Indian Institute of Technology Bhilai

IC202: Calculus II

2023-24-M Instructor: Dr. Raj Kumar Mistri

TUTORIAL SHEET-4

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1. Find all the first order partial derivatives of the following functions.

   (a) $f(x,y) = 2x^2 - xy + 2y^2$.

   (b) $f(x,y,z) = y\sin(xz)$.

   (c) $f(x,y) = x^y$.

   (d) $f(x,y) = x^3y + e^{xy^2}$.

2. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$f(x,y) = \begin{cases} 1, & \text{if } (x,y) \neq (0,0); \\ 0, & \text{if } (x,y) = (0,0). \end{cases}$$

   Show that $f_x(0,0)$ as well as $f_y(0,0)$ does not exist.

3. (a) Let

$$f(x,y) = \begin{cases} \frac{xy^3}{x^2+y^2}, & \text{if } (x,y) \neq (0,0); \\ 0, & \text{if } (x,y) = (0,0). \end{cases}$$

   Show that f_x and f_y exist at $(0,0)$ and f_x is continuous at $(0,0)$. Hence decide the differentiability and continuity of f at $(0,0)$.

   (b) Show that the function

$$f(x,y) = \begin{cases} \frac{x^3-y^3}{x^2+y^2}, & \text{if } (x,y) \neq (0,0); \\ 0, & \text{if } (x,y) = (0,0). \end{cases}$$

   possesses both the first order partial derivative at the point $(0,0)$, but it is not differentiable at $(0,0)$.

4. Show that the following function f is not differentiable at $(0,0)$.

$$f(x,y) = \begin{cases} \frac{x^5}{x^4+y^4}, & \text{if } (x,y) \neq (0,0); \\ 3, & \text{if } (x,y) = (0,0). \end{cases}$$

5. Find all the second order partial derivatives of the following functions.

   (a) $f(x,y) = x^5 + y^4\sin(x^6)$.

   (b) $f(x,y,z) = \sin(xy) + \sin(yz) + \cos(xz)$.

6. (a) Find the derivative (total derivative) of the function $f : \mathbb{R}^2 \to \mathbb{R} : f(x,y) = x^2 + y^2$ at the point $(1,2)$.

   (b) Find the derivative (total derivative) of the function $f : \mathbb{R}^3 \to \mathbb{R}^2 : f(x,y,z) = (x^2y^2z, y + \sin z)$ at the point $(1,2,0)$.

   (c) Find the derivative (total derivative) of the function $f\mathbb{R}^3 \to \mathbb{R} : f(x,y,z) = \frac{x^2+y^2+z^2}{x^2+1}$ at the arbitrary point $(x,y,z) \in \mathbb{R}^3$.

7. For the following questions, use chain rule.

   (a) Find the derivative (total derivative) of the function $f \circ g$ at the point $(1,1) \in \mathbb{R}^2$, where $f : \mathbb{R}^2 \to \mathbb{R}^3 : f(u,v) = (u+v, u, v^2)$ and $g : \mathbb{R}^2 \to \mathbb{R}^2 : g(x,y) = (x^2+1, y^2)$.

   (b) Find the derivative (total derivative) of the function $f \circ g$ at the arbitrary point $(x,y) \in \mathbb{R}^2$, where $f : \mathbb{R}^2 \to \mathbb{R} : f(u,v) = uv$ and $g : \mathbb{R}^2 \to \mathbb{R}^2 : g(x,y) = (x^2 - y^2, x^2 + y^2)$.

8. Let $r = r(x,y,z) = \sqrt{x^2+y^2+z^2}$ for all $(x,y,z) \in \mathbb{R}^3$. Show that

   (a) $\nabla(r) = \left(\frac{x}{r}, \frac{y}{r}, \frac{z}{r}\right)$.

   (b) $\nabla(\frac{1}{r}) = \left(-\frac{x}{r^3}, -\frac{y}{r^3}, -\frac{z}{r^3}\right)$, if $r \neq 0$.