MOT1421
Economic Foundations
Week Three

Production, Efficiency, Choice of Technique

S. STORM

LECTURE NOTE MOT1421-W-3B

The Lecture Note MOT1421-W-3A is part of the exam materials.
The required reading for Week 3 consists of:

- This Lecture Note MOT1421 W-3A and Lecture Note MOT1421 W-3B.

Supporting videos:

- https://www.youtube.com/watch?v=xLSRMt-wWAM production functions and diminishing marginal returns (to labour).
- https://www.youtube.com/watch?v=lT8eSU9pxcw discussion of isoquants and the optimal choice of production technique.
- https://www.youtube.com/watch?v=hqcHyuR-JF8 useful explanation of (neutral, capital-saving and labour-saving) technological progress.

Production, Efficiency, Choice of Technique

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Production, Efficiency, Choice of Technique

This Lecture Note examines some of the ways in which neoclassical economics analyses efficiency at the level of the firm. The neoclassical approach to 'efficiency' focuses on a firm's choice between available alternatives **techniques of production**. We set out the neoclassical theory of the (optimal) choice of production techniques, which is located at the level of the representative profit-maximising firm.

The basis of the neoclassical theory of production is the **production function**. The production function is a purely technical – engineering – relation between factor inputs and outputs. It describes the laws of proportion, that is, the transformation of factor inputs into products (outputs) in any particular time period. The production function represents the **technology** of a firm (or an industry) and it includes (as we shall see below) all the technically efficient methods of production.

The general mathematical form of the production function is:

(1) $$x = f(L, K, R, A; \vartheta, \gamma)$$

where *x* = output; *L* = labour input; *K* = capital input (= machines); *R* = raw materials; *A* = land; ϑ = returns to scale; γ = efficiency parameter. A widely used two-factor production function is the **Cobb-Douglas production function**, because it is easiest to handle mathematically (see **box 1**):

(2) $$x = a \times L^{\alpha} \times K^{\beta}$$

Different combinations of labour *L* and machine-capital *K* yield different levels of output, depending on the magnitude of *a* = the efficiency parameter, and the exponents. Differentiating eq. (2) with respect to labour *L* gives the marginal product of labour MP_L:

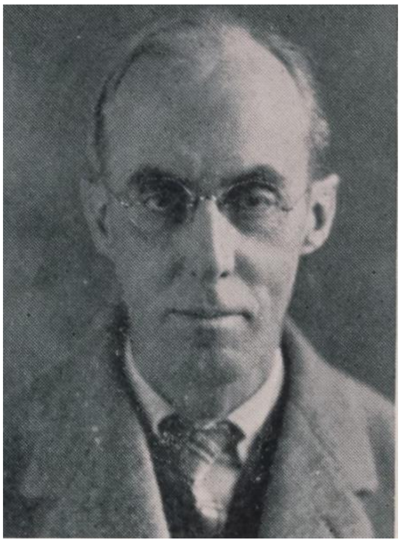
(3) $$MP_L = \frac{\partial x}{\partial L} = \alpha \times a \times L^{\alpha-1} \times K^{\beta} = \alpha \times \frac{a \times L^{\alpha} \times K^{\beta}}{L} = \alpha \times \frac{x}{L} > 0$$

The MP_L is the extra output which the firm will obtain when it hires one more unit of labour. The marginal product of labour is always positive: hiring one extra worker will always generate additional output. But the extra output generated by one extra unit of labour will decline, the more labour the firm is employing. This can be seen by differentiating eq. (3) with respect to labour:

$$(4) \qquad \frac{\partial\, MP_L}{\partial\, L} = -\alpha \times \frac{x}{L^2} < 0$$

Eq. (4) shows that there are diminishing marginal returns to the increased use of labour.

Box 1



Charles W. Cobb
(1875-1945)
American mathematician and economist

Paul H. Douglas
(1892-1976)
American economist and senator,
busy on the telephone.

Professors Cobb and Douglas published their production function in 1928, in an article entitled "A Theory of Production", *American Economic Review* 18 (Supplement): pp. 139–165.

Likewise, we can derive the MP_K:

$$(5) \qquad MP_K = \frac{\partial\, x}{\partial\, K} = \beta \times a \times L^{\alpha} \times K^{\beta-1} = \beta \times \frac{a \times L^{\alpha} \times K^{\beta}}{K} = \beta \times \frac{x}{K} > 0$$

The MP_K is the extra output which the firm will obtain when it uses one additional unit of capital. Differentiating eq. (5) with respect to capital gives:

$$(6) \qquad \frac{\partial\, MP_K}{\partial\, K} = -\beta \times \frac{x}{K^2} < 0$$

Eq. (6) shows that there are diminishing marginal returns to the increased use of machines.

Returning to the production function of eq. (2), we note that this production function exhibits **constant returns to scale** if $\alpha + \beta = 1$. What does this mean? It means that if the inputs of both labour and capital are raised by (say) 10%, output increases by 10% as well. Here is the proof. Let *L* and *K* increase by a factor *k* > 1. We then have:

$$(7) \quad x^* = a \times (k \times L)^\alpha \times (k \times K)^\beta = a \times L^\alpha \times K^\beta \times k^{\alpha+\beta} = k^{\alpha+\beta} \times x$$

If $\alpha + \beta = 1$, output will rise by a factor *k* in response to a rise in the inputs L and *K* by the same factor *k*. Output rises in proportion to the scale of the inputs labour and capital.

The production function will exhibit **increasing returns to scale**, if $\alpha + \beta > 1$. From eq. (7), it can be seen that output will rise by more than the factor *k* in response to a rise in the inputs L and *K* by the same factor *k*. Output rises more than proportionally to the scale of the inputs labour and capital.

The production function will exhibit **decreasing returns to scale**, if $\alpha + \beta < 1$. From eq. (7), it can be seen that output will rise by less than the factor *k* in response to a rise in the inputs L and *K* by the same factor *k*. Output rises less than proportionally to the scale of the inputs labour and capital.

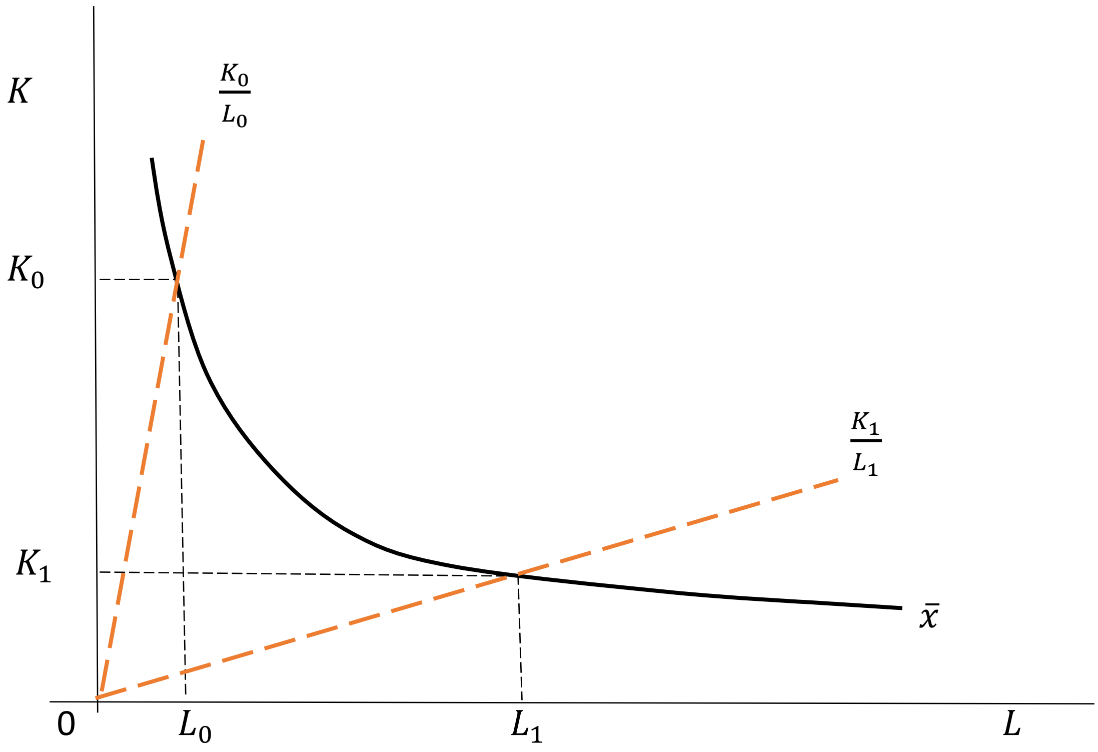
The production function can be depicted in the form of **a set of production isoquants**. (The idea is similar to that of deriving indifference curves from a utility function.). The production isoquant can be derived by setting output x at a particular level $\bar{x}$ and the re-writing eq. (2):

$$(8) \qquad \bar{x} = a \times L^\alpha \times K^\beta \rightarrow K = (\frac{\bar{x}}{a})^{\frac{1}{\beta}} \times L^{-\frac{\alpha}{\beta}}$$

The production isoquant is illustrated in Figure 1. For the firm, the combination of L_0 workers and K_0 machines generates a level of output $\bar{x}$. The same level of output can be produced with more workers L_1 and fewer machines K_1. This means that the combinations L_0, K_0 and L_1, K_1 are strictly comparable in terms of output, but the first combination is more capital-intensive, whereas the second combination is more labour-intensive. In general, each combination of capital and labour on the isoquant constitutes a **technique of production**, defined by a particular capital intensity (the ratio *K/L*). **The isoquant itself is the envelop which contains all available techniques of production, from which the**

firm must make a choice. The analogy with the consumer choosing the utility-maximising consumption bundle should be clear.

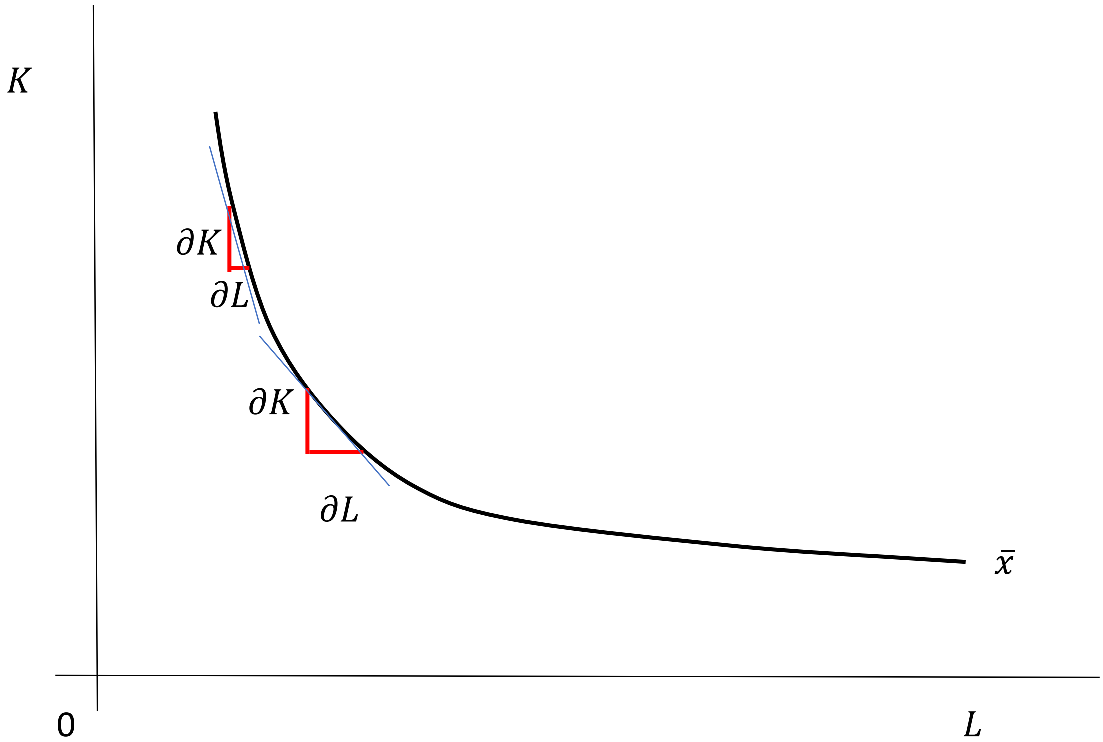
Figure 1
A production isoquant



Three further points concerning Figure 1 have to be made. First, if we move down the isoquant, from L_0, K_0 to L_1, K_1, we are lowering the input of machines and raising the input of labour, while keeping output constant. This process is called the process of **capital-labour substitution**. In this case, we are substituting capital for labour. If the firm decides to stop producing with the capital-intensive technique of production L_0, K_0 and hires more workers to produce with the other technique of production L_1, K_1, this change is called **technical change**, and it is based on capital-labour substitution.

The slope of the isoquant ($\frac{\partial K}{\partial L}$) defines the degree of substitutability of the two factors of production. The slope of the isoquant decreases (in absolute terms) as we move downwards along the isoquant, showing the increasing difficulty in substituting *K* for *L*. The slope of the isoquant is called the **marginal rate of technical substitution** (MRTS) of the two factors – see Figure 2.

Figure 2
The marginal rate of technical substitution (MRTS)



The MRTS is the slope of the isoquant: $\frac{\partial K}{\partial L}$. To determine the slope, note that the level of output $\bar{x}$ is constant along the isoquant. This means that if we increase the input of *L*, we have to reduce the input of *K*. The production function is $\bar{x} = f(L, K)$. Totally differentiating this production function gives: $\partial x = 0 = \partial \mathrm{L} \times \frac{\partial \mathrm{x}}{\partial \mathrm{L}} + \partial \mathrm{K} \times \frac{\partial \mathrm{x}}{\partial \mathrm{K}}$, which may be written as: $\partial K \times MP_K = -\partial L \times MP_L$. Solving for $\frac{\partial K}{\partial L}$, we obtain: $\frac{\partial K}{\partial L} = -\frac{MP_L}{MP_K}$. The MRTS is equal to the ratio of the marginal products of the factors of production. The marginal product of labour is $MP_L = \alpha \times \frac{x}{L}$ in eq. (3). The marginal product of capital is $MP_K = \beta \times \frac{x}{K}$ in eq. (5). Accordingly, the MRTS = $\frac{\partial K}{\partial L} = -\frac{MP_L}{MP_K} = -\frac{\alpha}{\beta} \times \frac{K}{L}$.

We can, *of course*, derive the MRTS directly by differentiating eq. (8), the equation for the isoquant, with respect to labour *L*:

$$\frac{\partial K}{\partial L} = -\frac{\alpha}{\beta} \times \left(\frac{x}{a}\right)^{\frac{1}{\beta}} \times L^{-\frac{\alpha}{\beta}-1} = -\frac{\alpha}{\beta} \times a^{-\frac{1}{\beta}} \times a^{\frac{1}{\beta}} \times L^{\frac{\alpha}{\beta}} \times K \times L^{-\frac{\alpha}{\beta}-1} = -\frac{\alpha}{\beta} \times \frac{K}{L}$$

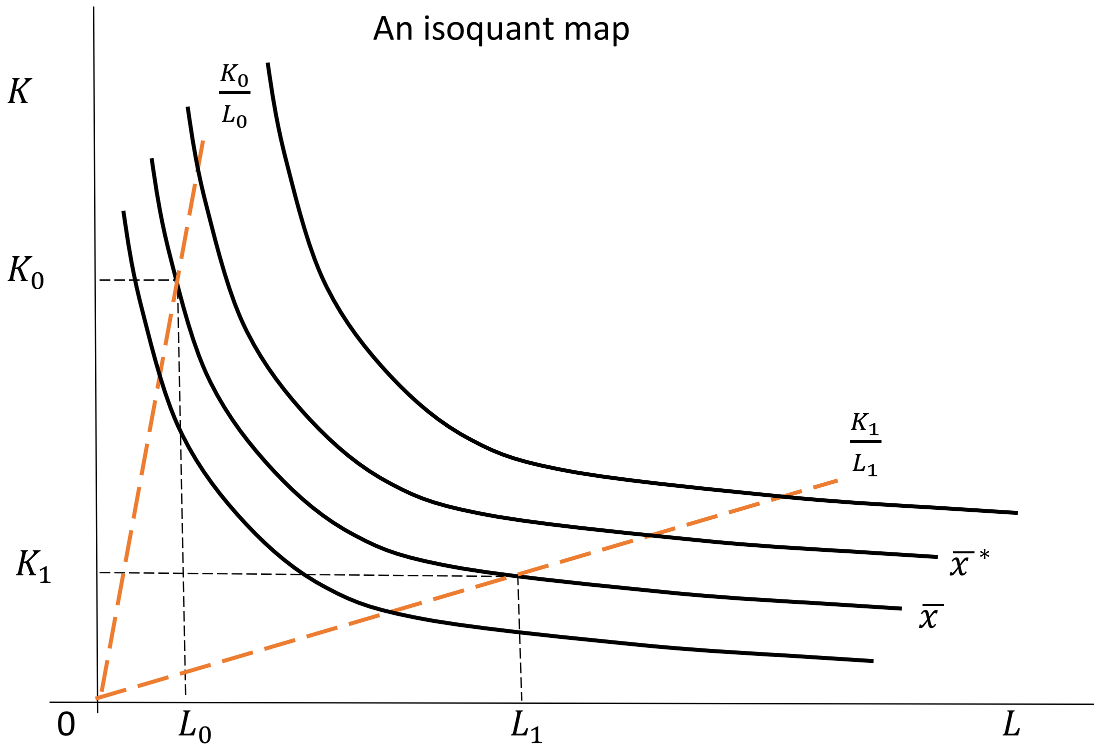
This gives the same result.

The second point concerning Figure 1 is the following. For the two combinations L_0, K_0 and L_1, K_1 to be strictly comparable, both combinations have to be **technically efficient**. This means that these combinations represent the minimum inputs of labour and capital required to generate output level $\bar{x}$ (from an engineering perspective). A production isoquant therefore implies **technical efficiency**.

Third, we can draw an isoquant map, as in Figure 3. The further away from the origin (the higher up in the graph) is the isoquant, the higher is the level of output *x* associated with that isoquant.

Figure 3

An isoquant map



Let us suppose that the profit-maximising output of the representative firm is $\overline{x}$. From Figures 1 and 2, we can see that the firm can produce this level of output $\overline{x}$ using a large variety of production techniques, all given by the production isoquant. The question facing the (instrumentally rational) firm thus is: which technique of production (= which combination of labour and capital) is the best or optimal one? The answer to this conundrum is: **the optimal technique of production is the one that maximises firm profits**.

Let us approach the problem in a more formal manner. Define firm profits Π as the difference between total revenue *TR* and total cost *TC*:

(9) $$\Pi = TR - TC = P \times x - (W \times L + R \times K)$$

where *P* = the price of the good produced by the firm; total revenue $TR = P \times x$; $TC = W \times L + R \times K$, where *W* = the wage paid by the firm; and *R* = the price of a capital good. Note that $W \times L =$ wage costs and $R \times K =$ capital costs. The output price *P*, the wage *W*, and the price of a capital good *R* are exogenous; the firm has no market power to influence these prices.

Eq. (9) is the profit function. The (instrumentally rational) firm is assumed to maximise profits – hence, we can use eq. (9) to determine maximum firm profits by investigating the first-order condition for a maximum. The firm has two (decision) instruments (which it can vary) to maximise profits, namely the input of labour *L* and the input of machines *K*. Starting with the latter, the firm will continue to increase the input of machines until the value of the marginal product of capital (see eq. (5)) is equal to the marginal cost (of adding one extra machine to the production process). We can see this by differentiating eq. (9) with respect to *K* and setting the resulting expression to zero:

(10) $$\frac{\partial\,\Pi}{\partial\,K} = P \times \frac{\partial\,\mathrm{x}}{\partial\,K} - R = P \times MP_K - R = 0 \rightarrow P \times MP_K = R$$

$P \times MP_K =$ the value of the marginal product of capital; $R =$ the marginal cost of capital. As long as $P \times MP_K > R$, the firm obtains more extra revenue from using one more machine than what it costs. From eq. (5), we know that $MP_K = \beta \times \frac{x}{K}$; substituting this expression into eq. (10) and re-arranging gives the following result:

(11) $$K = \beta \times x \times (\frac{R}{P})^{-1}$$

Eq. (11) is the capital demand function of the firm; the demand for machines increases if the level of output *x* rises, and it falls if the price of capital goods rises relative to the output price ($\frac{R}{P}$)

We can also determine the profit-maximising input of labour by differentiating eq. (9) with respect to *L* and setting the resulting expression to zero:

(12) $$\frac{\partial \, \Pi}{\partial \, L} = P \times \frac{\partial \, \mathrm{x}}{\partial \, L} - W = P \times MP_L - W = 0 \rightarrow P \times MP_L = W$$

$P \times MP_L =$ the value of the marginal product of labour; $W =$ the marginal cost of capital. As long as $P \times MP_L > W$, the firm obtains more extra revenue from employing one more worker than what she/he costs (which is the wage). From eq. (3), we know that $MP_L = \alpha \times \frac{x}{L}$; substituting this expression into eq. (12) and re-arranging gives the following result:

(13) $$L = \alpha \times x \times (\frac{W}{P})^{-1}$$

Eq. (13) is the labour demand function of the firm; the demand for labour increases if the level of output *x* rises, and it falls if the wage rate rises relative to the output price ($\frac{W}{P}$).

Using equations (11) and (13), we can answer the question: which technique of production is the optimal (= profit-maximising) one for the representative firm? Dividing eq. (11) by eq. (13), we obtain:

(14) $$\frac{K}{L} = \frac{\beta \times x \times \left(\frac{R}{P}\right)^{-1}}{\alpha \times x \times \left(\frac{W}{P}\right)^{-1}} = \frac{\beta}{\alpha} \times \frac{W}{R}$$

The optimal technique of production, defined by the profit-maximising ratio of capital to labour $\frac{K}{L}$, depends on the ratio of the exponents of the Cobb-Douglas production function $\frac{\beta}{\alpha}$ and the ratio of the wage rate and the price of capital goods $\frac{W}{R}$. What eq. (14) states is that the profit-maximising firm will (instrumentally rationally) respond to an increase in the wage rate *W*, while *R* stays constant, by increasing the capital-intensity $\frac{K}{L}$ of production, using more machines per worker. <u>Capital is substituted for labour</u> – and we <u>move up</u> the production isoquant in Figure 1. Conversely, the profit-maximising firm will respond to a decline in the ratio $\frac{W}{R}$ by lowering capital-intensity, using fewer machines per worker than before the relative price change. <u>Labour is substituted for capital</u> – and we now <u>move down</u> the isoquant in Figure 1. The essence of eq. (14) is capital-labour substitution by the profit-maximising firm, in response to changes in the relative price of labour $\frac{W}{R}$.

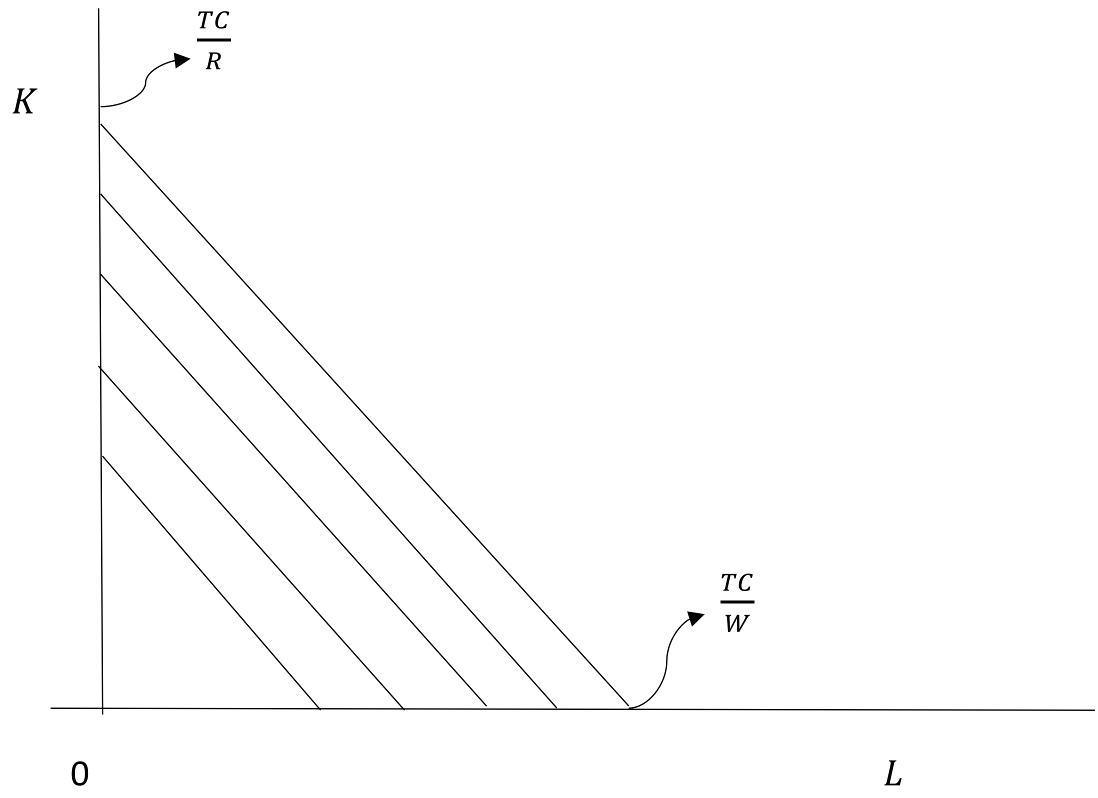
To illustrate the choice of the optimal production technique graphically, we must bring the total cost function into Figure 1. The total cost function $TC = W \times L + R \times K$ can be rewritten as follows:

(15) $$K = \frac{TC}{R} - \frac{W}{R} \times L$$

We assume that *TC* is exogenously determined. The total cost function is illustrated in Figure 4. We have a set of (parallel) cost curves; curves closer to the origin show a lower total-cost outlay. The cost lines are parallel, because they are drawn on the assumption of constant prices of factors *W* and *R*. This means that all cost curves have an identical slope $-\frac{W}{R}$.

Figure 4

The total cost functions of the firm



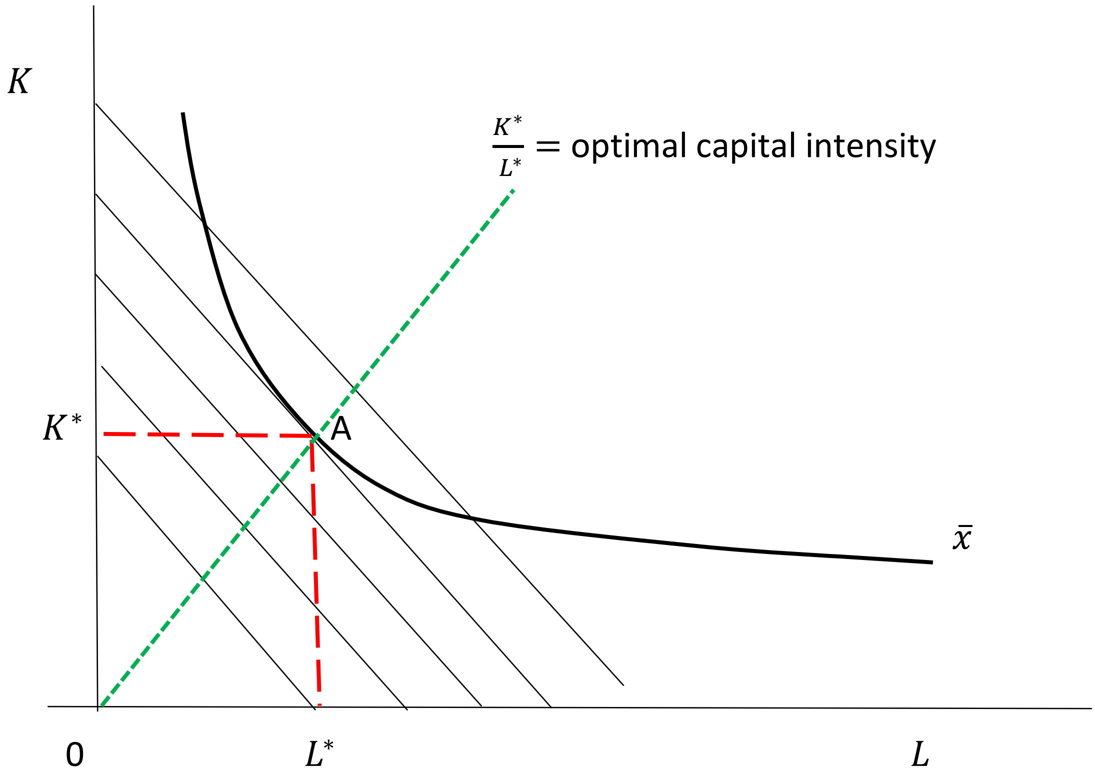
The firm is in equilibrium when it maximises its profits given its total cost outlay and the prices of the production factors, *W* and *R*. Maximum profits are attained when the firm uses that combination of capital and labour which is defined by the tangency of the total-cost line and the highest reachable isoquant – point A in Figure 5. In this point of tangency, the slopes of the total-cost curve and the isoquant are the same:

slope of isoquant = slope of total-cost curve

or: $$MRTS = \frac{\partial K}{\partial L} = -\frac{MP_L}{MP_K} = -\frac{W}{R}$$

This gives the following solution for $\frac{K}{L} = \frac{\beta}{\alpha} \times \frac{W}{R}$, which is the same as in eq. (14).

Figure 5
The optimal choice of technique

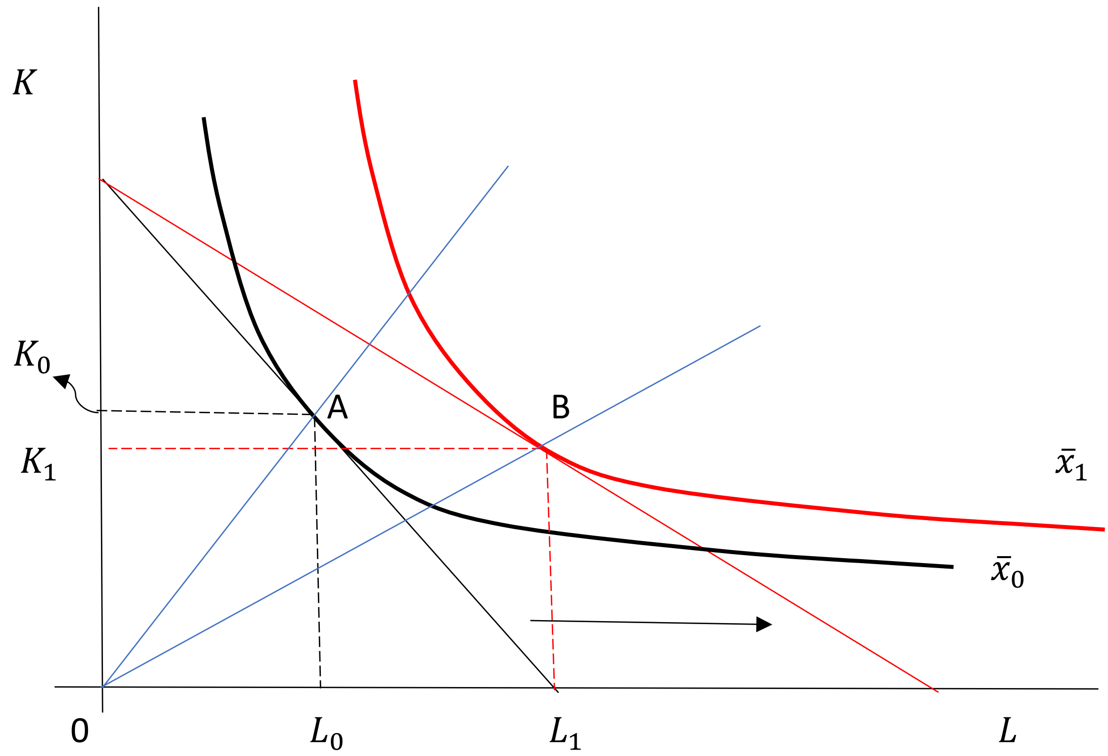


Let us finally consider what happens when the price of labour (the wage *W*) declines relative to the price of capital *R*. This is illustrated in Figure 6.

The initial profit-maximising choice of technique is given by point A – the point of tangency between the isoquant $\bar{x}_0$ and the original total-cost line. Due to the decline in *W/R*, the total-cost curve rotates; the red total-cost line is the new total-cost line; its slope $\frac{W}{R}$ has declined (in absolute terms) and the cost curve declines less steeply. With the same total cost outlay, the firm can now hire more workers and/or use more machines; as a result, the firm can produce more, which is illustrated by its move upwards to the higher isoquant $\bar{x}_1$. The new optimal choice of technique is given by point B. It can be seen that the new optimal technique of production is less capital-intensive than the original one: the relative wage decline has led to an increase of labour relative to the use of machines. This is capital-labour substitution (= a movement along the isoquant; in this case: down the isoquant, in favour of *L*.)

Figure 6

Technical change in response to a lower wage



In the **neoclassical theory of the firm**, the firm chooses the profit-maximising **technique of production**, defined by **capital intensity** $\frac{K}{L}$, in response to the cost of labour relative to the cost of capital $\frac{W}{R}$. A higher relative wage induces a profit-maximising substitution of labour for capital – which raises capital intensity. This is called **technical change**. The process of **capital-labour substitution** (moving up and down the isoquant) presupposes that the firm can choose the optimal technique of production from a continuous set of production techniques, as defined by the **isoquant** (and the underlying **production function**). The optimal technique of production is **technically efficient** and **economically efficient** (because it is the one technique which maximises firm's profits). Technical efficiency is a necessary but not a sufficient condition for economic efficiency.

Technological progress and the production function

The choice of technique is a static optimisation problem: choosing the profit-maximising production technique out of an available set of blueprints (as described by the isoquant). Because the underlying production function is constant (unchanging), the technology of production is constant. Firms, as it were, choose the best technique of production within a given overarching technology (or technological paradigm).

But as science advances, firms innovate and knowledge of new and more efficient methods of production become available, technology changes – and hence the production will change. This constitutes **technological progress**. Graphically, the effect of technological progress is shown by a change in the shape and/or the location of the production isoquant.

We may distinguish three types of technological progress: (1) neutral technological progress; (2) labour-saving technological progress; and (3) capital-saving technological progress. Let us first consider the case of neutral technological progress.

Neutral technological progress

In the case of neutral technological progress, the firm is able to produce the same level of output with fewer inputs of labour and capital; crucially, the ratio of capital to labour remains unchanged (while assuming that the relative price ratio *W/R* remains constant as well).

Let us consider the standard Cobb-Douglas production function:

(16) $$x = a \times L^{\alpha} \times K^{\beta}$$

Suppose $\alpha = \beta = \frac{1}{2}$. We assume that *a* = 1. We further assume that the ratio $\frac{W}{R} = \frac{4}{25}$. The optimal technique of production is determined by $\frac{K}{L} = \frac{\beta}{\alpha} \times \frac{W}{R} = \frac{0.5}{0.5} \times \frac{4}{25} = \frac{4}{25}$; this gives $K = \frac{4}{25} \times L$. The profit-maximising level of output is assumed to be *x* = 40 units. We can determine the optimal input of labour by substituting $K = \frac{4}{25} \times L$ in the production function:

(17) $$x = 40 = 1 \times L^{0.5} \times (\frac{4}{25} \times L)^{0.5} \rightarrow L = 40 \times (\frac{4}{25})^{-0.5} = 100$$

and *K* = 16. The firm is producing 40 units with an optimal capital intensity of $\frac{16}{100} = \frac{4}{25}$.

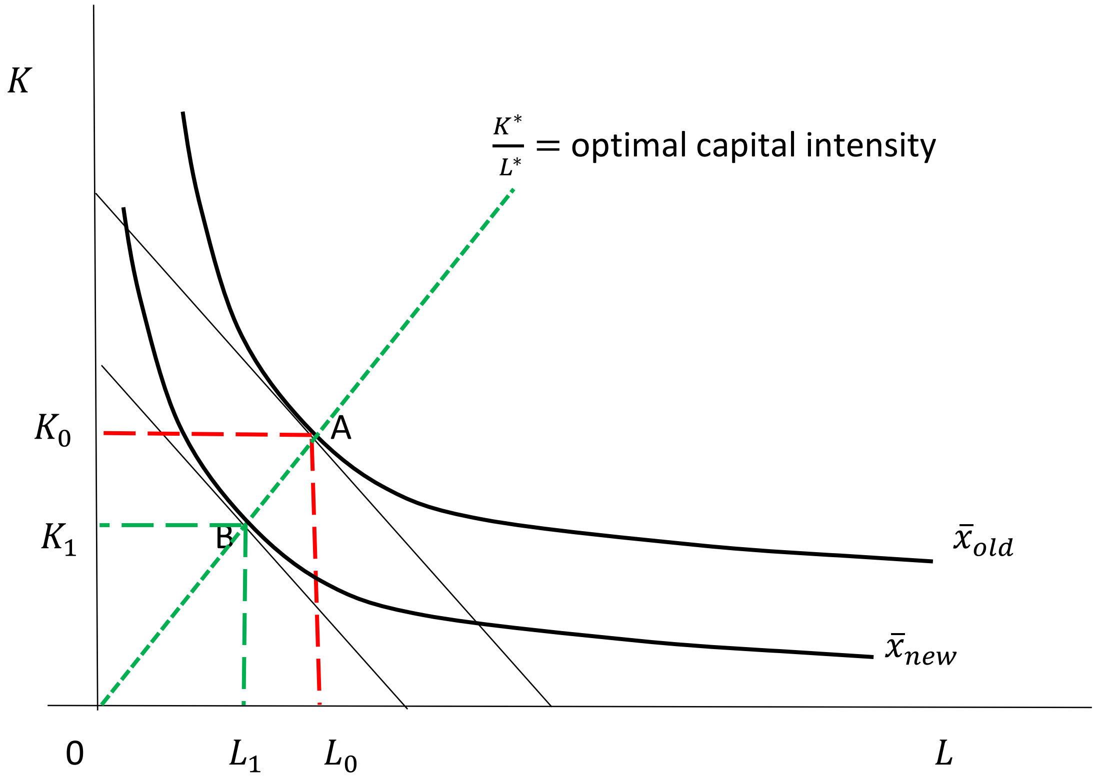
If there is **neutral technological progress**, this will show up in an increase in the efficiency parameter *a*. Let us assume that *a* doubles, hence *a* = 2. Due to this neutral technological advance, the firm can produce 40 units of output with fewer inputs of labour and fewer inputs of capital. We know that optimal capital intensity is not affected by the rise in *a*; after all, $\frac{K}{L} = \frac{\beta}{\alpha} \times \frac{W}{R} = \frac{0.5}{0.5} \times \frac{4}{25} = \frac{4}{25}$. We can compute the optimal input of labour by substituting $K = \frac{4}{25} \times L$ in the new production function:

(18) $$x = 40 = 2 \times L^{0.5} \times (\frac{4}{25} \times L)^{0.5} \rightarrow L = 20 \times (\frac{4}{25})^{-0.5} = 50$$

and $K = 8$. In the case of neutral technological progress, the firm can produce the same level of output, using fewer inputs of labour and capital; labour can be economised in proportion to capital – and optimal capital intensity stays the same. Figure 7 illustrates what happens. Due to the rise in efficiency parameter *a*, the isoquant shifts (in parallel fashion) towards the origin. The level of output of isoquant $\bar{x}_{old}$ is the same as the level of output associated with isoquant $\bar{x}_{new}$. The old optimum A shifts to the new optimum B. Optimal capital intensity did not change. For this reason, this type of technological progress is called **neutral**.

Figure 7

Neutral technological progress



Labour-saving technological progress

The second type of technological progress is called labour-saving, because due to the technological advancement, optimal capital intensity of production increases. In terms of the production function, labour-saving technological progress will manifest itself in a rise in the ratio $\frac{\beta}{\alpha}$. Let us return to the numerical example above: $\alpha = \beta = \frac{1}{2}$ and *a* = 1. Output is 40; *L* = 100; *K* = 16. The ratio $\frac{W}{R} = \frac{4}{25}$. The firm is producing 40 units with an optimal capital intensity of $\frac{4}{25} = 0.16$.

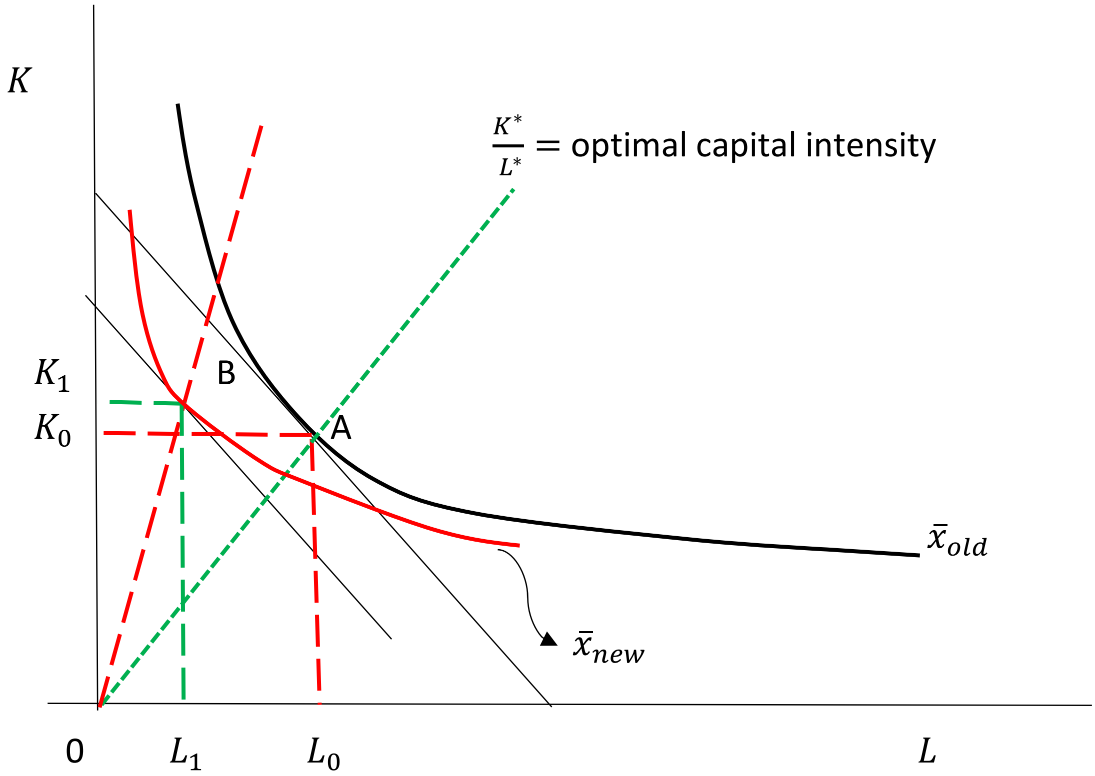
To illustrate **labour-saving technological progress**, we assume that $\alpha = 0.4$ and $\beta = 0.6$. As a result, optimal capital intensity will increase: $\frac{K}{L} = \frac{\beta}{\alpha} \times \frac{W}{R} = \frac{0.6}{0.4} \times \frac{4}{25} = \frac{6}{25}$, while we continue to keep the ratio $\frac{W}{R} = \frac{4}{25}$ constant. (The firm is consequently not moving up the original isoquant, but rather jumping to the new – changed – isoquant; see Figure 8) We get $K = \frac{6}{25} \times L$. The profit-maximising level of output remains *x* = 40 units. We determine the optimal input of labour by substituting $K = \frac{6}{25} \times L$ in the production function:

(19) $$x = 40 = 1 \times L^{0.4} \times (\frac{6}{25} \times L)^{0.6} \rightarrow L = 40 \times (\frac{6}{25})^{-0.6} \approx 94.2$$

and *K* = 22.6. The firm has economised on the input of labour, reducing the number of workers from 100 full-time workers to 94 full-time workers and one person working 20% of full time. The input of machines has increased from 16 robots to 22 robots + 60% of a robot 😊. Following the labour-saving technological progress, the firm is producing 40 units with a **higher** optimal capital intensity of $\frac{6}{25} = 0.24$.

Figure 8

Labour-saving technological progress



In Figure 8, the firm shifts from the old optimum A on the original isoquant $\bar{x}_{old}$ to the new optimum B on the new isoquant $\bar{x}_{new}$. Capital intensity rises, after the labour-saving technological progress. Labour-saving technological progress is the dominant type of technological progress in real life.

Capital-saving technological progress

The third type of technological progress is called capital-saving, because due to the technological advancement, optimal capital intensity of production decreases. In terms of the production function, capital-saving technological progress will manifest itself in a decline in the ratio $\frac{\beta}{\alpha}$. Let us return to the numerical example above: $\alpha = \beta = \frac{1}{2}$ and *a* = 1. Output is 40; *L* = 100; *K* = 16. The ratio $\frac{W}{R} = \frac{4}{25}$. The firm is producing 40 units with an optimal capital intensity of $\frac{4}{25} = 0.16$.

To illustrate **capital-saving technological progress**, we assume that $\alpha = 0.6$ and $\beta = 0.4$. As a result, optimal capital intensity will decrease: $\frac{K}{L} = \frac{\beta}{\alpha} \times \frac{W}{R} = \frac{0.4}{0.6} \times \frac{4}{25} = \frac{8}{75}$, while we continue to keep the ratio $\frac{W}{R} = \frac{4}{25}$ constant. (The firm is consequently not moving down the original isoquant, but rather jumping to the new – changed – isoquant.) We get $K = \frac{8}{75} \times L$. The profit-maximising level of output remains *x* = 40 units. We determine the optimal input of labour by substituting $K = \frac{8}{75} \times L$ in the production function:

(20) $$x = 40 = 1 \times L^{0.6} \times (\frac{8}{75} \times L)^{0.4} \rightarrow L = 40 \times (\frac{8}{75})^{-0.4} \approx 97.9$$

and *K* = 10.4. The firm has decreased the input of labour, reducing the number of workers from 100 full-time workers to 97 full-time workers and one person working 90% of full time. The input of machines has decreased from 16 robots to 10 robots + 40% of a robot 😊. The input of machines has declined more strongly than the input of labour, and hence, the K/L ratio has declined. The firm is producing in a more labour-intensive manner. Following the capital-saving technological progress, the firm is producing 40 units with a **lower** optimal capital intensity of $\frac{8}{75} \approx 0.11$.

Dynamic efficiency

The neoclassical approach to technological change treats **technological progress as exogenous** to the model of the firm. The neoclassical theory of the firm is concerned with **static economic efficiency**: led by its wish to maximise profits, the firm chooses that technique of production out of a set of already available techniques of production, which maximises profits. Any change in technology, be it neutral, labour-saving or capital-saving technological change, originates from factors outside the model.

Many (non-neoclassical) economists would argue instead that technological progress should be treated as an endogenous variable: a variable that is explained in the model itself. These economists emphasize **dynamic economic efficiency**: the fact that the firm itself can, over time, influence its production function through R&D and process innovation – so that the isoquants can no longer be assumed to be constant/unchanging.

In Lecture Note W-2B, we discussed static versus dynamic efficiency. If (monopolistic) firms use profits to innovate (and reduce cost of production by economising on the inputs of labour and capital), then they ensure dynamic efficiency. But such technological progress requires funding – and internal (accumulated) profits can be a useful source of financing innovation. This could lead to progress: lower costs, cheaper goods, new goods – which is what we call dynamic efficiency. This is beneficial to consumers in the long run, if the monopolist is entrepreneurial.

Joseph Schumpeter pointed to the conflict between static efficiency and dynamic efficiency which exists in perfect competition and in monopoly.

- In perfect competition, $P = AC$ in long-run equilibrium. There are no super-normal profits. This benefits consumers (cheap goods) but it deprives firms from internal sources of finance to fund innovation. Perfect competition may achieve static efficiency (the lowest prices for consumers), but it fails to bring about dynamic efficiency (i.e. allow for profits to finance R&D, product development and innovation). Consumers may be best off in perfect competition in the short run, but not in the long run.
- In monopoly, $P > AC$. Consumers pay high prices (because of the presence of super-normal profits) and the supply of goods is restricted. Static efficiency for consumers is compromised. But if the profits are used to finance innovation, consumers will benefit in the longer run from better, new, and cheaper products & services. Schumpeter argued that dynamic efficiency is of more importance to improvements in living standards and welfare than static efficiency.

This Schumpeterian logic underpins the legal intervention of patents. A patent provides an innovating firm with a temporary monopoly (roughly for 15-18 years), in which this firm can make super-normal profits to recoup the cost of the earlier innovation and finance further innovations. The trade-off between static and dynamic efficiency is resolved by giving priority to innovation – and dynamic efficiency of firms.

Questions

1. What is a production function?
2. What does the Cobb-Douglas production function look like?
3. What is the marginal product of labour?
4. What is the marginal product of capital?
5. When does a production function exhibit (a) constant returns to scale; (b) increasing returns to scale; and (c) decreasing returns to scale?
6. Explain the notion of a production isoquant.
7. What is the marginal rate of technical substitution?
8. Explain Figure 5.
9. What happens to optimal capital intensity when the wage declines? Use Figure 6 in your answer.
10. What is the difference between technical change and technological change?
11. Explain the notion of neutral technological progress.
12. Explain labour-saving technological progress – and use Figure 8 in your answer.
13. What is capital-saving technological change?
14. What is the difference between static and dynamic efficiency?

The answers to the questions can be found in this Lecture Note 😊

EXERCISES

Exercise 1

What is the difference between "static efficiency" and "dynamic efficiency"? Why is the difference importance? (Motivate your answer).

Exercise 2

Consider the following production function: $X = 10\sqrt{KL}$
Assume further that *W* =5 and *R* = 5. Hence, total cost *TC* = 5*L* + 5*K*

1. Show that the above production function exhibits constant returns to scale.

2. Suppose *X* = 40. Draw the corresponding isoquant in (*L,K*) space.

3. Draw the total-cost curve for *TC* = 40 in (*L,K*) space.

4. What is the profit-maximising combination of *L* and *K*? Note: the firm faces a total cost constraint of 40.

5. Suppose that technological progress changes the firm's production function to: $X = 20\sqrt{KL}$
   Draw the new isoquant (for *X* = 40) in the graph constructed under (c) and compare it to old isoquant. Suppose *K* = 4. How many workers per unit of *K* does it take to produce an output of 40 with the old technology as compared to the new technology?

6. What is the nature of the technological progress under (5): labour-saving, neutral or capital-saving?

7. Suppose now that technological progress changes the production to: $X = 20K^{0.6}L^{0.4}$. Draw the new isoquant in the graph constructed under (3) and (6).

8. Show that the technical progress under (7) has been labour-saving.

Exercise 3

In lengthy Exercise 1 we explore the neoclassical theory of production and of labour-saving technological progress. We will look at the concept "production isoquant" and use profit-maximisation to derive the formula for the optimal choice of production technique. We will finally consider the difference between shifts along the production isoquant (= capital-labour substitution) and shifts of the production isoquant (caused by labour-saving technological progress). We will use numerical example to explore the neoclassical theory of production.

Consider the following Cobb-Douglas (constant-returns to scale) production function (which describes the production process of all firms in the economy):

$$x = L^{0.5} \times K^{0.5}$$

where *x* = real GDP; *L* = number of workers; and *K* = number of robots in the production process. The production function is a description of how inputs of labour and of robots combine to produce a particular level of output. If we fix the level of output at (say) $\bar{x} = 10$, then we can identify all combinations of (*L*, *K*) which together generate an output level of 10 units.

1. Using the production function and assuming $\bar{x} = 10$, calculate at least 10 combinations of (*L*, *K*) which together generate an output level of 10 units. Plot these 10 combinations in a graph with *L* on the horizontal axis and *K* on the vertical axis; this convex downward-sloping curve in (*L*, *K*) plane is called a *production-isoquant*. Write *K* as a function of *x* and *L*. (This is the expression for the production isoquant.)
2. Now do the same as in question 1.1 but assuming that $\bar{x} = 16$. Plot the 10 combinations in the graph of question 1.1. You now have two production isoquants: one associated with a level of output of $\bar{x} = 16$ and one associated with a level of output $\bar{x} = 10$. You can see that the two isoquants have the same shape and (negative) slope. We can draw as many isoquants as we want; but note the following: a higher isoquant is associated with a higher level of output.
3. Suppose the economy as a whole has a total cost budget (determined by the level of economic activity) equal to $TC = W \times L + P_K \times K$, where *W* = the wage rate per worker; and P_K = the price of one robot. The wage

rate and the robot price are exogenous; $W = 1$ and $P_K = 4$. The total cost budget is 40. Hence, we can write: $TC = 40 = L + 4 \times K$. Write the TC function in terms of *K* being a function of *L*. Draw this *TC*-curve in your graph.

4. The production isoquant $\bar{x} = 10$ and the TC-function share one point of tangency. Determine the (*L, K*) coordinates of this point of tangency and call this point 'point A'. Draw a straight line from the origin (0, 0) through the point of tangency. This line gives you the optimal (profit-maximising) ratio of *K* to *L*. How large is this ratio?
5. We will now consider the process of capital-labour substitution, which means that we will be moving along the production isoquant. The production isoquant remains the same: $\bar{x} = 10$. The TC-function changes as follows: $TC = 2 \times L + 4 \times K$. That is, the wage rate *W* doubles from *W* =1 to *W* = 2. The input of labour becomes relatively more expensive (compared to the unchanged $P_K = 4$) and hence, profit-maximising firms will substitute robots for workers. The optimal (profit-maximising) ratio of *K* to *L* will rise. To illustrate this in your graph, assume that *TC* = 56.56854. The new *TC*-function thus becomes: $56.56854 = 2 \times L + 4 \times K$. Draw the new *TC*-curve in your graph and identify the point of tangency between the old isoquant (with $\bar{x} = 10$) and the new *TC*-curve. Draw a straight line from the origin (0, 0) through the new point of tangency, which we will call point B. This line gives you the new optimal (profit-maximising) ratio of *K* to *L*. Did the *K*-*L* ratio change as you expected?
6. This shift along the production isoquant reflects capital-labour substitution. We will now analyse this process in a more formal manner. To do so, we first define the profit function of firms:

$$\Pi = TR - TC = p \times x - (W \times L + P_K \times K)$$

where *p* = the general price level (which is here assumed to be exogenous); and *TR* = total revenue. Substituting the production function into the profit function gives:

$$\Pi = p \times L^{0.5} \times K^{0.5} - (W \times L + P_K \times K)$$

Firms are assumed to maximise profits and they have two instruments to do this: the input of *L* and the input of *K*. Starting with *K*, firms will continue to add robots to the stock of robots as long as the marginal revenue generated by the extra robot exceeds the marginal cost of the additional robot. We can identify the optimal number of robots in operation by differentiating the profit function with respect to *K* and setting the derivative equal to zero (= the first-order condition for a maximum):

$\frac{d\,\Pi}{d\,K} = 0.5 \times p \times L^{0.5} \times K^{-0.5} - P_K = 0 \rightarrow K = 0.5 \times x \times (\frac{P_K}{p})^{-1}$

The optimal (profit-maximising) input of robots increases as the level of output *x* rises and the price of one robot and declines as the price of one robot rises relative to the general price level $\frac{P_K}{p}$.

Similarly, you can identify the optimal number of workers by differentiating the profit function with respect to *L* and setting the derivative equal to zero (= the first-order condition for a maximum). Derive the outcome for *L*.

7. Divide the equation $K = 0.5 \times x \times (\frac{P_K}{p})^{-1}$ by the equation you obtained for the optimal input of *L* and simplify. What is the resulting equation for the optimal K-L ratio (or capital intensity of production)?
8. Suppose $W = 1$ and $P_K = 4$. What is the optimal capital-labour ratio of production? Next suppose $W = 2$ and $P_K = 4$: what is the optimal capital labour ratio of production? Compare your answers to the *K-L* ratios given by the two straight lines (through the origin) in your graph. The wage rate doubled? What happened to optimal capital intensity? How does the process of capital-labour substitution work?
9. In economic theory, a production function represents the technology of production. If there is technological progress, the production function changes. Let us assume that there is labour-saving technological progress and that the original production function $x = L^{0.5} \times K^{0.5}$ changes into the following novel production function: $x = L^{0.4} \times K^{0.6}$. (The change concerns the two exponents.) Derive the formula for the production isoquant associated with an output level $\bar{x} = 10$ and draw this production isoquant in your graph. How does labour-saving technological progress change the isoquant (compared to the original one)?

10. To illustrate this in your graph, assume that *TC* = 45.032. The new *TC*-function thus becomes: $45.032 = 1 \times L + 4 \times K$. Draw the new *TC*-curve in your graph and identify the point of tangency between the new isoquant (with $\bar{x} = 10$) and the new *TC*-curve. Draw a straight line from the origin (0, 0) through the new point of tangency, which we will call point C. This line gives you the new optimal (profit-maximising) ratio of *K* to *L*. The change from optimal point A to optimal point C is caused by induced labour-saving technological progress (due to which the production function and the production isoquant changed). Did the *K-L* ratio change as you expected?

Answers to the Exercises

Exercise 1

Static efficiency: the firm choses the profit-maximising technique of production (defined by the ratio K/L) out of a GIVEN SET of available production techniques, as defined by the production isoquant and hence the production function. The technology is constant or exogenous.

Dynamic efficiency: the firm invests in R&D in order to improve the production process through innovating. If successful, this will mean that the production function will change ("*a*", "α" or "*β*" will alter) – and as a result, the production isoquant will shift down towards the origin (in the case of neutral technological progress) or it will change its form (in case of labour-saving or capital-saving technological progress). The technology itself is changing as firms are economizing on the inputs of labour and capital.

Exercise 2

1. Constant returns to scale: A production function exhibits constant returns to scale when a proportionate increase in each input produces the same proportionate increase in output. Here: $\alpha + \beta = 1$.

2. Suppose X = 40. Then we can calculate the following combinations of (K,L) which together form the isoquant X = 40.

X	K	L
40	16	1
40	8	2
40	4	4
40	2	8
40	1.6	10

3. Assume: TC = 5K + 5L (R = 5 and W = 5). The isocost-curve for TC = 40 is: K = 8 - L

TC	K	L
40	8	0
40	7	1
40	6	2
40	4	4
40	2	6
40	0	8

4. The profit maximising (K,L) ratio: $\frac{K}{L} = \frac{\beta}{\alpha} \times \frac{W}{R} = \frac{0.5}{0.5} \times \frac{5}{5} = 1$
   This means: K = L. Substitution into the cost-curve K = 8 – L then gives the profit-maximising values of K = 4 and L = 4.

5. The new isoquant for X = 40 becomes: K = 4/L

X	K	L
40	4	1
40	2	2
40	1	4

With the old technology, one required 4 units of L and 4 units of K to produce X = 40. Hence, the old L/K ratio = 1. With the new technology, one requires only 1 unit of L and 4 units of K to produce X = 40. The new L/K ratio = 1/4. We cannot however conclude that the technological change has been labour-saving, because the factor price ratio (W/R) is not the same in the old (K=4, L=4) point as compared to the new (K=4, L =1) point.

6. The technological change has been neutral, because - at the given factor price ratio (W/R = 1) - the (K,L) does not change. This can be seen from the equation for the optimal (profit-maximising) (K,L) ratio:
$\frac{K}{L} = \frac{\beta}{\alpha} \times \frac{W}{R} = \frac{0.5}{0.5} \times \frac{5}{5} = 1$ β and α have not changed!

7. The new isoquant:

X	K	L
40	3.2	1
40	2.0	2
40	1.3	4
40	0.8	8
40	0.7	10

8. This time, the technological change affects β and α, and hence the optimal (profit-maximising) ratio of K to L is changed at unchanged relative factor prices (W/R):

$$\frac{K}{L} = \frac{\beta}{\alpha} \times \frac{W}{R} = \frac{0.6}{0.4} \times \frac{5}{5} = 1.5$$

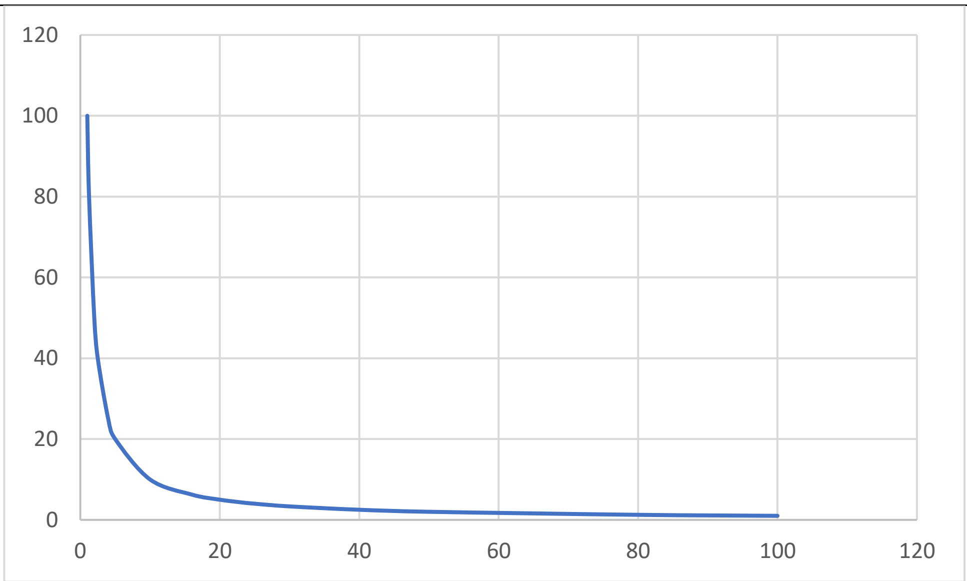
The (K,L) ratio increases from 1.0 in the old situation to 1.5 after technological progress. The technological progress has thus been labour-saving.

Exercise 3

1. The production function is $x = L^{0.5} \times K^{0.5}$ and we fix $\bar{x} = 10$ We can then derive the following equation for the isoquant: $K = \frac{100}{L}$. Using this formula, we can calculate the pairs of (L, K) which yield an output level of 10 in the following table. The corresponding production-isoquant is drawn in Figure 1 (next to the table; with *L* on the horizontal axis and *K* on the vertical axis).

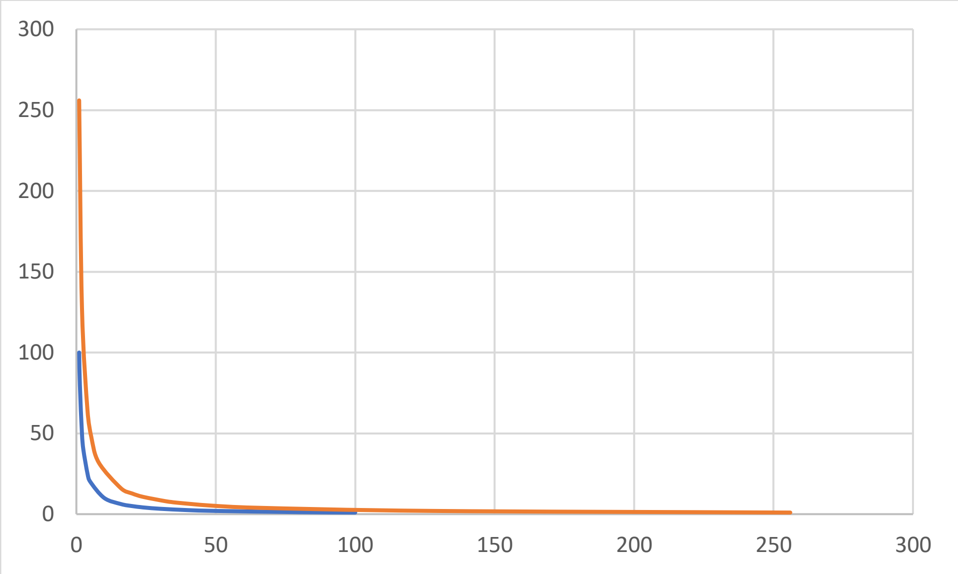
Figure 1

L	K
1	100
1,25	80
2	50
2,5	40
4	25
5	20
10	10
16	6,25
20	5
25	4
40	2,5
50	2
80	1,25
100	1

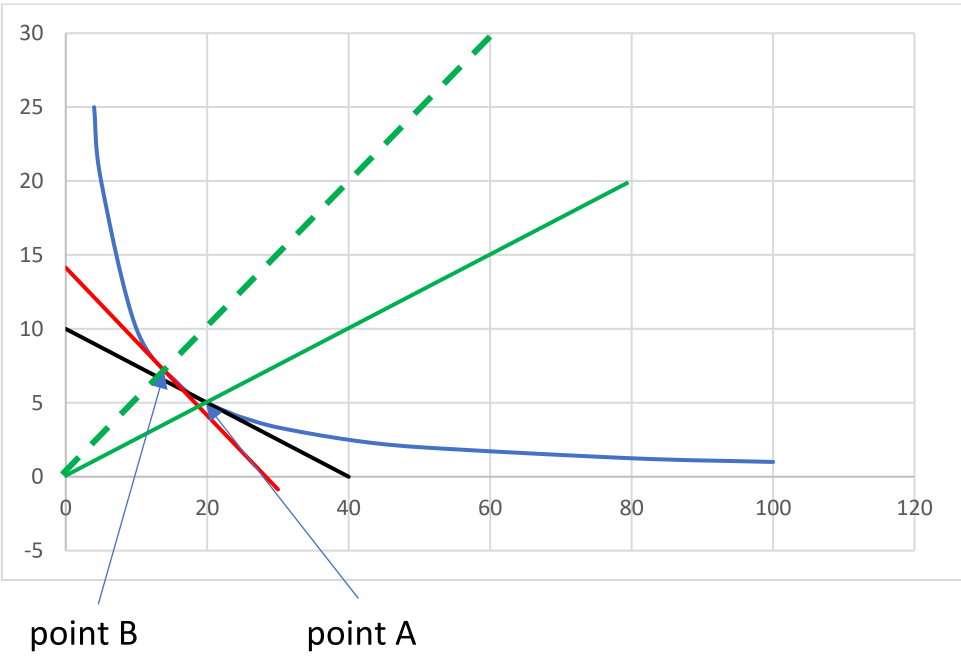


2. Figure 2 (with *L* on the horizontal axis and *K* on the vertical axis):

L	K
1	256
2	128
4	64
5,333333	48
8	32
16	16
20	12,8
24	10,66667
32	8
36	7,111111
48	5,333333
64	4
128	2
256	1



3. $TC = 40 = L + 4 \times K \rightarrow K = 10 - 0.25 \times L$. This is the black curve in the following graph (with *L* on the horizontal axis and *K* on the vertical axis):



4. The point of tangency is (*L* = 20; *K* = 5). This is point A (on the green curve). The capital-labour ratio is ¼.

5. The new TC-curve becomes: $K = \frac{56.56854}{4} - 0.25 \times L$ This is the red TC-curve in the graph. The new point of tangency with the isoquant is (*L* = 14.14; *K* = 7.07). This is point B (on the green dashed curve). The capital-labour ratio has increased. This is what we expected: the wage rate doubled, the capital price stayed unchanged; hence, firms economise on labour and start to use capital instead. The K/L ratio increases to ½.
6. To identify the optimal number of workers we differentiate the profit function with respect to *L* and set the first derivative equal to zero (= the first-order condition for a maximum):

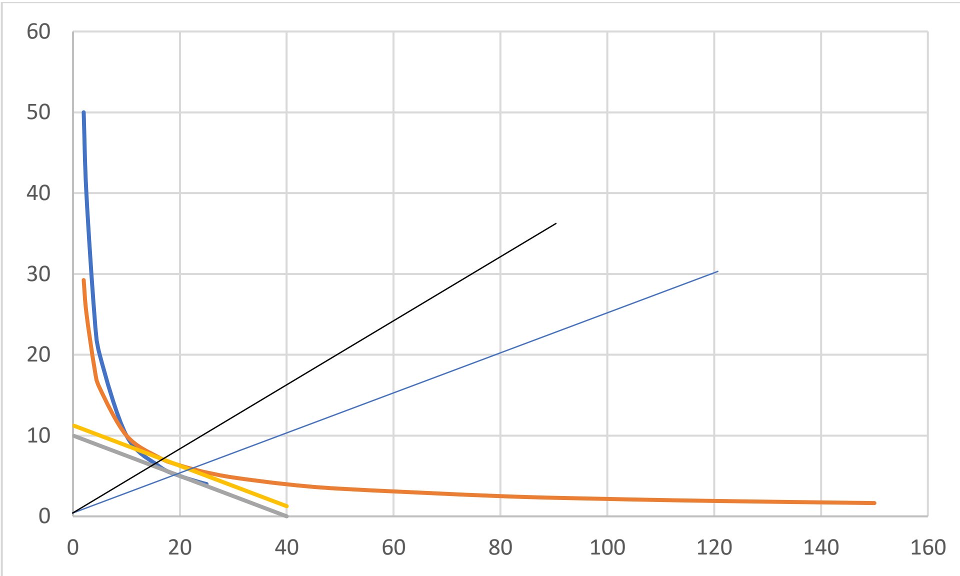
   $\frac{d\,\Pi}{d\,L} = 0.5 \times p \times L^{-0.5} \times K^{0.5} - W = 0 \rightarrow L = 0.5 \times x \times (\frac{W}{p})^{-1}$

   Labour demand increases as output rises; labour demanded declines when the real wage $\frac{W}{p}$ rises, which means that nominal wage rate increases more than the general price level.

7. $\frac{K}{L} = \frac{0.5 \times x \times (\frac{P_K}{p})^{-1}}{0.5 \times x \times (\frac{W}{p})^{-1}} \rightarrow \frac{K}{L} = \frac{W}{P_K}$

   The optimal (= profit-maximising) capital-labour ratio depends on the ratio of the nominal wage rate and the nominal price of a robot.

8. We can check this finding using our two numerical examples. If $W = 1$ and $P_K = 4$, the optimal $\frac{K}{L} = 0.25$ (point A). If $W = 2$ and $P_K = 4$, the optimal $\frac{K}{L} = 0.5$ (point B). We can now predict what will happen if $W = 3$ and $P_K = 4$. The optimal capital-labour ratio will increase to 0.75. Capital-labour substitution means that firms will shift up along the isoquant when the relative wage rate rises; they will shift downward (to the right) when the relative wage rate declines. What matters for substitution is the relative cost of labour to the cost of capital goods: $\frac{W}{P_K}$.
9. The new production function, after the labour-saving technological progress occurred, is $x = L^{0.4} \times K^{0.6}$. The old isoquant and the new isoquant are pictured in the following graph (with *L* on the horizontal axis and *K* on the vertical axis):



The new isoquant declines less steeply. The equation for the new isoquant is: $K = x^{5/3} \times L^{-2/3}$.

10. Using we what have learned in sub-question 7, we can calculate the optimal capital-labour ratio with the new isoquant as follows:

$$\frac{K}{L} = \frac{0.6 \times x \times (\frac{P_K}{p})^{-1}}{0.4 \times x \times (\frac{W}{p})^{-1}} \rightarrow \frac{K}{L} = \frac{3}{2} \times \frac{W}{P_K} = \frac{3}{2} \times \frac{1}{4} = 0.375$$

since we assume that $W = 1$ and $P_K = 4$. The optimal $\frac{K}{L}$ was 0.25 and increases to 0.375. Production becomes more capital-intensive at the unchanged ratio $\frac{W}{P_K} = \frac{1}{4}$. This is therefore not a shift along the isoquant (caused by a change in $\frac{W}{P_K}$), but this increase in capital-intensity occurred due to **labour-saving technological progress**. Per robot we now produce with fewer workers. The new $TC = 45.032 = 1 \times L + 4 \times K$. We know that $K = 0.375 \times L$, and hence we get: $45.032 = 1 \times L + 4 \times 0.375 \times L$. This gives *L* = 18.01 and *K* = 6.75.