

PART A : Stability and structure of stars

1. Calculate the collapse time-scale for the Sun (mass $\approx 2 \times 10^{30}$ kg, radius $\approx 7 \times 10^5$ km), if it collapses freely (no-pressure support).
2. Find out the total mass of the Universe from the web and calculate the collapse time-scale for the Universe if it were to collapse to a point freely.
3. Why the ordinary gas pressure would not be able to stop the collapse of the Sun if there is no fusion?
4. Assume if there is no fusion and the Sun has lost its entire gravitational potential energy (because it is negative) through radiation that we receive then what is the current age of the Sun (although it would not be the actual age). Assume the luminosity of the Sun to be a constant (check its value from the web). Compare the age you calculated with the age of the Earth.
5. How would you derive/find-out the current actual age of the Sun?
6. Consider the hypothetical situation that density (ρ) of a main-sequence star is constant (does not change with radius, r). Calculate the mass, M as a function of r and the pressure, P , as a function of r , and plot them.
7. Write down a polytropic equation of state for hydrogen using variables P and T . Write down the equation of state for radiation/photons using variables P and T (or you can derive it from black-body Planck spectrum/law). Describe an 'Eddington solar model': what is the equation of state for it?

PART-B: End state of stars

1. Following as was taught in the class, plot the mass-radius relation for W-D stars (on a log-log plot and use the solar mass and radius as units). Then also calculate and plot the maximum velocity of electrons (from the uncertainty principle) as a function of mass (log-log plot, velocity unit: speed of light, mass unit: solar mass). On these two plots, illustrate at what mass the velocity becomes equal to the speed of light.
2. Repeat the calculation of quantum stars described in the class for stars composed of only neutrons (assume there are no electrons and protons), and get the degeneracy pressure using the uncertainty principle. Use that to calculate the mass radius relationship for such stars. Calculate the radius for such a neutron star of mass equal to that of Sun, and compare the radius for a white-dwarf star of the same mass (as was calculated in the class).
3. Explain (try to show mathematically or with plots) to the tutorial instructor/class if degeneracy pressure can stop the star from collapsing to a point/singularity then how/why would a black hole form at all?