

# Summary

## ODE Review

### Separation of Variables

### Linear First Order ODE

### Constant Coefficients 2nd Order

### Cauchy-Euler ODE

### Power Series Method

### Singular Points

- Classification of Singular Points
- Frobenius method for RSP
- Bessel Equation

## PDES

### Heat Equation

- Finite Difference Method
  - Discretizing
  - Difference Schemes
- Boundary conditions
  - Dirichlet
    - $\lambda_n = -\left(\frac{n\pi}{L}\right)^2, \quad \phi_n = \sin\left(\frac{n\pi x}{L}\right)$
  - Neumann
    - $\lambda_n = -\left(\frac{n\pi}{L}\right)^2, \quad \phi_n = \cos\left(\frac{n\pi x}{L}\right)$
  - Periodic
    - $\lambda_n = -\left(\frac{n\pi}{L}\right)^2, \quad \phi_n = \begin{cases} \cos\left(\frac{n\pi x}{L}\right) \\ \sin\left(\frac{n\pi x}{L}\right) \end{cases}$
- Mixed Type A
  - $\lambda_n = -\left(\frac{(2n+1)\pi}{2L}\right)^2, \quad \phi_n = \sin\left(\frac{(2n+1)\pi x}{2L}\right)$

- Mixed Type B

$$\lambda_n = -\left(\frac{(2n+1)\pi}{2L}\right)^2, \quad \phi_n = \cos\left(\frac{(2n+1)\pi x}{2L}\right)$$

- Separation of Variables
  - Fourier Series
  - Orthogonality
  - Heat Equation in a ring
  - Non homogeneous equations
    - Particular solutions
    - Steady state problems

## Wave Equation

- Finite Difference Method
  - Discretizing
  - Difference Schemes
- Boundary conditions
  - Same as heat equation
- Separation of Variables
  - Fourier Series
  - Orthogonality
  - Heat Equation in a ring
  - Non homogeneous equations
    - Particular solutions
    - Steady state problems
- d'Alembert's Solution

## Laplace Equation

- Finite Difference Method
  - Discretizing
  - Difference Schemes
  - Jacobian Iteration Scheme
- Boundary conditions
  - Same as heat equation
- Domains
  - Rectangular domain
  - Disk
  - Semi Infinite Strip
- Separation of Variables
  - Fourier Series
  - Orthogonality

- Heat Equation in a ring
- Non homogeneous equations
  - Particular solutions
  - Steady state problems

## Sturm-Liouville

- $\mathcal{L}y = -\frac{d}{dx}\left(p(x)\frac{dy}{dx}\right) + q(x)y(x)$ , the SL form
- Periodic boundary conditions are not SL problems
- Regular SL problems
  - Eigenvalues
  - Eigenfunctions
- Lagrange's Identity
- Robin Type Boundaries for Heat Equation