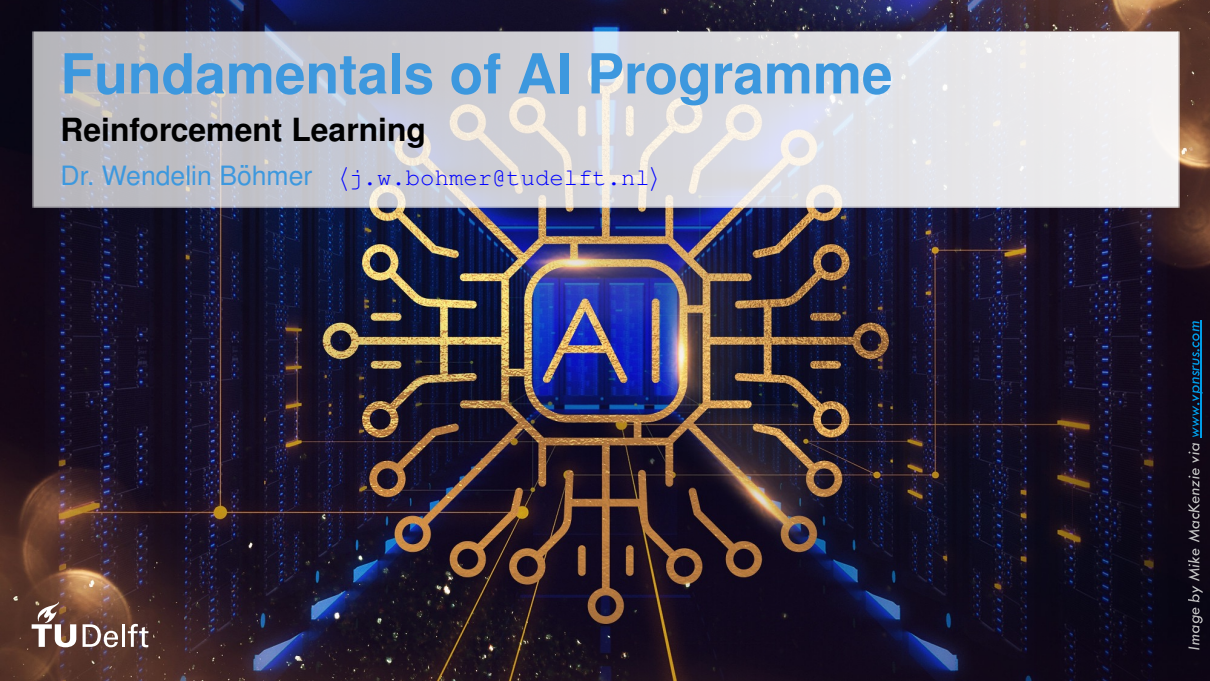
Fundamentals of AI Programme

Reinforcement Learning

Dr. Wendelin Böhmer ⟨j.w.bohmer@tudelft.nl⟩



TU Delft

Image by Mike MacKenzie via www.vpnsrus.com

Content of this lecture

1. Learning to Interact
2. Tabular Q-learning
3. Deep Reinforcement Learning
4. State-of-the-Art and Limits

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Reinforcement Learning: Learning to Interact

AI methods for interaction

- Search & optimization
  - task: reach goal with minimal costs
  - given: dynamics model of environment
  - prediction: action sequence that reaches goal

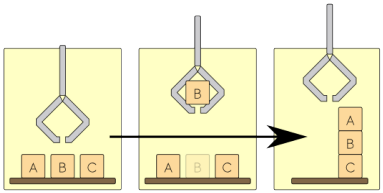

image source: www.wikipedia.org

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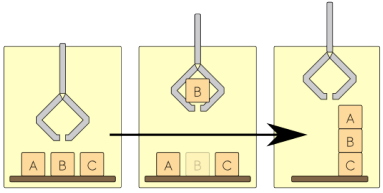


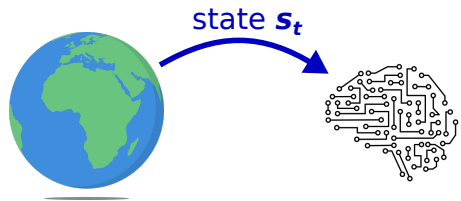
- But what if we do **not know** the environment?
  - go boldly where no one has gone before
  - uncertain dynamics or observations
  - unknown dynamics, e.g. humans in the loop

- Sometimes we need to **learn** from interaction!

image source: Steven Shorrock

Markov decision processes

- MDP $M = \langle \mathcal{S}, \mathcal{A}, \rho, P, r \rangle$ consists of:
  - state space $\mathcal{S}$ of environment configurations
  - distribution $\rho(s_0)$ of initial states $s_0 \in \mathcal{S}$



see Chapter 3 in Sutton and Barto (2018) for a more formal introduction

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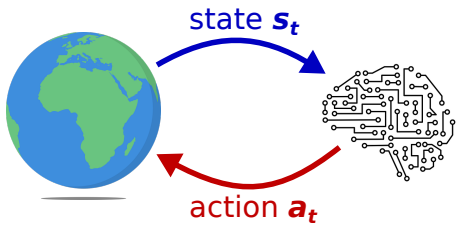

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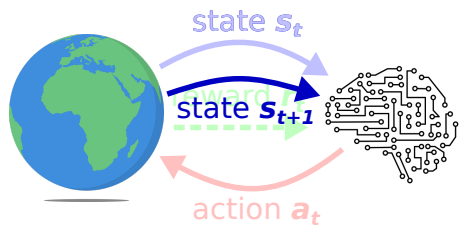

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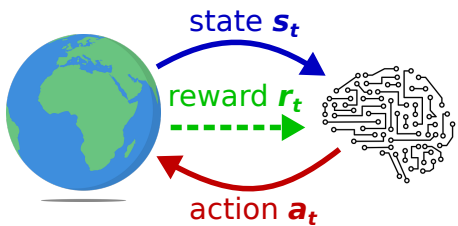
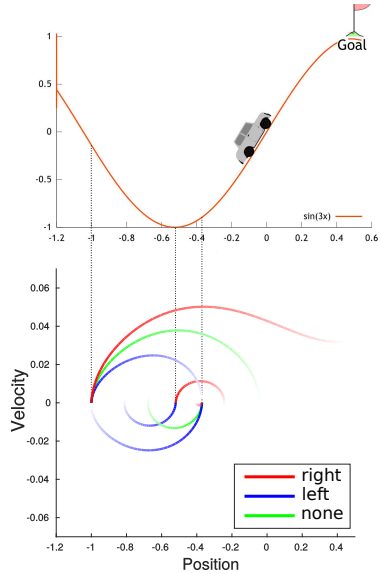


- Reinforcement learning aims for **actions** that maximize **all future reward**
  - future reward depends where we end up in the future → complex problem
  - no prior knowledge of reward or transition model

see Chapter 3 in Sutton and Barto (2018) for a more formal introduction

Example: MountainCar

- Underactuated car in a valley between mountains
  - state is position and velocity: $\boldsymbol{s}_t = [x_t, v_t] \in \mathcal{S} \subset \mathbb{R}^2$
  - actions accelerate car $\boldsymbol{a}_t \in \mathcal{A} = \{\text{left}, \text{none}, \text{right}\}$
  - physics dynamics P: gravitation, but no friction



a reference implementation can be found at www.gymlibrary.ml

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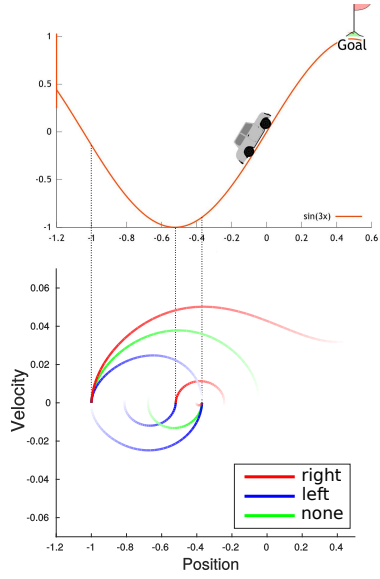

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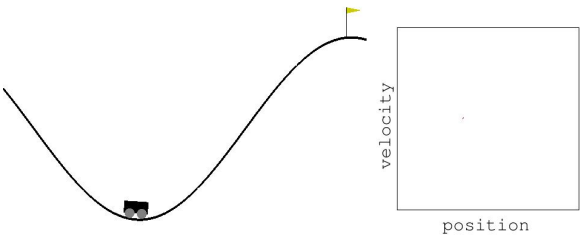
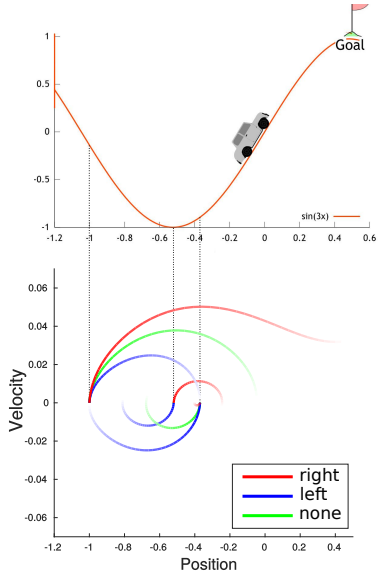
  - physics dynamics P: gravitation, but no friction
  - reward of 1 for reaching the flag, otherwise 0
  - underactuated car needs to pick up speed first





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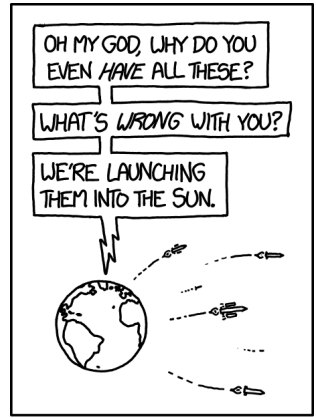
Learning to interact

- In this video we have:
  - discussed under which situations we need to learn interaction
  - learned how a stochastic environment is formalized as a MDP
  - familiarized ourselves with the MountainCar environment



THE MOMENT THE COMPUTERS CONTROLLING OUR NUCLEAR ARSENALS BECAME SENTIENT

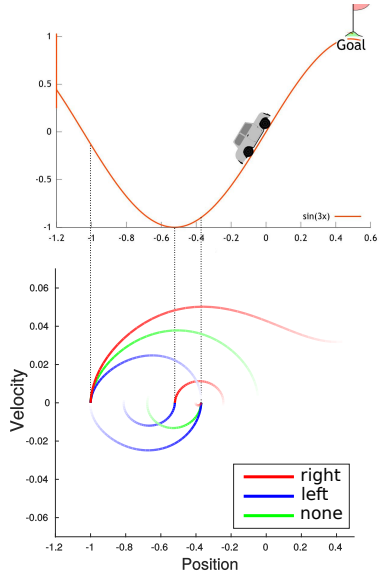
image source: xkcd.com

2

Reinforcement Learning: Tabular Q-learning

Agent policies

- Policy π defines agent's behavior
  - $\pi(a|s)$ probability the agent chooses $a \in \mathcal{A}$ in $s \in \mathcal{S}$



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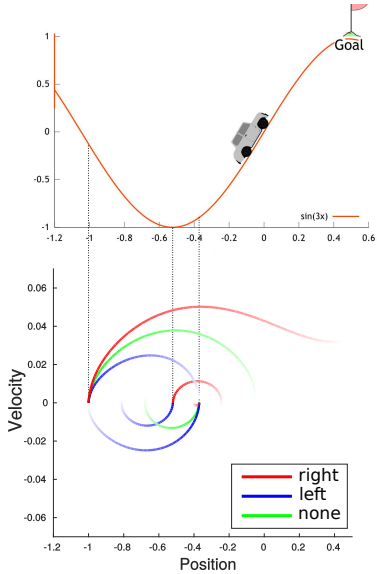
  - allows expectations over all possible futures

$$\mathbb{E}_\pi\Big[\dots\Big] \quad := \quad \mathbb{E}\Big[\dots \Big| \begin{matrix} s_0 \sim \rho(\cdot), \quad r_t = r(s_t, a_t) \\ a_t \sim \pi(\cdot|s_t),\ s_{t+1} \sim P(\cdot|s_t, a_t) \end{matrix}, \ \forall t \geq 0\Big]$$



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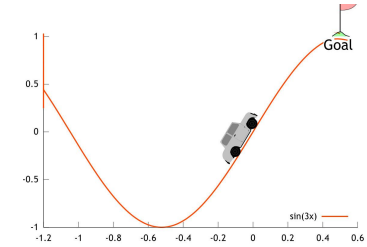
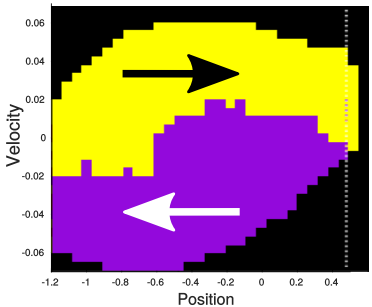
- Discretize continuous states $s \in \mathcal{S}$
  - policy table/matrix $\Pi \in \mathbb{R}^{|\mathcal{S}| \times |\mathcal{A}|}$ with $\Pi_{sa} := \pi(a|s)$





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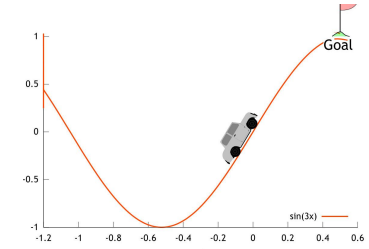
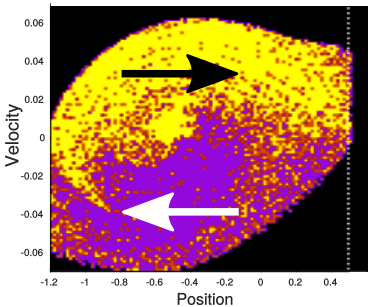
- resolution determines control precision

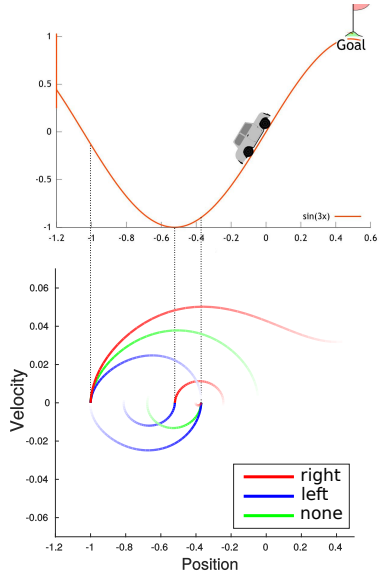




Policy evaluation

- Policy should maximize the **value** V^π
  - expected sum of discounted future rewards

$$V^\pi(s) = \mathbb{E}_\pi\left[\sum_{t=0}^{\infty}\gamma^t r_t \middle| s_0=s\right] \qquad ,\gamma \in [0,1)$$



the infinite sum is a geometric series with $\sum_{t=0}^{\infty}\gamma^t = \frac{1}{1-\gamma}$

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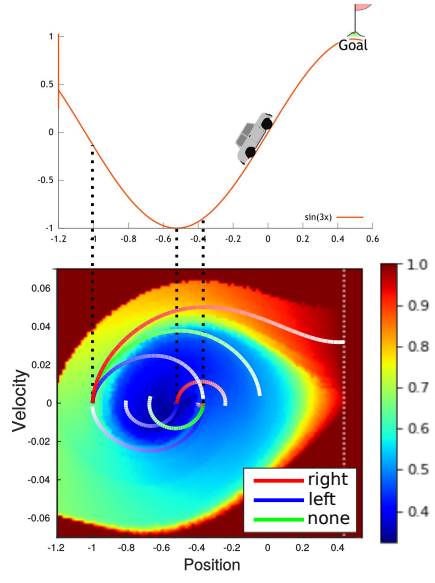

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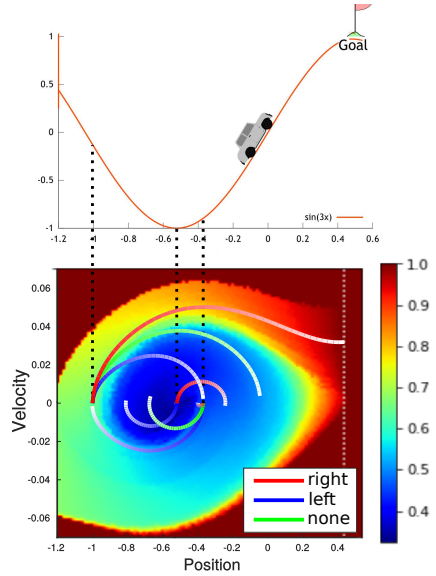
$$= \mathbb{E}_\pi\left[r_0 + \underbrace{\mathbb{E}_\pi[\gamma r_1 + \gamma^2 r_2 + \ldots \,|s_1]}_{\gamma V^\pi(s_1)} \,\middle|\, s_0{=}s\right]$$



this equation goes back to Richard Bellman (1957), see Section 3.5 in Sutton and Barto (2018)

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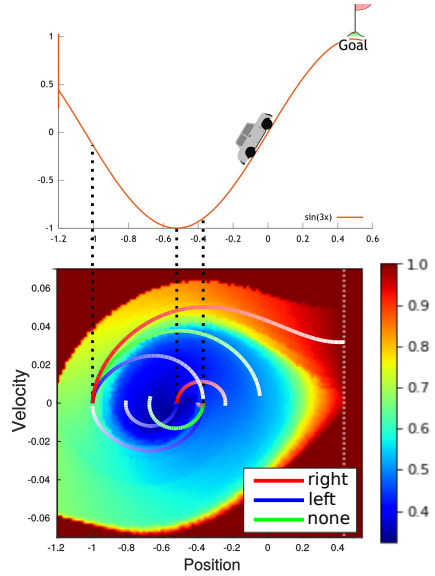
&= \mathbb{E}_\pi\left[r_0 + \gamma V^\pi(s_1) \,\middle|\, s_0{=}s\right]
\end{aligned}
$$

- Recursive **Bellman equation** of the state value



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The optimal Bellman equation

$$\max_{\pi} V^{\pi}(s) \quad = \quad \max_{\pi} \mathbb{E}_{\pi}\left[\sum_{t=0}^{\infty} \gamma^t r(s_t, a_t) \,\middle|\, s_0 = s\right]$$

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$$\overset{(*)}{=} \max_{a} \mathbb{E}\left[r(s_0,a_0) + \gamma \max_{\pi} V^{\pi}(s_1)\,\middle|\, \begin{matrix} s_0{=}s \\ a_0{=}a \end{matrix}\right]$$

$^{(*)}$ $\max_\pi \mathbb{E}[\ldots] = \mathbb{E}[\max_\pi \ldots]$ only holds because a policy π^* exists that is optimal in all states: $V^{\pi^*}(s) \geq V^{\pi}(s),\ \forall \pi, \forall s \in \mathcal{S}$.

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\end{aligned}
$$

- Optimal state-action **Q-values**:

$$
Q^*(s,a) = r(s,a) + \gamma\,\mathbb{E}\left[\max_{a'} Q^*(s',a')\,\middle|\, s'\sim P(\cdot|s,a)\right]
$$

see Section 3.6 in Sutton and Barto (2018)

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\max_{\pi} V^{\pi}(s) &= \max_{\pi} \mathbb{E}_{\pi}\left[\sum_{t=0}^{\infty}\gamma^t r(s_t,a_t)\,\middle|\, s_0{=}s\right] = \overbrace{\max_{a}\max_{\pi} \mathbb{E}_{\pi}\left[\sum_{t=0}^{\infty}\gamma^t r(s_t,a_t)\,\middle|\, \begin{smallmatrix} s_0{=}s \\ a_0{=}a \end{smallmatrix}\right]}^{Q^*(s,a)} \\
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&= \max_{a} \mathbb{E}\left[r(s_0,a_0) + \gamma \max_{a'} \underbrace{\max_{\pi} \mathbb{E}_{\pi}\left[\sum_{t=0}^{\infty}\gamma^t r(s'_t,a'_t)\,\middle|\, \begin{smallmatrix} s'_0{=}s_1 \\ a'_0{=}a' \end{smallmatrix}\right]}_{Q^*(s_1,a')}\,\middle|\, \begin{smallmatrix} s_0{=}s \\ a_0{=}a \end{smallmatrix}\right]
\end{aligned}
$$

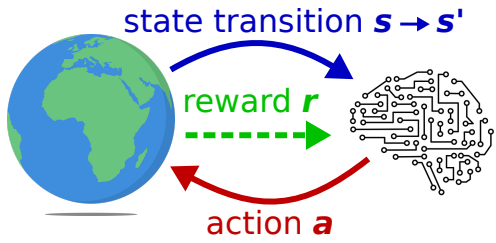
- Optimal state-action **Q-values** and **policy**:

$$
\begin{aligned}
Q^*(s,a) &= r(s,a) + \gamma\, \mathbb{E}\left[\max_{a'} Q^*(s',a')\,\middle|\, s'\sim P(\cdot|s,a)\right] \\
\pi^*(a|s) &= \begin{cases} 1, & \text{if } \; a = \arg\max_{a'} Q^*(s,a') \\ 0, & \text{otherwise} \end{cases}
\end{aligned}
$$

see Section 3.6 in Sutton and Barto (2018)

Tabular Q-learning

- Estimate optimal Q-value Q^*
  - while interacting with environment
  - MDP is unknown, can only be sampled



tabular Q-learning has been introduced by Watkins and Dayan (1992)

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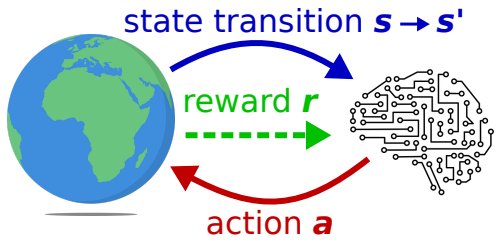
  - same discretization as policy



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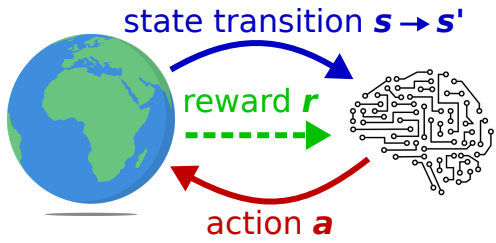

- Update table $\mathbf{Q}$ after observing transition $s, a \to r, s'$
  - temporal averaging with learning rate $\alpha \in (0, 1]$

$$Q_{sa} \leftarrow (1-\alpha)\underbrace{Q_{sa}}_{\text{old value}} + \alpha\underbrace{\left(r + \gamma \max_{a'} Q_{s'a'}\right)}_{\text{Bellman update}}$$

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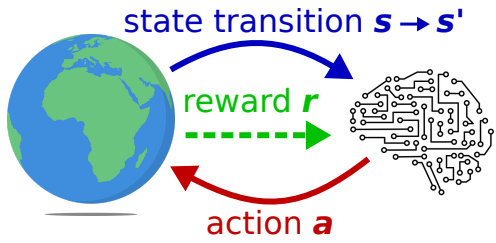

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```
1 s' = environment.reset()
2 while not done:
3     s, a = s', argmax(Q[s'])
4     s', r, done = environment.step(a)
5     td = r + γ * max(Q[s']) - Q[s, a]
6     Q[s, a] += α * td
```

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Exploration and exploitation

- Tabular Q-learning converges to optimal Q^*
  - if all state-action pairs are visited often enough
  - how can we guarantee that we **explore** enough?

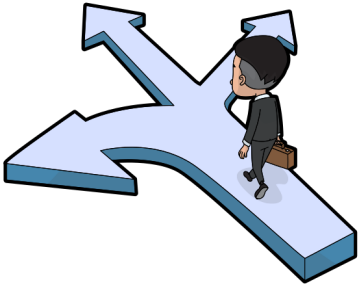

image source: wikipedia.org

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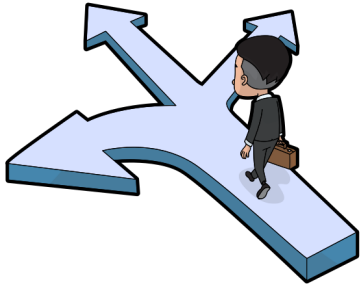
1. ϵ-greedy: explore randomly with probability ϵ

$$\pi_\epsilon(a|s) \quad := \quad (1-\epsilon)\underbrace{\pi^*(a|s)}_{\text{greedy}} + \epsilon\underbrace{\frac{1}{|\mathcal{A}|}}_{\text{random}}$$



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2. optimistic initialization: overestimate values
   - initialize $Q_{sa} > \max_{s',a'} Q^*(s',a')$
   - updates reduce table entries towards $Q^*(s,a)$
   - under-explored actions are more attractive

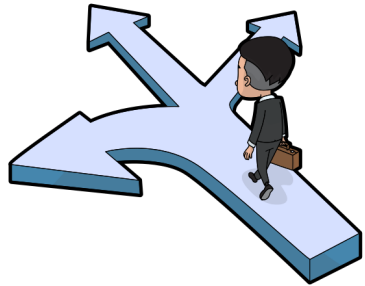

image source: wikipedia.org

Tabular Q-learning

- In this video we have:
  - defined the agent's behavior as policy π
  - defined how to evaluate a policy V^{π}
  - derived the value Q^{*} of the optimal policy
  - introduced the tabular Q-learning algorithm
  - discussed how to guarantee sufficient exploration

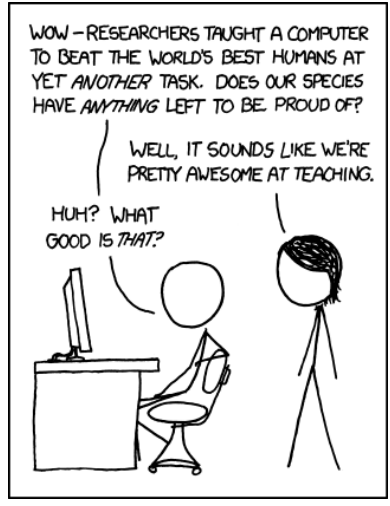

image source: xkcd.com

3

Reinforcement Learning: Deep Reinforcement Learning

Scaling up reinforcement learning

- Tabular reinforcement learning is restricted
  - small state-action spaces
  - curse of dimensionality

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  - curse of dimensionality
- How can we learn to e.g. play ATARI games
  - no model available for planning
  - image is high dimensional input
  - no feasible discretization

image source: wikipedia.com

Scaling up reinforcement learning

- Tabular reinforcement learning is restricted
  - small state-action spaces
  - curse of dimensionality
- How can we learn to e.g. play ATARI games
  - no model available for planning
  - image is high dimensional input
  - no feasible discretization
- Can we replace the Q-table with a neural network?

image source: wikipedia.com

Deep Q-networks (DQN)

- Convolutional neural network $q_\theta(\boldsymbol{s}) \in \mathbb{R}^{|\mathcal{A}|}$ with one output per action

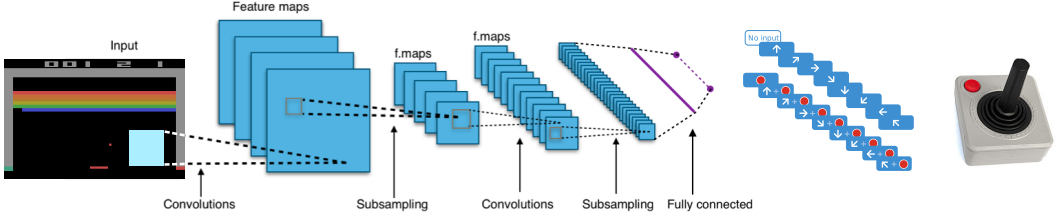

image modified from Apex34 and Bilby (wikipedia.com)

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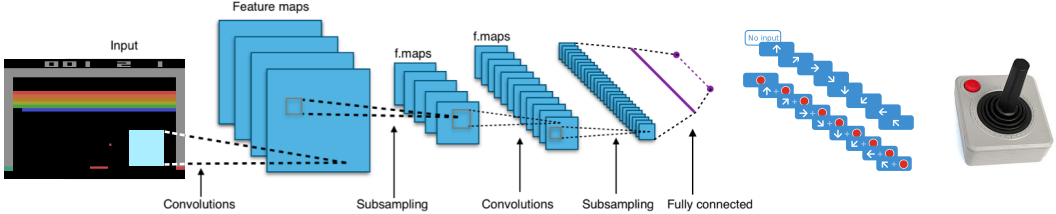
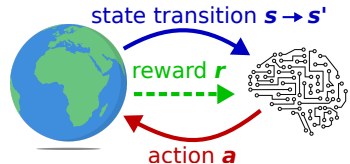


- For transitions $s, a \rightarrow r, s'$ we update paramteres $\boldsymbol{\theta}$
  - gradient descent on mean-squared TD error

$$\boldsymbol{\theta} \quad \leftarrow \quad \boldsymbol{\theta} \; - \; \alpha \nabla_\theta \underbrace{\left(r + \gamma \max_{a'} \left(q_\theta(\boldsymbol{s}')\right)_{a'} - \left(q_\theta(\boldsymbol{s})\right)_a \right)^2}_{\text{TD-error}}$$



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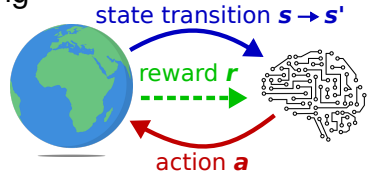
RL is not supervised learning!

- Q-learning violates assumptions of supervised learning
  - TD-error: regression targets are *non-stationary*
  - exploration: the state-distribution is *non-stationary*
  - sampling: transitions are not visited *i.i.d.*



$$\boldsymbol{\theta} \quad \leftarrow \quad \boldsymbol{\theta} \; - \; \alpha \, \nabla_\theta \Big(\overbrace{r + \gamma \max_{a'} \big(q_\theta(\boldsymbol{s}')\big)_{a'}}^{\text{regression target}} - \big(q_\theta(\boldsymbol{s})\big)_a \Big)^2$$

TD-error

see Mnih et al. (2013, 2015) for details on stabilization techniques; see Badia et al. (2020) for a modern version; image modified from Designmodo

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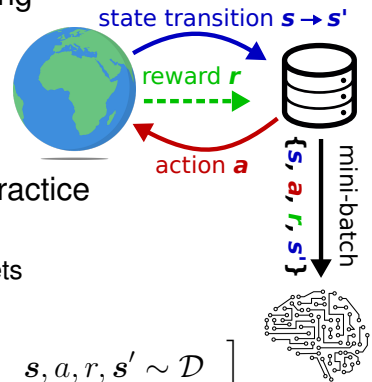
- Q-learning violates assumptions of supervised learning
  - TD-error: regression targets are *non-stationary*
  - exploration: the state-distribution is *non-stationary*
  - sampling: transitions are not visited *i.i.d.*
- DQN must be stabilized with many tricks to work in practice
  - draw mini-batches i.i.d. from *experience replay buffer* $\mathcal{D}$
  - use slowly changing *target network* $q_{\theta'}$ for regression targets



$$\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} - \alpha \nabla_\theta \mathbb{E}\Big[\underbrace{\Big(r + \gamma \max_{a'} \big(\overbrace{q_{\theta'}(\boldsymbol{s}')}^{\text{target network}} \big)_{a'} - \big(q_\theta(\boldsymbol{s}) \big)_a \Big)^2}_{\text{TD error}} \Big| \underbrace{\boldsymbol{s}, a, r, \boldsymbol{s}' \sim \mathcal{D}}_{\text{from experience replay buffer}} \Big]$$

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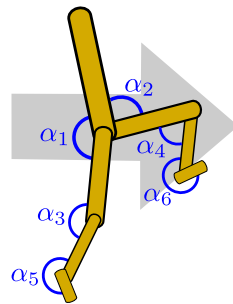
Continuous actions

- Applications like robotics have continuous actions
  - states are limb angles, momentum, pressure sensors, etc.
    e.g. $\boldsymbol{s} = [\alpha_1, \dot{\alpha}_1, \ldots, \alpha_6, \dot{\alpha}_6]$
  - actions are motor torques, e.g. $\boldsymbol{a} = [\ddot{\alpha}_1, \ldots, \ddot{\alpha}_6] \in \mathbb{R}^{|\boldsymbol{a}|}$

$$\max_{\boldsymbol{a}} q_\theta(\boldsymbol{s}, \boldsymbol{a}) \quad = \quad ???$$



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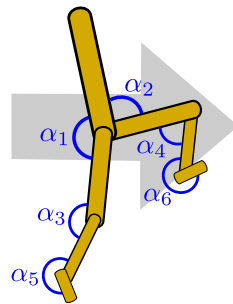
- Neural network outputs distribution over actions
  - e.g. a Gaussian distribution $\pi_\theta(\boldsymbol{a}|\boldsymbol{s}) := \mathcal{N}\big(\boldsymbol{a}|\mu_\theta(\boldsymbol{s}), \sigma^2_\theta(\boldsymbol{s})\big)$
  - $\mu_\theta(\boldsymbol{s}) \in \mathbb{R}^{|\boldsymbol{a}|}$ and $\sigma_\theta(\boldsymbol{s}) \in \mathbb{R}^{|\boldsymbol{a}|}$ are neural networks



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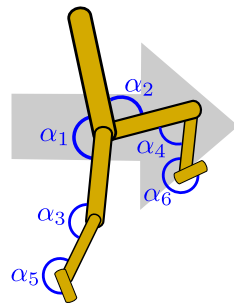
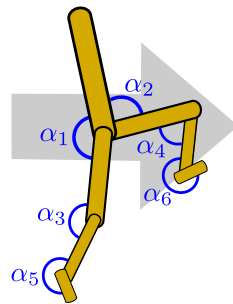
  - $\mu_\theta(\boldsymbol{s}) \in \mathbb{R}^{|\boldsymbol{a}|}$ and $\sigma_\theta(\boldsymbol{s}) \in \mathbb{R}^{|\boldsymbol{a}|}$ are neural networks
- How can we learn the parameters θ of the policy?



Policy gradients

- Directly maximize expected return $R_t := \sum_{k=0}^{\infty} \gamma^k r_{t+k}$
  - starting a the first state
  - gradient reinforce actions that led to good returns

$$\nabla_\theta \mathbb{E}_{\pi_\theta}[R_0] \quad = \quad \mathbb{E}_{\pi_\theta}\left[\sum_{t=0}^{\infty} \gamma^t R_t \, \nabla_\theta \ln \pi_\theta(a_t|s_t)\right]$$



see the *policy gradient theorem* (originally by Sutton et al., 1999) in Section 13.2 of Sutton and Barto (2018)

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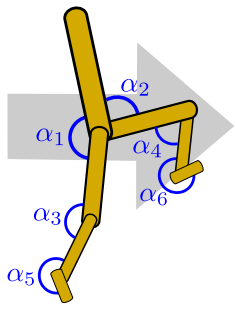
- REINFORCE algorithm (Williams, 1992)
  - sample n trajectories per policy gradient
  - slow and not very stable



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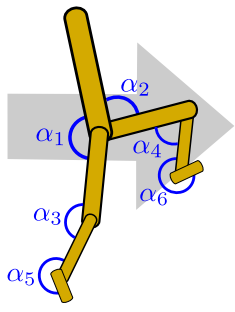
- Modern variations faster and more stable
  - PPO (Schulman et al., 2017)
  - SAC (Haarnoja et al., 2018a,b)



Deep reinforcement learning

- In this video we have:
  - discussed why tabular RL does not scale
  - seen that Q-learning can be scaled up to play video games
  - learned how neural networks represent continuous policies
  - seen how continuous policies can be learned

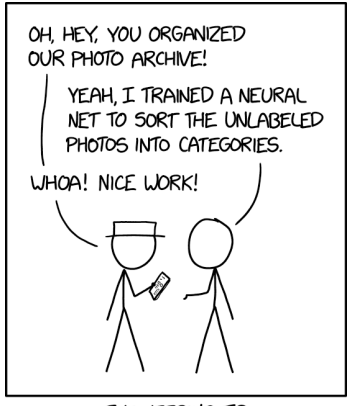


image source: xkcd.com

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  - discussed why tabular RL does not scale
  - seen that Q-learning can be scaled up to play video games
  - learned how neural networks represent continuous policies
  - seen how continuous policies can be learned
- But be careful, RL is not supervised learning!
- Deep RL is an active research topic

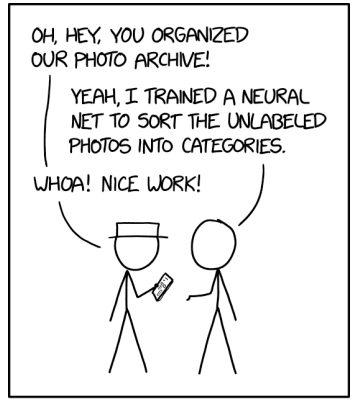


image source: xkcd.com

4

Reinforcement Learning: State-of-the-Art and Limits

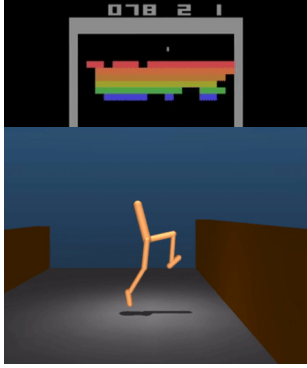
Recent RL success stories

- Playing video games (2013/2015)
  - *challenge:* high dimensional pixel input
  - one algorithm can learn each Atari game

see Mnih et al. (2013, 2015); image source: youtube.com

Recent RL success stories

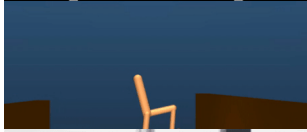
- Playing video games (2013/2015)
- Fast moving simulated robots (2016/2017)
  - *challenge:* continuous actions with complex dynamics
  - running robots pass random obstacle courses
  - only in simulators so far

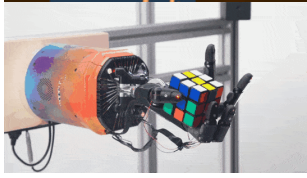


see Heess et al. (2017); image source: deepmind.com

Recent RL success stories

- Playing video games (2013/2015)
- Fast moving simulated robots (2016/2017)
- Real robotic hand solves Rubik's cubes (2019)
  - *challenge:* transfer from simulator to reality
  - works only in the lab so far





see OpenAI et al. (2019); image source: openai.com

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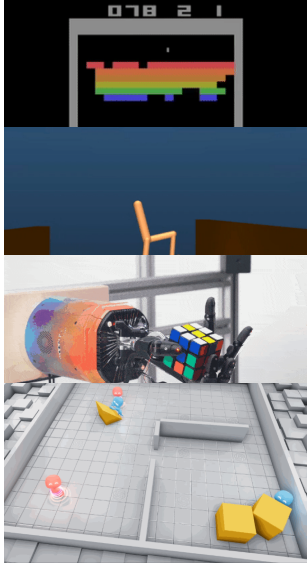
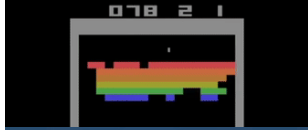
- Fast moving simulated robots (2016/2017)
- Real robotic hand solves Rubik's cubes (2019)
- Multi-agent RL in competitive games (2019/2020)
  - *challenge:* many agents interact with each other
  - surprisingly complex and seemingly intelligent behavior
  - RL beats professional players in StarCraft and MOBA's

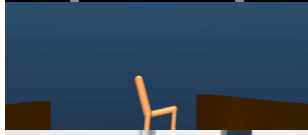


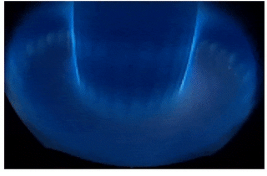
see Vinyals et al. (2019, StarCraft), Berner et al. (2019, Dota) and Baker et al. (2020, hide and seek); image source: openai.com

Recent RL success stories

- Playing video games (2013/2015)
- Fast moving simulated robots (2016/2017)
- Real robotic hand solves Rubik's cubes (2019)
- Multi-agent RL in competitive games (2019/2020)
- Magnetic control of a Tokamak fusion reactor (2022)
  - *challenge:* keep the plasma bubble from bursting
  - simulations transferred to real fusion reactor







see Degrave et al. (2022); image source: deepmind.com

Recent RL success stories

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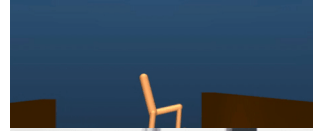
- Multi-agent RL in competitive games (2019/2020)
- Magnetic control of a Tokamak fusion reactor (2022)
- Most experts expect breakthrough in RL soon
  - industry applications within a decade or so
  - might impact **your** future job!



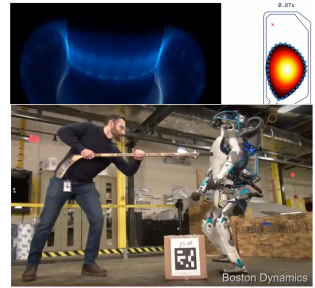


What is possible?

- Now: learn to control unknown dynamics
  - environment must be stationary and completely explorable
  - many interactions with the environment (e.g. simulator/game)
  - no catastrophic/irreversible actions (e.g. crashing the car)

image source: Tokamak fusion reactor from deepmind.com

What is possible?



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- Soon: safe transfer of learned control
  - learn in a simulation, act safely in the real world
  - learn to anticipate behavior of other learning agents
  - integrate existing knowledge/constraints into the training

image source: robot stability test from Boston Dynamics

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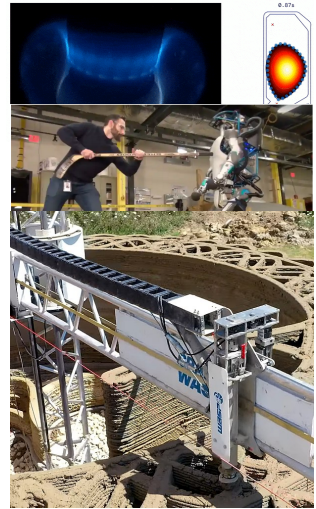

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- Possible applications
  - infinite sampling: computer games, web-crawling, video processing
  - limited autonomy: construction, factories, power plants/grids, driver assistance

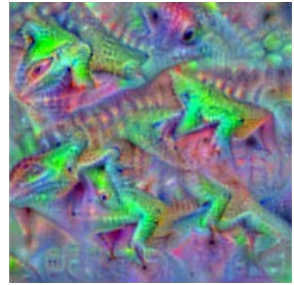
image source: 3D-printed construction by Alfredo Milano (from wikipedia)

What is impossible?

- RL agents do **not represent** the world as we do
  - only patterns: no objects, relationships, movements
  - only training data: no intuition of what is *similar*

What you see:

What a neural net sees:



visualizing “archetypical inputs” for neural nets; image source: Girish Gowda and Farhad Dalirani

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  - no imagination: all reaction, no prediction

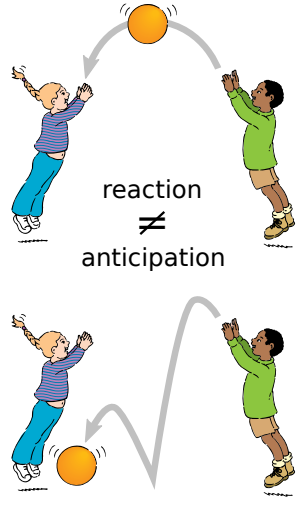

image source: modified from freesvg.org

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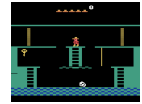
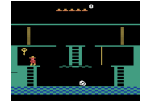
- RL agents **only learn** the task they are given
  - only looks smart: memorizes what yielded reward
  - misspecification: RL can act dumb to get reward

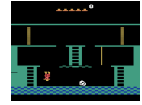
1. start



2. go get the key

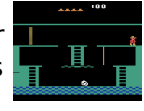


3. **die**



4. respawn near door

5. open door get points



example of the Atari game *Montezuma's Revenge*, available in OpenAI's `gym library`

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  - misspecification: RL can act dumb to get reward

- RL agents are **not intelligent** like humans! Don't expect them to be!

image source: *The Simpsons* (20th Television Animation)

When to use reinforcement learning?

1. Is there no heuristic that already solves the task almost optimal?
2. Is your task virtual or do you have a simulator of your tasks?
3. Are there no “forbidden” actions that would destroy the system?
4. Can the system cause **no** serious harm to people?
5. Is the variety of situations the agent can face limited?
6. Do you have a lot of compute to train the agent for all situations?
7. Are you willing to spend 6 month fiddling with hyper-parameters?

State-of-the-art and limits

- In this video we have:
  - seen some recent success stories of RL
  - discussed what RL *can* learn
  - discussed what RL *cannot* learn
  - discussed when to use RL

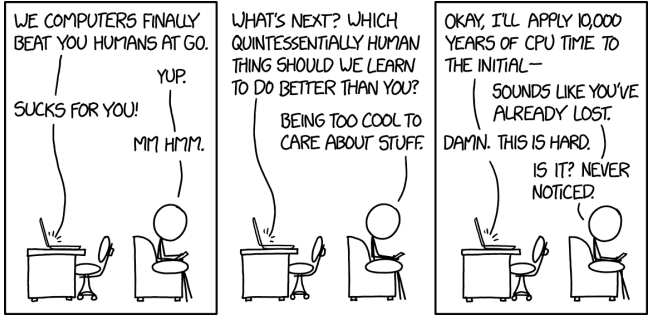

image source: xkcd.com

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