Summary

- **Differential calculus**: Rate of change, basic concepts: limits, computational tools, relation to approximation, tangent lines, use in min/max, graphing
- **Integral calculus**: Accumulation/Summation summing an infinite number of infinitesimally small things, basic concepts: limits, relation to area, volume, averages, computational techniques
- FTC: $F(x) = \int_{t=0}^{x} f(t)dt$, $\frac{dF}{dx} = f(x)$
- **Vector** is a quantity with magnitude and direction
- Distance between points (x_1, y_1, z_1) and (x_2, y_2, z_2) is given by $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$
- Arc length $l = \int_{t=t_{start}}^{t_{end}} |\vec{r}'(t)|\, dt$ assuming we only traverse the curve once
- If we have $F(x, y, z) = 0$ we can differentiate z implicitly without solving, get $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$, $\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}$
- Example $c^2 = a^2 + b^2 - 2ab\cos\theta$ then we get $F = a^2 + b^2 - 2ab\cos\theta - c^2$ then $F_\theta = 2ab\sin\theta$, $F_a = 2a - 2b\cos\theta$, and $F_c = -2c$ then $\frac{\partial\theta}{\partial c} = -\frac{F_c}{F_\theta} = \frac{c}{ab\sin\theta}$ and $\frac{\partial\theta}{\partial a} = -\frac{F_a}{F_\theta} = \frac{b\cos\theta - a}{ab\sin\theta}$
- Tangent plance equation $z = z_0 + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$
- Directional derivative: $(D_{\vec{u}} f)(x_0, y_0)$ gives the rate of change of f at x_0, y_0 in the direction of $\vec{u}$
- $(D_{\vec{u}} f)(x_0, y_0) = (\nabla f)(x_0, y_0) \cdot \vec{u}$
- When traveling along a contour line then the directional derivative is going to be 0 ∇f is perpendicular to contour lines
- $D = f_{xx} f_{yy} - f_{xy}^2$
- if (x_0, y_0) is a critical point then:
- If $D(x_0, y_0) < 0$ saddle point
- If $D(x_0, y_0) > 0$ if $f_{xx} > 0$ local min and if $f_{yy} < 0$ local max
- To find min/max of f constrained to a curve we can parameterize to get one variable problem
- **LAGRANGE** $\begin{cases} f_x = \lambda g_x \\ f_y = \lambda g_y \\ g(x, y) = C \end{cases}$
- EXAM 2 MATERIAL ALREADY REVIEWED