Tutorial 1

12 September 2023 08:37

1. Consider the following model and problem:
   Model: There are 102 coins on a table, 98 are showing heads, and 4 are showing tails. There are two legal moves:
   (a) flip over any ten coins, or,
   (b) if n is the current number of heads showing, you can place n+1 additional coins on the table, all showing tails.
   Problem: Choose a sequence of moves so that eventually there is exactly one coin showing heads.
   Your task: Prove that there is no sequence of moves that will solve the problem.

2. **Model:** The numbers 1,2,3,4,5 are written on a sheet of paper. In one step, an algorithm picks any two written numbers x and y, writes the value of |x-y| on the paper, and erases x and y.
   Problem: Choose a sequence of steps so that eventually only 0's are written on the paper.
   Your task: to prove that It is impossible for any algorithm to solve this problem.

3. Prove by well ordering property that srqt(2) is irrational.
4. Prove by well ordering principle that n^3-n is always divisible by 6 for all n.
5. Prove by well ordering principle that $\sum_{i=1}^{n} i^3 = \frac{(n(n+1))^2}{4}$
6. Using the encoding scheme discussed in the class, draw the unlabelled tree whose binary encoding is 1010101011110000.
7. Solve the following recurrences using master method.

   1. $T(n) = 3T(n/2) + n^2$

   2. $T(n) = 4T(n/2) + n^2$

   3. $T(n) = T(n/2) + 2^n$

   4. $T(n) = 2^n T(n/2) + n^n$

   5. $T(n) = 16T(n/4) + n$

8. Consider the following theorem and a proof attempt to prove the theorem. Is the proof correct? Justify.

Theorem: All horses are the same color.

Proof: (by induction on n)

Induction hypothesis:

$P(n)$ ::= any set of n horses have the same color

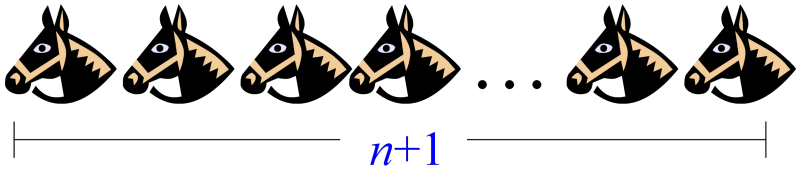
Base case (n=0):

No horses so *obviously* true!

(Inductive case)

Assume any n horses have the same color.

Prove that any n+1 horses have the same color.



(Inductive case)

Assume any n horses have the same color.

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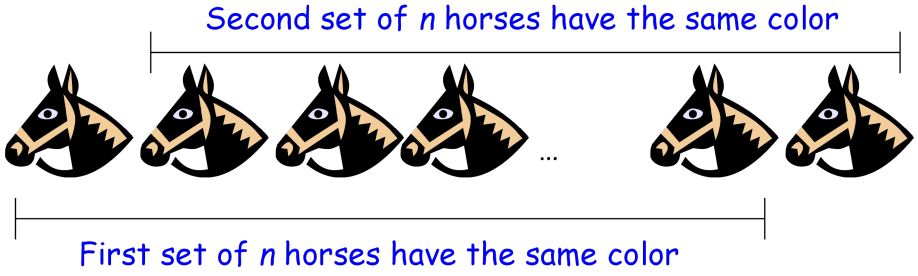

Therefore the set of n+1 have the same color