

Integral (CV) forms of law of mechanics :-
(Use of RTT) :-

$$\frac{d\vec{B}_{\text{sys}}}{dt} = \frac{d}{dt} \int_{\text{cv}} \rho \vec{b} dV + \int_{\text{cs}} \rho (\vec{v} \cdot \vec{n}) dA \quad [\text{RTT}]$$
$$=$$

Conservation of mass :- $B = m$, $b = \frac{dB}{dm} = \frac{dm}{dm} = 1$

↓
[system]

~~mass~~ $m_{\text{sys}} = \text{constant}$
[Fixed quantity of mass].

$$\left[\frac{dm_{\text{sys}}}{dt} = 0 \right] \quad \left[\frac{dB_{\text{sys}}}{dt} \right] \quad \left[\frac{dB_{\text{cv}}}{dt} \right]$$

$$\left[\frac{dm_{\text{sys}}}{dt} = 0 \right]$$

$$\left[\frac{dB_{\text{sys}}}{dt} \right]$$

$$\left[\frac{dB_{\text{cv}}}{dt} \right]$$

$$[RTT]$$

$$\frac{dm_{\text{sys}}}{dt} = \frac{d}{dt} \int_{\text{cv}} \rho dV + \int_{\text{cs}} \rho (\vec{v} \cdot \vec{n}) dA = 0$$

$$\left[0 = \frac{d}{dt} \int_{\text{cv}} \rho dV + \int_{\text{cs}} \rho (\vec{v} \cdot \vec{n}) dA \right]$$

$$\left[0 - \left(\frac{d}{dt} \int_{CV} \rho dV \right) + \int_{CS} \rho (\vec{V} \cdot \vec{n}) dA \right] = 0$$

① CV fixed :-

Leibnitz Integral rule :-

$$\frac{d}{dx} \left[\int_{a(x)}^{b(x)} f(x, t) dx \right] = \int_{a(x)}^{b(x)} \frac{\partial}{\partial x} f(x, t) dx$$

$$+ f(x, b(x)) \frac{d}{dx} [b(x)] - f(x, a(x)) \frac{d}{dx} [a(x)]$$

$$\left[0 = \int_{CV} \frac{\partial \rho}{\partial t} dV + \int_{CS} \rho (\vec{V} \cdot \vec{n}) dA \right] \Rightarrow \left[\text{Integral mass conservation law for fixed CV} \right]$$



$$0 = \left(\frac{d}{dt} \right)_{cv} \int_{cv} \rho dV + \int_{cs} \rho (\vec{V} \cdot \vec{n}) dA$$

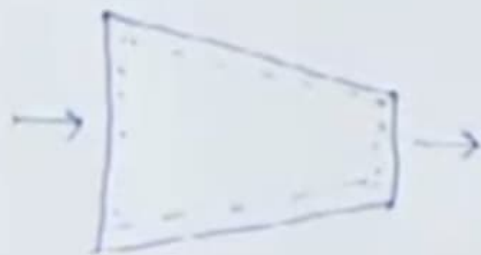
⑦ CV Fixed :-

Leibnitz Integral rule:-

$$\frac{d}{dt} \left[\int_{a(x)}^{b(x)} f(x, t) dx \right] = \int_{a(x)}^{b(x)} \frac{\partial}{\partial t} f(x, t) dx$$

$$+ f(x, b(x)) \frac{d}{dx} [b(x)]$$

$$- f(x, a(x)) \frac{d}{dx} [a(x)]$$



$$\left[0 = \int_{cv} \frac{\partial \rho}{\partial t} dV + \int_{cs} \rho (\vec{V} \cdot \vec{n}) dA \right]$$

→ Integrals mass
conservation law
for fixed CV.

$$+ f(x, b(x)) \frac{d}{dx} [b(x)] \\ - f(x, a(x)) \frac{d}{dx} [a(x)]$$

$$\left[0 = \int_{CV} \frac{\partial \rho}{\partial t} \cdot dV + \int_{CS} \rho (\vec{V} \cdot \vec{n}) dA \right]$$

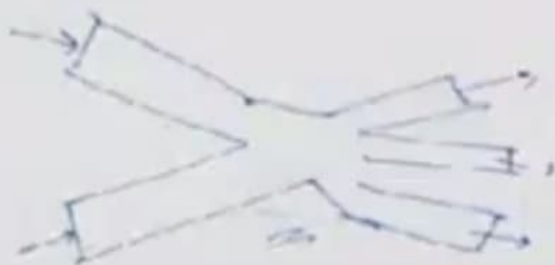
Integral mass
conservation law
for fixed CV

Special case -

(1) one-dimensional inlet & outlet in the CV

$$\int_{CV} \frac{\partial \rho}{\partial t} \cdot dV + \sum_i (\rho_i A_i V_i)_{out} - \sum_i (\rho_i A_i V_i)_{in} = 0$$

$$\int_{CV} \frac{\partial \rho}{\partial t} dV + \sum_i (\rho_i A_i V_i)_{out} - \sum_j (\rho_j A_j V_j)_{in} = 0$$



② 1D inlet & outlet & steady flow :-

$$\frac{\partial \rho}{\partial t} = 0$$

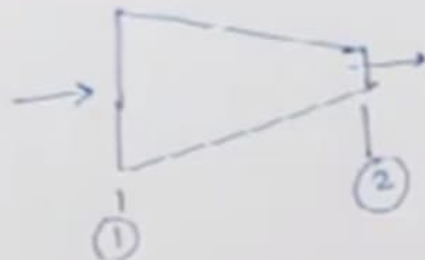
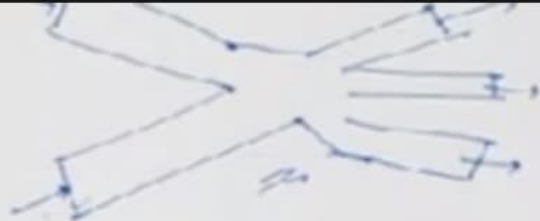
$$\sum_i (\rho_i A_i V_i)_{out} = \sum_j (\rho_j A_j V_j)_{in}$$

(2) 1D inlet & outlet & steady flow :-

$$\frac{\partial P}{\partial t} = 0.$$

$$\sum_i (\rho_i A_i V_i)_{out} = \sum_j (\rho_j A_j V_j)_{in}$$

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

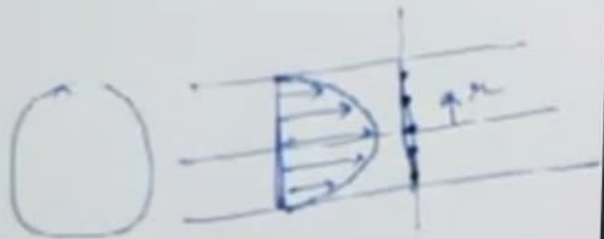
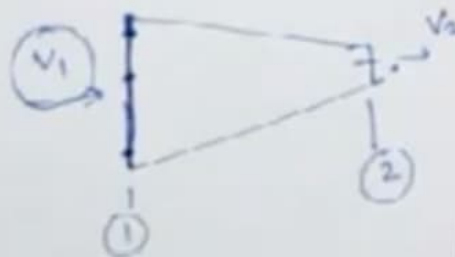


$$\sum_i (P_i A_i V_i)_{out} = \sum_j (P_j A_j V_j)_{in}$$

$$P_1 A_1 V_1 = P_2 A_2 V_2$$

$$P_1 = P_2$$

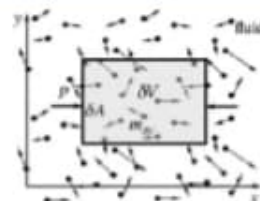
$$A_1 V_1 = A_2 V_2$$



Continuum Hypothesis

➤ Microscopic Uncertainty:

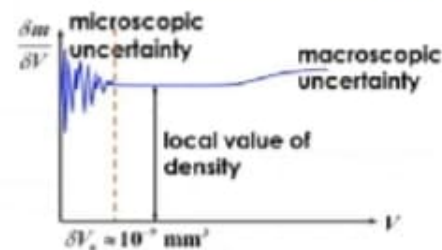
- Due to microscopic variation (random molecular motion).
- Should not generate significant fluctuations of the averaged quantities.
- **Size of a representative sampling volume must be large enough to erase the microscopic fluctuations ($\delta V \gg \delta V_0$)**



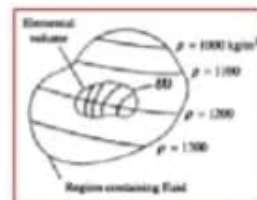
$$\rho = \lim_{\delta V \rightarrow \delta V_0} \frac{\delta m}{\delta V}$$

➤ Macroscopic Uncertainty:

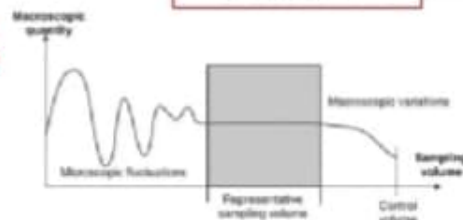
- Due to aggregate variation.
- Variation associated with spatial distribution of density, velocity or pressure.
- **Size of a representative sampling volume must also be small enough (not $\delta V \gg \delta V_0$) to point out the macroscopic variations (such as velocity or pressure gradients of interest in the control volume)**



(3×10^7 molecules at sea level, 15°C, 1atm)



- ✓ **Sampling a volume containing 10,000 molecules leads to 1% statistical fluctuations in the macroscopic quantities** (Karniadakis and Beskok, 2002).



Fluid Properties

- ❑ **Property:** Any characteristic of a system is called property.
 - ❑ State of a system is defined by its properties. Specifying a certain number of properties is sufficient to fix a state.

 - ❑ **Intensive properties:** independent of the mass of a system
 - ❑ Temperature (T)
 - ❑ Pressure (P)
 - ❑ Density (ρ)
 - ❑ **Extensive properties:** Properties whose values depend on the size/extent of the system.
 - ❑ Total mass (m)
 - ❑ Total volume (V)
 - ❑ Total momentum
 - ❑ Total energy (E)
- | |
|--------|
| m |
| V |
| T |
| P |
| ρ |

➔

$\frac{1}{2}m$	$\frac{1}{2}m$
$\frac{1}{2}V$	$\frac{1}{2}V$
T	T
P	P
ρ	ρ

} Extensive properties

} Intensive properties
- ❑ Extensive properties per unit mass are called **specific properties**.
 - ❑ Specific volume ($v=V/m$)
 - ❑ Specific total energy ($e=E/m$)

Fluid Properties

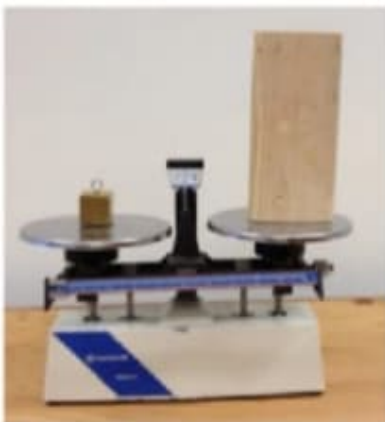
Density

- ❑ **Density:** Mass per unit volume $\rho = \frac{m}{V}$ kg/m^3
- ❑ **Specific volume:** Volume per unit mass (reciprocal of density)

$$v = \frac{V}{m} = \frac{1}{\rho} \quad \text{m}^3/\text{kg}$$

- ❑ Density is an important characteristic of substances. It is crucial, for example, in determining whether an object sinks or floats in a fluid.

*A block of
brass and a
block of
wood*



Density

- ❑ The *densities of gases are much less than those of liquids and solids*, because the atoms in gases are separated by large amounts of empty space.

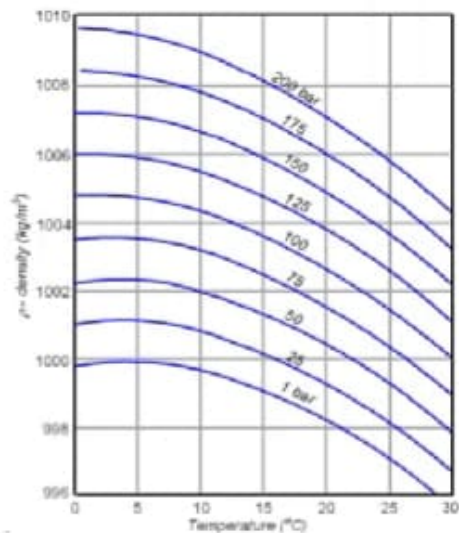
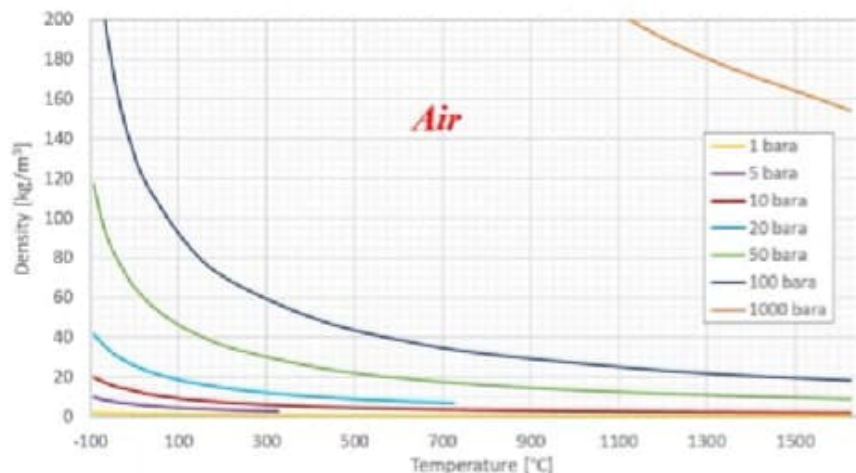
Solids		Liquids		Gases	
(0.0 °C)		(0.0 °C)		(0.0 °C, 101.3 kPa)	
<i>Densities of some common substances</i>					
Substance	$\rho(\text{kg/m}^3)$	Substance	$\rho(\text{kg/m}^3)$	Substance	$\rho(\text{kg/m}^3)$
Aluminum	2.70×10^3	Benzene	8.79×10^2	Air	1.29×10^0
Bone	1.90×10^3	Blood	1.05×10^3	Carbon dioxide	1.98×10^0
Brass	8.44×10^3	Ethyl alcohol	8.06×10^2	Carbon monoxide	1.25×10^0
Concrete	2.40×10^3	Gasoline	6.80×10^2	Helium	1.80×10^{-1}
Copper	8.92×10^3	Glycerin	1.26×10^3	Hydrogen	9.00×10^{-2}
Cork	2.40×10^2	Mercury	1.36×10^4	Methane	7.20×10^{-2}
Earth's crust	3.30×10^3	Olive oil	9.20×10^2	Nitrogen	1.25×10^0
Glass	2.60×10^3			Nitrous oxide	1.98×10^0
Gold	1.93×10^4			Oxygen	1.43×10^0

Fluid Properties

Density

- ❑ In general, $\rho = f(T, P)$ $\rho \propto P; \rho \propto \frac{1}{T}$
- ❑ For Gases: *Strong dependence on pressure and temperature*

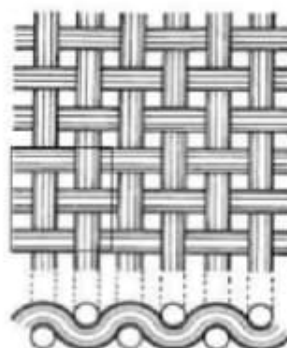
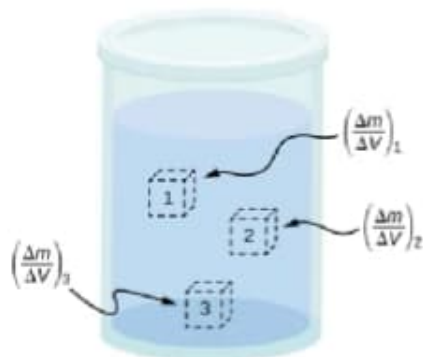
- ❑ For Liquids and Solids: *Strong dependence on temperature; dependence on pressure is usually negligible (essentially incompressible substance)*



Fluid Properties

Density

- ❑ Density of a substance is not necessarily constant throughout the volume of a substance.
- ❑ **Homogeneous substance:** density is constant throughout a substance (equal to its average density).
- ❑ **Heterogeneous substance:** density is not constant throughout a substance.
- ❑ The density at a specific location within a heterogeneous material is called **local density**, and is given as a function of location, $\rho = \rho(x, y, z)$



Fluid Properties

Density of Ideal Gases

- **Equation of state:** Any equation relating pressure, temperature and specific volume (density) of a substance.

- **Ideal-gas equation of state:** Simplest equation of state for gases

$$Pv = RT$$

$$P = \rho RT$$

$$N = \frac{m}{M}$$

$$R = \frac{R_u}{M}$$

$$PV = mRT = NR_uT$$

P: absolute pressure

v: specific volume

ρ : density

T: absolute temperature

R: gas constant

R_u : universal gas constant

M: molar mass (molecular weight) of gas

N: number of moles

- Real gases at **low pressures and high temperatures** (at lower density) approximate ideal gas behaviour.

- Practically, many familiar gases such as air, N₂, O₂, H₂, He, Ar etc. can be treated as ideal gas with negligible error (< 1 %).

Fluid Properties

Specific gravity

- Sometimes density of a substance is given relative to the density of a well-known substance.
- **Specific gravity or relative density:** the ratio of the **density of a substance to the density of some standard substance at a specified temperature** (usually water at 4°C and 1 atm pressure).

$$SG = \frac{\rho}{\rho_{\text{water}}}$$

$$\rho_{\text{water}} = 1000 \text{ kg/m}^3$$

- **Specific gravity:** often used to compare densities of two substances (**dimensionless quantity**).

- $SG < 1.0$: lighter than water
- $SG > 1.0$: heavier than water

Substance	SG
Water	1.0
Blood	1.05
Seawater	1.025
Gasoline	0.7
Ethyl alcohol	0.79
Mercury	13.6
Wood	0.3–0.9
Gold	19.2
Bones	1.7–2.0
Ice	0.92
Air (at 1 atm)	0.0013

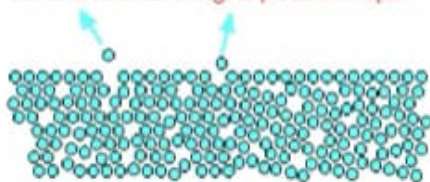
Fluid Properties

Vapour Pressure

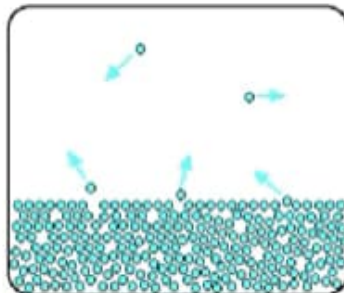
- ❑ **Vapour pressure (P_v):** Pressure exerted by its vapour in phase **equilibrium** with its liquid at a given temperature.
- ❑ P_v is a property of the pure substance.
- ❑ $P_v = P_{\text{sat}}$ (Saturation pressure of the liquid)
- ❑ $P_v = f(T)$

In equilibrium, when these particles hit the walls of the container, they exert a pressure. This pressure is called the vapour pressure of the liquid.

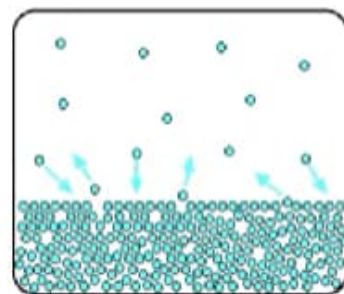
Some of the more energetic particles escape.



Some of the more energetic particles on the surface of the liquid can be moving fast enough to escape from the attractive forces holding the liquid together. They evaporate



Particles continue to break away from the surface of the liquid - but this time they are trapped in the space above the liquid



As the gaseous particles bounce around, some of them will hit the surface of the liquid again, and be trapped there. There will rapidly be an equilibrium set up.

*No. of particles leaving the surface =
No. rejoining it.*

Fluid Properties

Vapour Pressure

❑ **Liquid evaporates:** Liquid pressure $> P_v$ (only exchange between liquid and vapour is at the interface)

❑ **Liquid boils:** Liquid pressure $< P_v$ (vapour bubbles begin to appear in the liquid)

❑ *At atmospheric conditions (30 °C) for water:*

❑ Liquid pressure = $P_{atm} = 101325 \text{ Pa}$; $P_v = 4248 \text{ Pa}$

❑ Liquid pressure $> P_v$

❑ **Water can only evaporate!**

❑ *When water is heated to 100 °C under atmospheric conditions:*

❑ Liquid pressure = $P_{atm} = 101325 \text{ Pa}$; $P_v = 101421 \text{ Pa}$

❑ Liquid pressure $< P_v$

❑ **Water starts boiling and vapour bubbles can be seen!**

❑ *At Mount Everest Summit (72 °C):*

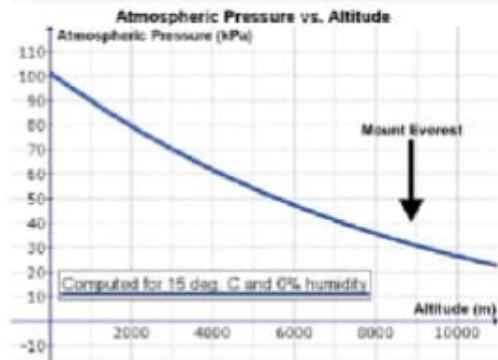
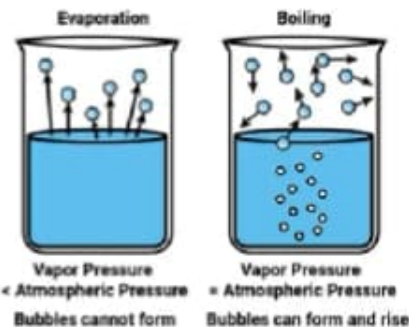
❑ Liquid pressure = $P_{atm} = 33700 \text{ Pa}$; $P_v = 34002 \text{ Pa}$

❑ Liquid pressure $< P_v$

❑ **Water starts boiling and vapour bubbles can be seen!**

10/5/2021

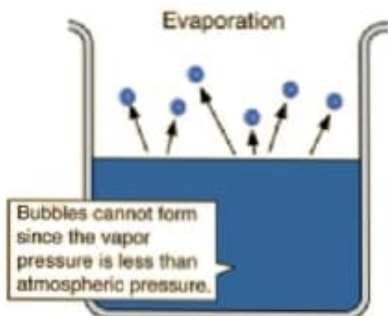
Dr. Udayraj, Department of Mechanical Engineering



Fluid Properties

Evaporation

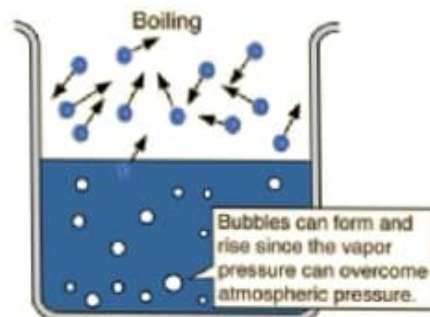
- Occurs at any temperature.
- Takes place on the surface of the liquid.
- No bubbles are formed.
- Happens slowly.
- Thermal energy supplied by the surrounding.
- Causes cooling.
- Condition: **Liquid pressure $> P_v$**
- Decreasing the P_{atm} increases the rate of evaporation.



❑ Evaporation and boiling require latent heat of vaporization.

Boiling

- Occurs at a definite temperature (T_b).
- Takes place throughout the liquid.
- Bubbles of vapours formed.
- Happens quickly.
- Thermal energy supplied by an energy source.
- Temp. remains constant during boiling.
- Condition: **Liquid pressure $< P_v$**
- Decreasing the P_{atm} lowers the boiling point.



Fluid Properties

Cavitation

- ❑ Local vapourization (or bubble formation) takes place in liquid flow systems if pressure drops below the vapour pressure.
- ❑ *The vapour bubbles (cavitation bubbles) collapse as they are swept away from the low-pressure regions.*
- ❑ It results in the generation of highly destructive, extremely high-pressure waves.
- ❑ This phenomenon is called **cavitation**.
- ❑ Cavitation results in **drop in performance** and even **corrosion of the impeller blades**.
- ❑ Cavitation is **noisy** and can cause **structural vibrations**.
- ❑ So, it is one of an **important criterion or consideration in the design of hydraulic turbines and pumps**.

