Morning → 9:30 – 11:30 am

Algebra

Night

Sp. Classes

Part of your course

Questions | LRDI sets

Diff

11:50 pm

Sequence & Series - 5

Summation → Intg.

Special Series : 17^{th}

I. Use of Integration: → $\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n}$ =

$$\frac{1}{1}+\frac{1}{2}+\cdots+\frac{1}{100} = \sum_{1}^{n}\frac{1}{n} \text{ form} = \int_{1}^{100}\frac{1}{x}$$

I. $\int \frac{1}{x} = \ln x = \log_e x$

e^x

$e \simeq 2.7$

II. $\log_e x = 2.3 \times \log_{10} x$

III. $\int_a^b \frac{1}{x} = \ln b - \ln a$

(Q) $S = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \cdots\cdots + \frac{1}{100} = ?$

Sol. $\sum_{1}^{100} \frac{1}{x}$ form. $= \int_{1}^{100} \frac{1}{x} = \ln 100 - \ln 1.$

$= \log_e 100 - 0$

$= 2.3 \times \log_{10} 100$

$= 2.3 \times \log_{10} 10^2$

$= 2.3 \times 2 \log_{10} 10$

$= \underline{4.6}.$ A

(Q) $S = \frac{1}{1} + \frac{1}{2} + \cdots + \frac{1}{1000} = ?$

Sol.

$$\int_1^{1000} \frac{1}{x} = \ln 1000 - \ln 1$$

$$= \log_e 1000 - 0$$

$$= 2.3 \times \log_{10} 1000$$

$$= 2.3 \times 3 = \underline{6.9}\ \text{A.}$$

(Q) $\frac{1}{10} + \frac{1}{11} + \cdots + \frac{1}{500} = ?$

$$\int_{10}^{500} \frac{1}{x} = \ln 500 - \ln 10$$

$$= 2.3 \log_{10} 500 - 2.3 \log_{10} 10$$

$$= 2.3 \times 2.7 - 2.3$$

$$= 2.3 \times 1.7$$

$$= \underline{\underline{3.91}}\ .\ \checkmark$$

$\underline{87 \times 3}$

$\underline{\text{Tables}}$

$$\log_{10}(5 \times 100) = \log_{10}\left(\frac{1000}{2}\right)$$

$$= \log_{10} 1000 - \log_{10} 2$$

$$= 3 - 0.301 = 2.699$$

$$\simeq 2.7$$

Speed Math Task.

II. A.G.P : Arithmetico - Geometric Series.

$$\Rightarrow S = \underline{1} + \underline{4}\left(\frac{1}{5}\right) + \underline{7}\left(\frac{1}{5}\right)^2 + \underline{10}\left(\frac{1}{5}\right)^3 + \cdots\cdots \infty.$$

G.P.

A.P

Std. way.

$$S = 1 + 4\left(\frac{1}{5}\right) + 7\left(\frac{1}{5}\right)^2 + 10\left(\frac{1}{5}\right)^3 + \cdots\cdots \infty \quad -(1)$$

$\times r$

$$S \times \frac{1}{5} = \frac{1}{5} + 4\left(\frac{1}{5}\right)^2 + 7\left(\frac{1}{5}\right)^3 + \cdots\cdots \infty \quad -(2)$$

$$\frac{4S}{5} = 1 + 3\left(\frac{1}{5}\right) + 3\left(\frac{1}{5}\right)^2 + 3\left(\frac{1}{5}\right)^3 + \cdots \infty$$

$$\frac{4S}{5} = 1 + 3\left(\frac{1}{5} + \left(\frac{1}{5}\right)^2 + \left(\frac{1}{5}\right)^3 + \cdots \infty\right)$$

$$\frac{4S}{5} = 1 + 3\left(\frac{1/5}{1 - 1/5}\right)$$

$$\frac{4S}{5} = 1 + 3\left(\frac{1}{4}\right) = \frac{7}{4}$$

$$\frac{4S}{5} = \frac{7}{4}$$

$$S = \frac{35}{16} \quad \cdot \; A \cdot$$

Q) $1 + 4x + 7x^2 + 10x^3 + \dots\dots \infty = \frac{35}{16}$; $x = ?$

$|x| < 1$

A.P

G.P

Sol.

$S = 1 + 4x + 7x^2 + 10x^3 + \dots \infty$ — (1)

$\times r = x$

$Sx = x + 4x^2 + 7x^3 + \dots\dots \infty$

$S(1-x) = 1 + 3x + 3x^2 + 3x^3 + \dots \infty$

$S(1-x) = 1 + 3(x + x^2 + x^3 + \dots \infty)$

$S(1-x) = 1 + 3\left(\frac{x}{1-x}\right) \rightarrow S = 35/16$

$\frac{35}{16}(1-x) = 1 + \frac{3x}{1-x}$

$\frac{35-35x}{16} = \frac{1-x+3x}{1-x}$

$\vdots$

$x = 1/5$. A

G.P : $a, ar, ar^2, ar^3, \text{------}, ar^{n-1}$

$\times r \quad \times r \quad \times r$

$$\boxed{t_n = ar^{n-1}}$$

$-1 < r < 1$

1) $S_\infty = a + ar^2 + ar^3 + ar^4 + \text{----}\infty = \dfrac{a}{1-r}$; $|r| < 1$

$3 + 6 + 12 + 24 + 48 + \text{---} = \infty$

$\times 2 \quad \times 2 \quad \times 2 \quad \times 2$

$300 + 100 + 33.33 + 11.11 + \text{----} \Rightarrow S_\infty = \dfrac{a}{1-r}$

$\times \frac{1}{3} \quad \times \frac{1}{3}$

$r = 1/3$

$= \dfrac{300}{1-\frac{1}{3}} = \underline{450}$

A

2) $S_n = a + ar^2 + ar^n + \cdots + ar^{n-1}$

$$= \frac{a(1-r^n)}{(1-r)} \quad ; \; r < 1$$

$$= \frac{a(r^n-1)}{(r-1)} \quad ; r > 1$$

(Q) $1 + \frac{8}{19} + \frac{88}{19^2} + \frac{888}{19^3} + \dots \infty = ?$

$57/41$ $\quad$ $157/181$ $\quad$ $167/71$ $\quad$ $157/91$ $\quad$ $152/81$

(Q) N is the set of natural nos. is partitioned into groups –, $S_1 = \{1\}$, $S_2 = \{2, 3\}$, $S_3 = \{4, 5, 6\}$ - - - -

Find Sum of numbers in S_{50}.

62525 $\quad$ 31625 $\quad$ 15625 $\quad$ 75625 $\quad$ none of these

$S_1 = 1$

$S_2 = 2, 3 \leftarrow 1+2$

$S_3 = 4, 5, 6 \leftarrow 1+2+3$

$S_4 = 7, 8, 9, 10 \leftarrow 1+2+3+4$

$\vdots$

$S_{50} = \left(1226, 1227, 1228 - - - - - - -, \dfrac{1275}{50^{th}}\right) \leftarrow 1+2+3+ \dots +50 = \dfrac{50 \times 51}{2} = 1275$

$= \dfrac{50}{2}\left(1226 + 1275\right) = 25 \times 2501$

$= \underline{62525}$. Ans.

$\dfrac{n}{2}(a+l)$

1) $1 + \frac{8}{19} + \frac{88}{19^2} + \frac{888}{19^3} + \cdots$ (Type)

$\frac{1}{19} \xrightarrow{\times \frac{1}{19}} \frac{1}{19}^2 \xrightarrow{\times \frac{1}{19}} \frac{1}{19^3}$

$S = 1 + \frac{8}{19} + \frac{88}{19^2} + \frac{888}{19^3} + \cdots \infty \quad -(1)$

$\times r = \frac{1}{19}$

$\frac{S}{19} = \frac{1}{19} + \frac{8}{19^2} + \frac{88}{19^3} + \cdots \infty \quad -(2)$

$\frac{18S}{19} = 1 + \frac{7}{19} + \left[\frac{80}{19^2} + \frac{800}{19^3} + \frac{8000}{19^4} + \cdots \infty\right]$

$\frac{80}{19^2} \xrightarrow{\times 10/19} \frac{800}{19^3} \xrightarrow{\times 10/19} \frac{8000}{19^4}$

$$\frac{18S}{19} = 1 + \frac{7}{19} + \left[\frac{80/19^2}{1 - 10/19} \right]$$

$$\frac{18S}{19} = \frac{26}{19} + \frac{80}{19 \times 9}$$

$$18S = 26 + \frac{80}{9 \times \cancel{19}} \times \cancel{19}$$

$$18S = 26 + \frac{80}{9}$$

$$S = \frac{314}{9 \times 18} = \frac{157}{81} \quad A$$

Q) $\dfrac{2^3-1}{2^3+1} \times \dfrac{3^3-1}{3^3+1} \times \dfrac{4^3-1}{4^3+1} \ldots \times \dfrac{20^3-1}{20^3+1} = ?$

a) 411/600
b) 321/530
c) 421/630
d) 521/730
e) 511/750

Sol. $\dfrac{7}{9} \times \dfrac{26}{28} \times \dfrac{63}{65} \times - - -$ Wrong way (no pattern).

$t_n = \dfrac{n^3-1}{n^3+1} = \dfrac{(n-1)(n^2+n+1)}{(n+1)(n^2-n+1)}$

n=2. n=3 n=4 n=19 n=20

$\dfrac{1 \times 7}{3 \times 3} \times \dfrac{2 \times 13}{4 \times 7} \times \dfrac{3 \times 21}{5 \times 13} \times \ldots \times \dfrac{18 \times 381}{20 \times 343} \times \dfrac{19 \times 421}{21 \times 381}$

$\left(\dfrac{1}{\cancel{3}} \times \dfrac{2}{\cancel{4}} \times \dfrac{\cancel{3}}{\cancel{5}} \times \dfrac{\cancel{17}}{\cancel{19}} \times \dfrac{\cancel{18}}{20} \times \dfrac{\cancel{19}}{21}\right) \times \left(\dfrac{\cancel{7}}{3} \times \dfrac{\cancel{13}}{\cancel{7}} \times \dfrac{\cancel{21}}{\cancel{13}} \times \ldots \times \dfrac{\cancel{381}}{\cancel{343}} \times \dfrac{421}{\cancel{381}}\right)$

$$= \frac{1 \times \cancel{2}}{\underset{10}{\cancel{20}} \times 21} \times \frac{421}{3} = \frac{421}{630}$$

A.

(Q) $1^3 - 2^3 + 3^3 - 4^3 + 5^3 - 6^3 + \ldots + 51^3 = ?$

Sol. $\left[1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 + \ldots + 51^3\right] - 2^3 \times 2 - 4^3 \times 2 - 6^3 \times 2 - \ldots - 50^3 \times 2$

$$= \sum_{1}^{51} n^3 - 2 \times 2^3 \left[1^3 + 2^3 + 3^3 + \ldots + 25^3\right]$$

$$\sum n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

$$= \sum_{1}^{51} n^3 - 16 \sum_{1}^{25} n^3$$

$$= \left(\frac{51 \times 52}{2}\right)^2 - 16 \times \left(\frac{25 \times 26}{2}\right)^2$$

20 Qn → Ans. 1
2
3
4
1
1
1
1
3

Quant Assignm-t

Error free

Modul

Ineq + Ma. M.

Log

(Q) $1^2 - 2^2 + 3^2 - 4^2 + \cdots - + n^2$

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