Fourier Series

- When you solve the heat equation you get a function $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right)$ which is a Fourier series

> **Definition**
> A function is piecewise continuous in the interval $[a, b]$ if it has a finite number of discontinuities in $[a, b]$ and has finite left and right limit at the points of discontinuity

- Fourier series applies to the periodic extension of $f(x)$, denoted $f_{\text{per}}(x)$
- 1 Even extension, Neumann Boundary condition we mirror what happens from $0 \to L$ is mirrored across the axes to $0 \to -L$
- 2 Odd extension , Dirichlet Boundary Condition, what happens from $0 \to L$ would be flipped from $0 \to -L$
- 3 Periodic Extension, periodic boundary conditions, what happens from $0 \to L$ is the same as what happens from $-L \to 0$
- Convergence if $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right)$ Fourier series converges to $f(x)$ at all x where $f(x)$ is continuous and to $\frac{1}{2}[f(x^+ + f(x^-))]$
- At the point of discontinuity the Fourier series struggles and oscillates, this is called the Gibbs phenomenon
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