theorem for the elliptical orbit. (Hint: you will have to calculate the K.E. & P.E. averaged over time period)

(2) Prove that the total gravitational potential energy of Sun is $-\frac{3GM^2}{5R}$ where M is the mass of Sun and R the radius

(3) Use the virial theorem argument to calculate the temperature of the sun if

(I) it expands to 10 times its current size

(II) it shrinks to a radius of 10 km

(4) We found that the interior of sun is very hot (> million kelvin) but the surface is not that hot (only around 5000 K). So there must be temperature gradient in successive shells. Using this information; try to explain Limb darkening in words.