Fraction Series

AGP

Splitting of D^r

Sequence | Series | Progressions – 7

(Q.1) $\frac{5}{1^2 \times 2^2} + \frac{11}{2^2 \times 3^2} + \frac{19}{3^2 \times 4^2} + \frac{29}{4^2 \times 5^2} + \cdots \infty$?

a) 1 b) 3/2 c) 9/4 d) 9/2 e) ~~none~~ of these. (2)

(Q.2) $\frac{1^2}{1} + \frac{1^2 + 2^2}{1+2} + \frac{1^2 + 2^2 + 3^2}{1+2+3} + \cdots$ upto 42 terms?

a) 512 b) 616 c) 726 d) 582 e) none of these.

(Q.3) 2 distinct 2-digit numbers 'p' & 'q' have integral A.M & G.M — A & G respectively, such that A can be obtained by interchanging digits of G & vice-versa. Find sum of digits of p+q.

a) 130 b) 120 c) 90 d) 8 e) ~~none of these~~ (4)

130
= 1+3+0
= 4

3) Sol

AM

$p, q \to \dfrac{p+q}{2} = \overline{ab} = 10a + b$ ✓

$\dfrac{10+20}{2} = 15$

Gm

$p, q \to (pq)^{1/2} = 10b + a \leftarrow \overline{ba}$ pq.

✓ $p + q = 2(10a+b)$ | $p+q = ?$

✓ $pq = (10b+a)^2$

$(p-q)^2 = p^2 + q^2 - 2pq.$

$= p^2 + q^2 + 2pq - 2pq - 2pq$

$(p-q)^2 = (p+q)^2 - 4pq$

$p - q =$ integer.

$(p-q)^2 = (20a+2b)^2 - 4(10b+a)^2$

$= 400a^2 + 4b^2 + \cancel{80ab} - 400b^2 - 4a^2 - \cancel{80ab}.$

$= 396(a^2 - b^2)$

$p - q = \sqrt{396}\sqrt{a^2-b^2} = 6\sqrt{11}\sqrt{a^2-b^2}$

$p - q = 6\sqrt{11}\sqrt{a^2 - b^2}$

$= 6\sqrt{11}\,\sqrt{11}$

$= 6 \times 11 = 66$

$a+b > a-b$

$p - q =$ integer.

$\downarrow$

$\sqrt{\ }$ free

; $a^2 - b^2 = 11$

$(a+b)(a-b) = 11 \times 1$

$1^2 = 1$

$2^2 = 4$

$3^2 = 9$

$4^2 = 16$

$5^2 = 25$

$6^2 = 36$

$a + b = 11$

$a - b = 1$

$2a = 12$

$$\boxed{\begin{matrix} a = 6 \\ b = 5 \end{matrix}}$$

$p + q = 2(10a + b)$

$= 2(10 \times 6 + 5)$

$= 130$

a, b

$ab \rightarrow 0$ to 9

2) Sol. $t_n = \dfrac{1^2 + 2^2 + \cdots + n^2}{1 + 2 + \cdots + n} = \dfrac{\dfrac{\cancel{n(n+1)}(2n+1)}{\cancel{6}\,3}}{\dfrac{\cancel{n(n+1)}}{\cancel{2}}} = \dfrac{2n+1}{3}$

$S_n = \Sigma t_n = \Sigma\left(\dfrac{2n+1}{3}\right) = \dfrac{1}{3}\left(\Sigma 2n + \Sigma 1\right)$

$= \dfrac{1}{3}\left(2\sum_1^n n + \sum_1^n 1\right)$ ← $1+1+\ldots n$ tim

$\dfrac{1}{3} + \dfrac{2}{3} + \dfrac{3}{3} + \cdots$

$\dfrac{1}{3}\left[1 + 2 + 3 + \cdots\right]$

$= \dfrac{1}{3}\left[2 \cdot \dfrac{n(n+1)}{2} + n\right]$

$= \dfrac{1}{\cancel{3}} \times \left[\dfrac{\cancel{2} \times 42 \times 43}{\cancel{2}} + \cancel{42}\,14\right]$

$= \underline{616}$. Ans.

1) Sol.

$$\frac{5}{1^2 \times 2^2} \qquad \frac{11}{2^2 \times 3^2} \qquad \frac{19}{3^2 \times 4^2}$$

$$\frac{(1+2)+(1\times 2)}{1^2\times 2^2} \qquad \frac{(2+3)+(2\times 3)}{2^2\times 3^2} \qquad \frac{(3+4)+(3\times 4)}{3^2\times 4^2} \; \ldots$$

Spcl. series

$$= \left(\frac{1+2}{1^2\times 2^2}+\frac{1}{1\times 2}\right)+\left(\frac{2+3}{2^2\times 3^2}+\frac{1}{2\times 3}\right)+\left(\frac{3+4}{3^2\times 4^2}+\frac{1}{3\times 4}\right)+\cdots$$

$$\frac{1}{1\times 2}+\frac{1}{2\times 3}+\frac{1}{3\times 4}+\cdots\infty = \left(\frac{1}{1}-\cancel{\frac{1}{2}}\right)+\left(\cancel{\frac{1}{2}}-\cancel{\frac{1}{3}}\right)+\left(\cancel{\frac{1}{3}}-\cancel{\frac{1}{4}}\right)\ldots$$

$$= 1.$$

$$\frac{3}{1^2\times 2^2}+\frac{5}{2^2\times 3^2}+\frac{7}{3^2\times 4^2}+\cdots = \frac{3}{4}+\frac{5}{36}+\frac{7}{144}+\cdots$$

$$= \frac{3}{1\times4} + \frac{5}{4\times9} + \frac{7}{9\times16} + \cdots$$

$$= 3\times\left(\frac{1}{1\times4}\right)$$

$$= \cancel{3}\times\frac{1}{\cancel{3}}\left[\frac{1}{1}-\cancel{\frac{1}{4}}\right]+\left(\cancel{\frac{1}{4}}-\cancel{\frac{1}{9}}\right)+\left(\cancel{\frac{1}{9}}-\cancel{\frac{1}{16}}\right)+\cdots\infty$$

$$= 1$$

Ans. $1+1=\underline{2}$. A.

$$\frac{1}{36}=\frac{1}{\underbrace{4\times9}_{5}}$$

$$=\frac{1}{5}\left[\frac{1}{4}-\frac{1}{9}\right]$$

$$\frac{1}{4}=\frac{1}{\underbrace{1\times4}_{3}}$$

$$=\frac{1}{3}\left[\frac{1}{1}-\frac{1}{4}\right]$$

(Q4) The sum of 1st n (n > 3) terms of an A.P is 513. If 1st term of the sequence is an integer and the common difference of the series is 2, then find out the number of possible values of n.

a) 6
b) 7
c) 5
d) 8
e) 9

Sol.

$S_n = 513$

$\frac{n}{2}\left[2a + (n-1)d\right] = 513$

$\frac{n}{\cancel{2}}\left[\cancel{2}a + (n-1)\cancel{2}\right] = 513$

$n\left[a + (n-1)\right] = 19 \times 27 = 3^3 \times 19^1$

$3^0, 3^1, 3^2, 3^3 \quad 19^0, 19^1 \rightarrow 4 \times 2 = 8$ f.

$n = 1$ ✗
$n = 3$ ✗ $\} - 2$

$8 - 2 = 6$ Ans.

~~1~~, ~~3~~, 9, 19, 57, 27, 171, 513 ← 6 Ans.

Q5) $\frac{7xy}{x+y} = 12$, $\frac{9yz}{y+z} = 20$, $\frac{8xz}{z+x} = 15$; $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are in

a) A.P b) G.P ~~c) H.P~~ d) none of these.

Sol: $\frac{xy}{x+y} = \frac{12}{7}$ | $\frac{y+z}{yz} = \frac{9}{20}$ | $\frac{z+x}{xz} = \frac{8}{15}$

$\frac{x+y}{x.y} = \frac{7}{12}$ | $\frac{1}{z} + \frac{1}{y} = \frac{9}{20}$ | $\frac{1}{z} + \frac{1}{x} = \frac{8}{15}$

minus

$\frac{1}{y} + \frac{1}{x} = \frac{7}{12}$ —(1)

$\frac{1}{z} + \frac{1}{y} = \frac{9}{20}$ —(2)

$\frac{1}{z} + \frac{1}{x} = \frac{8}{15}$ —(3)

Add: $\cancel{2}\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right) = \frac{7}{12} + \frac{9}{20} + \frac{8}{15}$

$= \frac{35 + 27 + 32}{60 \times 2} = \frac{47}{60}$

$\frac{1}{z} = \frac{47}{60} - \frac{7}{12} = \frac{1}{5}$

$\frac{1}{x} = \frac{1}{3}$ | $\frac{1}{y} = \frac{1}{4}$ | $x = 3$, $y = 4$, $z = 5$ $\Big\}$ x, y, z A.P

Q) $x = 1111\ldots\ldots 24$ times. Find sum of digits in final no. of

$y = 3333\ldots\ldots 12$ times.

$z = 4444\ldots\ldots 12$ times.

$$\frac{x - 3y}{z}.$$

a) 93
b) 97
c) 94
d) 95
e) none of these.

Sol. $A = 1111\ldots\ldots 12$ times

$y = 3A$.

$z = 4A$

$$\frac{x-3y}{z} = \frac{(10^{12}A + A) - 3.3A}{4A} = \frac{\cancel{A}(10^{12}+1) - 9\cancel{A}}{4\cancel{A}} = \frac{10^{12}-8}{4}$$

$$= \frac{\cancel{10^2}^{25}.10^{10} - \cancel{8}^{2}}{\cancel{4}} = 25\times 10^{10} - 2 = \underbrace{24999999998}. = 95.\ A.$$

$25 \times 10^{10} - 2 = ?$

$25 \times 10 - 2 = 248$

$25 \times 10^{2} - 2 = 2498$

$25 \times 10^{3} - 2 = 24998$

$\times 10^{4} = 24\underline{9998}$

$A = 11.$

$$\underline{1111} = \overbrace{10^3\times 1 + 10^2\times 1} + \overbrace{10^1\times 1 + 10^0\times 1}$$

$$= 10^2(11) + 10^0(11).$$

$$= \underline{10^2(A)} + \underline{A.}$$

$$A = \overbrace{111\,---\,1}^{12\text{ times}}$$

$$\overbrace{1111\,---\,-----\,1}^{24\text{ times}}$$

$$= 10^{12}A + A.$$

$$\overbrace{1\times 10^{23} --- + 1\times 10^{14} + 1\times 10^{13} + 1\times 10^{12}} + 1\times 10^{11} - + 1\times 10^{2} + 1\times 10^{1} + 1\times 10^{0}$$

$$10^{12}(\underbrace{11111\ldots 1}_{12\text{ times}}) + 10^{0}(\underbrace{111\ldots 1}_{12\text{ times}}) = 10^{12}A + A$$

Functions (Start Tomorrow)