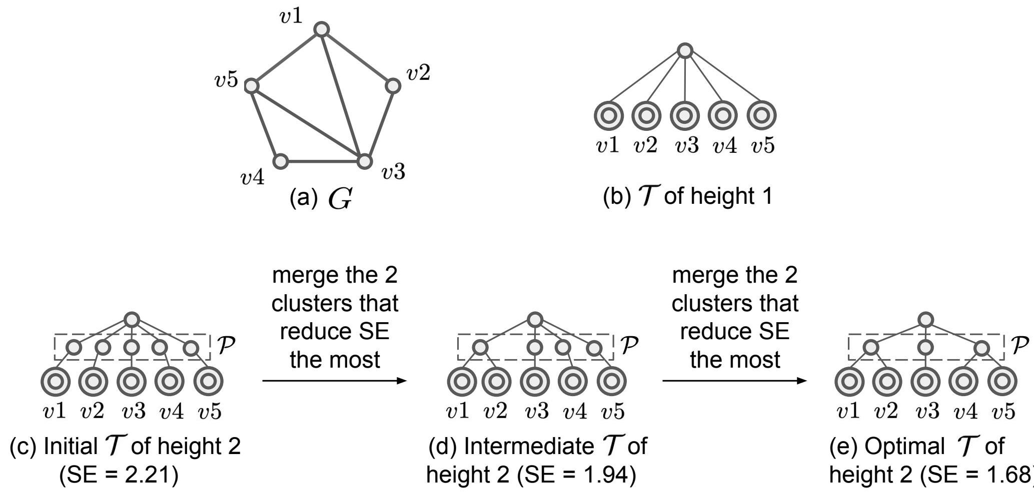

Figure 4: Examples of single-relational graph, encoding tree, and 2D SE minimization. (a) is a single-relational graph G. (b) is the encoding tree of height 1, which encodes the 1st-order structures, i.e., nodes, in G. (c) - (e) demonstrate how 2D SE minimization detects the 2nd-order structures, i.e., communities, in G. Initially, each node in G is assigned to its own cluster. $\mathcal{P}$ in (c) shows the initial clusters. Following the vanilla greedy 2D SE minimization algorithm, at each step, any two clusters that would reduce SE the most are merged. Eventually, the optimal encoding tree of height 2, as shown in (e), is associated with the minimum possible SE value, and encodes the communities, in G. $\mathcal{P}$ in (e) shows the detected communities.

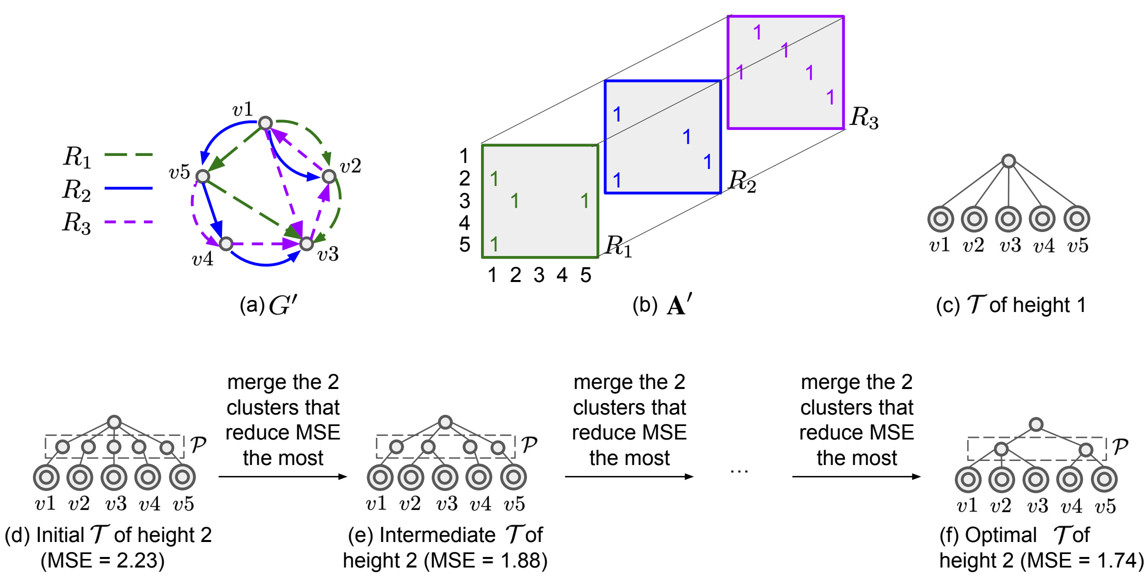

Figure 5: Examples of multi-relational graph, encoding tree, and 2D MSE minimization. (a) is a multi-relational graph G'. (b) is the adjacency tensor of G'. (c) is the encoding tree of height 1, which encodes the 1st-order structures, i.e., nodes, in G'. (d) - (f) demonstrate how 2D MSE minimization detects the 2nd-order structures, i.e., communities, in G'. Initially, each node in G' is assigned to its own cluster. $\mathcal{P}$ in (d) shows the initial clusters. Following our proposed 2D MSE minimization algorithm, at each step, any two clusters that would reduce MSE the most are merged. Eventually, the optimal encoding tree of height 2, as shown in (f), is associated with the minimum possible MSE value, and encodes the communities, in G'. $\mathcal{P}$ in (f) shows the detected communities.