Review for midterm 2

Torsion

Part 1

- Assume isotropic material
- the cross sections rotate but remain planar in the cylinder
- L is the length of the shaft
- r is an arbitrary radius
- T is the applied torque
- ϕ is the angle by which one end of the shaft rotates
- γ is the shear strain experienced at some radius r
- $L\gamma_{surf} = r_{surf}\phi$ or for an arbitrary radius, $L\gamma = r\phi$
- Using Hooke's law for shear, $\tau = G\gamma = \frac{Gr}{L}\phi$
- $\phi = \frac{TL}{GJ}$, this is for cylinders since J is the polar moment of area
- $J \equiv \begin{cases} \frac{\pi}{2} r_{surf}^4 & \text{Solid shaft} \\ \frac{\pi}{2}(r_{surf}^4 - r_{in}^4) & \text{Hollow shaft any thickness} \\ 2\pi t r_{surf}^3 & \text{thickness} \frac{t}{r_{surf}} < 0.01 \end{cases}$
- $\frac{\tau}{r} = \frac{T}{J} = \frac{G}{L}\phi$
- $\gamma(r) = \frac{\tau(r)}{G}$
- $\delta = \frac{FL}{AE}$
- $\phi = \frac{TL}{JG}$

Part 2

- $F_{||} = \frac{T}{r}$
- $\frac{T_1}{r_1} = \frac{T_2}{r_2}$
- $\delta_1 = -\delta_2 \Rightarrow r_1\phi_1 = -r_2\phi_2$
- In cases of various properties and loading remember to make cuts

Shear And Bending Moment diagrams

- $V(z)$ is the shear force at z
- $M(z)$ is the bending moment at z
- Positive shear pulls the right side down
- Positive moment turns region into a smile

- Let $w(z)$ be a distributed force then $V(z) = \int_0^z w_{\text{upward}}(z)\,dz + \sum_0^z F_{\text{upward}}(z)$ and $M(z) = \int_0^z V(z)dz + \sum_0^z T_{\text{cw}}(z)$

Pure Bending

- For some loaded beam, a point z is said to be in a state of pure bending if $V(z) = 0$ and $M(z) \neq 0$
- Thee neutral layer is the one layer in the beam that has not changed length
- z horizontal coordinate along beam
- ρ radius of curvature, positive for smile deformation
- y distance above the neutral layer
- y^* co-ordinate system parallel to y but not necessarily having 0 at neutral axis
- y^*_{NA} location of the neutral axis in the y^* coordinate system
- $\varepsilon_z = -\frac{y}{\rho}$
- $\sigma_z = E\varepsilon_z = -E \cdot \frac{y}{\rho}$
- I_x second moment of inertia which in solid mechanics represents how strongly the cross section resists bending about the x axis
- $\frac{\sigma_z}{-y} = \frac{M_x}{I_x} = \frac{E}{\rho}$
- The peak bending stress occurs at the maximum $|y|$
- For composite shapes:
    - $y^*_{\text{NA}} = \frac{1}{A_{\text{total}}} \sum_{i=1}^{n_{\text{components}}} y^*_{\text{NA},i} A_i$
    - $I_x = \sum_{i=1}^{n_{\text{components}}} I_{x,i} + \sum_{i=1}^{n_{\text{components}}} A_i (y^*_{\text{NA},\,i} - y^*_{\text{NA}})^2$

Working With Moments of Area

- In the xy plane,
    - $I_x = \int_{A_\perp} y^2 dA$
    - $I_y = \int_{A_\perp} x^2 dA$
    - $J = I_x + I_y$
- Material father from the axis resist bending/torsion more strongly

Bending of Composite Beams

- Increasing E at some point is the same as increasing local width
- To solve these questions we choose a reference material and then we adjust the axis parallel to the neutral axis to $L_{\text{new}} = L \cdot \frac{E}{E_{\text{ref}}}$
- After doing this, to get stresses we must use $\sigma_{\text{actual}} = \sigma \cdot \frac{E}{E_{ref}}$