Control System

EE 202

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EE202 Control Systems –I (6 Credits)

Mathematical Modelling and Transfer Function, Signal Flow Graph, Feedback System, Time response analysis, Performance Indices, Frequency Response (Polar Plots), Stability Analysis (Routh-Hurwitz, Bode, Nyquist, Root Locus), Compensator Design (Lead, Lag, Lead-lag), PIO Controller. Introduction to MATLAB Control System Toolbox.

Introduction to State space and state variables, Eigen Vector, Canonical Forms, Observability and Controllability. MIMO systems.

Suggested Books

Control Systems Engineering by I. J. Nagrath and M. Gopal, New Age Int Pvt Ltd
Automatic Control System by B. C. Kuo, Willey Publishing House
Modern Control Engineering by K. Ogata, Pearson India

Evaluation Pattern

Mid term (50)			End Term (50)
Quizzes	Mid Term	Assignments/ Class interaction	End Term Exam
10	30	20	40

Topics

- What is Control System?
- Open-loop, Closed-loop and Feed back System
- Transfer Function, Order, Type and Poles and Zeroes
- Modeling of Physical System

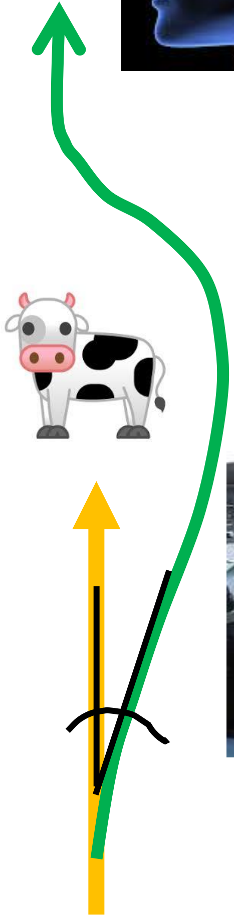
Control System: Introduction

To control what?

A control system is to control….

Some processes?

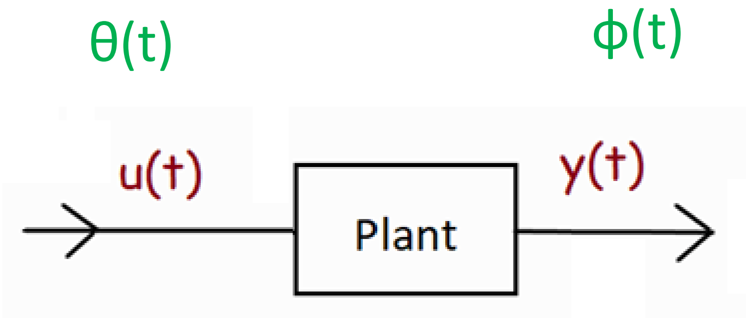
Or a system?

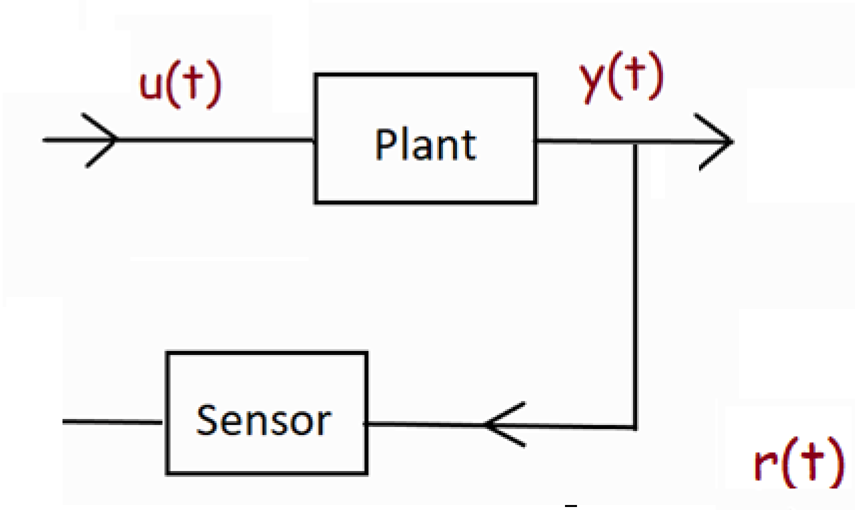


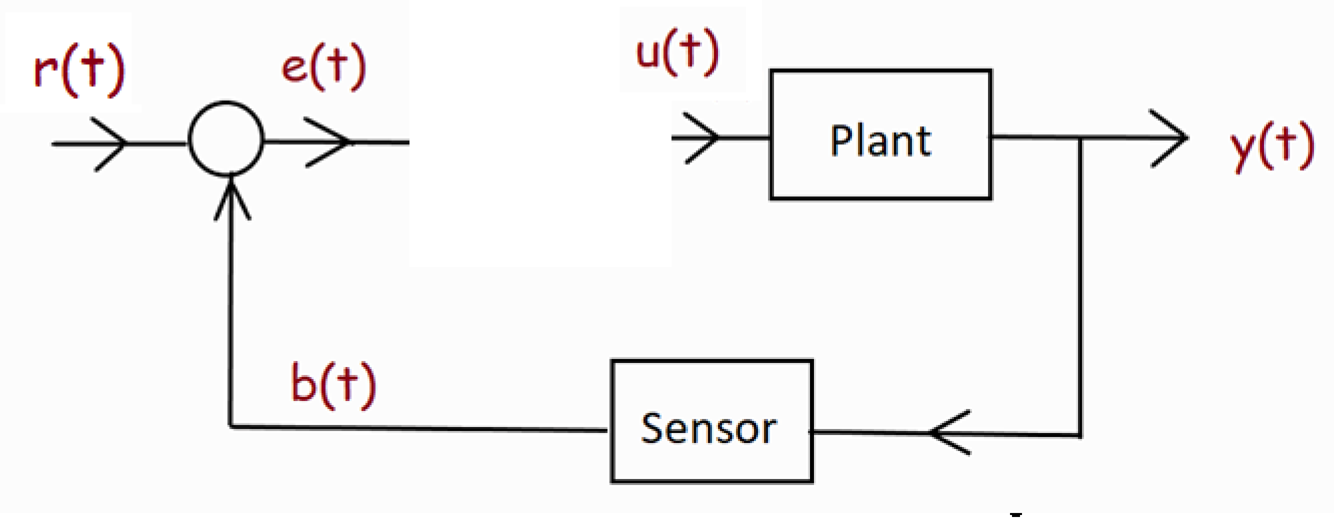
Some events?

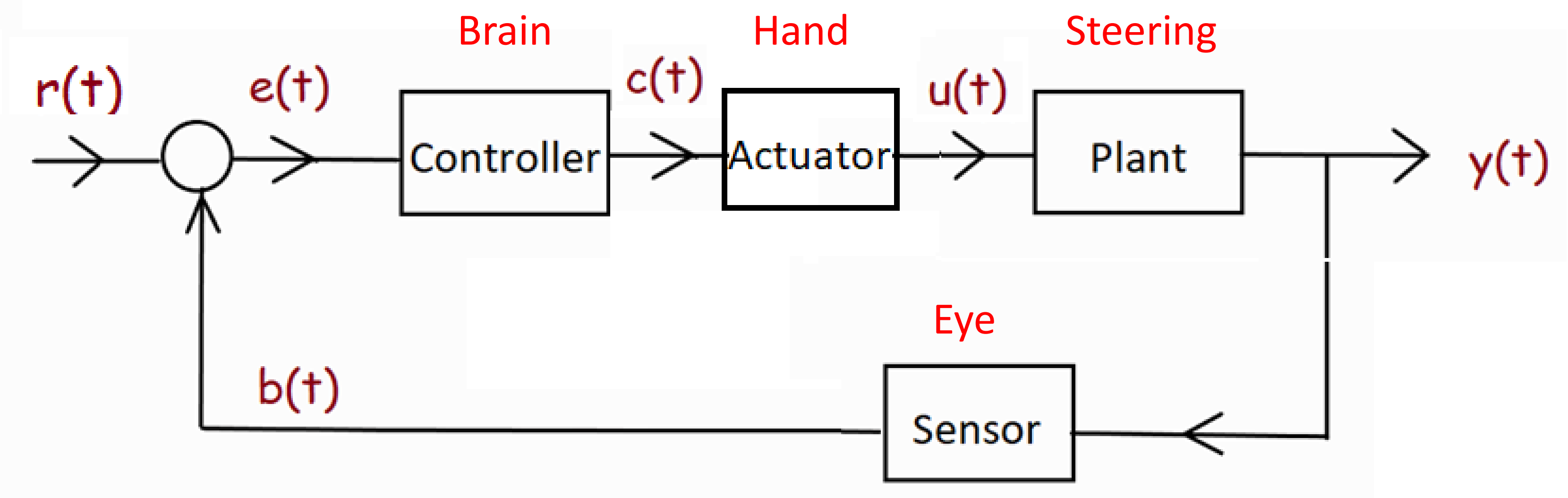
All of These

Control System: Basic Blocks



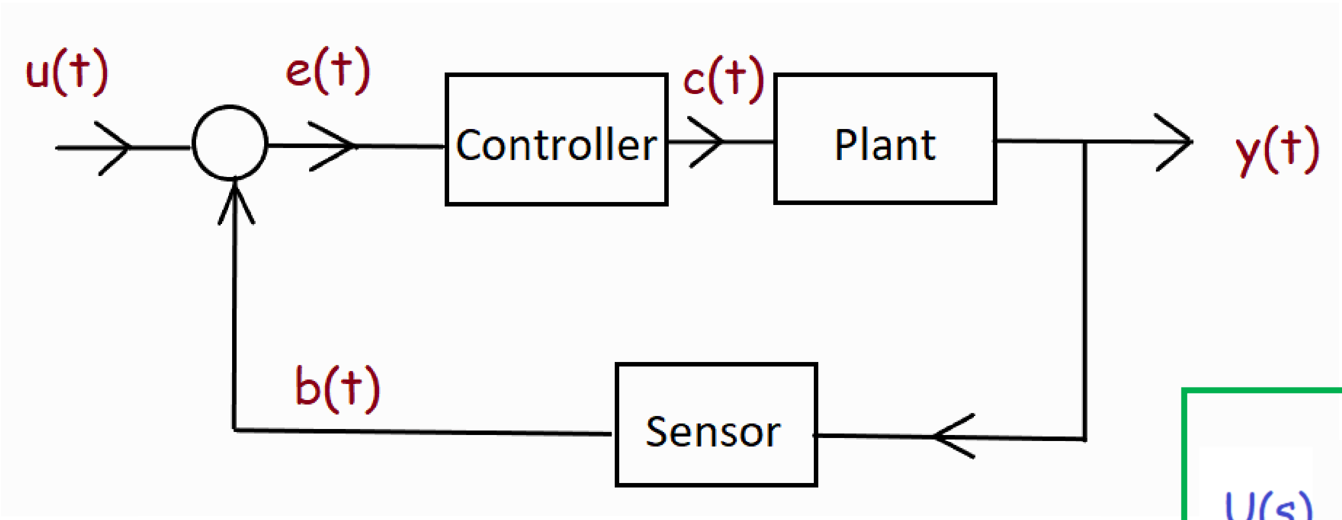






Closed Loop System

Control System: Transfer Function



$$Y(s) = G_P(s)G_C(s)E(s)$$

$$Y(s) = G_P(s)G_C(s)(U(s)-B(s))$$

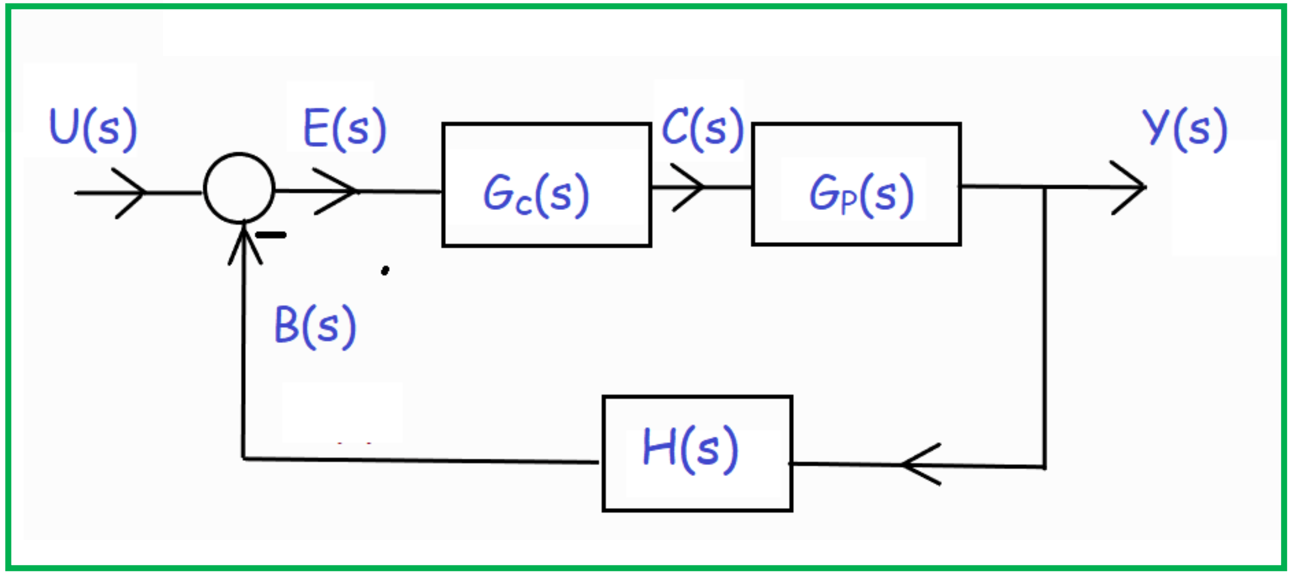
$$Y(s) = G_P(s)G_C(s)(U(s)-H(s)Y(s))$$

$$Y(s) + G_P(s)G_C(s)H(s)Y(s) = G_P(s)G_C(s)U(s)$$

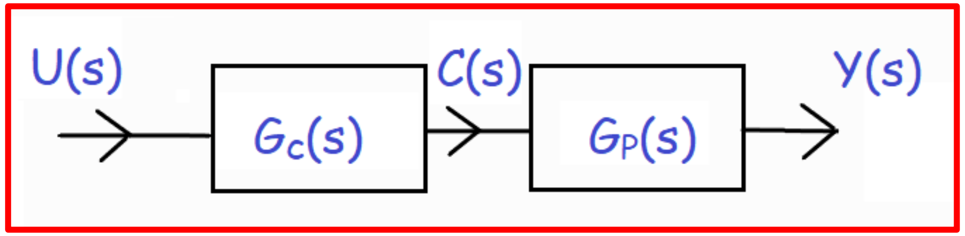
$$\frac{Y(s)}{U(s)} = \frac{G_P(s)G_C(s)}{1+G_P(s)G_C(s)H(s)}$$

$$= \frac{G(s)}{1+G(s)H(s)} = T(s) \quad \Big| \quad G(s) = G_P(s)G_C(s)$$

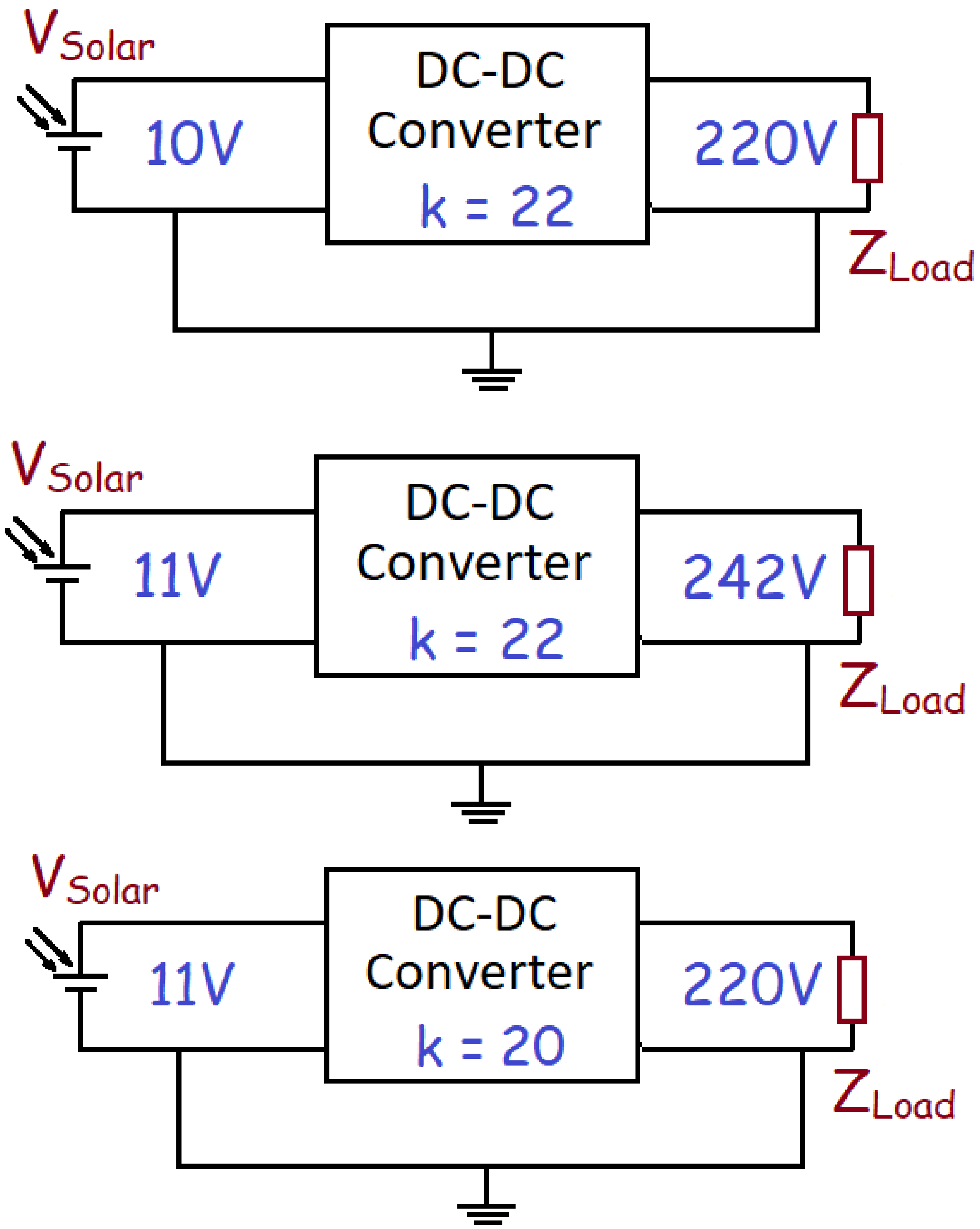
Closed Loop System

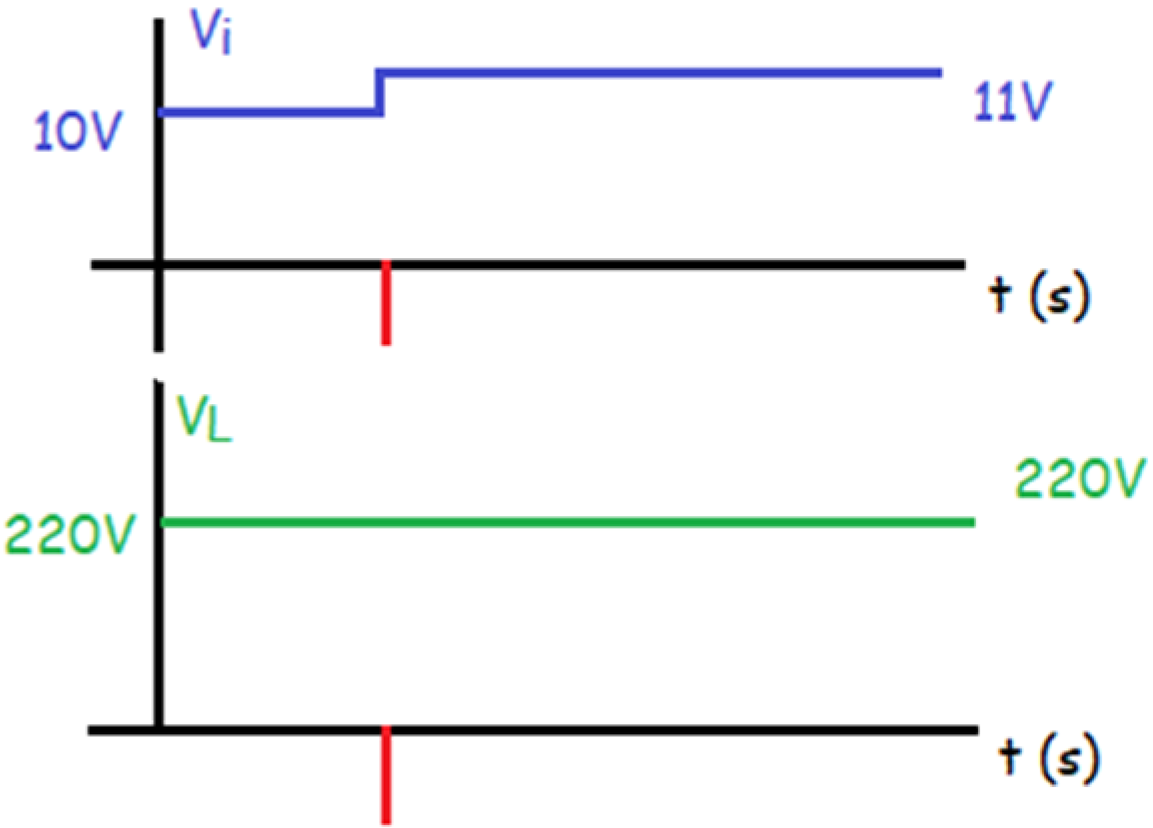


Open Loop System



Why Control System?

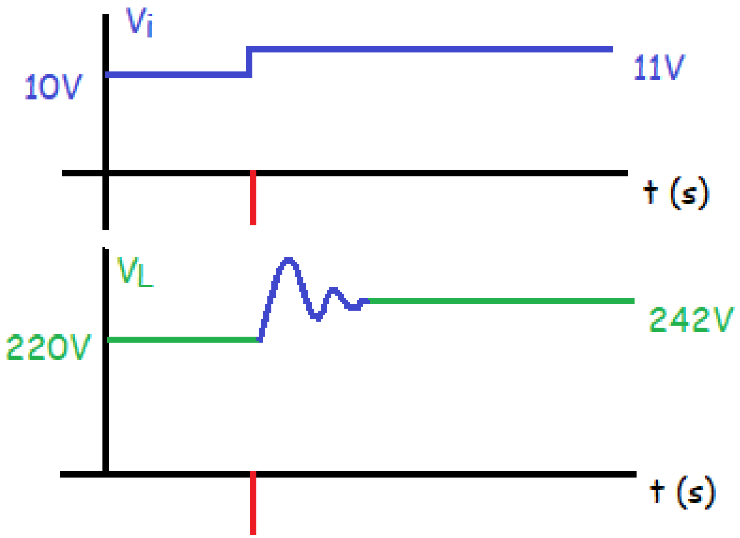


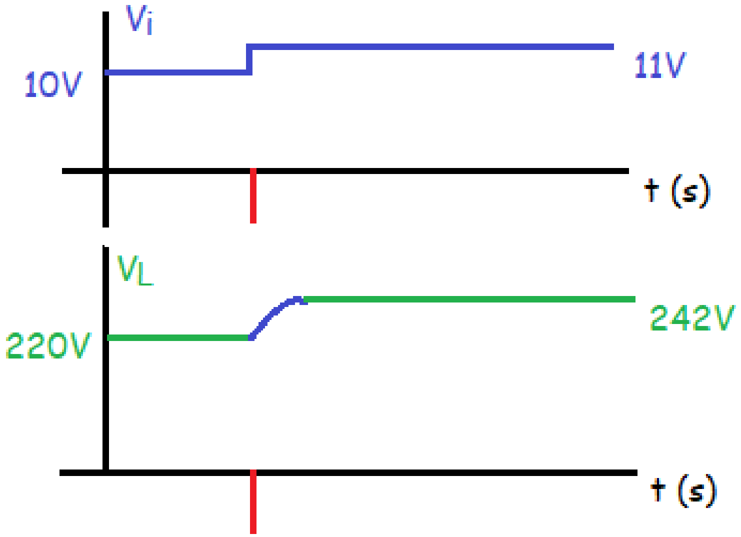


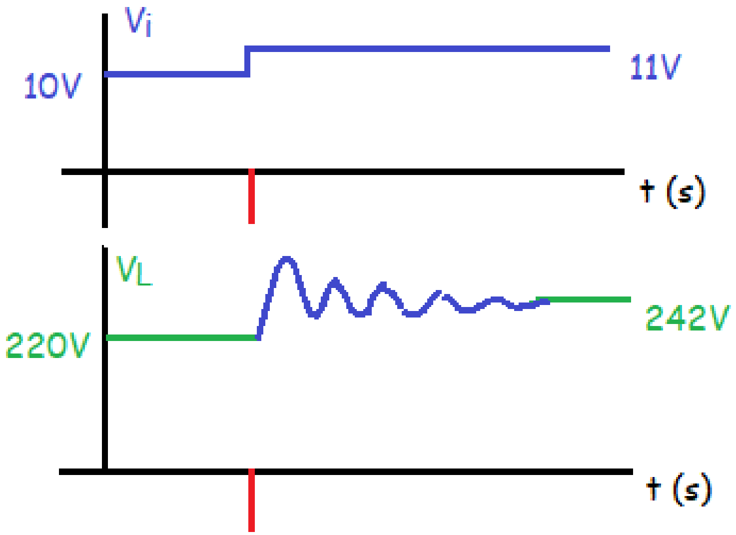
The requirement: Output shall not be affected by any change in input

The Reality

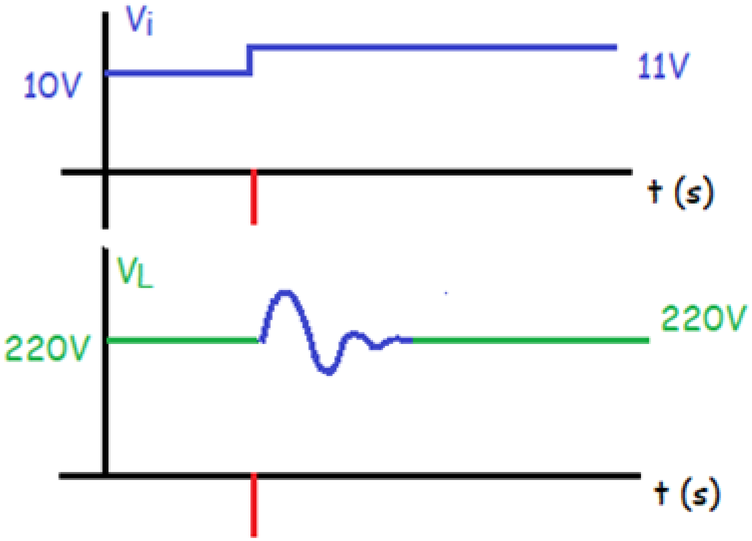
Uncontrolled System







Controlled System



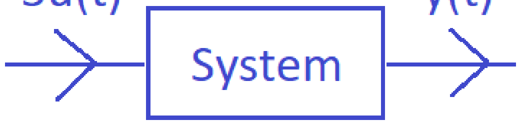
Transient State

Steady State

Transfer Function

Transfer function is the ratio of the output of a system to the input of the system, both expressed in Laplace domain, given all initial conditions are zero

$x(t) = 5u(t)$ $\qquad$ $y(t) = 10(1-e^{-2t})u(t)$



$X(s) = 5/s$

$$Y(s) = 10(1/s-1/(s+2)) = 20/s(s+2)$$

$$G(s) = \frac{Y(s)}{X(s)} = \frac{20}{s(s+2)}\,\frac{s}{5} = \frac{4}{(s+2)}$$

Derive transfer function of following system ($y(t)$: output, $u(t)$ input)

$$\dot{u}(t) + 10u(t) = \ddot{y}(t) + 101\dot{y}(t) + 100\,y(t)$$

$$y(0) = 1, \dot{y}(0) = 0, u(0) = 1$$

$$sU(s) + 10U(s) = s^2y(s) + 101sY(s) + 100Y(s)$$

$$sU(s) - u(0) + 10u(s) = (s^2Y(s) - sy(0) - \dot{y}(0)) + 101(sY(s)-y(0)) + 100Y(s)$$

$$sU(s) + 10U(s) = s^2Y(s) + 101sY(s) + 100Y(s)$$

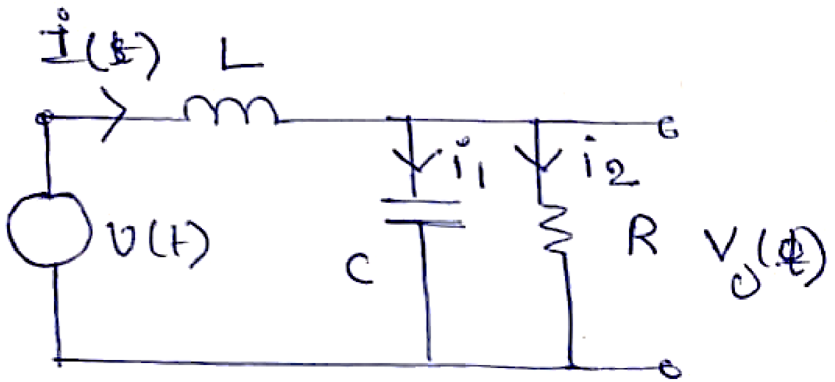
$$\frac{Y(s)}{U(s)} = \frac{s+10}{s^2+101s+100} = \frac{s+10}{(s+1)(s+100)}$$

zero = -10
pole = -1, -100

Order = 2, Type = 0

RLC circuit

Find T.F



$$V_o(s) = R I_2(s)$$

$$= R\, I(s) \times \frac{\frac{1}{Cs}}{R + \frac{1}{Cs}}$$

$$= R \frac{V(s)}{SL + R \parallel \frac{1}{Cs}} \times \frac{1}{RCs+1}$$

$$= \frac{R}{RCs+1} \cdot \frac{V(s)}{SL + \frac{R\frac{1}{Cs}}{R+\frac{1}{Cs}}}$$

$$= \frac{R}{RCS+1} \cdot \frac{V(S)}{SL + \frac{R}{RCS+1}} = \frac{R\,V(S)}{S^2RCL + SL + R}$$

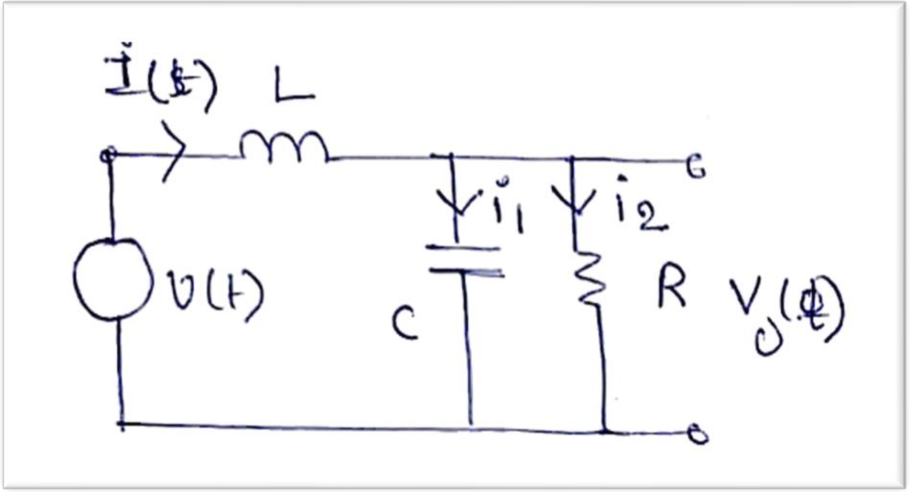
$$\Rightarrow \frac{V_0(S)}{V(S)} = \frac{R}{S^2RLC + SL + R} = \frac{\frac{1}{LC}}{S^2 + S\cdot\frac{1}{RC} + \frac{1}{LC}}$$

$$\therefore G(S) = \frac{\frac{1}{LC}}{S^2 + S\left(\frac{1}{RC}\right) + \frac{1}{LC}} = \frac{K\omega_n^2}{S^2 + 2\xi\omega_n S + \omega_n^2}$$

$$\Rightarrow \omega_n = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10\times10^{-3}\times10^{-6}}} = \frac{1}{\sqrt{10^{-8}}} = 10^4 \text{ rad/s}$$

$$2\xi\omega_n = \frac{1}{RC} = \frac{10^6}{100} = 10^4$$

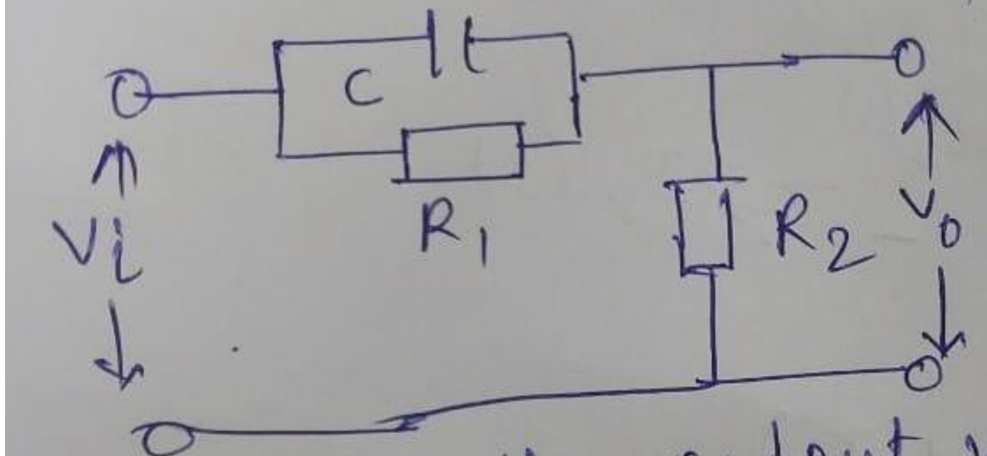
$$\therefore \xi = \frac{10^4}{2\times10^4} = \frac{1}{2}, \quad K = 1.$$



If L = 1mH, C = 1 uF, R = 100 Ohm,

Find out damping constant and natural frequency

Que Derive the transfer function of the circuit (9)



$v = 8 \sin 10t$

$R_1 = 50k\Omega$, $R_2 = 5k\Omega$

$C = 1\mu F$

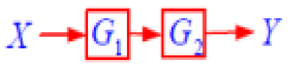
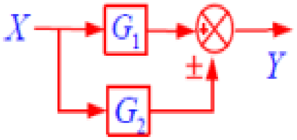
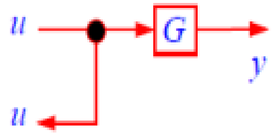
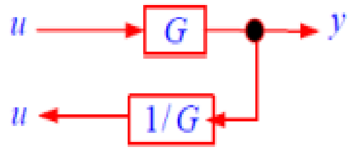
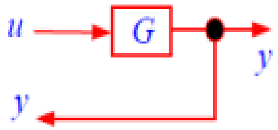
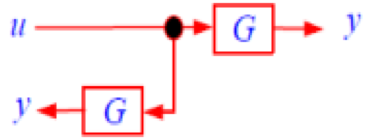
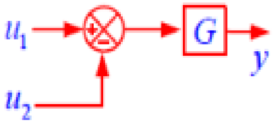
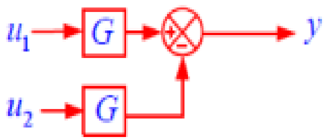
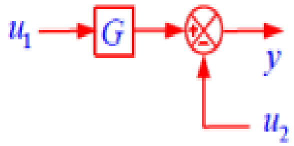
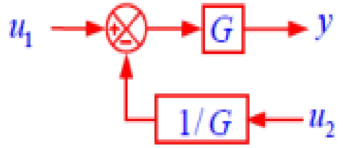
calculate the output voltage in magnitude and in phase relative to input voltage. $s = j\omega$.

Block Diagram Reduction Rules

Any complex system can be represented by a block diagram, and these block diagrams can be reduced to very simple block diagrams by using block diagrams reduction technique.

Components of a block diagram:

- Blocks to represent components
- Arrows to indicate direction of signal flow
- Summing points to show merging signals
- Take off points to indicate branching of signals

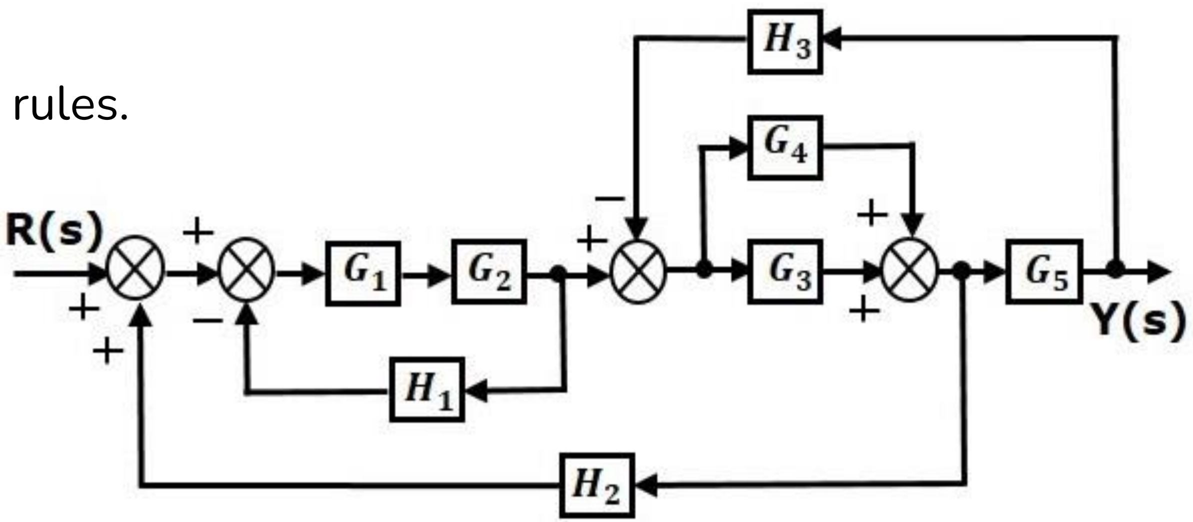
	Manipulation	Original Block Diagram	Equivalent Block Diagram	Equation
1	Combining Blocks in Cascade			$Y=(G_1G_2)X$
2	Combining Blocks in Parallel; or Eliminating a Forward Loop			$Y=(G_1\pm G_2)X$
3	Moving a pickoff point behind a block			$y = Gu$ $u = \frac{1}{G}y$
4	Moving a pickoff point ahead of a block			$y=Gu$
5	Moving a summing point behind a block			$e_2=G(u_1-u_2)$
6	Moving a summing point ahead of a block			$y=Gu_1-u_2$

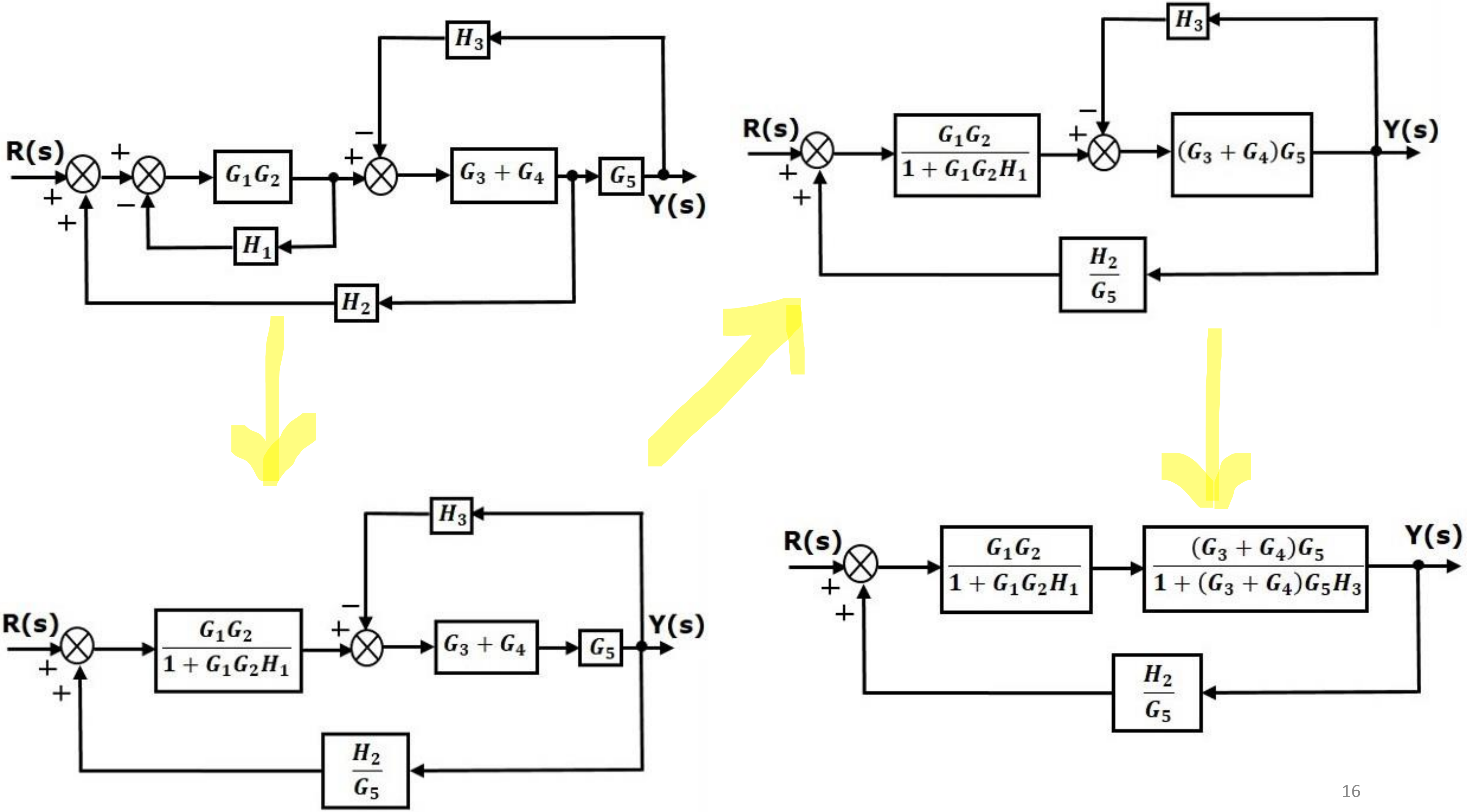
Other Rules to Solve the Block Diagram

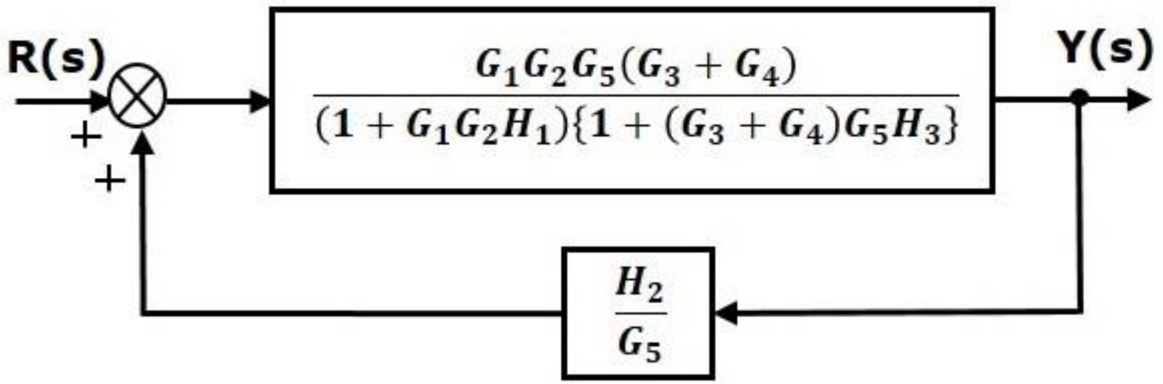
- **Rule 1** – Check for the blocks connected in series and simplify.
- **Rule 2** – Check for the blocks connected in parallel and simplify.
- **Rule 3** – Check for the blocks connected in feedback loop and simplify.
- **Rule 4** – If there is difficulty with take-off point while simplifying, shift it towards right.
- **Rule 5** – If there is difficulty with summing point while simplifying, shift it towards left.
- **Rule 6** – Repeat the above steps till you get the simplified form, i.e., single block.

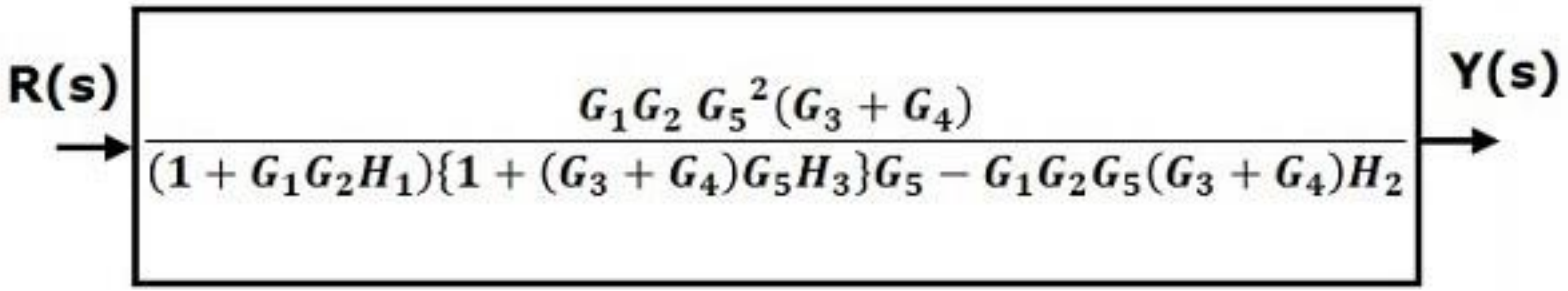
Problem No.1

Simplify the block diagram using the block diagram reduction rules.

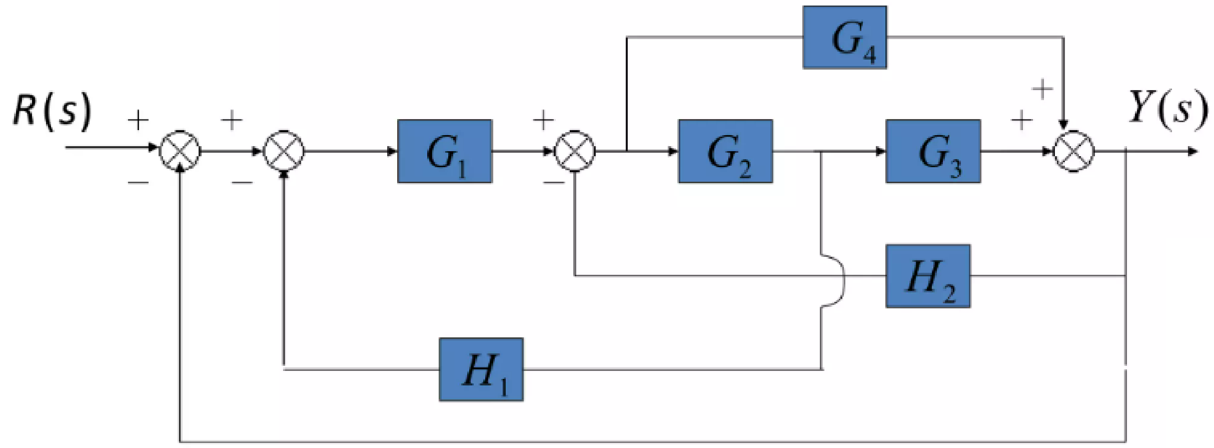



R(s)
Y(s)
G_1G_2
$G_3 + G_4$
G_5
H_1
H_2
H_3
$\frac{G_1G_2}{1 + G_1G_2H_1}$
$\frac{H_2}{G_5}$
$(G_3 + G_4)G_5$
$\frac{(G_3 + G_4)G_5}{1 + (G_3 + G_4)G_5H_3}$

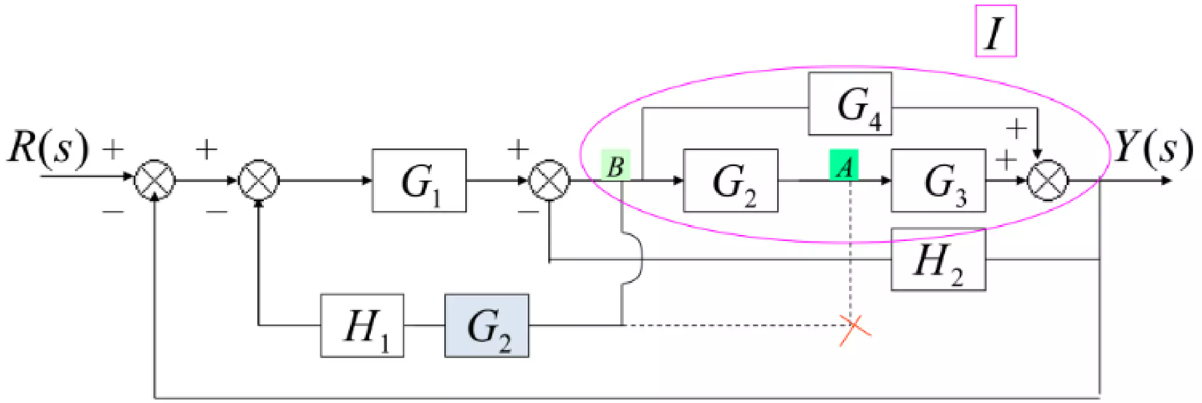

R(s)
+
+
$\frac{G_1G_2G_5(G_3+G_4)}{(1+G_1G_2H_1)\{1+(G_3+G_4)G_5H_3\}}$
Y(s)
$\frac{H_2}{G_5}$


R(s)
$\frac{G_1G_2\,{G_5}^2(G_3+G_4)}{(1+G_1G_2H_1)\{1+(G_3+G_4)G_5H_3\}G_5-G_1G_2G_5(G_3+G_4)H_2}$
Y(s)

Find the transfer function of the following block diagrams



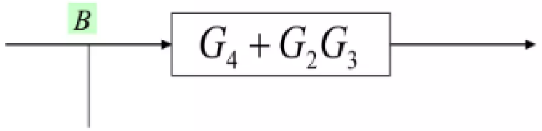
Problem No.2

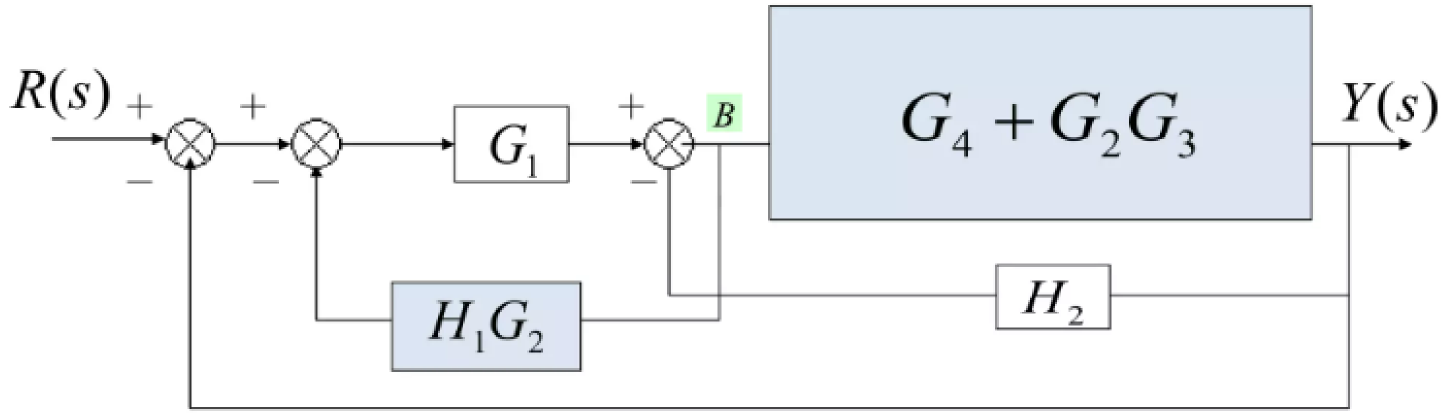


Solution:

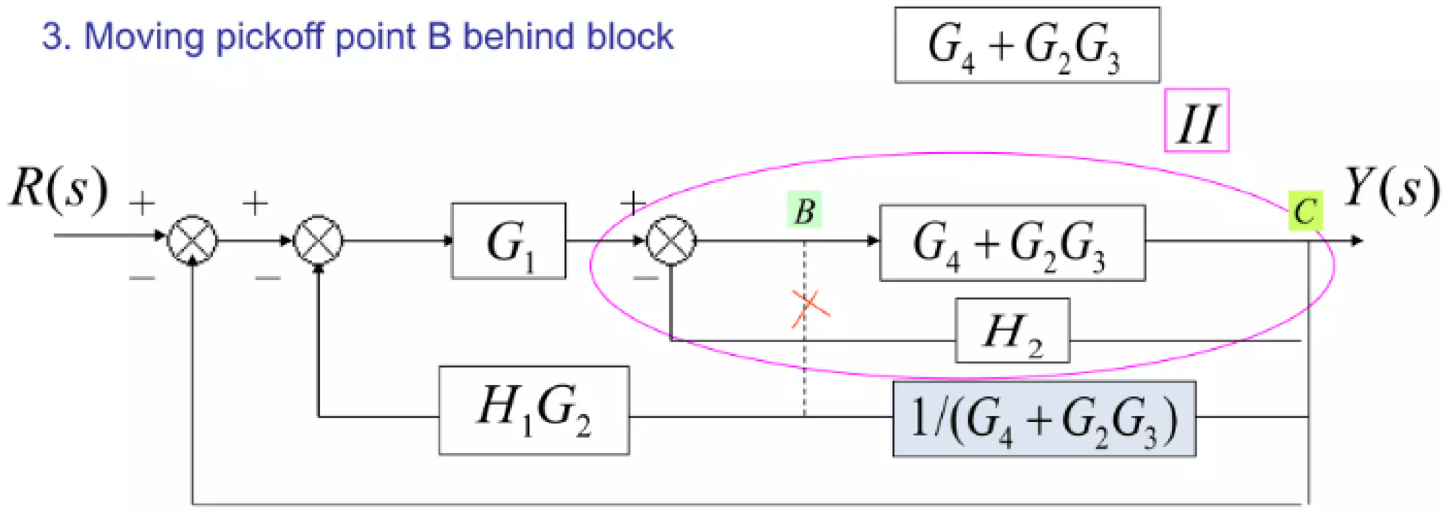
1. Moving pickoff point A ahead of block $\boxed{G_2}$

2. Eliminate loop I & simplify

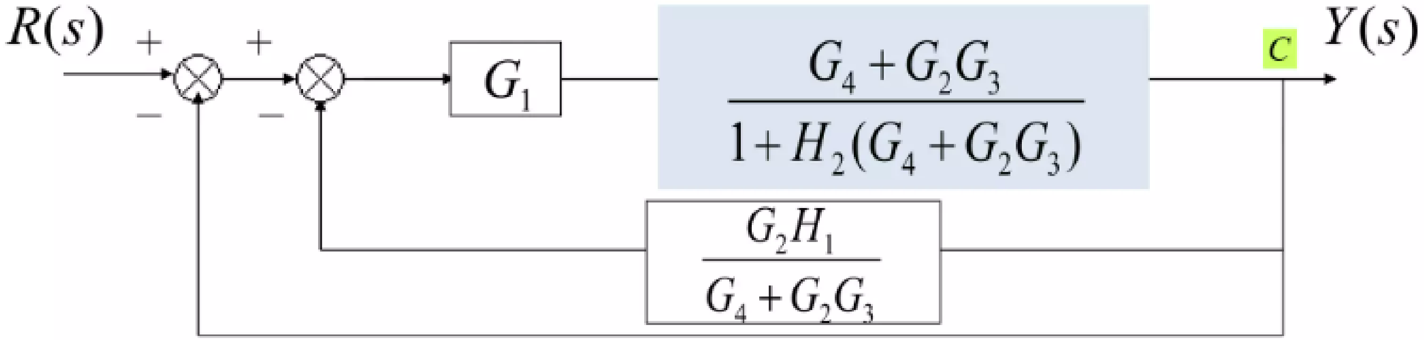




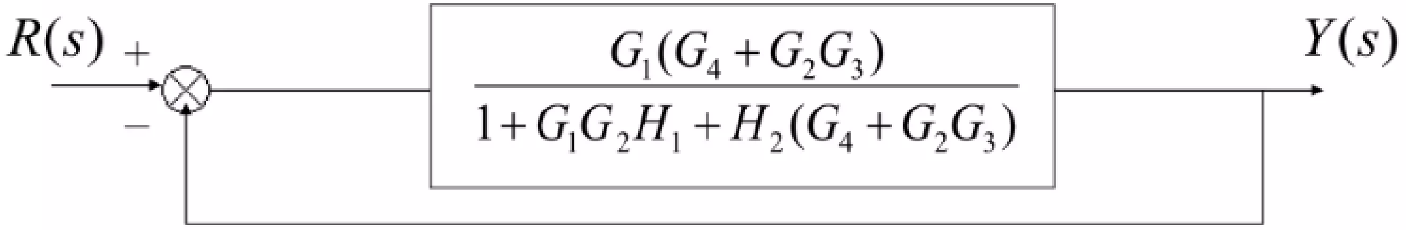
3. Moving pickoff point B behind block



4. Eliminate loop III

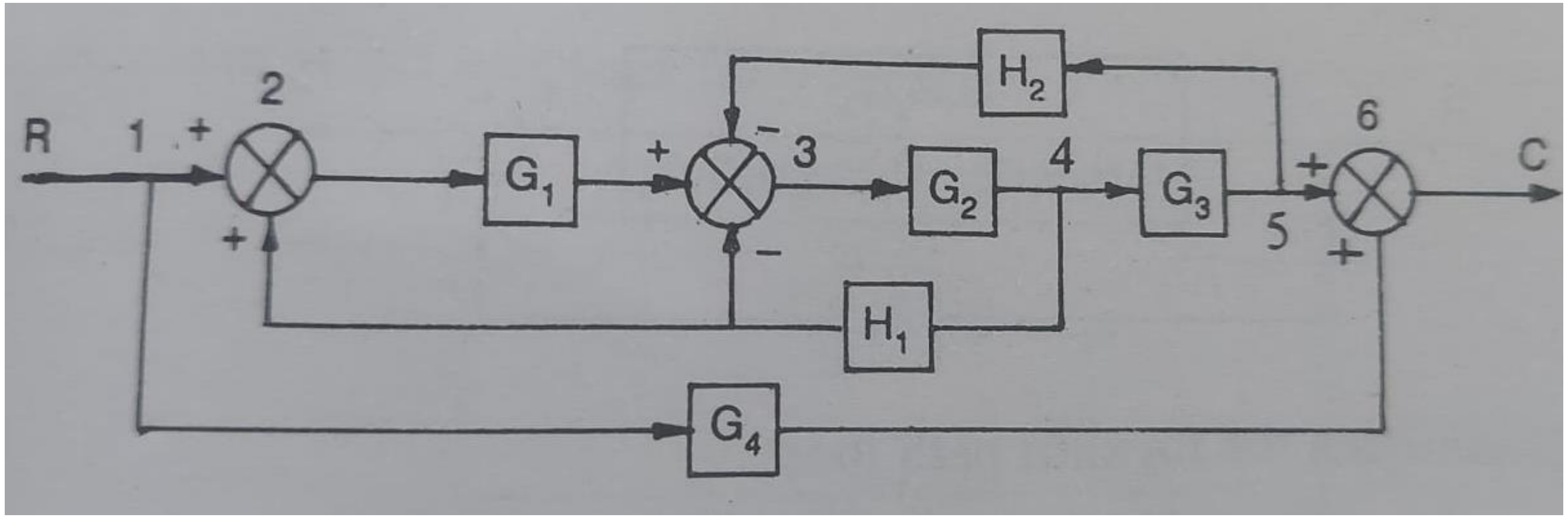


Using rule 6

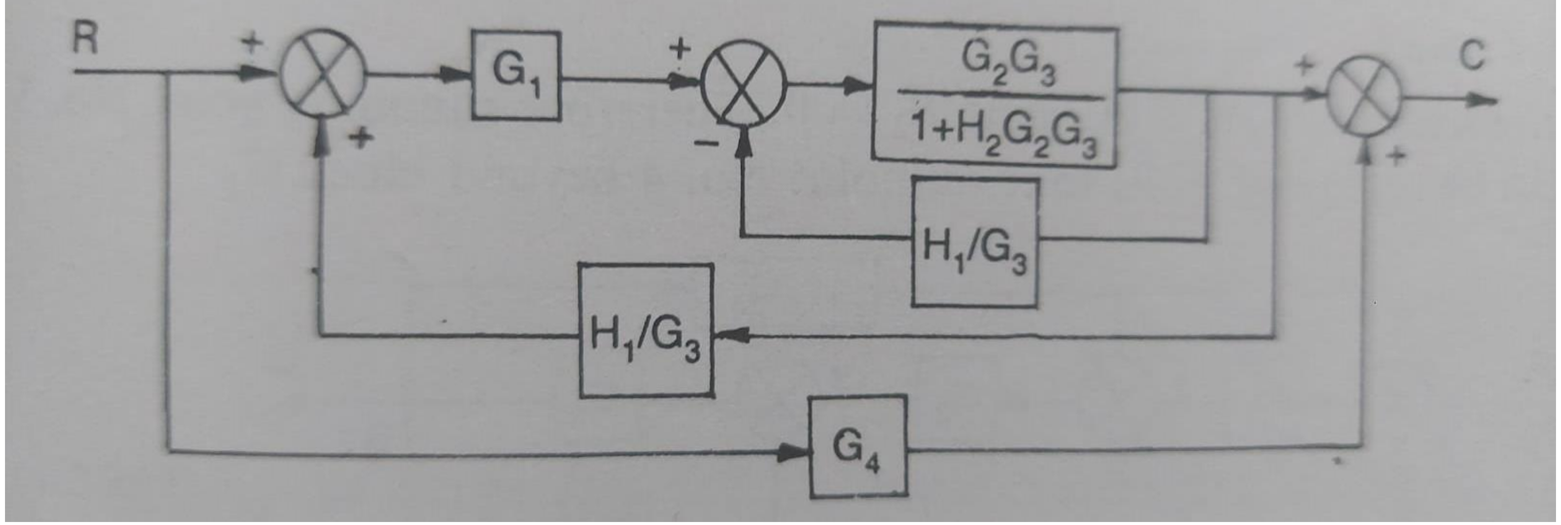


$$T(s) = \frac{Y(s)}{R(s)} = \frac{G_1(G_4 + G_2G_3)}{1 + G_1G_2H_1 + H_2(G_4 + G_2G_3) + G_1(G_4 + G_2G_3)}$$

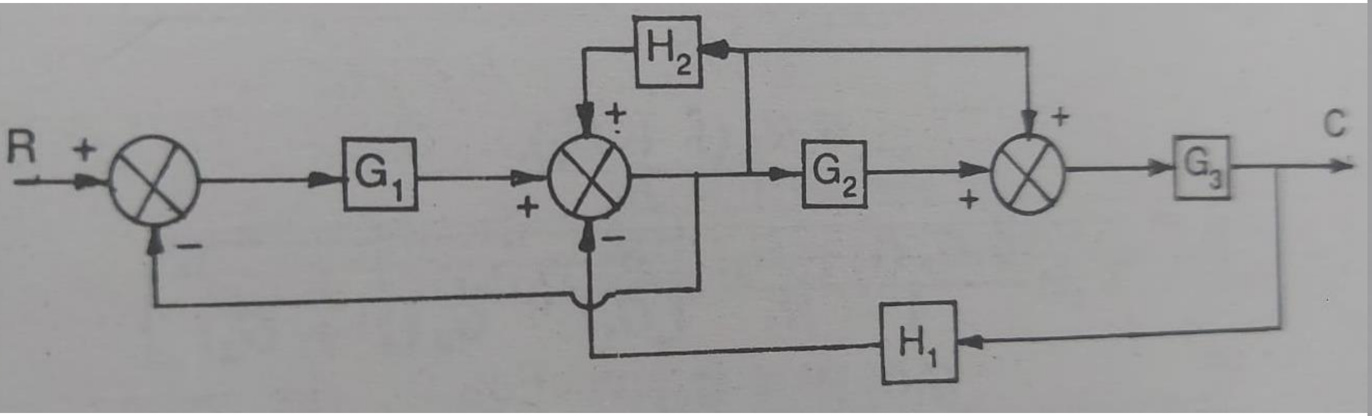
Problem No.3



Eliminating feedback having transfer function H_2



Problem No.4



Solution

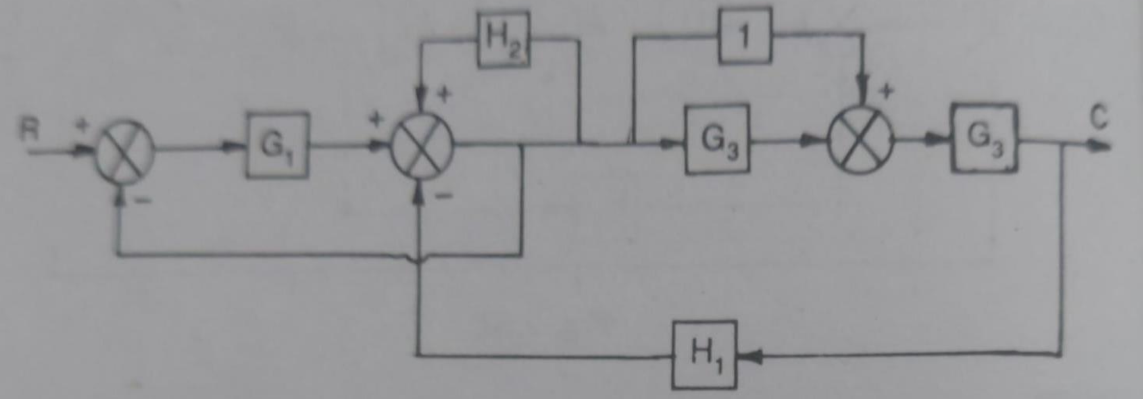


Fig. 2.66

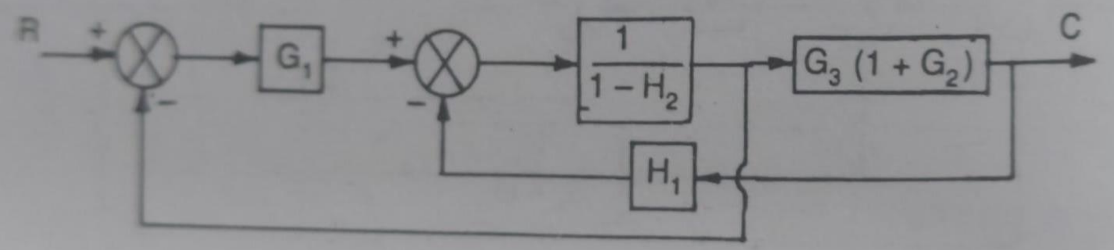
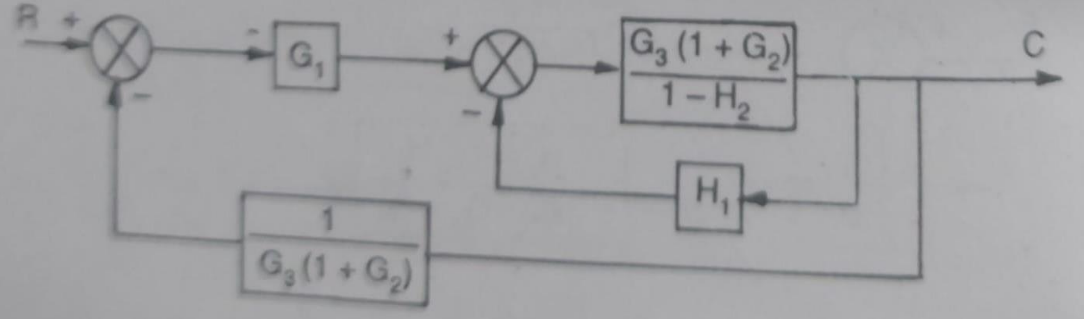
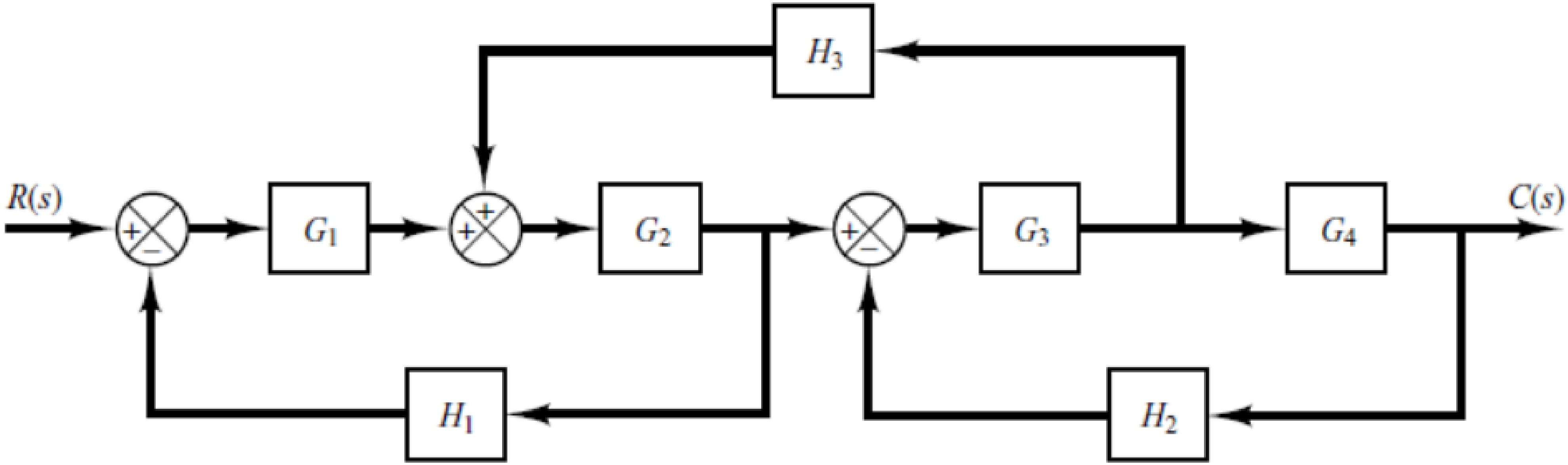


Fig. 2.67

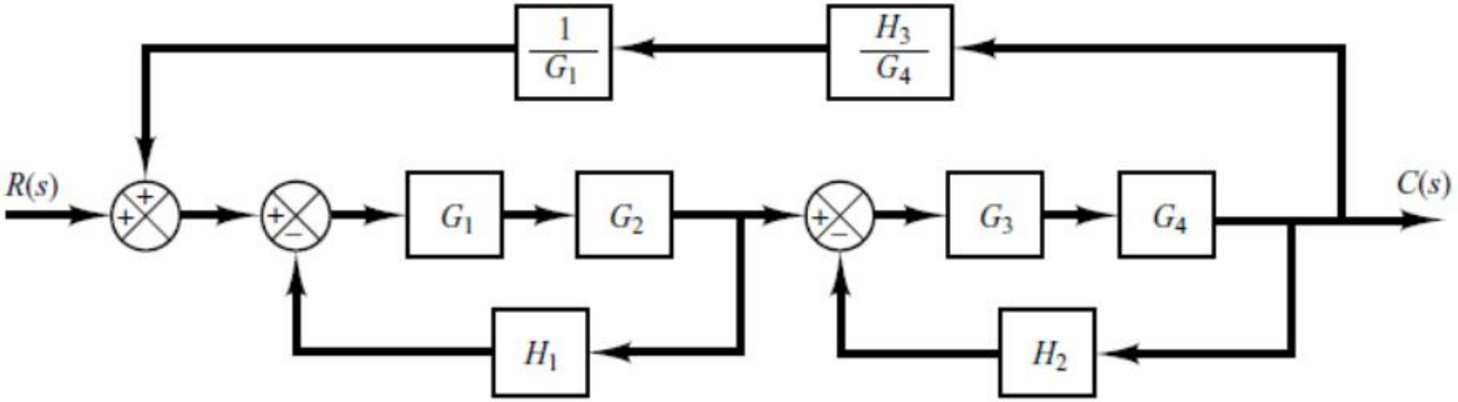


Problem No.5

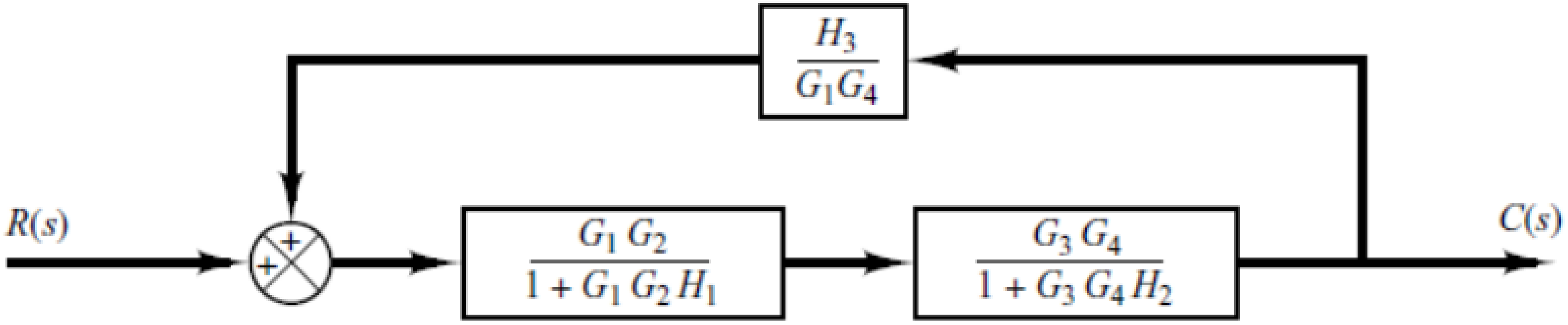
OBTAIN THE CLOSE-LOOP TRANSFER FUNCTION C(S)/R(S).



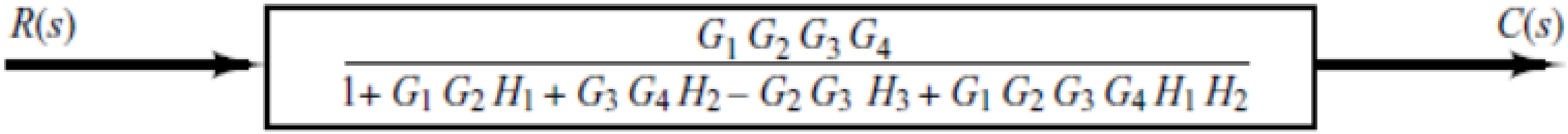
First move the branch point between G_3 and G_4 to the right-hand side of the loop containing G_3, G_4, and H_2. Then move the summing point between G_1 and G_2 to the left-hand side of the first summing point.



By simplifying each loop, the block diagram can be modified as



Further simplification results in

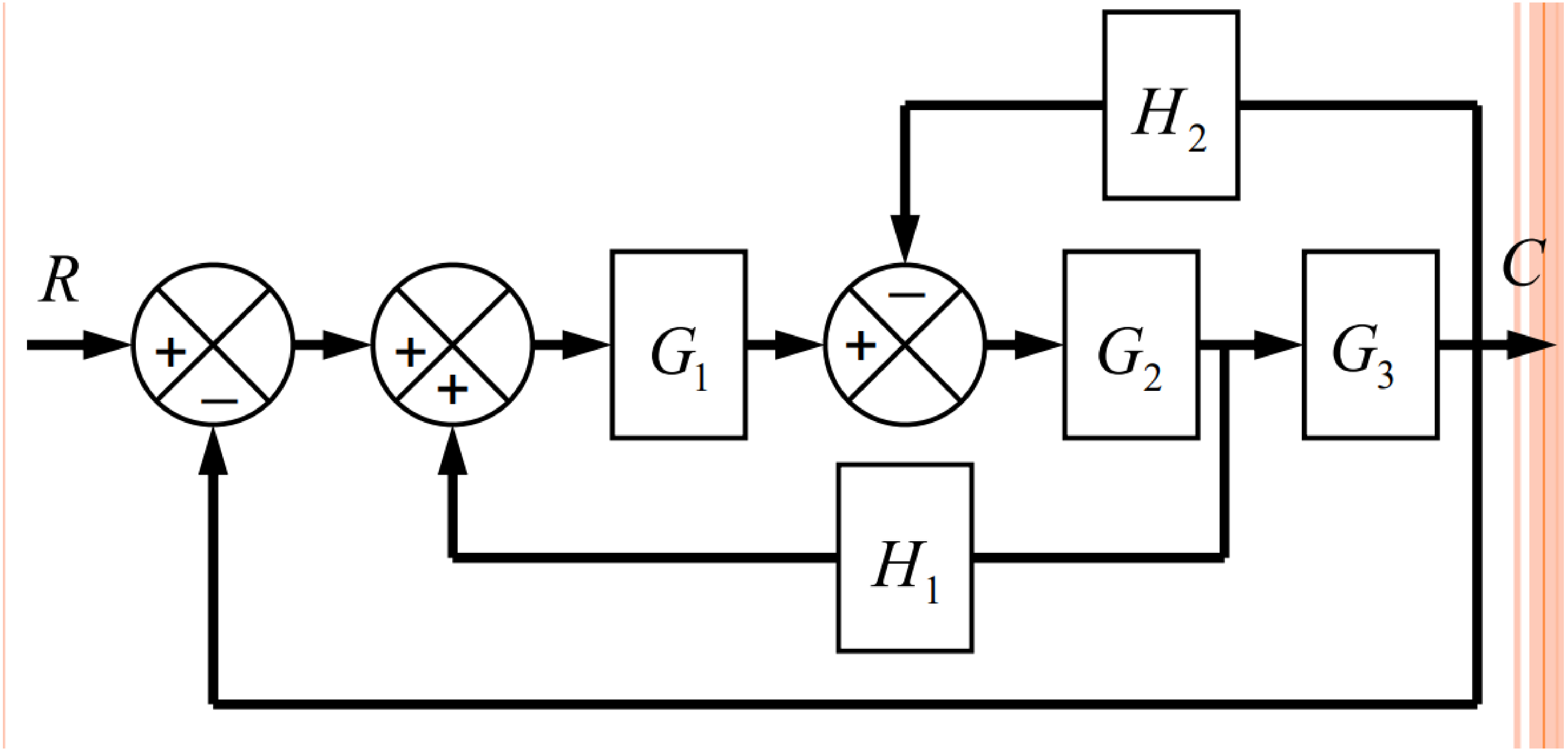


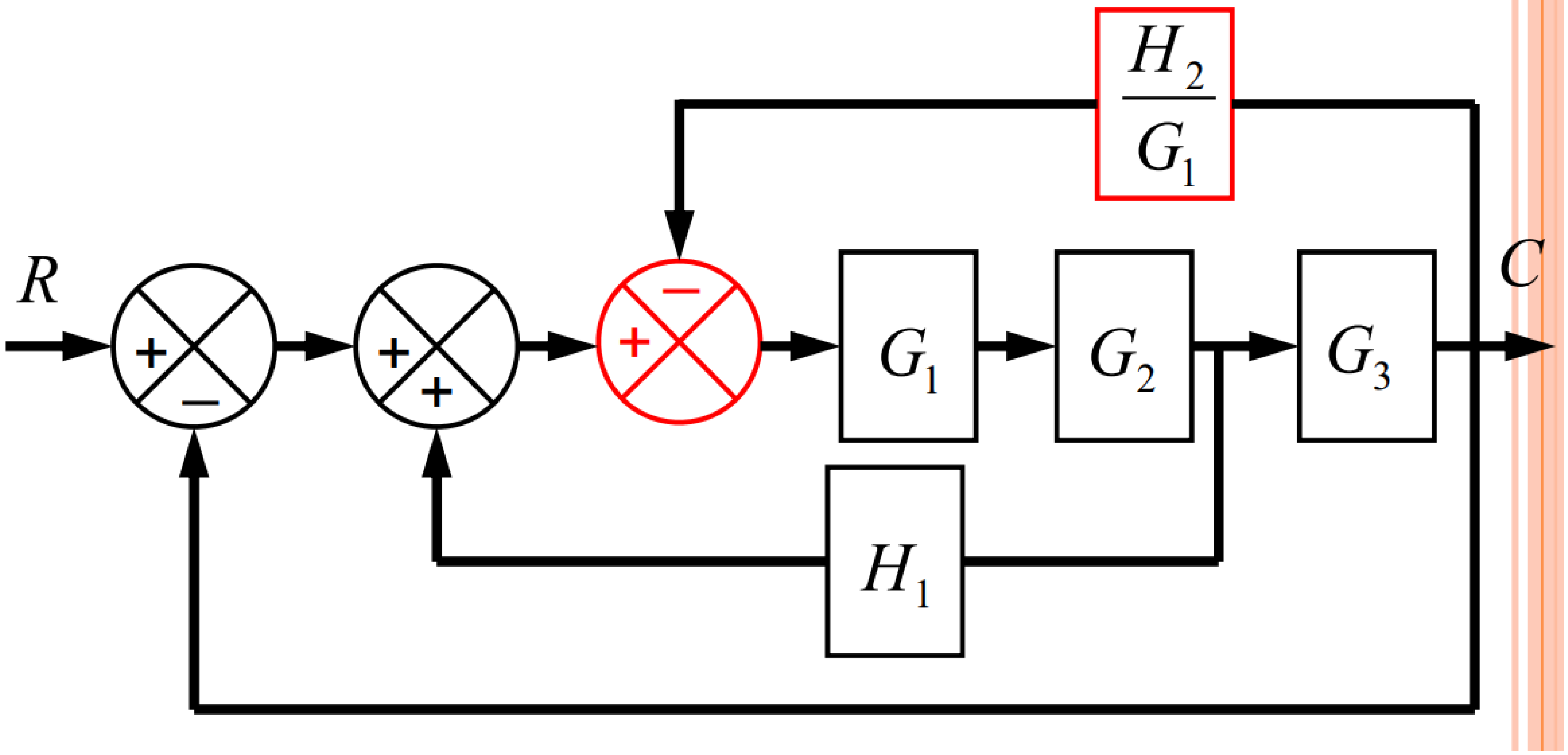
the closed-loop transfer function $C(s)/R(s)$ is obtained as

$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3G_4}{1 + G_1G_2H_1 + G_3G_4H_2 - G_2G_3H_3 + G_1G_2G_3G_4H_1H_2}$$

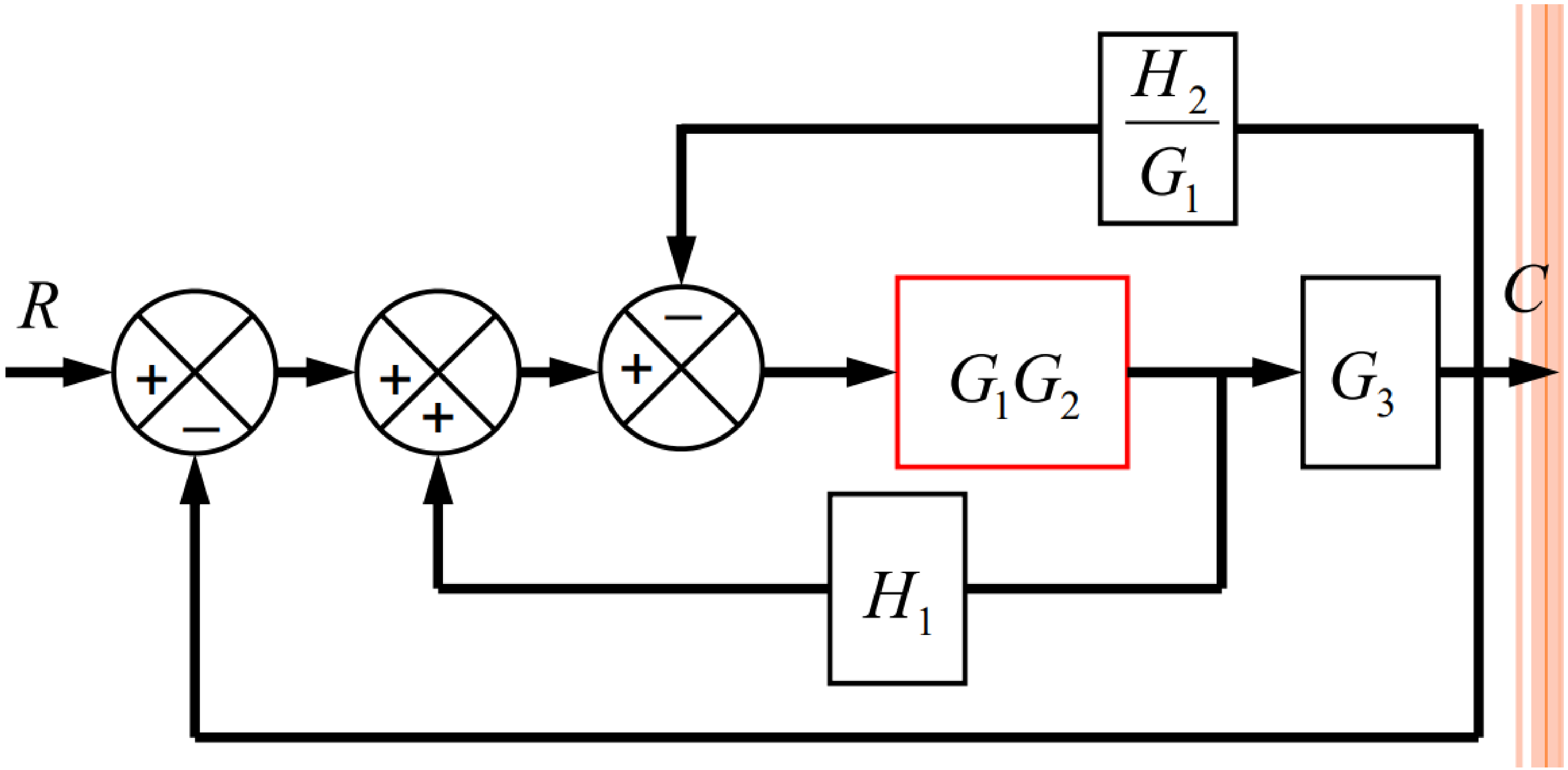
Problem No.6

Reduce the block Diagram

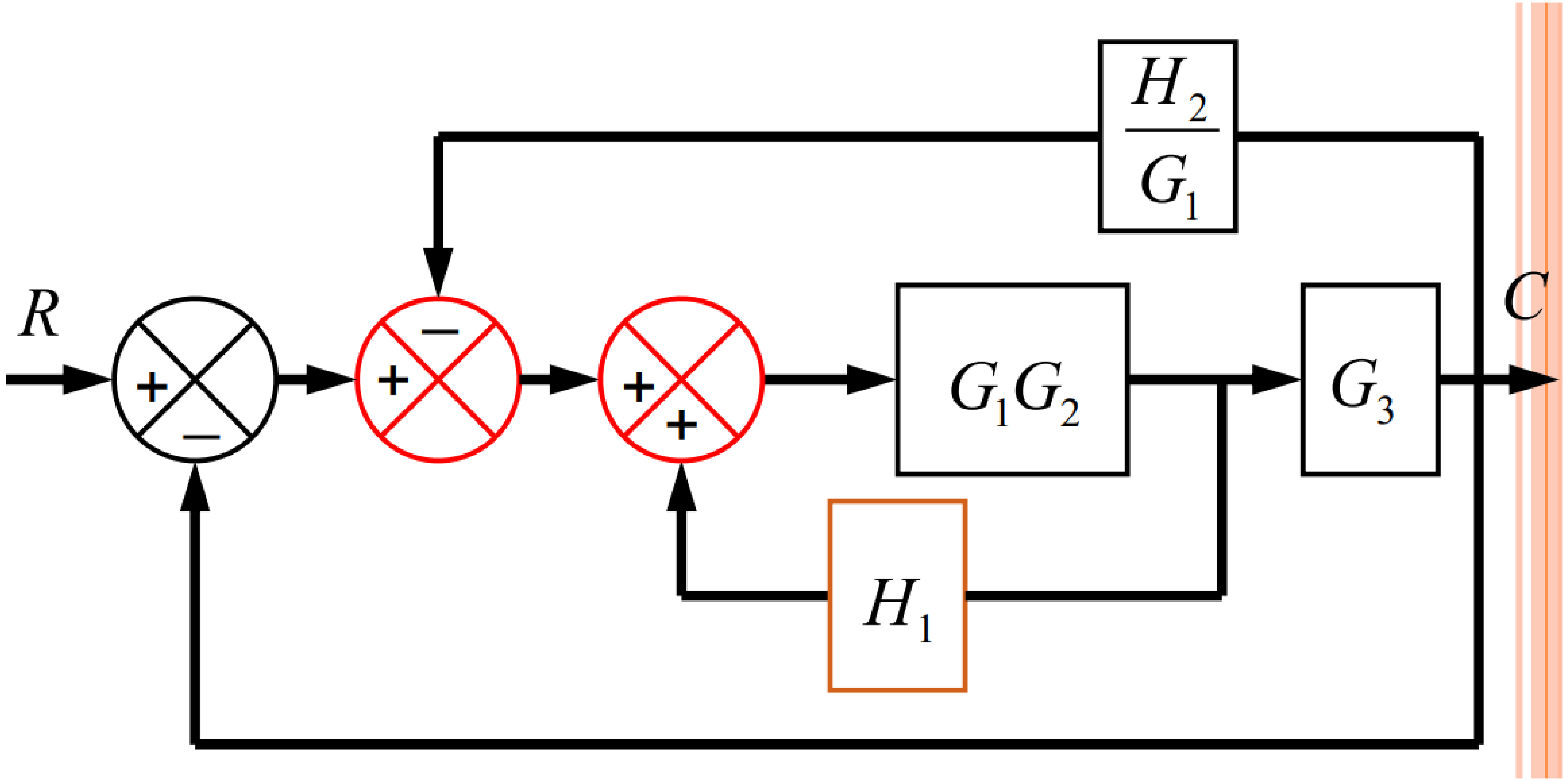




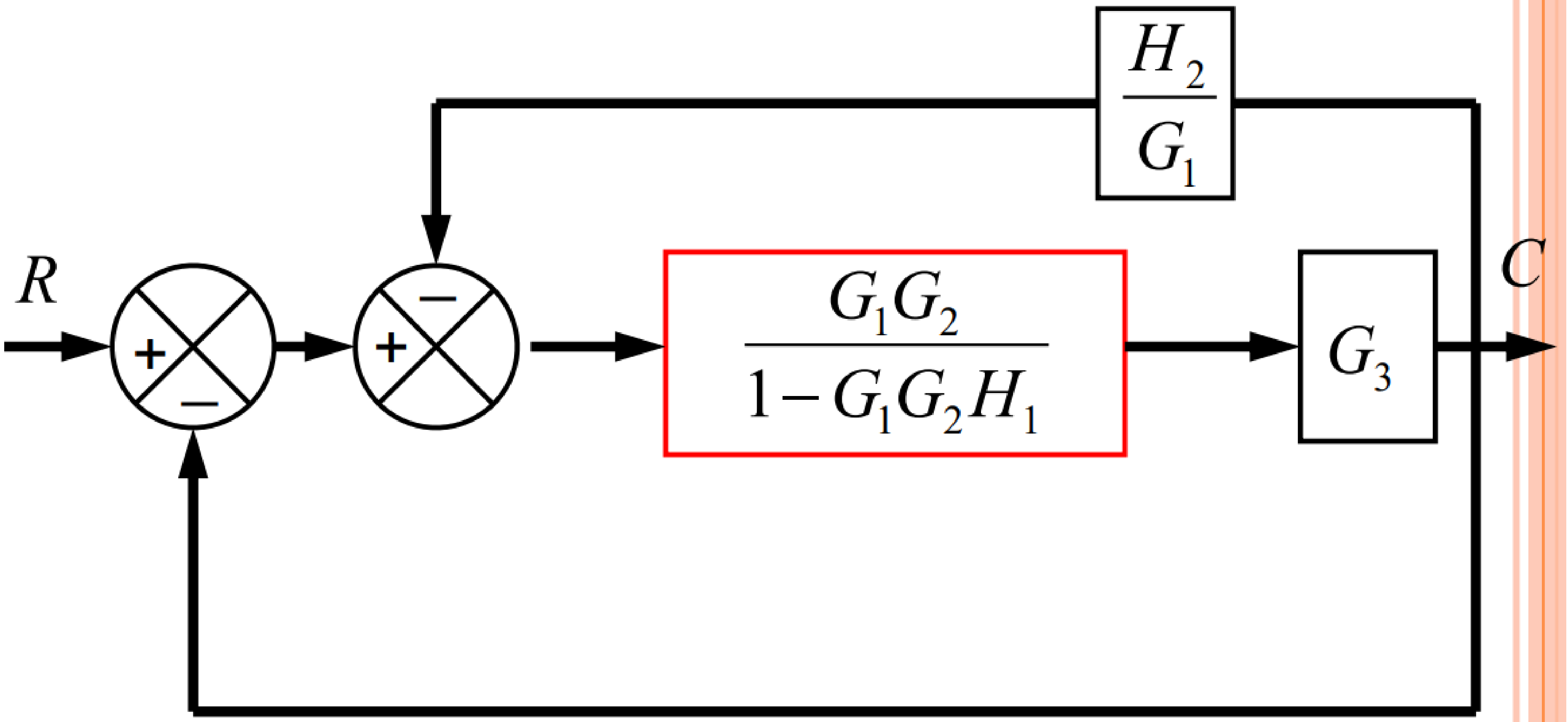
$\frac{H_2}{G_1}$
R
C
+
−
+
+
−
+
G_1
G_2
G_3
H_1



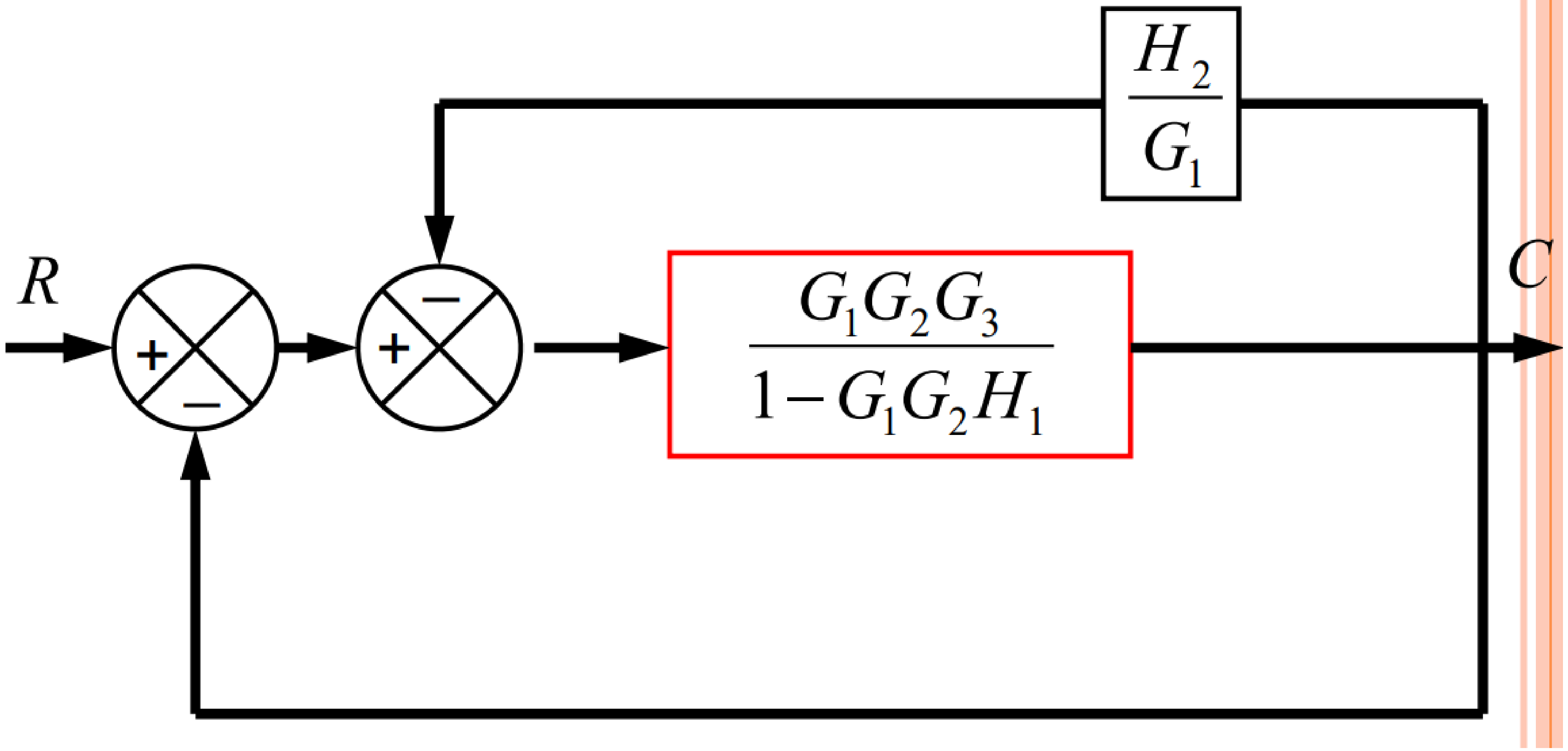
$\frac{H_2}{G_1}$
R
C
G_1G_2
G_3
H_1



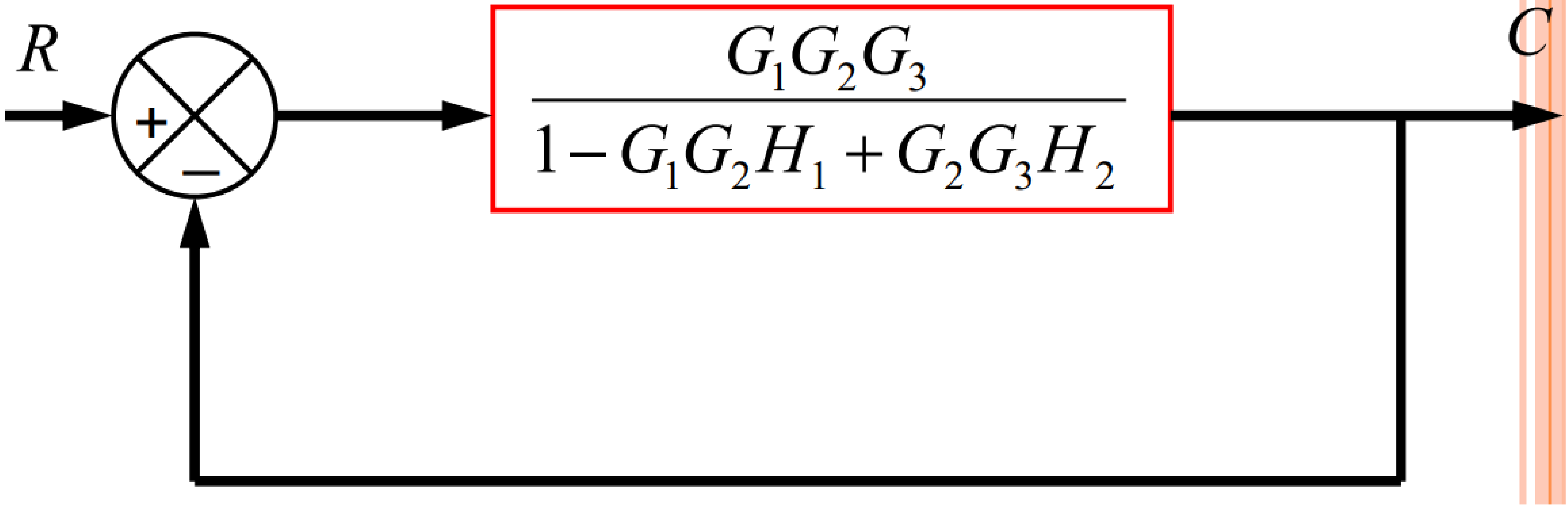
R
+
−
+
−
+
+
$\frac{H_2}{G_1}$
G_1G_2
G_3
H_1
C

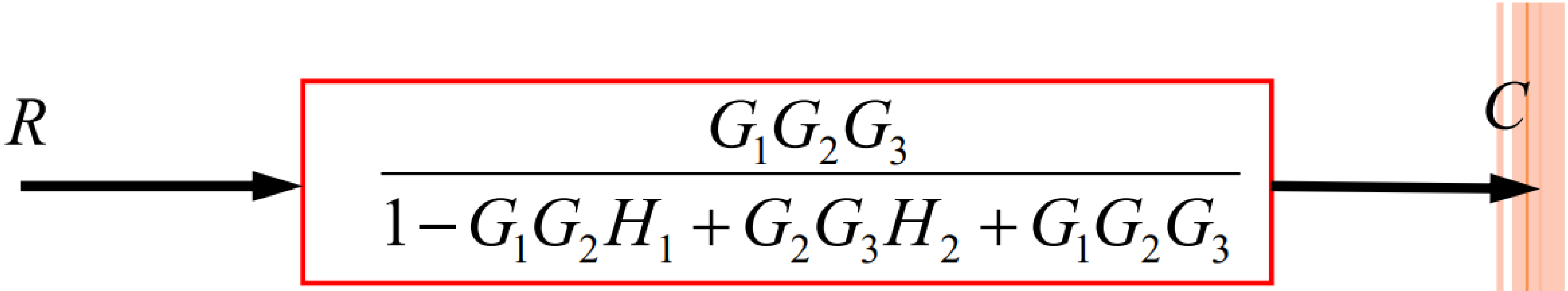


$\frac{H_2}{G_1}$
R
+
−
+
−
$\frac{G_1G_2}{1-G_1G_2H_1}$
G_3
C

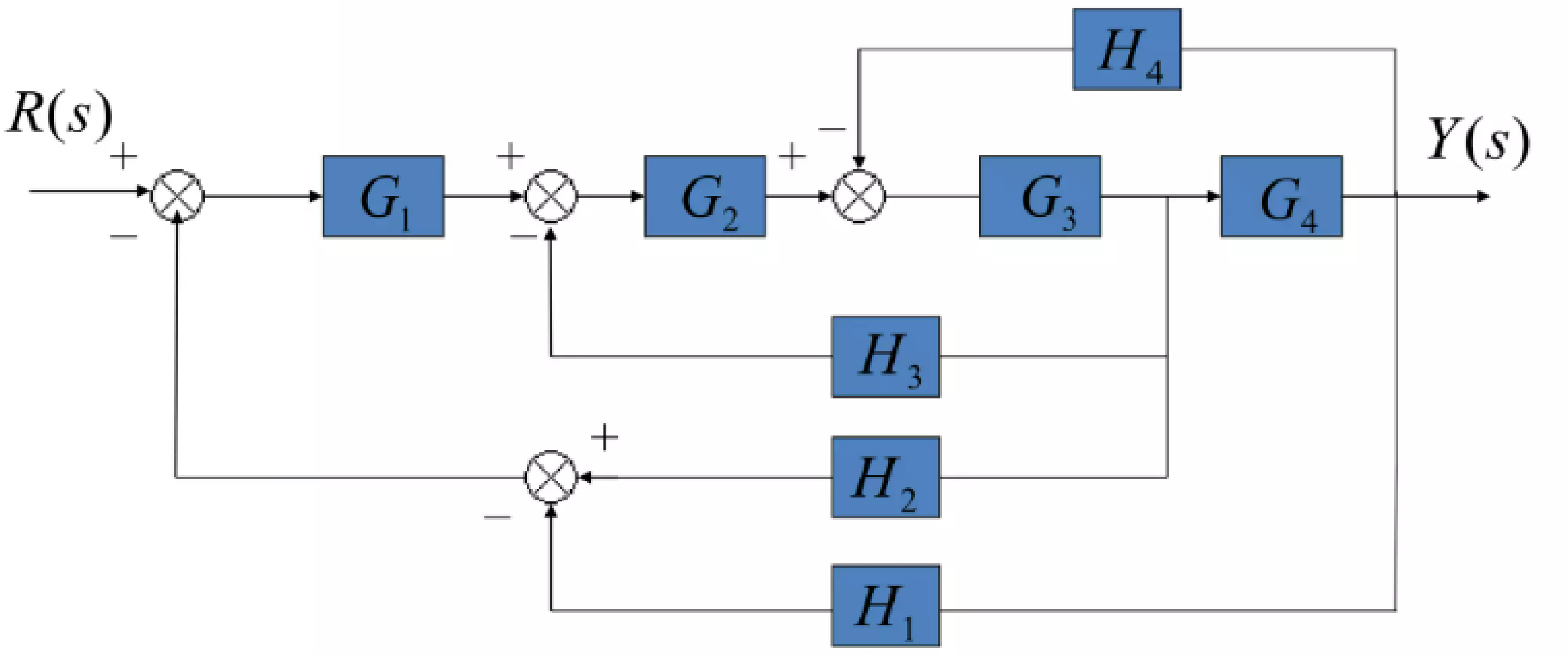


R
+
−
+
−
$\frac{H_2}{G_1}$
$\frac{G_1G_2G_3}{1-G_1G_2H_1}$
C


R
+
–
$\frac{G_1G_2G_3}{1-G_1G_2H_1+G_2G_3H_2}$
C


R
$\frac{G_1G_2G_3}{1-G_1G_2H_1+G_2G_3H_2+G_1G_2G_3}$
C

Find the transfer function of the following block diagrams

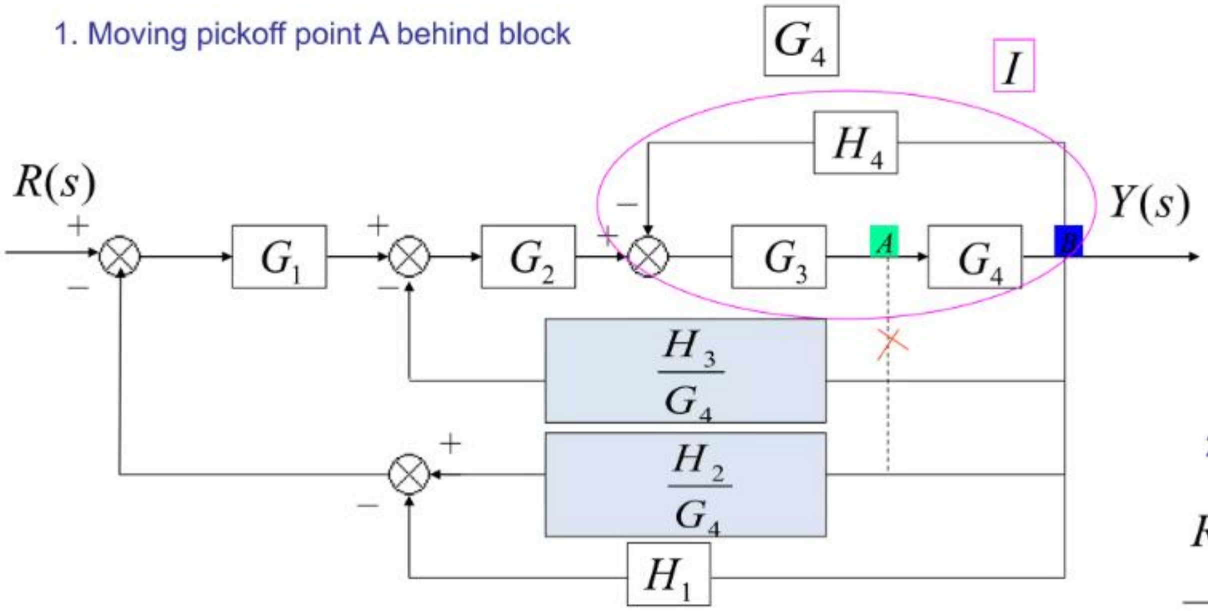


Problem No.7

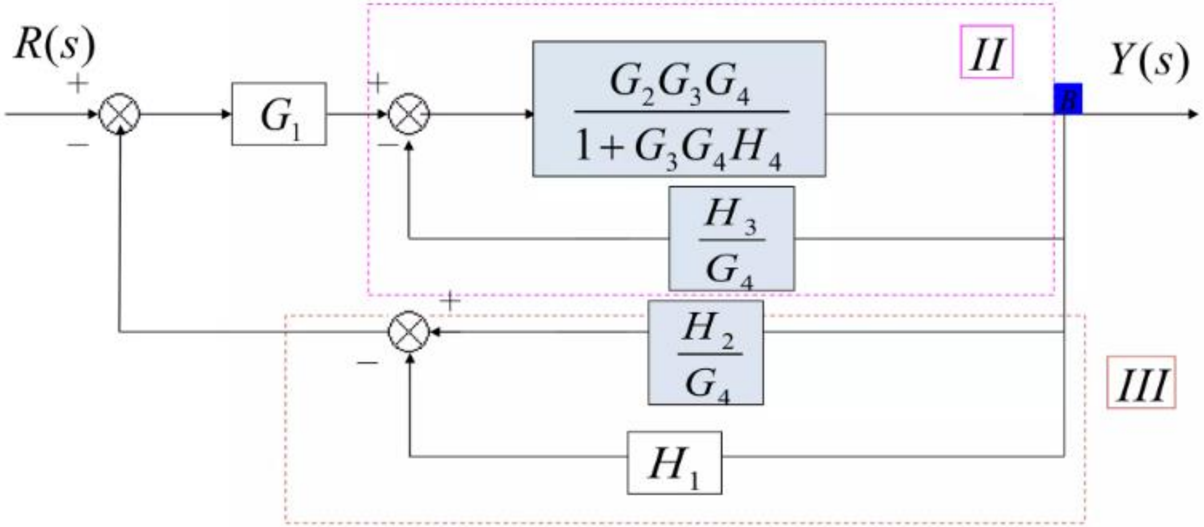
Reduce the block Diagram

Solution:

1. Moving pickoff point A behind block



2. Eliminate loop I and Simplify



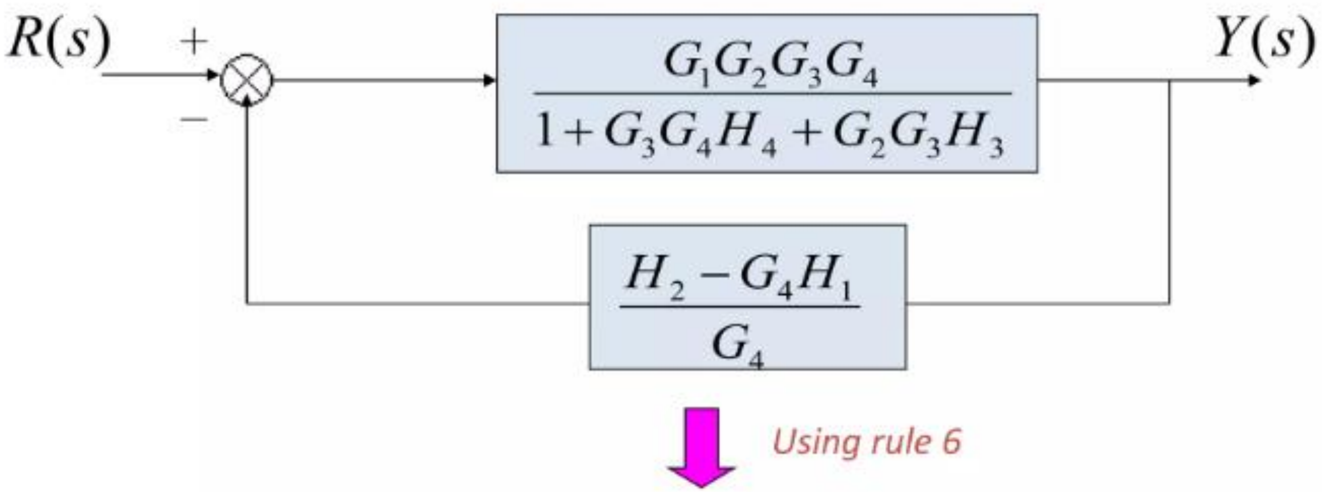
II feedback

$$\boxed{\frac{G_2G_3G_4}{1+G_3G_4H_4+G_2G_3H_3}}$$

III Not feedback

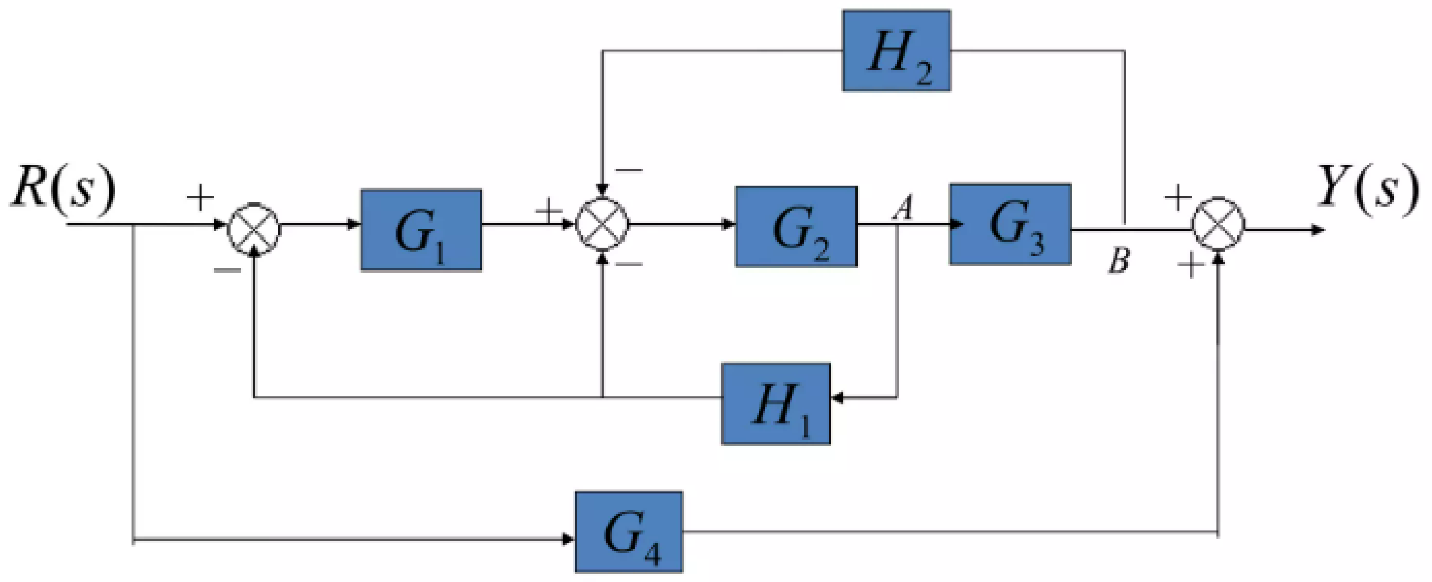
$$\boxed{\frac{H_2-G_4H_1}{G_4}}$$

3. Eliminate loop II & IIII



$$T(s)=\frac{Y(s)}{R(s)}=\frac{G_1G_2G_3G_4}{1+G_2G_3H_3+G_3G_4H_4+G_1G_2G_3H_2-G_1G_2G_3G_4H_1}$$

Find the transfer function of the following block diagrams

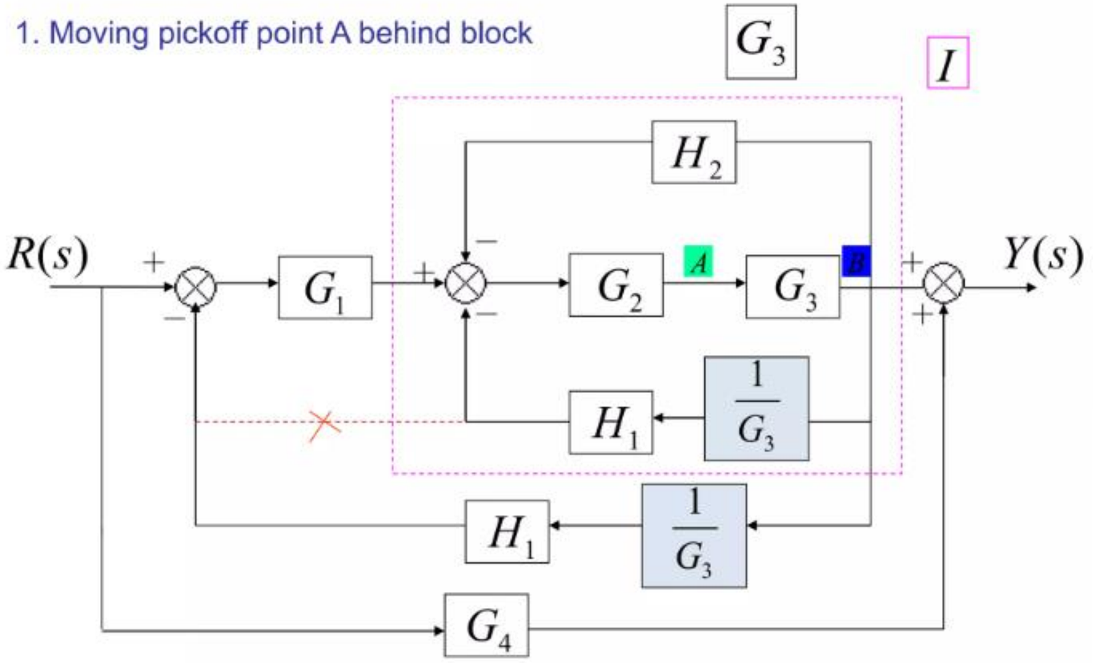


Problem No.8

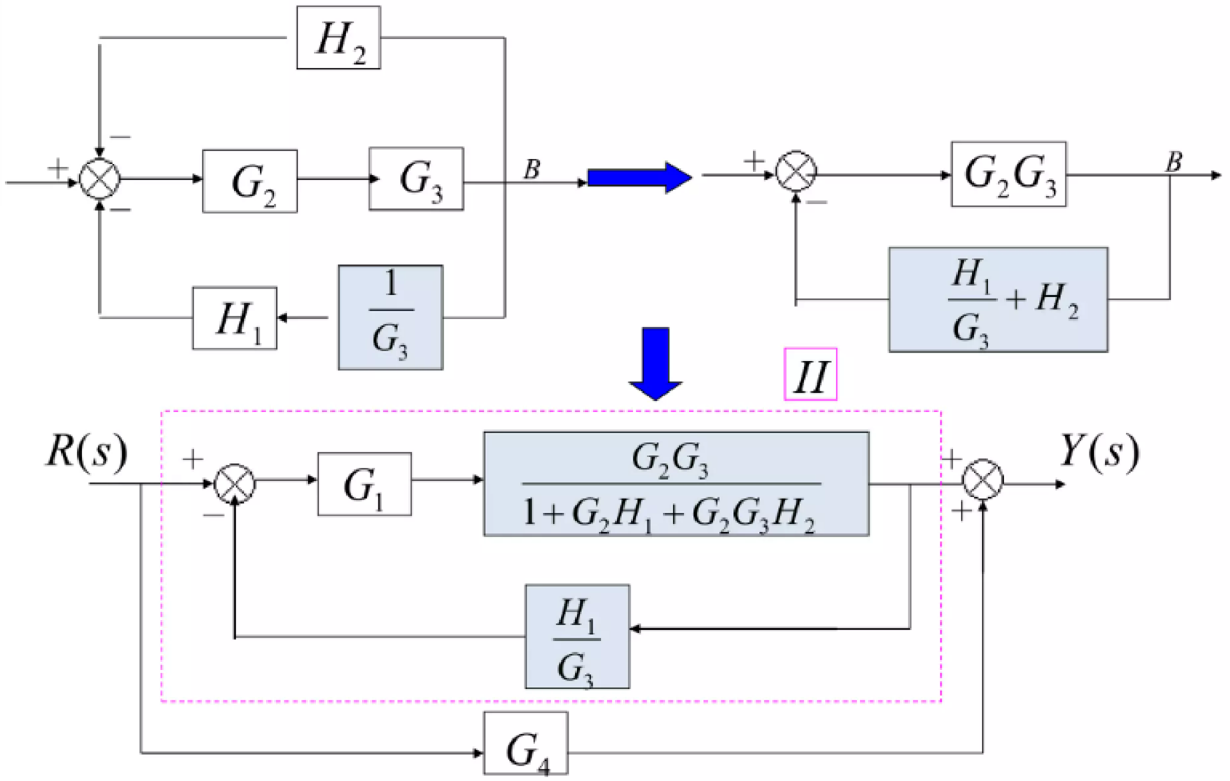
Reduce the block Diagram

Solution:

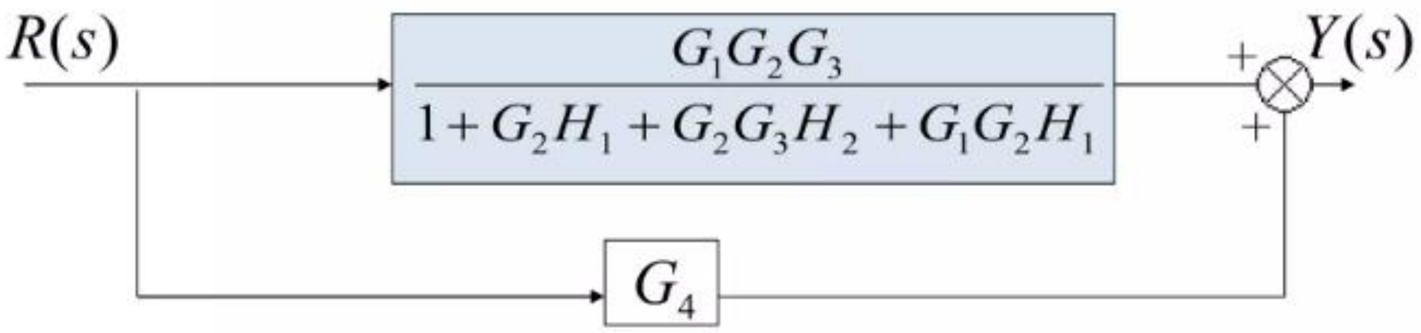
1. Moving pickoff point A behind block



2. Eliminate loop I & Simplify



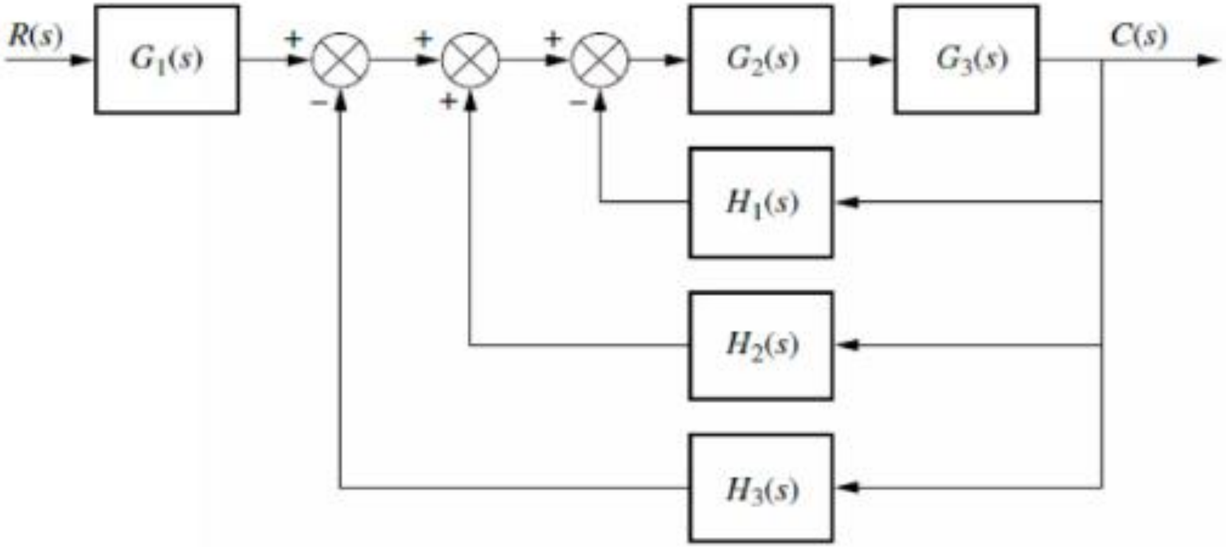
3. Eliminate loop II



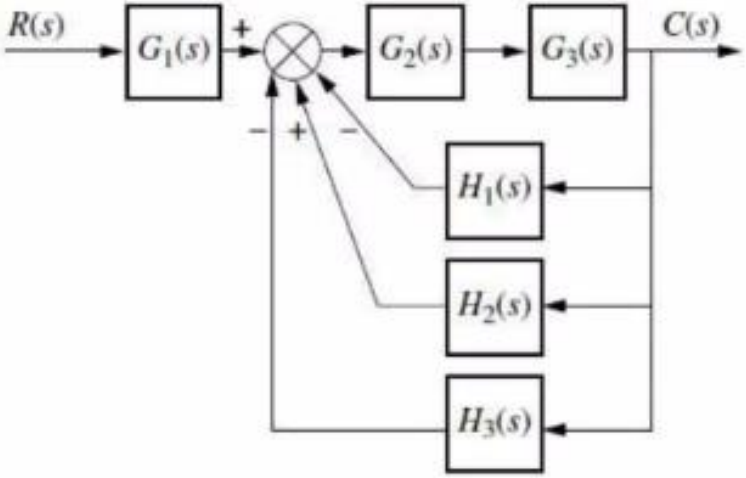
$$T(s) = \frac{Y(s)}{R(s)} = G_4 + \frac{G_1G_2G_3}{1 + G_2H_1 + G_2G_3H_2 + G_1G_2H_1}$$

Problem No.5

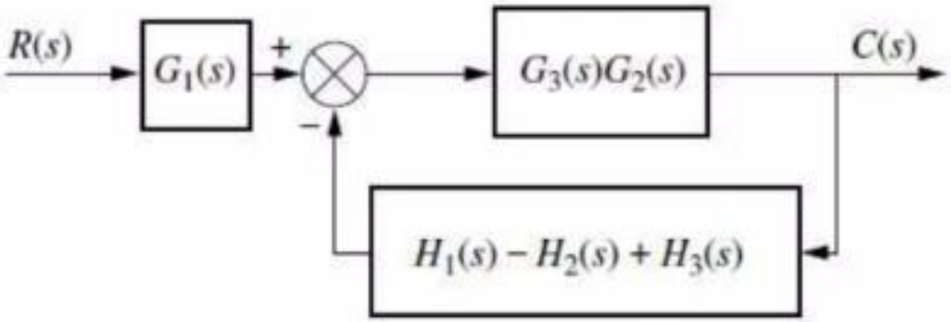
Reduce the block Diagram



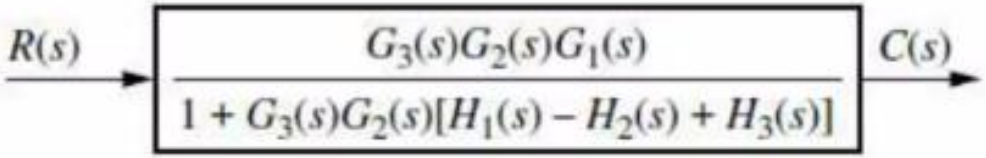
First, the three summing junctions can be collapsed into a single summing junction,



Second, recognize that the three feedback functions, $H_1(s)$, $H_2(s)$, and $H_3(s)$, are connected in parallel. They are fed from a common signal source, and their outputs are summed. Also recognize that $G_2(s)$ and $G_3(s)$ are connected in cascade.



Finally, the feedback system is reduced and multiplied by $G_1(s)$ to yield the equivalent transfer function shown in Figure



Thank You