EPA143A – Macroeconomics for Policy Analysis
Week Six

INPUT-OUTPUT ECONOMICS

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LECTURE NOTE W-6

The required reading for Week 6 includes:

- Lecture Note EPA143A Week 6
- Justin Kitzes. 2013. 'An Introduction to Environmentally-Extended Input-Output Analysis.' *Resources* vol. 2: 489-503.

Supporting video:

- Introduction to input-output analysis: https://www.youtube.com/watch?v=UheRnDJ4dgc

Lecture Note W-6 and the exercises of Week 6 are part of the exam materials.

Input-Output Analysis: Wassily Leontief

An influential multi-industry version of the Keynesian macro model is the Input-Output Model (IOM). Input-output models, which examine economic activity by studying the relationships between industries, were developed by economist Wassily Leontief in the 1940s. Wassily Leontief was born into a well-to-do academic family in St. Petersburg, Russia. Leontief enrolled at the University of Leningrad in 1921, at the age of fifteen. He started out in philosophy, but eventually moved on to economics, earning his MA in 1925. Finding times in Communist Russia difficult, his parents emigrated to Germany, bringing their son along with them. Leontief enrolled for graduate studies at the University of Berlin, obtaining his PhD in 1928 under Ladislaus von Bortkiewicz and Werner Sombart. Leontief joined the Kiel Institute in 1927, where the seeds of his input-output analysis were laid. Input-output was partly inspired by the Marxian and Walrasian analysis of general equilibrium via inter-industry flows - which in turn has ancestral origins in Quesnay's (1758) *Tableau Economique*. Leontief moved to the United States in 1931, and took up a position at Harvard University in 1932. In 1941 he published an empirical example of his input-output system - *Structure of American Industry*. Leontief followed up this work with a series of classical papers on input-output economics (collected in 1966). Input-output was novel and inspired large-scale empirical work. It has been and continues to be used for economic policymaking and planning throughout the world.

The input-output model (IOM)

The IOM is based on the input-output table of an economy. To understand the logic of the IOM, we use the hypothetical (2 industry x 2 industry) input-output table of Table 1. In the first row, we have recorded the sources of demand for the agricultural sector: agriculture's demand for goods produced by agriculture itself is €8 billion; manufacturing is demanding agricultural goods worth €5 billion (as intermediate inputs), and final demand accounts for €3 billion. Total demand for agricultural goods is €16 billion. In the columns, we have recorded the components of gross output. In the second column, we can read that to produce a gross output of €12 billion of manufacturing goods, the industrial sector needs €5 billion of intermediate inputs from agriculture and €2 billion of manufactured goods. Labour and capital add €5 billion of value to these intermediate inputs – to generate a manufacturing gross output of €12 billion. Real GDP in this economy is equal to € 9 billion = the sum of value added in agriculture and manufacturing, or, alternatively, the sum of final demand for agricultural and manufactured goods.

Table 1
An input-output table
(billions of euro)

	Agriculture	Manufacturing	Final demand	Total demand
Agriculture	8	5	3	16
Manufacturing	4	2	6	12
Value added	4	5		
Gross output	16	12		

From Table 1 we know that the sum across both the rows of the square inter-industry transactions matrix (**Z**) and the final demand vector (**y**) is equal to vector of gross output by industry (**x**).

(1) $$\mathbf{x} = \mathbf{Z}\,\mathbf{i} + \mathbf{y} = \begin{bmatrix} 8 & 5 \\ 4 & 2 \end{bmatrix} \times \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 3 \\ 6 \end{bmatrix} = \begin{bmatrix} 16 \\ 12 \end{bmatrix}$$

where **i** is a summation vector of ones. Matrix **Z** is called the matrix of intermediate input-output *flows* with typical element Z_{ij}. Z_{ij} = the quantity of intermediate inputs produced by industry *i* and sold to the using industry *j*. We calculate the direct intermediate input requirements matrix **A** by dividing the inter-industry transactions matrix **Z** by the gross output vector **x**:

(2) $$\mathbf{A} = \mathbf{Z}\,\hat{\mathbf{x}}^{-1} = \begin{bmatrix} 8 & 5 \\ 4 & 2 \end{bmatrix} \times \begin{bmatrix} 1/16 & 0 \\ 0 & 1/12 \end{bmatrix} = \begin{bmatrix} 8/16 & 5/12 \\ 4/16 & 2/12 \end{bmatrix} = \begin{bmatrix} 0.50 & 0.42 \\ 0.25 & 0.17 \end{bmatrix}$$

where $\hat{\mathbf{x}}^{-1}$ is a square matrix with inverse of each element in the gross output vector **x** on the diagonal and the rest of the elements equal to zero.

Matrix **A** $= \begin{bmatrix} 0.50 & 0.42 \\ 0.25 & 0.17 \end{bmatrix}$ is called the matrix of intermediate input-output coefficients with typical element α_{ij}. Technical coefficient α_{ij} = the quantity of intermediate inputs produced by industry *i* which are required to produce one unit of gross output of industry *j*, or $\alpha_{ij} = \frac{z_{ij}}{x_j}$. For example, $\alpha_{21} = 0.25$; this technical coefficient indicates that to produce one unit of gross output in agriculture (industry 1), farmers need 0.25 units of intermediate inputs produced by manufacturing (industry 2). Rearranging equation (2) yields:

(3) $$\mathbf{Z} = \mathbf{A}\,\hat{\mathbf{x}} = \begin{bmatrix} 0.50 & 0.42 \\ 0.25 & 0.17 \end{bmatrix} \times \begin{bmatrix} 16 & 0 \\ 0 & 12 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ 4 & 2 \end{bmatrix}$$

where $\hat{\mathbf{x}}$ is a diagonalized square matrix with the elements of the gross output vector **x** on the diagonal and zeros elsewhere. Substitution of equation (3) in equation (1) gives:

(4) $$\mathbf{x} = (\mathbf{A}\,\hat{\mathbf{x}})\,\mathbf{i} + \mathbf{y} = \begin{bmatrix} 8 & 5 \\ 4 & 2 \end{bmatrix} \times \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 3 \\ 6 \end{bmatrix}$$

or, alternatively:

(5) $$\mathbf{x} = \mathbf{A}\,\mathbf{x} + \mathbf{y} = \begin{bmatrix} 0.50 & 0.42 \\ 0.25 & 0.17 \end{bmatrix} \times \begin{bmatrix} 16 \\ 12 \end{bmatrix} + \begin{bmatrix} 3 \\ 6 \end{bmatrix} = \begin{bmatrix} 16 \\ 12 \end{bmatrix}$$

Solving equation (5) for gross output **x** yields as an intermediate step:

(6) $$[\mathbf{I} - \mathbf{A}]\,\mathbf{x} = \mathbf{y} \quad \rightarrow \quad [\mathbf{I} - \mathbf{A}] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.50 & 0.42 \\ 0.25 & 0.17 \end{bmatrix} = \begin{bmatrix} 0.50 & -0.42 \\ -0.25 & 0.83 \end{bmatrix}$$

and as the final reduced-form solution:

(7) $$\mathbf{x} = [\mathbf{I} - \mathbf{A}]^{-1} \times \mathbf{y} \quad \rightarrow \mathbf{L} = [\mathbf{I} - \mathbf{A}]^{-1} = \begin{bmatrix} 2.68 & 1.35 \\ 0.81 & 1.61 \end{bmatrix} = \text{the Leontief inverse.}$$

Hence, we obtain:

(8) $$\mathbf{x} = [\mathbf{I} - \mathbf{A}]^{-1} \times \mathbf{y} = \begin{bmatrix} 2.68 & 1.35 \\ 0.81 & 1.61 \end{bmatrix} \times \begin{bmatrix} 3 \\ 6 \end{bmatrix} = \begin{bmatrix} 2.68 \times 3 + 1.35 \times 6 \\ 0.81 \times 3 + 1.61 \times 6 \end{bmatrix} = \begin{bmatrix} 16 \\ 12 \end{bmatrix}$$

Equation (8) constitutes the solution to our (2 industry x 2 industry) input-output model.

In the IOM, gross output is a function of (exogenous) final demand and the Leontief inverse. The Leontief inverse is based on the technical coefficients of the A-matrix; it is a (Keynesian) multiplier matrix. The reduced-form equation can be written as follows:

(9) $$\Delta\mathbf{x} = [\mathbf{I} - \mathbf{A}]^{-1} \times \Delta\mathbf{y} = \frac{\mathrm{I}}{[\mathbf{I} - \mathbf{A}]} \times \Delta\mathbf{y}$$

If we assume that final demand changes by one unit ($\Delta\mathbf{y} = \mathbf{1}$), then $\Delta\mathbf{x} = [\mathbf{I} - \mathbf{A}]^{-\mathbf{1}}$. We assume that there is excess production capacity in all industries; gross output can increase in response to an increase in final demand.

The input-output model: linkages and multiplier analysis

The IOM can be used to estimate 'backward production linkages' between industries and to estimate the multiplier effects of (exogenous) changes in final demand.

1. Backward production linkages

The backward production linkage of industry j is defined as the column sum of the Leontief inverse for industry *j*. In our (numerical) example, the Leontief inverse is:

$$\mathbf{L} = [\mathbf{I} - \mathbf{A}]^{-\mathbf{1}} = \begin{bmatrix} 2.68 & 1.35 \\ 0.81 & 1.61 \end{bmatrix}$$

The column sum for agriculture (industry 1) is 3.49. The 'backward production linkage' tells us by how much total gross output in all industries combined will increase if final demand for agricultural goods increases by 1 unit. The column sum for manufacturing (industry 2) is 2.96, which means that gross output in all industries combined will increase by almost 3 units if final demand for manufactured goods increases by 1 unit. The more one industry is using intermediate inputs produced by other industries, the greater will be its column sum of the Leontief inverse – and the stronger are its backward production linkages.

Backward production linkages (or 'upstream' production linkages) refer to the intermediate-input linkages from a using industry to other industries along the (national or global) supply chain which supply the inputs. In the example, agriculture's backward production linkages refer to linkages from the farm to the part of the non-farm sector that provides inputs for agricultural production, for example agrochemicals.

Backward production linkages operate across national borders – since firms in our country will source intermediate inputs produced by firms in other industries in foreign countries. The international (cross-border) integration of production chains has increased considerably during the past three decades, but the following example of cross-border backward production linkages between Japan and the U.S.A., on the one hand, and Indonesia, Malaysia, Thailand and South Korea in 1975 illustrates that cross-border production networks (in East Asia and the Pacific) were closely integrated already in the 1960s and 1970s. Table 2 presents backward production linkages between the mentioned economies (derived from the Leontief inverse of an inter-national input-output model; see Yamazawa, Nohara & Osada 1986).[1]

[1] Ippei Yamazawa, Takashi Nohara and Hiroshi Osada. 1986. 'Economic interdependence in Pacific Asia: an international input-output analysis.' *The Developing Economies* volume 24 (2): 95-108.

Table 2
Backward production linkages in East Asia and the Pacific, 1975
(selected industries)

	Indonesia	Malaysia	Thailand	S. Korea	Japan	U.S.A.
Textiles	2.15	2.13	2.12	2.69	2.61	2.29
Domestic	1.68	1.68	1.91	2.30	2.53	2.26
Foreign	0.47	0.45	0.21	0.39	0.08	0.03
Iron & steel:	2.29	2.22	2.23	3.10	3.06	2.19
Domestic	1.55	1.87	1.69	2.17	2.98	2.14
Foreign	0.74	0.35	0.54	0.93	0.06	0.05
Industrial Machinery:	1.85	1.81	2.00	2.50	2.50	1.92
Domestic	1.21	1.42	1.48	1.98	2.45	1.86
Foreign	0.64	0.41	0.52	0.52	0.05	0.06
Motor vehicles:	1.98	2.15	2.00	2.44	2.67	2.40
Domestic	1.29	1.62	1.66	1.92	2.63	2.31
Foreign	0.69	0.57	0.34	0.52	0.04	0.09

Source: Yamazawa, Nohara & Osada (1986).

Consider the backward production linkage of the iron & steel industry in South Korea – which has a (high) value of 3.1. This means that an increase in final demand for iron & steel produced in South Korea (by 1 unit) did create 3.1 units of additional output in the economies of the East-Asian / Pacific regional economy in 1975. Gross output in South Korea itself did rise by 2.17 units and gross output in the other countries increased by 0.93 units; this constitutes a considerable cross-border 'spill-over effect' (since it is caused by an increase in final demand for iron & steel in South Korea of exactly 1 unit). One unit of final demand for iron & steel in South Korea was found to induce 0.67 units of production in Japan and 0.25 units of output in the U.S.A. This is a critical finding: it suggests that if Japan loses one unit of its iron & steel exports to a third country (say, Australia) to South Korea, Japan will recover 0.67 units of output through Korean demand for Japanese intermediate inputs.

Table 2 highlights other structural facts. First, the large backward production linkages indicate that East Asian supply chains were already quite internationalized in 1975, as cross-border production spill-over effects were found to be substantial. Second, international spill-over impacts are very small for Japan and the U.S.A.; this indicates the relatively strong domestic industrial structures of these two economies. Industries in Japan and the U.S. did not have to import intermediate inputs on a large scale, as their supply chains were almost completely domestic. Third, industries in Indonesia, Malaysia, Thailand and South Korea were relatively heavily dependent on imported intermediate inputs; hence, their backward production linkages include substantial cross-border spill-over impacts, which is an indication of their relatively weak (or incomplete) industrial structures.

2. Value-added multipliers

Let us return to our hypothetical example (of Table 1) and define the (1x2) vector of value added **va** per unit of gross output:

$$\mathbf{va} = [4/16 \quad 5/12] = [0.25 \quad 0.42]$$

va_i = the direct value-added per €1 billion of gross output in industry *i*. In agriculture, each extra €1 billion of gross output creates € 0.25 billion of additional income (or value added); in industry, each additional €1 billion of gross output creates €0.42 billion additional income (or value added). But each additional unit of (agricultural) gross output leads to additional output and income generation in the other industries. Hence, how much value added does one extra unit of final demand for industry *j* create <u>in total</u> (directly and indirectly)?

The answer is straightforward: if final demand for goods produced by industry j increases by €1 billion, total value added in the economy will (even must) increase by €1 billion as well. The reason is that (in ex-post equilibrium) final demand = value added. We can check this by pre-multiplying the Leontief inverse with the vector of value-added per unit of gross output:

$$\mathbf{VA} = \mathbf{va} \times [\mathbf{I} - \mathbf{A}]^{-1} = [0.25 \quad 0.42] \times \begin{bmatrix} 2.68 & 1.35 \\ 0.81 & 1.61 \end{bmatrix}$$
$$= [0.25 \times 2.68 + 0.42 \times 0.81 \quad 0.25 \times 1.35 + 0.42 \times 1.61] = [1 \quad 1]$$

This is not a trivial result, however. We have seen that each extra €1 billion of gross output in agriculture creates €0.25 billion of value added in agriculture itself (directly). <u>Indirectly</u>, through the backward production linkages, agriculture is creating an additional €0.75 billion of income – in manufacturing and in agriculture. These coefficients **VA** are called <u>value-added inducement coefficients</u>.

An increase in agricultural gross output by €1 billion the direct generates €0.25 billion in value added directly, but €1 billion in extra income in total. The direct impact (of €0.25 billion) is multiplied by a factor of four. This factor is called the <u>value-added multiplier</u>. It can be calculated as follows:

$$\mathbf{VAm} = \mathbf{va} \times [\mathbf{I} - \mathbf{A}]^{-1} \times \widehat{\mathbf{va}}^{-1} = [1 \quad 1] \times \begin{bmatrix} \frac{1}{0.25} & 0 \\ 0 & \frac{1}{0.42} \end{bmatrix} = [4 \quad 2.4]$$

where $\widehat{\mathbf{va}}$ = the diagonalized matrix of vector **va**. The value-added multiplier for manufacturing is 2.4; it means that the direct increase in manufacturing value-added will have a multiplier impact of 2.4 on value added in the whole economy (due to the presence of backward production linkages). The value-added multiplier of an industry will be higher, the higher are the backward production linkages of that industry.

Value-added inducement does not stop at the national border – since firms in our country will source intermediate inputs produced by firms in other industries in foreign countries. This is illustrated for East-Asia (in 1975) in Table 3.

Table 3

Value-added inducement coefficients in East Asia, 1975

	Indonesia	Malaysia	Thailand	S. Korea	Japan	U.S.A.
Indonesia	0.8170	0.0038	0.0008	0.0063	0.0054	0.0014
Malaysia	0.0005	0.7060	0.0002	0.0209	0.0004	0.0003
Thailand	0.0007	0.0093	0.7696	0.0017	0.0011	0.0001
South Korea	0.0008	0.0011	0.0011	0.7345	0.0020	0.0005
Japan	0.0460	0.0429	0.0410	0.0651	0.8830	0.0067
U.S.A.	0.0225	0.0266	0.0100	0.0626	0.0191	0.9142
Total East Asia & Pac.	0.0761	0.0934	0.0562	0.1386	0.0279	0.0095
Total foreign	0.1830	0.294	0.2304	0.2655	0.1170	0.0858

Source: Yamazawa, Nohara & Osada (1986).

The domestic value-added inducement (on the diagonal of Table 3) is no longer equal to 1, because industries import part of the intermediate inputs they require – and part of value added 'leaks' to abroad. In Indonesia, 18.3% of the increase in final demand and value-added leaks to foreign countries and in South Korea this leakage is 26.55%. Japan (the strongest economy in the region) benefits most from final demand growth in the other economies. For instance, an increase in final demand in South Korea by 1 unit will generate (via backward production linkages) around 0.065 units of value added in Japan. The economic relationship is not symmetrical: an increase in final demand in Japan by 1 unit will generate (via backward production linkages) around 0.002 units of value added in South Korea. This asymmetry means that the growth inducement of final demand growth in South Korea to Japan's GDP is 30 times larger than the growth inducement of final demand growth in Japan to Korea's GDP.

Another, more recent, example of cross-border value-added inducement appears in Table 4. The value-added inducement is measured here as an 'elasticity': the coefficient 0.185 expresses the percentage increase in real GDP in (say) Austria to an increase in final demand in Germany by 1 percentage point. It can be seen that the value-added induced by final demand growth in Germany is particularly high in its neighbouring economies: Austria (0.185), the Netherland (0.161) and Belgium (0.133). Final demand growth in Germany (by 1%-point) has only a modest impact on real GDP in Italy (+0.072%), Spain (+0.062%), Portugal (+0.066%) and Greece (only

+0.035%). These findings show that faster growth in Germany will not be able to bring about a strong recovery of real GDP in the Southern-European economies.

Table 4
Value-added inducement elasticities in the Eurozone, 2009
(caused by an increase of 1%-point of final demand in Germany)

Austria	Belgium	Spain	Finland	France	Greece	Ireland	Italy	NL	Portugal
0.185	0.133	0.062	0.064	0.064	0.035	0.121	0.072	0.161	0.066

Source: Table 2 in Oliver Picek and Enno Schröder. 'Spillover effects of Germany's final demand on Southern Europe.' *World Economy* 41: 2216–2242.

Note: The reported value-added inducement elasticities are based on the so-called closed IOM.

3. Employment multipliers

We can also use the IOM to investigate the structure of employment in an economy and the employment effects of changes in final demand. To do so, let us continue with our hypothetical example (of Table 1) and define the (1x2) vector of jobs ℓ_j per unit of gross output in industry *j*:

$$\boldsymbol{\ell} = [12/16 \quad 3/12] = [0.75 \quad 0.25]$$

ℓ_i = the number of jobs (in millions) per €1 billion of gross output in industry *i*. In agriculture, the job intensity is 0.75 million jobs per €1 billion of gross output; in industry, the job intensity is lower, with 0.25 million jobs per €1 billion of gross industrial output. The coefficients ℓ_i represent the direct job creation per €1 billion of gross output in industry *i*. But if industry *j* is growing, production – and hence jobs – in the other industry will rise. What can we say about the indirect job creation along the supply chain?

To estimate the total job creation **E** caused by an €1 billion increase in final demand and gross output of industry *j* we pre-multiply the Leontief inverse with vector $\boldsymbol{\ell}$ as follows:

$$\mathbf{E} = \boldsymbol{\ell} \times [\mathbf{I} - \mathbf{A}]^{-1} = [0.75 \quad 0.25] \times \begin{bmatrix} 2.68 & 1.35 \\ 0.81 & 1.61 \end{bmatrix} = [2.21 \quad 1.42]$$

The total increase in employment **E** caused by an increase in final demand for and gross output of agriculture by €1 billion is 2.21 million jobs. We know that the direct increase in employment due to this growth of agricultural gross output is 0.75 million jobs. The indirect increase in employment, triggered by a rise in agricultural gross output by €1 billion, is therefore 1.46 million jobs. The indirect job creation (along the supply chain) in this case is almost twice as large as the direct job creation; this shows the importance of the backward production linkages of agriculture. The employment multiplier of an increase in agricultural gross output by €1 billion is equal to (circa) 3 (= the total effect of 2.21 million jobs/the direct effect of 0.75 million jobs).

The total increase in employment caused by an increase in final demand for and gross output of manufacturing by €1 billion is 1.42 million jobs. The indirect increase in employment, triggered by a rise in manufacturing gross output by €1 billion, is therefore 1.17 million jobs; manufacturing expansion creates jobs in agriculture, because the demand for agricultural inputs of manufacturing firms increases (this is the backward production linkage). The employment multiplier of an increase in manufacturing gross output by €1 billion is equal to 5.7 (= the total effect of 1.42 million jobs/the direct effect of 0.25 million jobs).

The input-output analysis helps policy-makers to identify those industries where the potential for employment generation is largest. Based on the employment multipliers, policy-makers can also identify those industries with the largest indirect employment effects – which operate via the backward (upstream) production linkages.

4. Environmentally-Extended Input-Output Analysis

A final application of the input-output model is in the analysis of various environmental impacts of economic activity. We will here use the 'carbon footprint' of production as an example – and to do so, we define the (1 x 2) vector **f** of tonnes of CO_{2eq} emissions per unit of gross output as follows. Assume that activities in agriculture resulted in the emission of 8 million tonnes of carbon; manufacturing emitted 10 million tonnes of carbon. Note that total CO_{2eq} e emissions in the economy are 18 million tonnes of CO_2. Vector **f** then becomes:

$$\mathbf{f} = [8/16 \quad 10/12] = [0.50 \quad 0.83]$$

f_j is the direct emission intensity of industry *j*. f_1 = 0.50, which means that agriculture is emitting 0.5 million tonnes of CO_{2eq} per €1 billion of gross output and final demand for agricultural goods.

We next calculate the total (direct + indirect) emissions **F** associated with an increase in final demand for good *j* per unit of output of industry *j*, as follows:

$$\mathbf{F} = \mathbf{f} \times [\mathbf{I} - \mathbf{A}]^{-1} = [0.50 \quad 0.83] \times \begin{bmatrix} 2.68 & 1.35 \\ 0.81 & 1.61 \end{bmatrix}$$
$$= [0.50 \times 2.68 + 0.83 \times 0.81 \quad 0.50 \times 1.35 + 0.83 \times 1.61] = [2.0 \quad 2.0]$$

Element F_1 (= 2 million tonnes of CO_{2eq}) of F is the total (direct + indirect) carbon emissions associated with an increase in final demand for / gross output of agriculture by €1 billion. It follows that the indirect carbon emissions, which can be attributed to one unit of agricultural activity, equal 1.5 million tonnes of CO_{2eq}. Total carbon emissions per €1 billion of gross manufacturing output are 2 million tonnes of CO_{2eq} as well, but indirect CO_2 emissions, which can be attributed to one unit of industrial activity, are lower, equaling 1.17 million tonnes of CO_{2eq}.

If we post-multiply the diagonalized vector F by the vector of final demand **y**, we obtain the quantities P of CO_{2eq} emissions by industry of 'production':

$$\mathbf{P} = \hat{\mathbf{F}} \times \mathbf{y} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \times \begin{bmatrix} 3 \\ 6 \end{bmatrix} = \begin{bmatrix} 2 \times 3 \\ 2 \times 6 \end{bmatrix} = \begin{bmatrix} 6 \\ 12 \end{bmatrix}$$

We find that total CO_{2eq} emissions attributable to agriculture equal 6 million tonnes, whereas CO_{2eq} emissions for which manufacturing is responsible equal 12 million tonnes. Total carbon emissions remain 18 million tonnes of CO_{2eq}.

We must recall that recorded carbon emissions in agriculture were 8 million tonnes of CO_{2eq} and registered emissions from manufacturing amounted to 10 million tonnes of CO_{2eq}. These emissions can be called carbon emissions recorded by geographical location (a farm versus a factory). But we can now see that 2 million tonnes of carbon emitted by agriculture are caused by the production of agricultural inputs used by manufacturing; these emissions constitute carbon embodied in the intermediate inputs supplied by agriculture (the producing industry) to manufacturing (the using industry). If final demand for manufacturing increases by €1 billion, manufacturing will buy more inputs produced by agriculture; the carbon emissions by agriculture, needed to produce these inputs for manufacturing, are reclassified as being caused by manufacturing.

The Environmentally-Extended Input-Output model re-classifies a known quantity of total carbon emissions recorded at a particular geographical location to a 'production-chain-based' classification which transfers carbon embodied in the intermediate inputs supplied by agriculture (the producing industry) to manufacturing (the using industry). Manufacturing, in other words, is responsible for 12 million tonnes of CO_{2eq} emissions – directly (10 million tonnes of carbon) and indirectly (2 million tonnes of carbon, embodied in the intermediate inputs it purchased from agriculture).

The input-output model:
a (10 industry x 10 industry) illustration for Germany (2015)

We apply the IOM to the input-output matrix for Germany (in 2015) given in Table 3.

Table 3

Input-Output Table: Germany (2015) (billion Euros)

	Ag	Mi	Man	EGW	Con	WRT	Info	FIRE	Govt	Serv	FD	Demand
Ag	6	0	34	0	0	1	1	0	1	2	5	51
Mi	0	0	37	13	1	1	1	0	0	1	-43	12
Man	14	2	632	21	69	62	51	17	42	48	841	1798
EGW	1	1	33	28	3	9	14	9	10	10	70	187
Con	0	0	7	4	24	7	18	15	9	4	187	275
WRT	7	1	182	14	17	134	36	14	30	26	310	772
Info	2	1	90	16	22	78	187	58	124	42	526	1144
FIRE	1	0	38	7	8	51	70	103	38	26	307	650
Govt	0	0	1	0	0	0	1	0	1	0	751	755
Serv	1	0	34	5	2	18	36	23	32	48	495	695
VA	18	6	710	80	130	411	730	410	468	488	0	3451
GO	51	12	1798	187	275	772	1144	650	755	695	3451	9790

Source: OECD statistics.
Notes: Ag = agriculture; Mi = mining; Man = manufacturing; EGW = electricity, gas & water supply; Con = construction; WRT = wholesale trade, retail trade and transportation and storage; Info = ICT and information; FIRE = finance, insurance & real estate; Govt = public administration and defense; Serv = other services including education, health care, arts & entertainment and food and accommodation. VA = gross value added; GO = gross output or supply. FD = final demand.

The first step is to calculate the (10 industry x 10 industry) **A**-matrix (of technical coefficients) by dividing each cell in the first ten rows and columns by the corresponding column total (= gross output).

This gives us the following **A**-matrix:

0.110	0.002	0.019	0.001	0.002	0.002	0.001	0.000	0.001	0.003
0.003	0.022	0.021	0.069	0.004	0.001	0.001	0.001	0.001	0.001
0.278	0.180	0.351	0.114	0.250	0.080	0.044	0.026	0.055	0.069
0.014	0.050	0.019	0.150	0.009	0.012	0.012	0.014	0.013	0.014
0.007	0.024	0.004	0.020	0.087	0.009	0.016	0.023	0.013	0.006
0.145	0.090	0.101	0.075	0.062	0.173	0.032	0.021	0.040	0.037
0.034	0.068	0.050	0.083	0.079	0.101	0.163	0.089	0.164	0.060
0.029	0.028	0.021	0.038	0.029	0.066	0.061	0.159	0.050	0.038
0.000	0.001	0.000	0.001	0.001	0.001	0.001	0.001	0.001	0.000
0.019	0.019	0.019	0.024	0.006	0.024	0.032	0.036	0.042	0.069

Next we calculate the [**I** – **A**] matrix:

0.890	-0.002	-0.019	-0.001	-0.002	-0.002	-0.001	0.000	-0.001	-0.003
-0.003	0.978	-0.021	-0.069	-0.004	-0.001	-0.001	-0.001	-0.001	-0.001
-0.278	-0.180	0.649	-0.114	-0.250	-0.080	-0.044	-0.026	-0.055	-0.069
-0.014	-0.050	-0.019	0.850	-0.009	-0.012	-0.012	-0.014	-0.013	-0.014
-0.007	-0.024	-0.004	-0.020	0.913	-0.009	-0.016	-0.023	-0.013	-0.006
-0.145	-0.090	-0.101	-0.075	-0.062	0.827	-0.032	-0.021	-0.040	-0.037
-0.034	-0.068	-0.050	-0.083	-0.079	-0.101	0.837	-0.089	-0.164	-0.060
-0.029	-0.028	-0.021	-0.038	-0.029	-0.066	-0.061	0.841	-0.050	-0.038
0.000	-0.001	0.000	-0.001	-0.001	-0.001	-0.001	-0.001	0.999	0.000
-0.019	-0.019	-0.019	-0.024	-0.006	-0.024	-0.032	-0.036	-0.042	0.931

and finally we obtain the Leontief inverse $\mathbf{L} = [\mathbf{I} - \mathbf{A}]^{-1}$

1.137	0.011	0.036	0.008	0.013	0.007	0.004	0.003	0.005	0.007
0.018	1.037	0.038	0.091	0.017	0.008	0.005	0.005	0.006	0.006
0.560	0.360	1.628	0.296	0.479	0.194	0.120	0.093	0.139	0.150
0.041	0.076	0.046	1.196	0.030	0.028	0.024	0.026	0.026	0.026
0.020	0.037	0.016	0.037	1.105	0.020	0.026	0.035	0.023	0.013
0.285	0.179	0.225	0.170	0.159	1.252	0.072	0.057	0.085	0.079
0.135	0.151	0.147	0.183	0.168	0.184	1.231	0.152	0.234	0.109
0.092	0.076	0.077	0.095	0.079	0.121	0.103	1.212	0.092	0.069
0.001	0.001	0.001	0.002	0.001	0.001	0.001	0.001	1.002	0.001
0.051	0.044	0.050	0.054	0.031	0.048	0.051	0.057	0.063	1.087

Note that the elements on the diagonal of the Leontief inverse all greater than unity; the reason is that if final demand for industry j increases by ≥ 1unit, gross output in industry j will increase by (at least) 1 unit.

By taking the column sums of the Leontief inverse, we obtain the following backward production linkages for the 10 German industries (in 2015):

Agriculture	2.340
Mining	1.972
Manufacturing	2.263
Electricity, gas & water supply	2.131
Construction	2.082
Wholesale, retail, transportation	1.863
Information & other business services	1.639
Finance, Insurance & Real Estate (FIRE)	1.640
Public administration	1.676
Other services	1.546

It can be seen that the backward production linkages of the services sectors ("other services", "FIRE" and "Information & other business services") are less strong than those of manufacturing, electricity, gas & water supply and agriculture.

Table 3

Value-added intensity, job intensity and greenhouse gas (GHG) intensity: Germany (2015)

	va	$\boldsymbol{\ell}$	f
Agriculture	0.361	11.02	1491.7
Mining	0.514	7.11	625.8
Manufacturing	0.395	4.32	94.3
Electricity, gas & water supply	0.427	3.04	1926.4
Construction	0.471	9.89	40.3
Wholesale, retail, transportation	0.533	9.88	135.2
Information & other business services	0.638	2.99	10.6
Finance, insurance & real estate (FIRE)	0.631	2.25	4.4
Public administration	0.620	3.65	8.3
Other services	0.702	19.02	28.4

Source: OECD Statistics.

Table 3 presents industry-wise data of the direct value-added intensity **va**, the direct job intensity $\boldsymbol{\ell}$, and the direct Greenhouse Gas (GHG) intensity **f** of Germany's gross output.

The direct value-added intensity is dimensionless (because it is the value added in billions of euros divided by gross output in billions of euros). The direct value-added intensity is highest (0.702) in "other services" – meaning that an increase in final demand for and gross output of "other services" by €1 generates €0.70 in extra income. The direct value-added intensity is lowest for manufacturing (0.395) and agriculture (0.361) – indicating that an increase in final demand for and gross output of manufacturing and agriculture by €1 generates only €0.40 and €0.36 in additional income. Using the vector of direct value-added intensities and the Leontief inverse, we can next calculate the value-added multipliers, using this equation: $\mathbf{VAm} = \mathbf{va} \times [\mathbf{I} - \mathbf{A}]^{-1} \times \widehat{\mathbf{va}}^{-1}$ as we did before. The results appear in Table 4.

The value-added multipliers are highest for agriculture (2.772) and manufacturing (2.534), the two industries with the lowest direct value-added intensities and the strongest backward production linkages. If final demand and output increase in agriculture and manufacturing, this generates additional income in their supply chains. The value-added multipliers are relatively low in "other services", FIRE, "Information & business services" and public administration – all industries where backward production linkages are less strong.

The direct employment (or job) intensity (in Table 3) is defined in terms of jobs per €1 million of gross output. In Germany, direct job intensity is very high in "other services": an increase in final demand for and gross output of "other services" by €1 million generates 19 extra jobs. It is also high in agriculture, where an increase in final demand for and gross output of "other services" by of €1 million generates 11 extra jobs. The direct employment intensity is relatively low in "information & other business services" (2.99 jobs per €1 million of gross output) and FIRE (only 2.25 jobs per €1 million of final demand and gross output).

Total (direct + indirect) job creation per €1 million of final demand and gross output can be calculated by pre-multiplying the Leontief inverse by the vector of direct job intensities, or: $\mathbf{E} = \boldsymbol{\ell} \times [\mathbf{I} - \mathbf{A}]^{-1}$. The resulting total employment multipliers for German industries (in 2015) appear in Table 4. The total job multiplier is highest for "other services"(22.91 jobs per €1 million of final demand and gross output), agriculture (19.8 extra jobs per €1 million of final demand and gross output), construction (16.2 additional jobs per €1 million of gross output), and wholesale, retail & transportation (15.37 extra jobs per €1 million of gross output). Industries with low employment multipliers include FIRE and "Information & other business services". What is interesting is that growth in some industries, such as agriculture and manufacturing, leads to considerable indirect job growth. An increase by €1 million in final demand and gross output for agriculture and manufacturing indirectly generates 8.78 jobs and 7.46 jobs (along the supply chains), respectively. Growth in "other industries", in contrast, generates only 3.89 indirect jobs – in response to an €1 million increase in its gross output

Table 4

Value-added intensity, job intensity and greenhouse gas (GHG) intensity: Germany (2015)

	VAm	**total job multiplier**	**indirect job creation**	**total GHG multiplier**	**indirect GHG creation**
Agriculture	2.772	19.80	8.78	1881.3	389.6
Mining	1.945	12.87	5.76	874.4	248.5
Manufacturing	2.534	11.78	7.46	353.3	259.0
Electricity, gas & water supply	2.344	9.49	6.45	2428.6	502.2
Construction	2.124	16.20	6.31	202.5	162.1
Wholesale, retail, transportation	1.876	15.37	5.48	261.4	126.2
Information & oth. business serv.	1.567	6.54	3.55	92.3	81.7
FIRE	1.585	5.71	3.46	84.1	79.7
Public administration	1.613	7.62	3.97	100.2	91.8
Other services	1.425	22.91	3.89	121.5	93.1

Finally, we turn to the direct GHG intensity of gross output in Germany in Table 3. Direct GHG intensity is measured as tonnes of CO_{2eq} per €1 million of gross output. The direct GHG intensity high in agriculture (1492 tonnes of GHG per €1 million of gross output) and electricity generation (1926 tonnes of GHG per €1 million of gross output). Direct GHG intensity is very low in FIRE and public administration. The total (direct + indirect) GHG intensities are estimated using the vector of direct GHG intensities **f** and the Leontief inverse: $\mathbf{F} = \mathbf{f} \times [\mathbf{I} - \mathbf{A}]^{-1}$. F is the vector of total GHG intensity by industry in Table 4; it can be interpreted as a total GHG multiplier as well. Electricity has the highest GHG multiplier: 2429 tonnes of GHG are generated in the economy as a whole in response to an increase in electricity gross output of €1 million. Agriculture comes second: an increase in agricultural gross output of €1 million generates an additional 1883 tonnes of GHG along the supply chain. Indirect effects are relatively large (compared to direct GHG intensities) in manufacturing, construction and all services sectors.

Total GHG emissions in Germany in 2015 were 769.5 million tonnes of CO_{2eq} (see Table 5). These emissions were recorded at some geographical source – say a farm or factory – and then attributed to a particular industry – agriculture or manufacturing (in our example). Electricity (and gas & water supply) is the biggest GHG polluter, emitting 360.4 million tonnes of CO_{2eq}, followed by manufacturing with 169.6 million tonnes of CO_{2eq}. The services sectors such as FIRE and public administration have low GHG emissions.

An important part of the GHG emissions by the electricity sector is related to the generation of electricity used as intermediate input by the other industries. These carbon emissions are the responsibility of the using industries, and should be counted as part of their emissions. This is what we do if we post-multiply the diagonalized vector **F** by the vector of final demand **y** and obtain the quantities P of CO_{2eq} emissions by industry of 'production': $\mathbf{P} = \hat{\mathbf{F}} \times \mathbf{y}$. The results are presented in the third column of Table 5. The final column of Table 5 gives the difference between GHG emissions recorded at source and emissions by industry of usage.

The biggest difference occurs in manufacturing. Due to its strong backward production linkages, manufacturing is using GHG-intensive intermediate inputs (such as electricity) in a relatively high degree. If we re-allocate the GHG emissions embodied in these intermediate inputs to the manufacturing sector, we find that manufacturing is – directly and indirectly – responsible for 297.3 million tonnes of CO_{2eq}, making it the most carbon-intensive industry in Germany. The difference of 127.7 million tonnes of CO_{2eq} constitutes the carbon embodied in the intermediate inputs used by manufacturing.

GHG emissions of the services industries also increase significantly. The combined GHG emissions of "Information & professional business services", FIRE, public administration and "other services" are 41.1 million tonnes of CO_{2eq} (at geographical location), but when we include carbon embodied in the intermediate inputs used by these industries, total GHG emissions are 209.8 million tonnes of CO_{2eq}; the difference is 168.8 million tonnes of CO_{2eq} of GHG emissions embodied in inputs.

The "electricity, gas & water supply" sector is a main supplier of intermediate inputs to other (using) industries. The carbon embodied in the supply of these inputs is subtracted from the recorded GHG emissions of this industry. As a result, "electricity, gas & water supply" is responsible for only 171.1 million tonnes of CO_{2eq}, instead of 360.4 million tonnes of CO_{2eq}. Finally, we must note that re-allocated GHG emissions for mining are negative, because Germany is a net importer of mining products (oil and gas). Final demand for mining is negative (see Table 3), because the value of imports is larger than (*c* + *g* + *i* + *e*). The negative emissions for mining are an indication that Germany is a net 'carbon importer' from the rest of the world.

Table 5

Greenhouse gas (GHG) emissions: Germany (2015)

(million tonnes of CO_{2eq})

	recorded at geographical location	**reclassified according to responsible (using) industry**	**difference**
Agriculture	75.9	9.8	- 66.1
Mining	7.2	-37.5	-44.8
Manufacturing	169.6	297.3	127.7
Electricity, gas & water supply	360.4	171.1	-189.2
Construction	11.1	37.9	26.8
Wholesale, retail, transportation	104.3	81.1	-23.3
Information & oth. business serv.	12.1	48.6	36.5
FIRE	2.9	25.8	22.9
Public administration	6.3	75.2	68.9
Other services	19.8	60.2	40.4
Total	769.5	769.5	0.0