Algorithms I (CS202)

1. Arrange the following functions in asymptotic increasing order:
   (a) $n^{\sqrt{\log n}}$, $\log n^{(\log n)^2}$, $n^{(\log\log n)^2}$
   (b) $\log\log n$, $n!$, $n^{n!}$, $n^{\log n}$, $n\log n$, $\log\log\log\log n$, n^n, 2^n
   (c) $n^{1+\frac{1}{\log n}}$, $n^{\log\log n}$, $n^{\log n}$, $n^{\sqrt{\log n}}$, 2^n, $\sqrt{n}$, $n^{1+\sin n}$, $2^{\log n \log n}$, n^3, $2^{\sqrt{\log n}}$
   (d) $x!$, $2^{\sqrt{\log x}}$, $x^{1.5}$, $x^{1.7}$, $\log(\log x)$, $(\log x)^2$, $\sqrt{x}$, $x^{1.5}$
   (e) $x^{1.1}$, $\log x^{\log x}$, x, $\sqrt{\log(x)}$, $\log\log\log x$, $(\log x)^2$, $\log x$, $x^{1.2}$, x^2, $x^{1.3}$, $x^{0.9}$, $x\log x$, $2^{(\log x)^2}$, $x\sqrt{x}$, $2^{\frac{x}{100}}$

2. Determine whether the function $f(n) = n + \log n$ is $O(n)$, $O(\sqrt{n})$ or $O(logn)$.

3. Consider the following functions$f(n) = n^{\log n}, g(n) = 2^{(\log n)^2}$ Determine whether $f(n)$ is $O(g(n))$ or $g(n)$ is $O(f(n))$.

4. Compute the time complexity of the given pseudocode:

   for $(i = 1 \; ; \; i <= n \; ; \; i = i * 2)$ **do**
       for $(j = n \; ; \; j >= 2; \; j = \sqrt{j})$ **do**
           for $(k = n \; ; \; k >= 1 \; ; \; k = k/2)$ **do**
           end for
       end for
   end for

5. Compute the time complexity of the given pseudocode:

   $x \leftarrow 0$
   for $(i = 1; \; i <= n; \; i++)$ **do**
       for $(j = 1; \; j <= n; \; j++)$ **do**
           for $(k = 1; \; k <= n; \; k + j)$ **do**
               $x \leftarrow x + 1$
           end for
       end for
   end for

6. Determine True or False for the following statements:
   (a) $f(n) + O(f(n)) = O(f(n))$
   (b) $f(n) + \omega(f(n)) = o(f(n))$
   (c) $f(n) + o(f(n)) = \Theta(f(n))$
   (d) $f(n) + \Theta(f(n)) = o(f(n))$

7. If $f(n) = \log_{10} n$ and $g(n) = \log_y n^x$, where x and y are constants, which of the following is True and why?

(a) $f(n) = O(g(n))$ and $g(n) = O(f(n))$

(b) $f(n) = O(g(n))$ and $g(n) \neq O(f(n))$

(c) $f(n) \neq O(g(n))$ and $g(n) = O(f(n))$

(d) $f(n) \neq O(g(n))$ and $g(n) \neq O(f(n))$