Indian Institute of Technology Bhilai

IC202: Calculus II

2023-24-M — Instructor: Dr. Raj Kumar Mistri

TUTORIAL SHEET-2

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1. Find the domains of the following functions.

   (a) $f(x,y) = \frac{xy}{x^4+y^4}$.

   (b) $f(x,y) = \frac{x^2}{\sqrt{1-x^2-y^2}}$.

   (c) $f(x,y,z) = \frac{z^2}{x^2-y^2}$.

   (d) $f(x,y) = \frac{x^2-y^2}{x^2+y^2}$.

2. Find the level curves of the following functions.

   (a) $f(x,y) = 3x + 2y$.

   (b) $f(x,y) = 4x^2 + 9y^2$.

3. (a) Use level curves to sketch the graph of $f(x,y) = x^2 + y^2$.

   (b) Sketch the surface $z = x^2 + y^2 - 4x - 6y + 13$.

   (c) Sketch the level surfaces with values $1, 2, 3$ of the function $f(x,y,z) = x - y + z + 2$.

   (d) Sketch the level surface of $f(x,y,z) = x^2 + y^2 + z^2 - 8$ with value 1.

4. (a) Let $f(x,y) = x^2y$ and $g(x,y) = x^2y^3$. Find the sum, product and quotient of f and g. State their domains.

   (b) Let $f(x,y) = y\sin\frac{1}{x}$ and $g(x,y) = x\sin\frac{1}{y}$. Find the sum and product of f and g. State their domains.

   (c) Let $f(x,y) = (e^x\cos y, e^x\sin y)$ and $g(x,y) = (x^2, y^3)$. Find the sum of f and g.

5. Let $f : \mathbb{R}^2 \to \mathbb{R}$ and let $g : \mathbb{R} \to \mathbb{R}$ be two functions defined as

$$f(xy) = \begin{cases} \frac{|2xy|}{x^2+y^2} & \text{if } (x,y) \neq (0,0); \\ 0 & \text{if } (x,y) = (0,0), \end{cases}$$

   and $g(t) = \arcsin t$, respectively. Find $f \circ g$ and $g \circ f$, if they exist.

6. State whether or not the given set in $\mathbb{R}^2$ is open. Describe the boundary of the set also.

   (a) $[0,1] \times [0,1]$

   (b) $(0,1) \times (0,1)$

   (c) $[0,1] \times (0,1)$

   (d) $\{(x,y) \in \mathbb{R}^2 : y \geq x^2\}$

   (e) $\{(x,y) \in \mathbb{R}^2 : y > x^2\}$

7. Prove that every open ball in $\mathbb{R}^n$ is an open set.

8. Prove that

$$\lim_{(x,y)\to(x_0,y_0)} y = y_0 \quad \text{and} \quad \lim_{(x,y)\to(x_0,y_0)} k = k,$$

   where $k \in \mathbb{R}$ is a constant.

9. Prove that

   (a)
$$\lim_{(x,y)\to(0,0)} \frac{xy^2}{\sqrt{x^2+y^2}} = 0.$$

   (b)
$$\lim_{(x,y,z)\to(0,1,2)} \frac{x^2+3xyz-5z^2}{xy^3+5z^2-3xy+x^3} = -1.$$

   (c)
$$\lim_{(x,y)\to(0,0)} \frac{x\sin y}{2x^2+1} = 0.$$

   (d)
$$\lim_{(x,y)\to(0,0)} (x^2\sin\frac{1}{y} + y^2\cos\frac{1}{x}) = 0.$$