CSGI.GA.2270 – Computer Graphics

2022 Fall Assignments

Due date: 11.15.2022 23:59

Assignment 3 - Written Assignment

Question 1)

A bounding volume is a basic volumetric geometric primitive that encloses a set of points or triangles. The choice of a type of bounding volume is crucial for the efficiency to detect collisions. The cost of the collision detection consists of the cost to refit the bounding volume to the (possibly) changing underlying geometry, and the cost to test to bounding volumes for intersection. Further, tighter-fitting bounding volume correspond to less ’false positives’ (a ’false positive’ corresponds to the situation where two bounding volumes intersect, but not their enclosed geometries). For the moment, assume that the cost to test the bounding volumes for intersection is negligible in contrast to the cost of refitting the bounding volumes.

Sort the four bounding volume types

- Axis-aligned bounding boxes
- Oriented bounding boxes
- Bounding spheres

according to their collision detection efficiency for each of the following situations:

a. Testing whether a player object given by a single 3d-position intersects with a static world scenery (consider the bounding volumes for the static world).
b. Testing two rigid bodies for intersection.
c. Testing two deformable bodies for intersection.

Question 2)

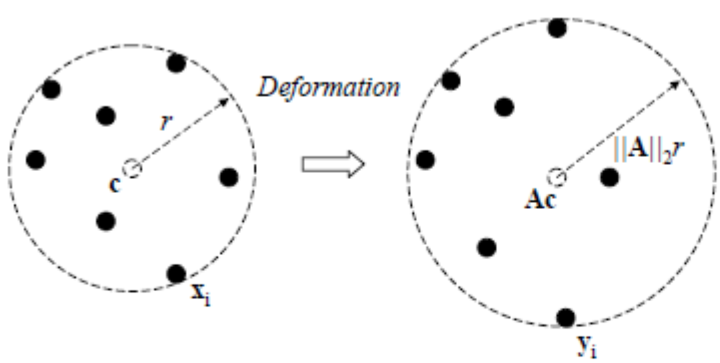
In this exercise, we build a bounding volume hierarchy for an object given as a triangulated surface. We choose axis-aligned bounding boxes (AABB) as bounding volume type. To construct the hierarchy, we start with the AABB enclosing the whole object. We then subdivide the enclosing geometry in two parts and compute the AABBs for each part. By proceeding recursively until arriving at the single triangles, we build the hierarchy layer by layer.

a) Assume that the object surface consists of n triangles. Give the order of the number of AABBs in the hierarchy tree. Hint: Proceed from the leaves to the root of the tree.

b) We now detect collisions between two such triangulated surfaces. Let c << n be the number of intersecting triangles. Give an upper bound on the number of AABB intersection tests.
Hint: First compute the length of a query path along the hierarchy.

Question 3)

Given is a geometry consisting of a set of points $\mathbf{x_i}$. For this set of points, we have constructed a bounding sphere enclosing the set of points. The bounding sphere is given by a center $\mathbf{c}$ and a radius r. This set of points is now linearly deformed. That means, we have a linear transformation matrix $\mathbf{A}$. By multiplying each position vector $\mathbf{x_i}$ with $\mathbf{A}$, we arrive at the deformed set of points with positions $\mathbf{y_i} = \mathbf{Ax_i}$.



Prove that by multiplying the center $\mathbf{c}$ of the sphere by $\mathbf{A}$, and the radius r of the sphere by $||A||_2$, the updated sphere still encloses all deformed points $\mathbf{y_i}$.

Hint: To accomplish this task, consider the precondition that the undeformed bounding sphere ($\mathbf{c}$; r) encloses all undeformed points $\mathbf{x_i}$, i. e. $||\mathbf{xi} - c||_2 \leq r$ for all points i. Now show that after deformation, the condition still holds if the sphere is updated accordingly. Use the Cauchy-Schwarz inequality.