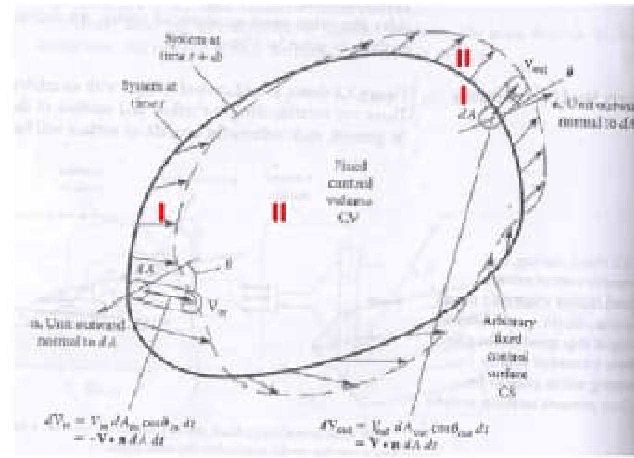
# Reynolds Transport Theorem (RTT)

- Can be applied to all the basic laws.

(Conservation of mass, momentum & energy)




***B:*** *any property of the fluid (vector or scalar)*
*(mass, momentum, energy, enthalpy etc.)*

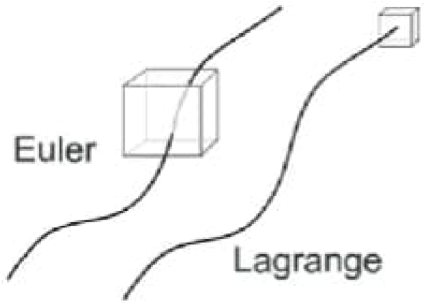
***b:*** *Intensive value (amount of B per unit mass in any small element of the fluid)*

$$b = \frac{dB}{dm}$$

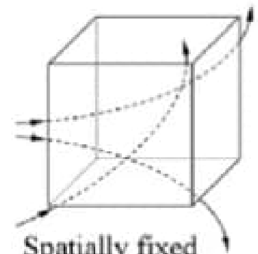
$$\boxed{\frac{dB_{syst}}{dt} = \frac{d}{dt}\left(\int_{CV} b\rho d\forall\right) + \int_{CS} b\rho(\vec{V}\cdot n)dA}$$

$$\frac{dm}{dt} = 0 \qquad F = ma = m\frac{dv}{dt} = \frac{d}{dt}(mV) \qquad \frac{dE}{dt} = \dot{Q} - \dot{W}$$

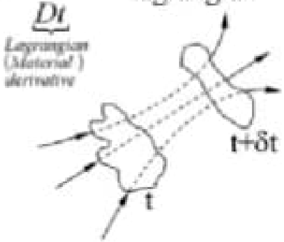
# Eulerian and Lagrangian Approach



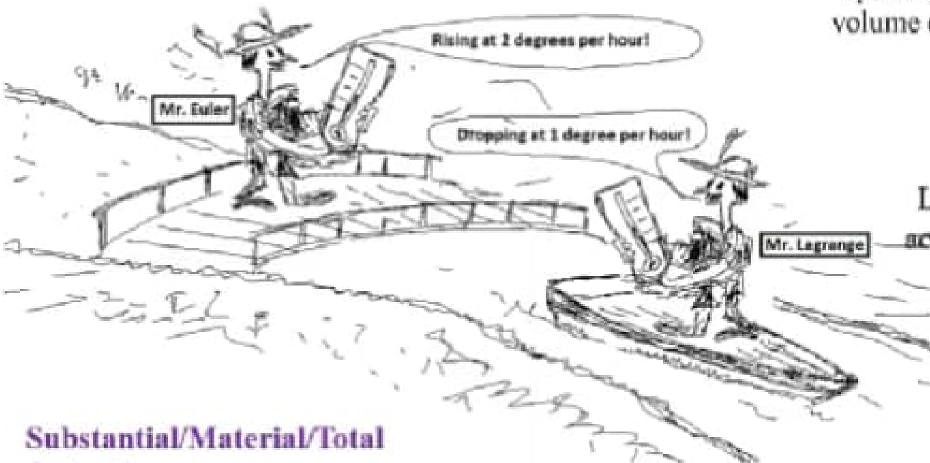

Eulerian $\quad \underbrace{\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla}_{\text{Eulerian derivative}} = \underbrace{\frac{D}{Dt}}_{\text{Lagrangian (Material) derivative}} \quad$ Lagrangian



Spatially fixed volume element



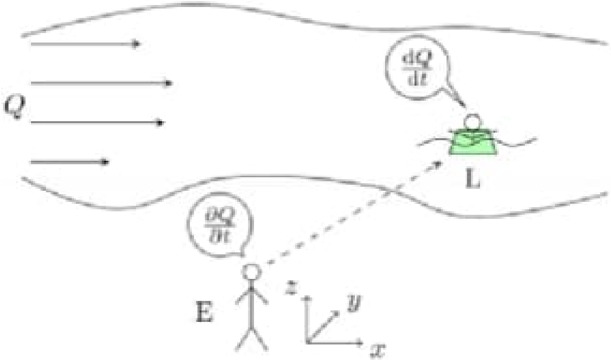

Following the motion of the fluid element




$$\underbrace{\frac{D\vec{v}}{Dt}}_{\text{Lagragian acceleration}} = \underbrace{\frac{\partial \vec{v}}{\partial t}}_{\text{Eulerian acceleration}} + \underbrace{\vec{v}\cdot\nabla\vec{v}}_{\text{Convective acceleration}}$$

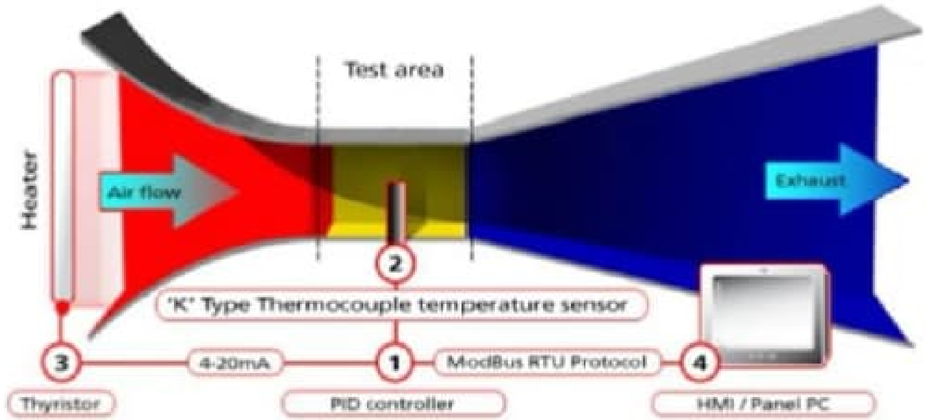
Substantial/Material/Total derivative

$$\frac{D}{Dt} = \underbrace{\frac{\partial}{\partial t}}_{\text{Local time derivative}} + \underbrace{u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}}_{\text{Convective derivative}}$$



# Eulerian and Lagrangian Approach

- We will be focusing more on Eulerian approach in this particular course.



# Eulerian and Lagrangian Approach

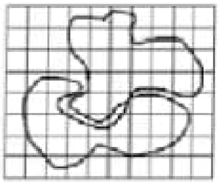
## Eulerian-Eulerian

Fluids

$$\rho \frac{DV}{DT} = \nabla \bullet \sigma + f$$

Solids

$$\rho \frac{DV}{DT} = \nabla \bullet \sigma + f$$



Merits:

1. Computational speed

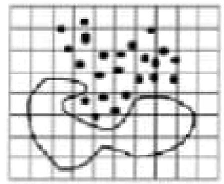
## Eulerian-Lagrangian

Fluids

$$\rho \frac{DV}{DT} = \nabla \bullet \sigma + f$$

Solids

$$F = ma$$

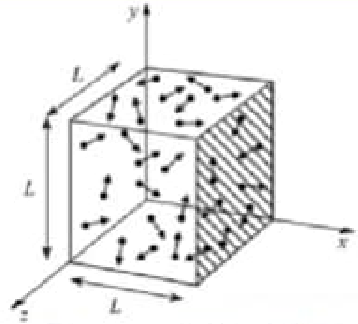


Merits:

1. High-resolution on particle positions
2. Can include more physics modes

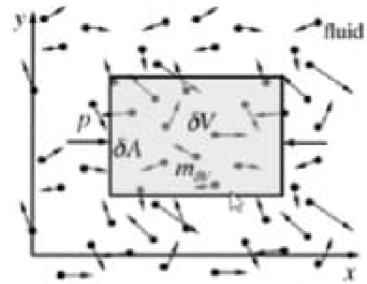
# Continuum Hypothesis

- At microscopic level, the ***molecules continuously interact with each other*** moving with random velocities.

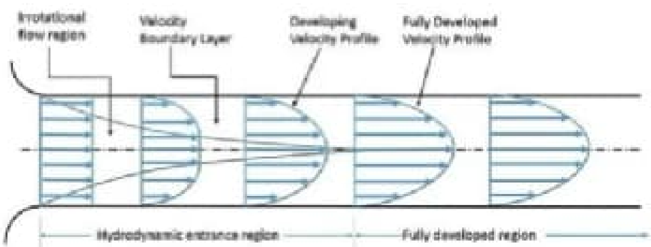


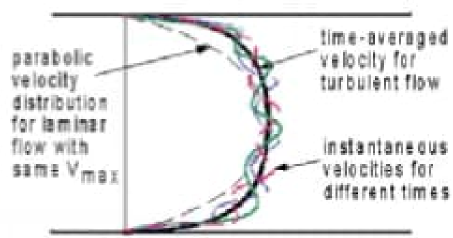

Schematic representation of the gas filled volume containing N molecules.

- The random motion does not allow to define a molecular velocity/density at a fixed spatial position (changing continuously).

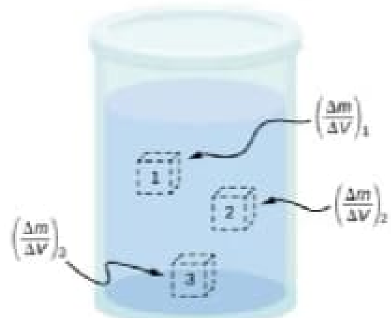



- ***Continuum assumption is very convenient since it erases the molecular discontinuities*** by averaging the microscopic quantities on a small sampling volume.



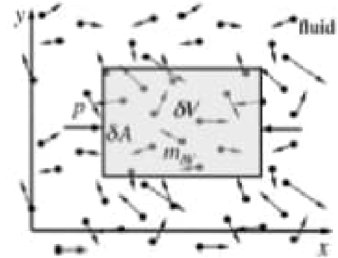

Eulerian velocities in pipe flow



# Continuum Hypothesis

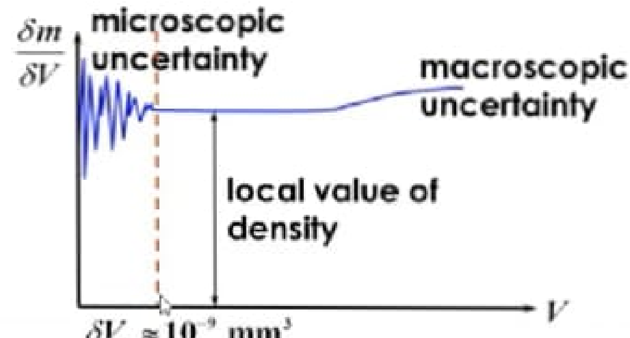
- Consider an example of density

$$Density = \frac{Mass\ of\ the\ particles\ occupying\ certain\ volume\ (CV)}{Volume\ (CV)}$$




$$\boxed{\rho = \lim_{\delta V \to \delta V_o} \frac{\delta m}{\delta V}}$$

- $\delta V_o$: Limiting volume (distinguishes continuum and non-continuum regions)
- Knudsen number, $Kn=\lambda/L$
- $\lambda$: Mean free path
- **Kn<0.01**: **Continuum** (bulk motion) can be assumed.
- **Kn>0.01**: **Boltzmann** eq. (random molecular motion) must be used.
- Continuum assumption becomes **invalid as pressure tends to zero.**

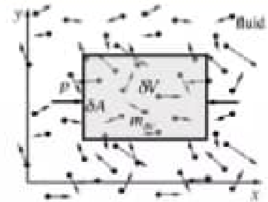



(3×10⁷ molecules at sea level, 15°C, 1atm)

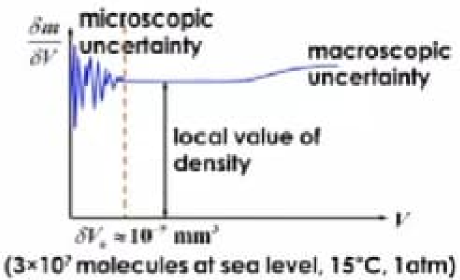
# Continuum Hypothesis

## ➢ Microscopic Uncertainty:

- Due to microscopic variation (random molecular motion).
- Should not generate significant fluctuations of the averaged quantities.
- **Size of a representative sampling volume must be large enough** to erase the microscopic fluctuations **($\delta V > \delta V_o$)**

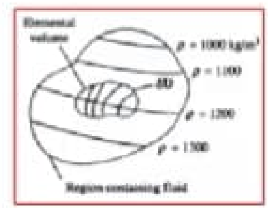



$$\boxed{\rho = \lim_{\delta V \to \delta V_o} \frac{\delta m}{\delta V}}$$




## ➢ Macroscopic Uncertainty:

- Due to aggregate variation.
- Variation associated with spatial distribution of density, velocity or pressure.
- **Size of a representative sampling volume must also be small enough (not $\delta V >>> \delta V_o$) to point out the macroscopic variations (such as velocity or pressure gradients of interest in the control volume** )




✓ **Sampling a volume containing 10,000 molecules leads to 1% statistical fluctuations in the macroscopic quantities** (Karniadakis and Beskok, 2002).