Review for Final

Uniaxial Loading

- δ is called the extension or deflection and is negative in compression
- $\sigma \equiv \frac{F_{tensile}}{A_\perp}$
- $\varepsilon \equiv \frac{\delta}{L_o}$
- $E = \frac{\sigma}{\varepsilon}$
- $\delta = \sigma \cdot \frac{L_0}{E}$

Equilibrium

- Relates forces to forces
- Make internal cuts to solve for forces

Trusses

- Members must be long and thin

Zero force members

- Consider one member pins
  - Identify all the pins that connect to one member
  - Eliminate those at which an external load or reaction acts
  - At each pin which remains, the connected member is a zero force member
- Consider two member pins
  - Identify all the pins that connect two members
  - Eliminate those at which an external load or reaction acts
  - Eliminate those at which the two connected members are parallel
  - At each pin which remains, the both of the members connected are zero force members
- Consider three member pins
  - Identify all the pins which connect three members
  - Eliminate those at which an external load or reaction acts
  - Eliminate those where none of the members are parallel
  - At each pin which remains, the non-parallel member connected is a zero force member

- **Repeat**

Poissons Ratio

- Values from -1 to 0.5
- Properties are the same in all directions for isotropic materials, not for anisotropic
- $\varepsilon = \frac{1}{E}[\sigma_{\parallel} - \nu(\sigma_{\perp_1} + \sigma_{\perp_2})]$
- $\sigma = \left(\frac{E}{(1+\nu)(1-2\nu)}\right)[(1-\nu)\varepsilon_{\parallel} + \nu(\varepsilon_{\perp_1} + \varepsilon_{\perp_2})]$
- For small strain we have $\varepsilon_V = \varepsilon_x + \varepsilon_y + \varepsilon_z$
- Modulus of rigidity $G = \frac{E}{2(1+\nu)}$
- Bulk modulus $K = \frac{E}{3(1-2\nu)}$
- $\sigma_0 = \frac{\sigma_x+\sigma_y+\sigma_z}{3}$
- $\sigma_0 = K\varepsilon_V$ not a needed equation
- For most engineering materials small stresses like atm are negligible

Shear

- $\tau = \frac{V}{A}$
- $\gamma \approx \frac{\delta}{L}$
- For shear stress the first subscript indicates the plane and the second the direction
- $\tau_{xy} = \tau_{yx}$
- $\tau = G\gamma$

Thermal Equations

- Small strain is valid up to 10%
- $\varepsilon = \frac{1}{E}[\sigma_{\parallel} - \nu(\sigma_{\perp_1} + \sigma_{\perp_2})] + \alpha_L \Delta T$
- $\sigma = \left(\frac{E}{(1+\nu)(1-2\nu)}\right)[(1-\nu)\varepsilon_{\parallel} + \nu(\varepsilon_{\perp_1} + \varepsilon_{\perp_2})] - \left(\frac{E}{1-2\nu}\right)\alpha_L \Delta T$
- Thermal stresses do not affect shear

Stress Concentration

- Stress distribution will be uniform far from all applied loads and geometry changes
- σ_{max}: maximum observable stress at any point
- σ_{nom}: the average stress at the smallest cross-section
- K: a dimensionless number relating the two stresses above $K \equiv \left(\frac{\sigma_{\text{max}}}{\sigma_{\text{nom}}}\right)$
- K depends only on geometry
- Ductile material $f_s = \frac{|\sigma_{\text{nom}}|}{\sigma_Y}$
- Brittle material $f_s = \frac{|\sigma_{\text{max}}|}{\sigma_{UT}}$ $f_s = \frac{|\sigma_{\text{max}}|}{\sigma_{UC}}$

- Failure when the factor of safety is less than one

Torsion

Part 1

- Assume isotropic material
- the cross sections rotate but remain planar in the cylinder
- L is the length of the shaft
- r is an arbitrary radius
- T is the applied torque
- ϕ is the angle by which one end of the shaft rotates
- γ is the shear strain experienced at some radius r
- $L\gamma_{surf} = r_{surf}\phi$ or for an arbitrary radius, $L\gamma = r\phi$
- Using Hooke's law for shear, $\tau = G\gamma = \frac{Gr}{L}\phi$
- $\phi = \frac{TL}{GJ}$, this is for cylinders since J is the polar moment of area
- $J \equiv \begin{cases} \frac{\pi}{2} r_{surf}^4 & \text{Solid shaft} \\ \frac{\pi}{2}(r_{surf}^4 - r_{in}^4) & \text{Hollow shaft any thickness} \\ 2\pi t r_{surf}^3 & \text{thickness} \frac{t}{r_{surf}} < 0.01 \end{cases}$
- $\frac{\tau}{r} = \frac{T}{J} = \frac{G}{L}\phi$
- $\gamma(r) = \frac{\tau(r)}{G}$
- $\delta = \frac{FL}{AE}$
- $\phi = \frac{TL}{JG}$

Part 2

- $F_{||} = \frac{T}{r}$
- $\frac{T_1}{r_1} = \frac{T_2}{r_2}$
- $\delta_1 = -\delta_2 \Rightarrow r_1\phi_1 = -r_2\phi_2$
- In cases of various properties and loading remember to make cuts

Shear And Bending Moment diagrams

- $V(z)$ is the shear force at z
- $M(z)$ is the bending moment at z
- Positive shear pulls the right side down
- Positive moment turns region into a smile
- Let $w(z)$ be a distributed force then $V(z) = \int_0^z w_{\text{upward}}(z)\,dz + \sum_0^z F_{\text{upward}}(z)$ and $M(z) = \int_0^z V(z)dz + \sum_0^z T_{\text{cw}}(z)$

Pure Bending

- For some loaded beam, a point z is said to be in a state of pure bending if $V(z) = 0$ and $M(z) \neq 0$
- Thee neutral layer is the one layer in the beam that has not changed length
- z horizontal coordinate along beam
- ρ radius of curvature, positive for smile deformation
- y distance above the neutral layer
- y^* co-ordinate system parallel to y but not necessarily having 0 at neutral axis
- y^*_{NA} location of the neutral axis in the y^* coordinate system
- $\varepsilon_z = -\frac{y}{\rho}$
- $\sigma_z = E\varepsilon_z = -E \cdot \frac{y}{\rho}$
- I_x second moment of inertia which in solid mechanics represents how strongly the cross section resists bending about the x axis
- $\frac{\sigma_z}{-y} = \frac{M_x}{I_x} = \frac{E}{\rho}$
- The peak bending stress occurs at the maximum $|y|$
- For composite shapes:
  - $y^*_{\text{NA}} = \frac{1}{A_{\text{total}}} \sum_{i=1}^{n_{\text{components}}} y^*_{\text{NA},i} A_i$
  - $I_x = \sum_{i=1}^{n_{\text{components}}} I_{x,i} + \sum_{i=1}^{n_{\text{components}}} A_i (y^*_{\text{NA},\,i} - y^*_{\text{NA}})^2$

Working With Moments of Area

- In the xy plane,
  - $I_x = \int_{A_\perp} y^2 dA$
  - $I_y = \int_{A_\perp} x^2 dA$
  - $J = I_x + I_y$
- Material father from the axis resist bending/torsion more strongly

Bending of Composite Beams

- Increasing E at some point is the same as increasing local width
- To solve these questions we choose a reference material and then we adjust the axis parallel to the neutral axis to $L_{\text{new}} = L \cdot \frac{E}{E_{\text{ref}}}$

Plane Stress Transformation

- Plane Stress: a system is in a state of plane stress if there is at least one coordinate system for which $\sigma_z = \tau_{zx} = \tau_{zy} = 0$
- Use Mohrs circle for the transformations

- Remember to double the angle when moving to mohrs circle

Principle Stresses

- At each point in an arbitrarily loaded material there exists at least one principle coordinate system where all shear stresses are zero
- The three normal stresses are called the principle stresses
- The standard notation is $\sigma_1 \geq \sigma_2 \geq \sigma_3$
- When all the principle stress are equal the 3D Mohrs circle is a dot and it is in hydrostatic stress

Maximum Shear & Tresca Criterion

- If the shear on any plane gets too large that plain will fail
- Consider a Mohrs circle, $f_s^{\text{Tresca}} = \frac{\sigma_{\frac{Y}{2}}}{\tau_{\max}}$
- The plane with the highest shear stress is called the critical plane which is the plane that is closest to failing

Von Mises and Mohr Criteria

- $\frac{1}{2}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2] = \left(\frac{\sigma_Y}{f_s^{\text{vM}}}\right)^2$ using the principle stresses
- The von Mises equivalent stress is a single value that quantifies how intense the loading is overall and is also applicable to isotropic ductile materials
- The von Mises is more accurate than the Tresca model but the Tresca model is still acceptable since it is conservative
- $f_s^{\text{vM}} \geq f_s^{\text{Tresca}}$, $f_s^{\text{vM}} \leq \left(\frac{2}{\sqrt{3}}\right) f_s^{\text{Tresca}}$ the maximum discrepancy is quite small
- The Mohr criterion is for brittle materials
- $\frac{\sigma_{\max}}{\sigma_{UT}} - \frac{\sigma_{\min}}{|\sigma_{UC}|} = \frac{1}{f_S^{\text{Mohr}}}$ this is a reasonable model for isotropic brittle materials
- **Selecting a Criterion**
  - Brittle $\longleftrightarrow$ Mohr
  - Ductile, Simple & Conservative $\longleftrightarrow$ Tresca
  - Ductile, Complex & Precise $\longleftrightarrow$ Von Mises

Thin Walled Pressure Vessels

- $P_{\text{gauge}} = P_{\text{abs}} - P_{\text{atm}}$
- $\sigma_{\text{gauge}} = \sigma_{\text{abs}} - \sigma_{\text{atm}}$
- For fluids, $\sigma > P_{\text{atm}} \implies$ negative absolute pressure; gases fail (pull apart), liquids metastable

- $\sigma < P_{\text{atm}} \implies$ positive absolute pressure; stable
- $\tau \neq 0$ failure(flowing)
- Mohrs dot is the only Mohrs circle that can fir inside the failure envelope
- In a pressure vessel, P is the Gauge pressure inside the tank $P = P_{\text{inside}} - P_{\text{outside}}$
- If the tank is pressurized from the outside we have $P < 0$
- The pressure in a liquid increases with depth according to $P = \rho g h$, this formula does not apply to gases
- **Thin walled** $\implies \frac{t}{r} \leq 0.1$
- Use inner radius
- σ_{ax}: the axial stress in a cylinder
- σ_θ^{cyl}: the "circumferential" or "hoop" stress in a cylinder
- σ_θ^{sph}: the "circumferential" or "hoop" stress in a sphere
- $\sigma_\theta^{cyl} = \frac{Pr}{t}$
- $\sigma_{ax} = \frac{Pr}{2t(1+\frac{t}{2r})} \approx \frac{Pr}{2t}$
- $\sigma_\theta^{sph} = \sigma_{ax}$
- For a cylinder $\varepsilon_V \approx 2\varepsilon_\theta + \varepsilon_{ax}$

Strain Transformation & Rosettes

- $\varepsilon_x' = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy} \sin\theta \cos\theta$ where θ is the angle to the x' direction from the reference