MOT1421
Economic Foundations
Week Three

MARKET CO-ORDINATION:
Oligopoly

S. STORM

LECTURE NOTE MOT1421-W-3A

The Lecture Note MOT1421-W-3A is part of the exam materials.
The required reading for Week 3 consists of:

- This Lecture Note MOT1421 W-3A and Lecture Note MOT1421 W-3B.

Supporting videos:

- https://www.youtube.com/watch?v=CmSlJ5XmwEU
- https://www.youtube.com/watch?v=N0L00FZnhtg
- https://www.youtube.com/watch?v=rQTf3oy0QUU

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Market Co-ordination: Oligopoly

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Oligopoly

Introduction

Most real-world markets resemble neither of the polar cases of perfect competition and monopoly. Most big corporations operate in a market structure of oligopoly. Oligopolistic markets are characterised by relatively few firms that have some power over the market price – they are not price takers – but no single firm exercises complete market power. What is crucial is that these big firms are strategically interdependent: they have to take note of the decisions and actions of their competitors (unlike firms in perfect competition and monopoly) in their attempt to maximize profits. This imperfectly competitive market is oligopolistic: it combines elements of competition and monopoly.

The oligopolistic market is a market in which supply is concentrated in only a few firms. The question, of course, is what we mean by 'a few'? Economics measure the degree of concentration of the supply-side of a market using concentration ratios – including the ratio C4 (= the combined market share of the four biggest firms in market) or the ratio C8 (= the combined market share of the top eight companies in a market). Table 1 presents the C4 and C8 concentration ratios for selected US industries.

Table 1
Market (Industry) Concentration Ratios
(in %): USA, 1992

Industry designation	C4	C8	Number of Firms
Cellulosic man-made fibre	98	100	5
Primary copper	98	99	11
Household laundry equipment	94	99	10
Cigarettes	93	100	8
Malt beverages (beer)	90	98	160
Electric lamp bulbs	86	94	76
Cereal breakfast foods	85	98	42
Motor vehicles	84	91	398
Small arms ammunition	84	95	55
Household refrigerators	82	98	52
Wood office furniture	26	34	611
Book publishing	23	38	2504
Fresh or frozen seafood	19	28	600
Misc. plastic products	5	8	7605

Source: Karl E. Case and Ray C. Fair (2002), Principles of Economics, Sixth Edition, Prentice Hall.

The C4 and C8 concentration ratios only give information about the combined shares of the largest few firms and discard information about the relative size of the smaller firms.

The Hirschman-Herfindahl Index (HHI) is an alternative indicator of market concentration, which combines information about the shares of all firms in the market, not just the largest 4 or 8 firms. The HHI is defined as follows:

$$HHI = s_1^2 + s_2^2 + s_3^2 + \ldots\ldots + s_N^2$$

where S_i = the market share of firm *i*. The HHI takes values between zero and 1. A HHI of 1 means that market supply is concentrated in only one firm (the monopolist). The closer the HHI is to zero, the lower is the degree of market concentration. The following table illustrates how the HHI is calculated.

Market shares	HHI
$s_1 = 1$ (monopoly)	HHI = 1
$s_1 = 0.5; s_2 = 0.5$	HHI = 0.5
$s_1 = (1/3); s_2 = (1/3); s_3 = (1/3)$	HHI = 1/3
$s_1 = \left(\frac{1}{4}\right); s_2 = \left(\frac{1}{4}\right); s_3 = \left(\frac{1}{4}\right);\ s_4 = \left(\frac{1}{4}\right)$	HHI = 0.25
$s_1 = 0.5; s_2 = 0.25; s_3 = 0.25$	HHI = 0.375

The HHI has the following important and interesting property: it is generally true that if there are *N* equal-sized firms, then the Herfindahl index H is (*1/N*).

Proof: Suppose there are *N* firms, each with a market share *1/N*. Then $H = (1/N)^2 + (1/N)^2 + \ldots\ldots + (1/N)^2 = N(1/N)^2 = (1/N)$. Accordingly, if there are three firms in the market with equal market shares, then HHI = 1/3. And if there are 25 firms in the market with equal market share, HHI = 1/25. This property of the HHI is helpful when we want to interpret the concentration of supply in markets in which firms have different market shares.

For instance, let us compare two different oligopolistic markets – as illustrated in the following table. Seven firms are operating market A. The biggest firm in Market A has a market share of 32%, the second largest firm has a share of 22%.

The HHI of Market A is 0.20 – which indicates that the degree of concentration in market A is comparable to the degree of concentration in a hypothetical market consisting of 5 firms with equal market shares. A HHI of 0.2 suggests a considerable degree of market concentration.

	Market A	Market B
s_1 (biggest firm)	0.32	0.61
s_2	0.22	0.10
s_3	0.17	0.09
s_4	0.11	0.08
s_5	0.08	0.07
s_6	0.07	0.03
s_7 (smallest firm)	0.03	0.02
HHI	0.20	0.40

There are also seven firms active in market B. Six of these firms are relatively small in terms of their market shares (all below 10%). There is one big firm holding 61% of supply. The HHI is 0.40 – which is very high. The degree of supply concentration in this market of seven firms is comparable to the degree of concentration in a hypothetical market consisting of 2½ firms with equal market shares. Using the HHI, we can conclude that the degree of market concentration is higher in market B than in market A.

Table 2 presents the HHI for a number of concentrated British industries in 2015. It can be seen that British tobacco manufacturing is a monopoly, while the degree of market concentration is very high in cement, metropolitan rail transport and mortgage financing.

Table 2

The HHI for selected British industries in 2015

Industry	HHI
Manufacture of macaroni, noodles, couscous and similar farinaceous products	0.87
Manufacture of tobacco products	0.99
Manufacture of ceramic sanitary fixtures	0.77
Manufacture of cement	0.80
Manufacture of basic iron and steel and of ferro-alloys	0.51
Manufacture of non-domestic cooling and ventilation equipment	0.60
Manufacture of railway locomotives and rolling stock	0.62
Manufacture of motorcycles	0.76
Urban, suburban or metropolitan area passenger railway transportation by underground, metro and similar systems	0.87
Activities of mortgage finance companies	0.92
Other credit granting companies not elsewhere classified (n.e.c.)	0.69

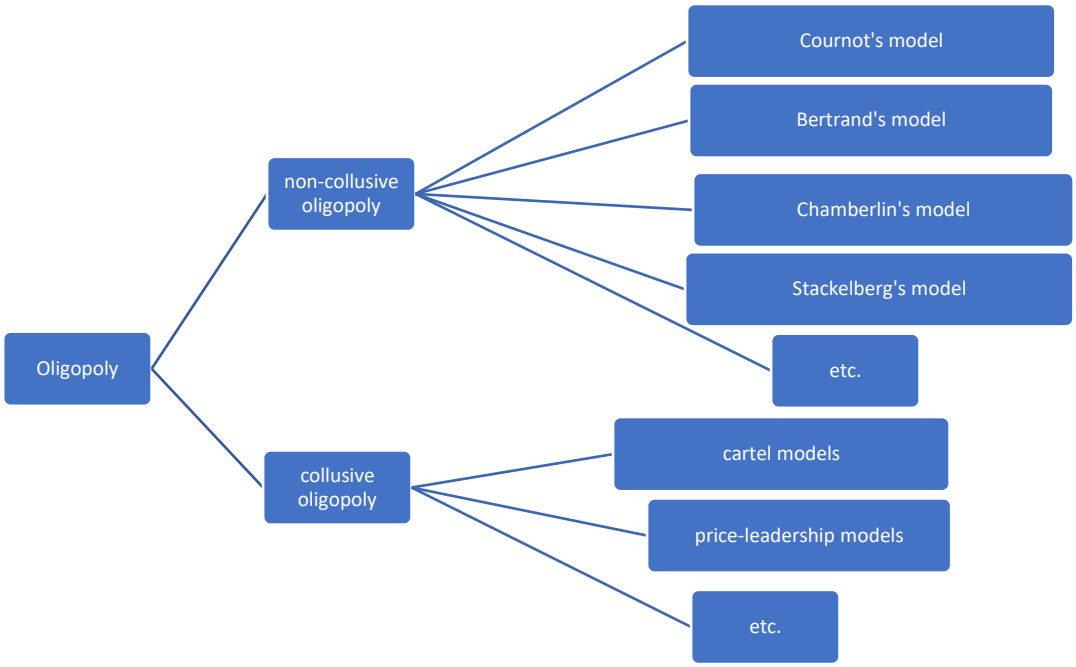
Oligopoly: The Recognition of Interdependence

There exists no generally accepted model of oligopolistic market behaviour. Figure 1 below illustrates the great variety in models of oligopolistic markets. Conduct (i.e. the price and output decisions by firms) in oligopoly depends on particular structural details of the specific market in question. Critical structural dimensions of oligopolistic markets include:

- the number of firms
- the importance of product differentiation[1]
- entry/exit conditions (is entry possible? easy? costly?)
- the legal conditions concerning collusion/cartels (antitrust law; competition policy)
- the uncertainty (concerning the reactions of the other firms) faced by the individual firm and the strategies they may adopt to cope with it.

[1] **Product differentiation** is the process of distinguishing a product or service from others, to make it more attractive to a particular target market. This involves differentiating it from competitors' products as well as a firm's own products. Product differentiation often acts as a barrier to entry of new firms into the industry. Advertising & marketing may enable a firm to achieve product differentiation.

Figure 1
Theories of Oligopoly



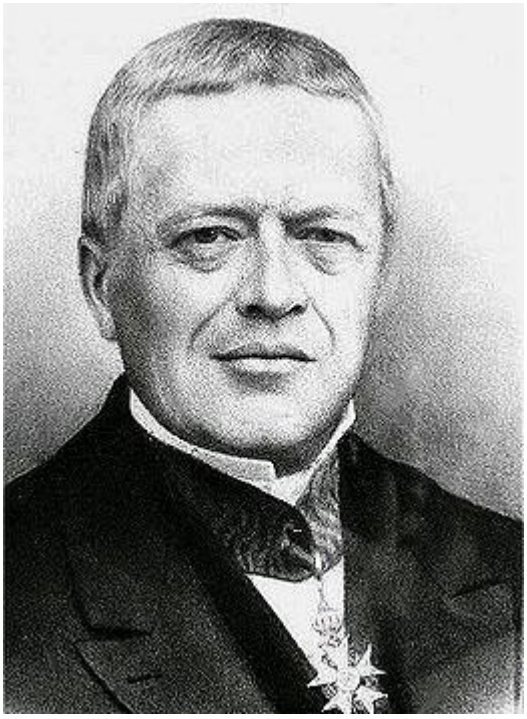
In this course we will analyse only one – canonical – model of oligopoly in detail: Cournot's model of non-cooperative oligopoly. We present this model for a duopoly (= a market with two firms), which is the limiting case of an oligopoly. We present a numerical version of Cournot's model – and use it to investigate the non-cooperative equilibrium outcome as well as the cooperative equilibrium outcome in which the two firms form a cartel (= an alliance of rivals). In a cartel agreement, firms act as one single (monopolistic) firm and coordinate their decisions to achieve monopoly profits. What they do is restrict output to the profit-maximizing level in the monopoly case. The cartel members have to agree upon the output levels of specific firms and the sharing of the monopoly profits earned by the cartel. Given the market demand curve and the industry's cost structure, profits will be as large as possible. We note that a cartel may be fundamentally unstable, because each member has an incentive to cheat on the agreement and expand its output to increase its own profits. Cartel agreements are also illegal: most jurisdictions consider forming a cartel anti-competitive behavior. Cartel behavior includes price fixing, bid rigging, and reductions in

output – to the disadvantage of consumers and other firms which are not members of the cartel.

Using Cournot's model we can highlight key features of oligopoly (in particular, the strategic interdependence between the two firms). Please note that Cournot's model is not THE model of oligopoly; as shown in Figure 1, there are many alternative versions of oligopolistic models.

Cournot's Model of Non-cooperative Oligopoly

The earliest oligopoly model was developed in 1838 by the sternly-looking French mathematician Antione Augustin Cournot.



Antoine Augustin Cournot (1801 – 1877), a French mathematician, occasional economist and philosopher. In 1838 he published his book *Researches on the Mathematical Principles of the Theory of Wealth*, in which he used mathematics in economic analysis. Cournot introduced the ideas of functions and probability into economic analysis. He derived the first formula for the rule of supply and demand as a function of price and in fact was the first to draw supply and demand curves on a graph, anticipating the work of Alfred Marshall by roughly thirty years. The Cournot duopoly model developed in his book also introduced the concept of a Nash equilibrium, reaction functions and best-response dynamics.

Assumptions

Cournot's model of oligopoly is based on the following assumptions:

- There are only a few sellers and there are many buyers. The total supply of the product is concentrated in a few firms.
- The product may be homogenous or differentiated. We assume here that the product is homogenous.

- Each firm has clearly defined (total, average and marginal) cost functions.
- The goal of the oligopolistic firms (*ex hypothesi*) is profit maximization. No other goals are pursued. The condition for maximum profits is: MR = MC.
- Entry of new firms is blocked by definition.
- Perfect knowledge, full transparency and complete information: the oligopolistic firms and all buyers have complete knowledge of the prevailing (and future) conditions of the market. Uncertainty is ruled out by assumption. There is also no asymmetric information, no insider knowledge etc.

Under the above assumptions we will examine the equilibrium of the monopoly market. We begin by assuming that the market-demand function is like this:

(1) $Q = 140 - P$

There are two firms, A and B, producing an identical good; we assume that there is no product differentiation. Total output $Q = Q_A + Q_B$.

We will first consider the dilemma facing firm A. Cournot assumes that firm A chooses its output level (Q_A) on the assumption that the output of firm B (Q_B) is fixed and will not be adjusted in response to firm A's actions. Hence, the demand curve facing firm A is:

(2) $Q_A = 140 - Q_B - P \rightarrow P = 140 - Q_A - Q_B$

Total revenue of firm A is by definition equal to:

(3) $TR_A = P*Q_A = [140 - Q_A - Q_B]\, Q_A = 140\, Q_A - (Q_A)^2 - Q_A Q_B$

It is important to recognize here that total revenue of firm A depends not just on Q_A (the level of output decided by firm A), but also on the level of output of firm B, Q_B. This shows the interdependence between the two oligopolists. If firm B raises its output, this will reduce total revenue of firm A. Why? Because of the downward-sloping market-demand curve in eq. (1). Greater supply to the market (in this case by firm B) creates an excess supply; this depresses the equilibrium price and this, in turn, lowers TR_A.

Based on eq. (3), we can derive marginal revenue for firm A:

(4) $MR_A = (dTR_A/dQ_A) = 140 - 2Q_A - Q_B$

Cournot assumed that the firms use the same technology to produce identical goods. (Cournot actually assumed the two firms each own a mineral well and operate with zero costs ……..) This means that the two firms have exactly similar total-cost functions. The total-cost function for firms A and B are assumed to look like as follows:

(5) $TC_A = 800 + 20Q_A$; and $TC_B = 800 + 20Q_B$

This means that marginal cost for enterprise A is:

(6) $MC_A = 20$

Profit maximisation by firm A requires that:

(7) $MR_A = MC_A$

This we get: $140 - 2Q_A - Q_B = 20$, which results in:

(8) $Q_A = 60 - ½ Q_B$

Eq. (8) is the reaction function of firm A. It gives firm A's profit-maximising output level as a function of the output of firm B. Equation (8) is called a reaction function because it specifies how firm A reacts (in terms of changes in Q_A) to firm'B's actions (in terms of Q_A). Note that it represents firm A's **optimal** (profit-maximizing) choice of output as a function of its **beliefs** about firm B's choice.

Firm B might perform a similar profit-maximization analysis and will then arrive at a reaction function that expresses Q_B as a function of Q_A

(9) $Q_B = 60 - ½ Q_A$

The reaction functions show that oligopolistic firms are not all powerful, but have to take into account the actions of competitors when trying to realize maximum profits. Specifically, if firm A decides (for whatever reason) to increase its output, the rational profit-maximizing response by firm B will be to lower its own output – rather than increase its output and flood the market even more with goods.

Cournot Equilibrium

Antoine Augustin Cournot showed that oligopolistic competition will converge to a stable and unique equilibrium outcome, in which each firm makes the correct assumption about what the other firm will produce and there is no reason for any firm to change its level of output. Algebraically, the solution to Cournot's model of non-cooperative oligopoly can be found by substituting firm B's reaction function into firm A's reaction function (or vice versa, of course):

(10) $Q_A = 60 - ½\ Q_B = 60 - ½\ (60 - ½\ Q_A) = 30 + ¼\ Q_A$

which gives as the solution: $Q_A = 40$; $Q_B = 40$. Using eq. (2), it follows that P = 60. Total revenue of firm A is $TR_A = P*Q_A = 60*40 = 2400$. $TC_A = 800 + 20*40 = 1600$. Hence firm A's super-normal profits are: 800. Because firm B is identical, $TR_B = 2400$; $TR_B = 1600$; and firm B's profits = 800. Joint profits (= the profits of firm A and firm B combined) are 1600.

The Cournot equilibrium is stable because each firm has adjusted its output to the actual level produced by the other firm. The beliefs of A and B are confirmed in equilibrium. At the output level of 40, neither of the two firms has any reason to change its production plans.

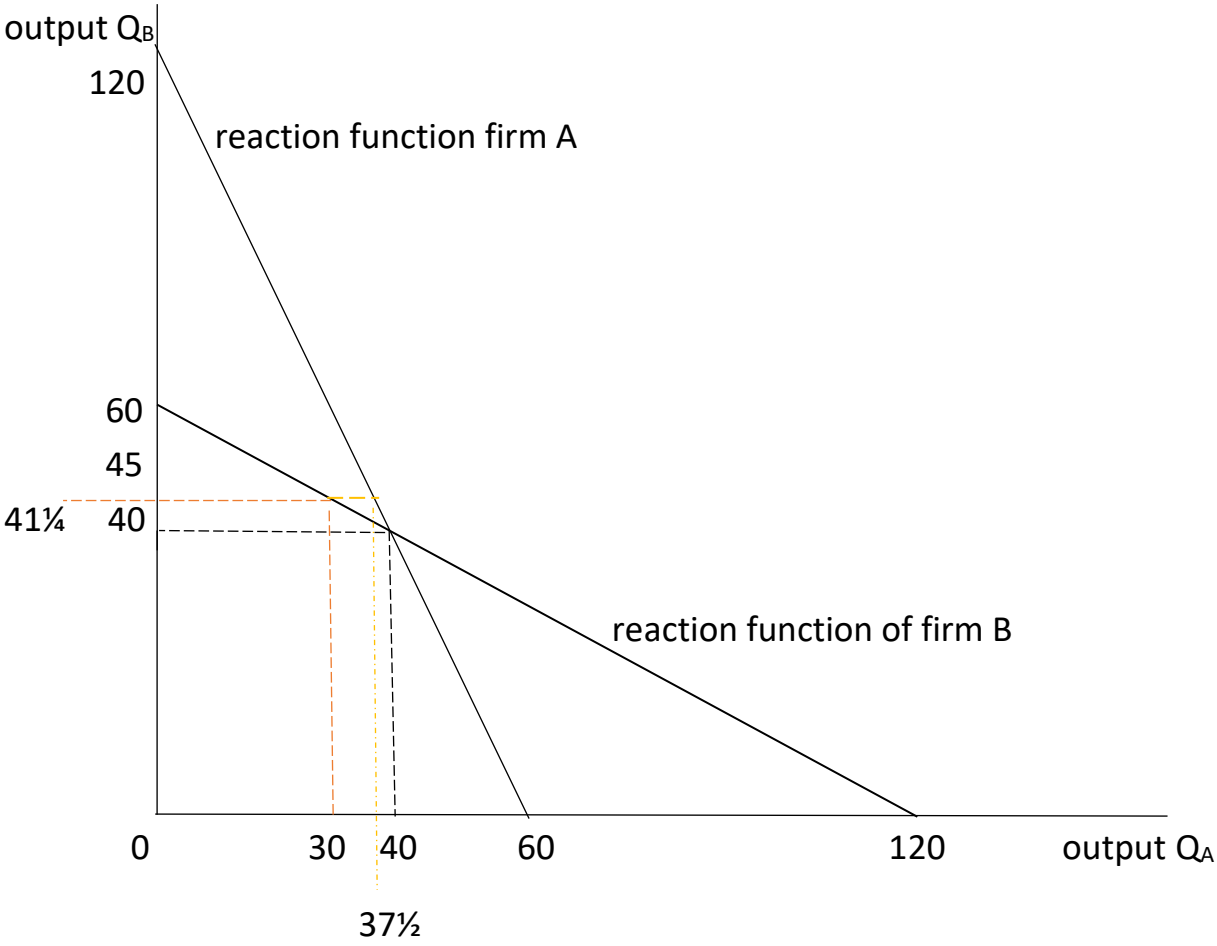
But how do we know if this equilibrium outcome is a stable outcome? Here we can follow Cournot's original logic – illustrated in Figure 2. In Figure 2, we have drawn the two reaction functions. The point of intersection between the two reaction functions determined the profit-maximizing levels of output (at 40) of firm A (on the horizontal axis) and firm B (on the vertical axis). How does this duopoly converge to the equilibrium outcome?

Let us look at what happens when the market is not in equilibrium. Let us suppose that firm A decides (unilaterally) to produce 30 units of output. Firm B will react to this decision by firm A: firm B's optimal (profit-maximizing) response is to produce 45 units of output; this follows from firm B's reaction function in eq. (9). In Figure 3, we can find this response by going from $Q_A = 30$ to firm B's reaction function and check the vertical co-ordinate ($Q_B = 45$).

Firm A will notice that $Q_B = 45$ and understand (from its reaction function in eq. (8)) that it is no longer optimal (profit-maximizing) to continue producing 30 units of output. Firm A will respond and produce 37½ units of output (in reaction

to Q_B = 45). We can find this in the figure by going from Q_B = 45 to firm A's reaction function. The adjustments process continues. Firm B will immediately discover that Q_A = 37½ instead of 30, and respond by reducing its own profit-maximizing level of output to Q_B = 41¼. Firm A will next respond. It will be clear that this adjustments process converges to the Cournot equilibrium in which Q_A = 40 and Q_B = 40. Q.E.D.

Figure 2



Collusion: what happens when the two firms form a cartel?

Collusion means that the two firms decide to cooperate. They form a cartel, an alliance of rivals. Cartel agreements are illegal in most jurisdictions, but they nevertheless exist. The cartel agreement stipulates what the cartel members have to do in order to create extra profits for the members. The heart of a cartel agreement is the agreement to restrict joint output of the members to the profit-maximizing output level of a monopolist. If the cartel agreement works, the (two) firms operate as if they are one monopolistic firm. The cartel outcome therefore is the same as the monopoly outcome.

Let us consider the cartel option for our duopoly consisting of firm A and firm B. In line with the cartel agreement, firm A and firm B act as if they are "one"- a monopolistic firm which we shall call firm AB. Firm AB wants to maximize profits. Firm AB faces the (unchanged) the market-demand function:

(11) $Q = 140 - P$

Firm AB has the following total cost function:

(12) $TC = 1600 + 20\,Q$

Variable cost (20Q) are the same for each firm. Firm A has fixed cost equal to 800, and so does firm B. Because fixed cost must be always paid, firm AB has fixed cost equal to 1600.

We can now derive the MR and MC of the monopolistic (cartel) firm AB. Clearly, MC = 20. What about MR?

Total revenue of cartel AB is: TR = P*Q. We know from eq. (11) that P = 140 – Q. Hence we get: TR = (140 – Q)*Q = 140Q – Q^2. MR then becomes: MR = 140 – 2Q.

The condition for maximum profits is: MR = MC → 140 – 2Q = 20 → Q = 60. This means that the cartel members have to restrict joint output to 60; each firm will produce 30 units of output; joint output in Cournot's non-cooperative oligopoly was 80. If Q = 60, P = 80. The cartel's total revenue TR = P*Q = 80*60 = 4800. TC = 1600 + 20*60 = 2800. The cartel profits are therefore equal to 2000; each cartel member gets 1000 as profit. The cartel profits (= the monopoly profits) are higher than the joint profits of 1600 in the case of the non-cooperative oligopoly. Clearly, the cartel benefits its members – at the cost of consumers (who have to

pay a higher price for a lower quantity of goods). It appears attractive for firms A and B to illegally set up a cartel.

Oligopoly, game theory and Nash equilibrium

Modern microeconomic models of oligopoly use the tools of game theory to describe and analyse the decision-making by firms in and the market outcomes of oligopoly. We can do this well. Consider the pay-off matrix in Figure 3. The pay-off (in terms of super-normal profits) for firm A (in the row) and firm B (in the column) is (1000, 1000). Joint profits are 2000 – the maximum monopoly profits which can made in this market (given the market-demand function). Cournot's non-cooperative equilibrium has a pay-off of (800, 800) as we have seen above; joint profits are 1600; it is the outcome which comes about when both firms violate the cartel agreement.

Figure 3
Pay-off matrix of a Cournot Duopoly

Firm B / Firm A	collude in a cartel	violate the cartel agreement
collude in a cartel	1000, 1000	550, 1225
violate the cartel agreement	1225, 550	800, 800

Let us now consider the two off-diagonal pay-offs. Suppose firm A decided to violate the cartel agreement; firm B is not aware (yet) of this violation and keeps to the agreement. Firm B thus produces 30 units of output. The profit-maximizing response of firm A to $Q_B = 30$, is to produce $Q_A = 45$. Joint output therefore is 75; P = 65. Profits of firm A become: $TR_A - TC_A = 65*45 – 800 – 20*45 = 1225$. This is higher (for firm A) than in the cartel outcome. Firm B ends up with lower profits (because the price decreased): $TR_B - TC_B = 65*30 – 800 – 20*30 = 550$. Firm B is worse off. Note that joint profits in this case are 1775 – lower than

in the cartel outcome, but higher than in Cournot's non-cooperative equilibrium. If firm A can do this, firm B could do it as well. The results will be similar, only this time firm B will earn 1225 in profit compared to 550 of profits for firm A.

From a game-theoretic point of view, firms A and B could try to identify the best strategy (best in terms of individual profits). Let us step in the shoes of firm A. Firm A has two options: (1) stick to the cartel agreement; (2) violate the cartel agreement. Firm A does not know what firm B will actually do. If we suppose that firm B decides to stick to the cartel agreement, what is the best choice for firm A? Looking at Table 3, it will be clear that if firm B adheres to the letter of the cartel agreement, it will be most profitable for A to violate the agreement.

Now let us suppose that firm B decides to cheat and violate the agreement. What should firm A do in this case? Based on Table 3, firm A will also violate the cartel agreement (the alternative, sticking to the agreement, has a lower profit pay-off). This leads us to a major insight: for firm A, it will be best (in terms of profits) to violate the cartel agreement, irrespective of what firm B decides to do. Violating the agreement is firm A's dominant strategy.

It is also, as ought to be clear, firm B's dominant strategy. This however means that both firms are likely to violate the cartel agreement – and in that case we end up Cournot's non-cooperative equilibrium. In terms of joint profits, this is the worst possible outcome for the two firms. This equilibrium is called the Nash equilibrium (named after mathematician John Nash, who developed the concept in a prisoner's dilemma game). The interesting point is that the lure of profit motivates each firm to violate the cartel agreement, making them jointly end up in the worst outcome for them.

In reality, there may be reasons why a cartel agreement may last for some time. But cartels tend to be inherently unstable. This is especially true when there is imperfect information (the market conditions are not transparent). In that case, firm A might try to cheat, hoping that it will get away with it, because firm B will not notice or notice only after a long time-lag. Firm B is hoping the same, so we end up in Cournot's equilibrium.

Limitations of the Theory of Oligopoly

Cournot's model may be criticized on several accounts.

- The behaviour and decision-making of firms is naïve. Firms do not learn from past miscalculations of competitors' reactions.
- We here assume that the two firms are identical, produce a homogenous good and have similar cost function. However, these assumptions can be relaxed without impairing the validity and usefulness of Cournot's model.
- In reality, oligopolistic firms try to differentiate their products – by innovation, advertising, and cost reductions. This is called non-price competition – and it does not feature in Cournot's model.
- The model can be extended to any number of firms. However, it remains a 'closed' model in that entry is not allowed: the number of firms that are assumed to be active in the market remains the same during the adjustment period.
- The model does not say how long the adjustment period will be. The adjustment process is also assumed to be costless (imagine how difficult it would be scale down production capacity in reality).
- The model assumes that the market is transparent and information is complete. There are no problems of asymmetric information.

Nevertheless, Cournot's reaction-curves approach remains a powerful method of analysis of firm behaviour in oligopolistic markets.

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Exercises and questions

Exercise 1

Suppose an industry has a duopoly structure.
Duopolist 1 has a cost function given by: $c_1(y_1) = (y_1)^2$ for $y_1 \geq 0$
Duopolist 2 has a cost function given by: $c_2(y_2) = 12y_2$ for $y_2 \geq 0$
The two firms do not cooperate and engage in Cournot (quantity-based) competition.
Denoting total output produced in the industry by $y = (y_1 + y_2)$, the demand function for the good produced in the industry is given by: $p = 100 - y$

(a) Find the reaction function of each duopolist.

(b) Using (a), obtain the output levels that will be produced in a Cournot equilibrium, and calculate the price level in such an equilibrium.

(c) Illustrate your solution in (b) above in a suitable diagram.

Exercise 2

Consider the following non-cooperative oligopolistic market consisting of two firms --- firm 1 and firm 2. Both firms produce the same homogeneous good. Let p be the market price of that commodity. c_1 represents the total cost of firm 1; c_2 is the total cost of firm 2. Assume that the aggregate market demand function and the costs functions of the two firms are:
$p = 100 - 0.5 (x_1 + x_2)$
$c_1 = 5\ x_1$
$c_2 = 0.5 (x_2)^2$
Both firms are assumed to maximize profits by means of variations in their levels of production. Doing so, they take note of their (oligopolistic) interdependence. Which one of the following statements is true?

(a) Supernormal profits of firm 1 are lower than those of firm 2.

(b) Joint supernormal profits of firms 1 and 2 are 4950 units.

(c) Supernormal profits of firm 1 are equal to those of firm 2.

(d) Firm 2 gets about 22% of total (joint) profits earned in this market.

Exercise 3

Suppose the relevant market for infrared binoculars is an oligopolistic market with two firms, a Dutch and a US-firm. The demand function for infrared binoculars in the relevant market is:

$p = 20 - 2\ (q_n + q_{usa})$

The cost functions of each firm:

$C_n = 8\ q_n + 2$ (Dutch firm)

$C_{usa} = 5\ q_{usa} + 2$ (US firm)

With: p = price of infrared binoculars in €;

q_n = quantity supplied by Dutch firm (in 1000s)

q_{usa} = quantity supplied by US firm (in 1000s)

C_n = total production costs of the Dutch firm

C_{usa} = total production costs of the US firm

Both firms aim at profit maximization and reveal Cournot behaviour.

(a) Calculate the price and quantities of both firms in the infrared binocular market. Show explicitly the reaction curves.

(b) Is welfare in a market with firms that perform Cournot behaviour higher or lower than in a market that would be dominated by one firm (a monopolist)? Motivate your answer. (*Hint: do not calculate. Economic reasoning will do)*

Exercise 4

Consider the following non-cooperative Cournot oligopoly consisting of two identical firms. Price (in euros) is determined according to the following market demand function:

$P = 300 - Q$. The cost functions of the two firms are: $C_1 = 60\ Q_1 + 500$ and

$C_2 = 60\ Q_2 + 500$. Consider the following two propositions.

Proposition I: In the non-cooperative Cournot equilibrium, total output will be 160 and profit for each firm will be 5900 euro.

Proposition II: When these firms form a cartel to maximise joint profits, they will restrict total output to 120 (and each firm will produce 60); joint profits with the cartel will be euro 1600 higher than in the non-cooperative Cournot case.

Which one of the following statements is true?

a. Both proposition I and proposition II are true.

b. Both proposition I and proposition II are false.

c. Proposition I is true and proposition II is false.

d. Proposition I is false and proposition II is true.

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Questions

1. What is the defining feature of an oligopoly?
2. What is the concentration ratio C4?
3. Explain the Hirschman-Herfindahl Index?
4. If HHI = 0.1, is this a high degree of market concentration?
5. Discuss three limitations of Cournot's model of oligopoly.

The answers to the questions can be found in this lecture note 😊.

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Answers to the exercises

Exercise 1

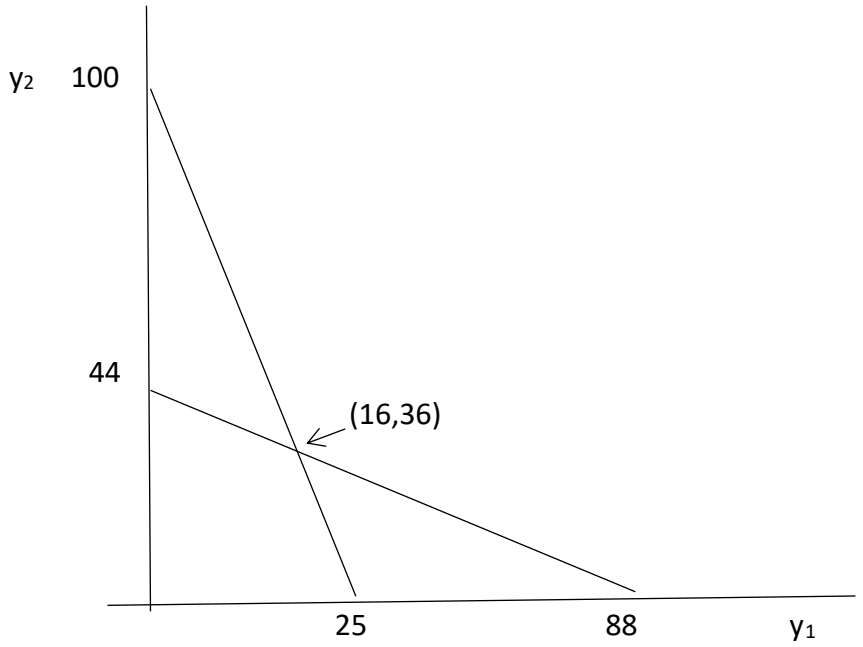
a. Reaction functions:

For Firm 1: max $\Pi_1 = (100 - y_1 - y_2)\, y_1 - (y_1)^2$ → $mr_1 = mc_1$
$mr_1 = 100 - 2y_1 - y_2$
$mc_1 = 2y_1$
$mr_1 = mc_1$ gives $4y_1 = 100 - y_2$ or $y_1 = 25 - \frac{1}{4}\, y_2$ reaction function of firm 1
For Firm 2: max $\Pi_2 = (100 - y_1 - y_2)\, y_2 - 12y_2$ → $mr_2 = mc_2$
$mr_2 = 100 - y_1 - 2y_2$
$mc_2 = 12$
$mr_2 = mc_2$ gives $2y_2 = 88 - y_1$ or $y_2 = 44 - \frac{1}{2}\, y_1$ reaction function of firm 2

b. Solving the system of reaction functions we just found, we obtain:

$y_1 = 16$, $y_2 = 36$ → $y = 52$; $p = 48$

c. Illustrate the solution.



Exercise 2

Answer D.

First derive the reactions functions of firm 1 and firm 2.

Firm 1:

$TR_1 = (100 - 0.5\ x_1 - 0.5\ x_2)\ x_1 = 100\ x_1 - 0.5\ (x_1)^2 - 0.5\ x_1\ x_2$

$MR_1 = 100\ - x_1 - 0.5\ x_2$

$MC_1 = 5 \rightarrow MR_1 = MC_1 \rightarrow 100\ - x_1 - 0.5\ x_2 = 5 \rightarrow x_1 = 95 - 0.5\ x_2$

Firm 2:

$TR_2 = (100 - 0.5\ x_1 - 0.5\ x_2)\ x_2 = 100\ x_2 - 0.5\ (x_2)^2 - 0.5\ x_1\ x_2$

$MR_1 = 100\ - x_2 - 0.5\ x_1$

$MC_2 = x_2 \rightarrow MR_2 = MC_2 \rightarrow 100\ - x_2 - 0.5\ x_1 = x_2 \rightarrow x_2 = 50 - 0.25\ x_1$

Then we substitute reaction function 2 into reaction function 1:

$x_1 = 95 - 0.5\ x_2 = 95 - 0.5\ (50 - 0.25\ x_1) = 70 + (1/8)\ x_1 \rightarrow x_1 = 80;\ x_2 = 30;\ p = 45$

Profits firm 1 = $TR_1 - TC_1 = 80*45 - 5*80 = 3200$
Profits firm 2 = $TR_2 - TC_2 = 30*45 - 0.5*900 = 900$.
Total profits in the market = 4100. Firm 2 gets about 22% of joint profits.

Exercise 3

a.

Reaction function of firm N:

$TR_n = p*q_n = (20 - 2\ q_n - 2q_{usa})\ q_n = 20\ q_n - 2\ (q_n)^2 - 2\ q_n\ q_{usa}$

$MR_n = 20\ - 4\ q_n - 2\ q_{usa} = MC_n = 8$
Hence, reaction function N is: $q_n = 3 - ½\ q_{usa}$

Reaction function of firm USA:

$TR_{usa} = p*q_{usa} = (20 - 2\ q_n - 2q_{usa})\ q_{usa} = 20\ q_{usa} - 2\ (q_{usa})^2 - 2\ q_n\ q_{usa}$

$MR_{usa} = 20\ - 4\ q_{usa} - 2\ q_n = MC_{usa} = 5$
Hence, reaction function N is: $q_{usa} = (15/4) - ½\ q_n$

Substitution the 2nd reaction function into the 1st gives:

$q_n = 3 - ½\ q_{usa} = 3 - ½\ ((15/4) - ½\ q_n) = 3 - (15/8) + (1/4)\ q_n \rightarrow$

$q_n = 1½$; $q_{usa} = 3$; $p = 11$

b. Consumer welfare is higher in a Cournot oligopoly than in monopoly (or a cartel). The monopolist is able to restrict output/supply more than the non-cooperating oligopolists; hence the price will be higher. Supply is more restricted and price is higher in a monopoly than in oligopoly; consumer welfare (consumer surplus) is lower.

Exercise 4.

Answer A: Proposition I = true; Proposition II = true.

We first derive the reaction functions of the 2 firms; we do this for firm 1.

Total revenue of firm 1:

$TR_1 = (300 – Q_1 – Q_2)*Q_1 = 300Q1 - Q_1\text{^}2 – Q_1Q_2$

This gives marginal revenue: $MR_1 = 300 – 2Q_1 – Q_2$

Marginal cost of firm 1 is: $MC_1 = 60$

The condition for maximum profits is $MR_1 = MC_1$ →

reaction function of firm 1: $300 - 2Q_1 - Q_2 = 60$ → $2Q_1 = 240 – Q_2$ → $Q_1 = 120 - 0.5Q_2$

Because the firms are identical, the reaction function of firm 2 is: $Q_2 = 120 – 0.5Q_1$

Substitution of reaction function 2 into reaction function 1 gives:

$Q_1 = 120 – 0.5 (120 – 0.5Q_1) = 60 + 0.25Qv$ → $Q_1 = 80 = Q_2$ (because the firms are identical)

P = 300 – 80 – 80 = 140.

Profits of firm 1 = 80*140 – 80*60 – 500 = 80*(140-60) - 500 = 5900

Profits of firm 2 = 5900 as well. Combined profits of firm 1 and 2 = 11800.

Suppose the two firms form a cartel.

Total revenue of the cartel: TR = 300Q – Q^2 →

MR = 300 – 2Q = MC = 60 → hence Q = 120 ; P = 180

TR = 180*120 = 21600 TC = 60* 120 + 1000 = 8200 Cartel profits = 13400