CHRIST UNIVERSITY,BENGALURU - 560029

End Semester Examination October - 2015

Bachelor of Computer Applications III SEMESTER

Code: BCA332 **Max.Marks: 100**

Subject: INTRODUCTORY ALGEBRA **Duration: 3Hrs**

SECTION A

Answer any 10 Questions. **10X3=30**

1 For integers a, b and c, if a|b and a|c, then a|(b+c). Prove or disprove!

2 Let m be a positive integer. The integers a and b are congruent modulo m if and only if there is an integer k such that a=b+km

3 The sum of any three consecutive integers is divisible by 3. Prove or disprove!

4 What is the decimal expansion of the integer $(101011111)_2$?

5 Find the prime factorization of 7007

6 Solve the congruence $2x \equiv 7 \pmod{11}$

7 Find an inverse of 144 modulo 233

8 Decrypt HDW GLP VXP encrypted using Caesar cipher

9 Let N be the set of all natural numbers and for a and b in N such that a*b=a+b+3. Is * an associative operation? Justify your answer

10 Show that set of all integers with the binary operation multiplication (x) is not a group

11 Let (A, *) be a semigroup such that, for all a, b in A, a*b=b*a implies that a=b. Show that for every x, y in A, x*y*x=x

12 Let (B, ♦) be group. Show that, for any two elements a and b in B, $(a♦b)^{-1} = b^{-1}♦a^{-1}$

SECTION B

Answer any 3 Questions. **3X7=21**

13 Find all solutions to the system of congruences $x \equiv 1 \pmod{2}$, $x \equiv 2 \pmod{3}$, $x \equiv 3 \pmod{5}$ and $x \equiv 4 \pmod{11}$

14 Solve $2^{340} \bmod 11$

15 What is the message produced from the message LOVE THY NEIGHBOUR using the function f(p)= (7p+3) mod 26? Is the message decryptable? Justify your answer

16 Encrypt the message FIRE using RSA cryptosystem with the key (13, 2537). Briefly explain the method of decryption

SECTION C

Answer any 4 Questions. **4X7=28**

17 Give an algorithm to find the number of Sundays in *n* consecutive days

18 Show that if a and b are congruent modulo 7, then 10a+13 and -4b+20 are also congruent modulo 7

19 Use the method of back substitution to find the least positive integer x such that when divided by 5, 6 and 7, it leaves the remainders 1, 2 and 3 respectively.

20 Prove that for positive integers a and b, lcm(a, b)gcd(a, b)=ab

21 If n is a composite integer, then show that n has a prime divisor $\leq \sqrt{n}$

SECTION D

Answer any 3 Questions **3X7=21**

22 Define (a) semigroup and (b) monoid. Give an example of a semigroup that is not a monoid. Give an example of a monoid that is not a group

23 Find all the symmetric transformation of a square

24 For a group, prove that any two left cosets are either identical or they are distinct

25 Show that if a group (G, ¤) is of order n and *a* in G such that a^m=e (identity) for some integer m less than or equal to n, then m|n