

MOT1421
Economic Foundations
Week Two

MARKET CO-ORDINATION:
Monopoly

S. STORM

LECTURE NOTE MOT1421-W-2B

The Lecture Note MOT1421-W-2B is part of the exam materials.

The required reading for Week 2 consists of:

- This Lecture Note MOT1421 W-2A and Lecture Note MOT1421 W-2B.

Supporting videos:

- <https://www.youtube.com/watch?v=PEFEns--mU>
- <https://www.youtube.com/watch?v=ZiuBWSFIfoU>

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Market Co-ordination: Monopoly

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Monopoly

Introduction

Once we leave the hypothetical world of perfect competition, the problems of competition, pricing, innovation and product differentiation become much more complicated (and realistic). In this lecture note, we analyse the monopoly market, dominated by one single producer/supplier – a big firm.

Monopoly is a market structure in which there is a single seller, there are no close substitutes for the commodity it produces and there are barriers to entry. The main causes of a monopoly include:

- Ownership of strategic materials. For instance, China holds a near-monopoly on rare-earth materials; Morocco holds a near-monopoly on phosphates.
- Exclusive knowledge of production techniques. For example, Dutch high-tech firm ASLM is the only firm in the world capable of producing Extreme Ultra Violet (EUV) lithography machines, which are needed to manufacture nano-computer chips. Intel has a market share of 80% in micro-processors.
- Patent rights for a product or a production process. This is a big issue in pharmaceutical products where producers (Big Pharma) hold temporary patent monopolies on often key drugs.
- Government licensing to exclude competitors. For example, Brazilian oil and gas producer Petrobras used to have a monopoly (until 200); state-owned Électricité de France (EdF) has a near-monopoly on power generation in France.
- The size of the market may be such as not to support more than production unit of optimal size. This may happen in transport, electricity and communications, where there are substantial economies of scale & scope.
- Incumbent firms may actively deter entry and competition in their markets (including by buying up potential competitors); this happens in

high-tech markets where positive network-externalities¹ are important. Examples of near-monopolists include Microsoft (which has a market share in operating systems of more than 75%), Google (which controls 70% of the web search market), Monsanto (more than 80 percent of the corn that is grown in the U.S. is Monsanto trademarked), and Facebook (controlling 60% of social media).

The difference between firms operating in perfect competition and the monopoly firm is that firms in perfect competition are price-takers, whereas the monopolistic firm is a price-setter. Firms in perfect competition are so small relative to aggregate supply, that their actions do not affect the equilibrium market price; they can therefore take the market price as a given. This is not the case in a monopoly in which one firm faces the downward-sloping demand curve on its own. This means that a monopolistic firm, by varying its output, can directly affect the price of its product.

Let us see how such a price-maker operates.

Assumptions

The model of monopoly is based on the following assumptions:

- There is only one seller and there are many buyers. The total supply of the product is concentrated in a single firm.
- The product may be homogenous or differentiated. We assume here that the product is homogenous.
- The firm has clearly defined (total, average and marginal) cost functions.
- The goal of the monopolistic firm (*ex hypothesi*) is profit maximization. No other goals are pursued. The condition for maximum profits is: $MR = MC$.

¹ Network externalities have to do with standardization. Network externalities are the effects a product or service has on a user while others are using the same or compatible products or services. Positive network externalities exist if the benefits are an increasing function of the number of other users. The more users use Microsoft, the more attractive this operating system becomes for potential users.

- Entry of new firms is blocked by definition (otherwise this would not be a monopoly).
- Perfect knowledge, full transparency and complete information: the monopolistic firm and all buyers have complete knowledge of the prevailing (and future) conditions of the market. Uncertainty is ruled out by assumption. There is also no asymmetric information, no insider knowledge etc.

Under the above assumptions we will examine the equilibrium of the monopoly market.

Equilibrium in monopoly

The monopolistic firm wants to make maximum profits. Profits Π are defined as the difference between total revenue (TR) and total cost (TC):

$$\Pi = TR - TC$$

By definition, total revenue TR is equal to: $TR = P \times Q$, where P = the market price, and Q is the quantity of output of the monopolist. Let us note here that Q is market supply.

We further assume that total cost is a function of the quantity of output, or:

$$TC = f(Q) = b_0 + b_1 \times Q$$

For ease of exposition, we define TC differently than in the case of perfect competition and assume that the total cost function is a linear function; the quadratic and cubic terms are ignored (assuming that $b_2 = b_3 = 0$). This simplifying assumption does not affect the (qualitative) conclusions of the analysis. It just makes life easier – and moreover, a linear total-cost function is probably more realistic than a cubic polynomial one.

It follows from the above that the monopolistic firm's profits are a function of its output, because we write:

$$\Pi = TR - TC = P \times Q - f(Q)$$

The monopolist aims at the maximization of its profits. Using the profit function to find maximum profits, the first-order condition for the maximization of a function tells us that its first derivative (with respect to Q in this case) be equal to zero. Differentiating the profit function and equating to zero we obtain:

$$\frac{\partial \Pi}{\partial Q} = \frac{\partial TR}{\partial Q} - \frac{\partial TC}{\partial Q} = 0 \rightarrow \frac{\partial TR}{\partial Q} = \frac{\partial TC}{\partial Q} \rightarrow MR = MC$$

The condition for maximum profits in monopoly is the same as that in perfect competition.

Marginal cost and average cost of the monopolist

Let us first consider the (marginal) cost side of the condition $MR = MC$. We defined total cost of the monopolist as follows:

$$TC = f(Q) = b_0 + b_1 \times Q$$

Hence, marginal cost is equal to

$$MC = \frac{dTC}{dQ} = b_1$$

which shows that the extra cost incurred to produce one more unit of output is constant (and equal to b_1). We can next define average cost of production as:

$$AC = \frac{TC}{Q} = \frac{b_0}{Q} + b_1$$

Average cost declines as Q increases (as fixed cost per unit of output goes down). For very high levels of Q , AC converges to MC :

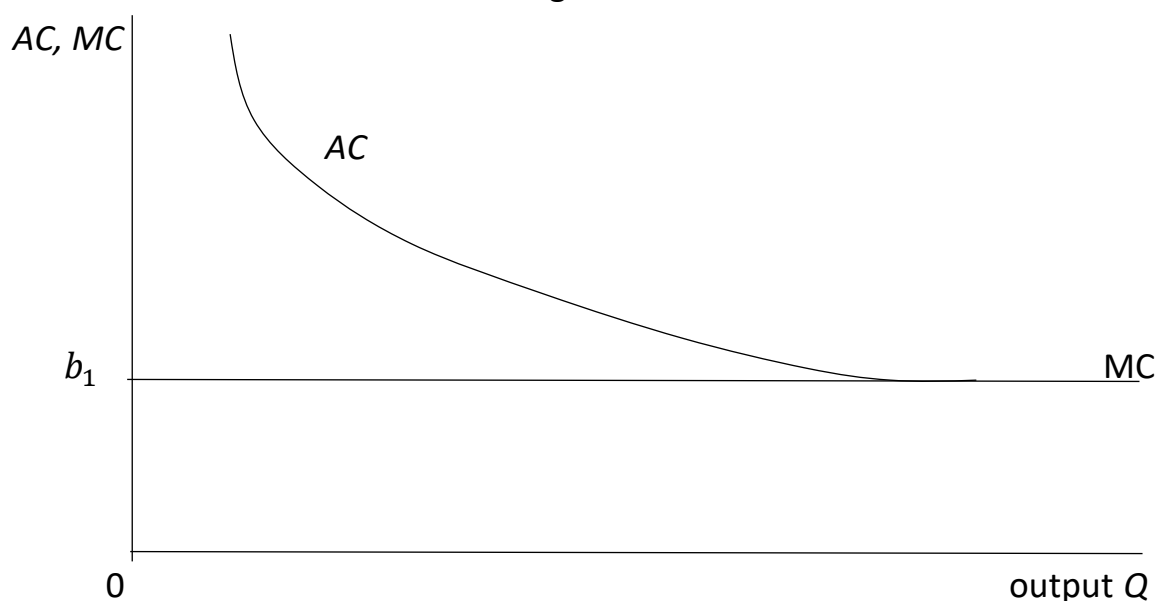
$$\lim_{Q \rightarrow \infty} AC = b_1 = MC$$

The marginal-cost curve and the average-cost curve of the monopolistic enterprise are illustrated in Figure 1. Let us before we proceed note the following. Average cost is, by definition, equal to TC/Q . This implies we can write that $TC = AC \times Q$. Using this definition, we obtain the following – useful – expression for the profits of the monopolist:

$$\Pi = TR - TC = P \times Q - AC \times Q = (P - AC) \times Q$$

Monopoly (super-normal) profits are bigger than zero if $P > AC$.

Figure 1



Demand, total revenue and marginal revenue

Because there is only one firm in the market, the market-demand curve is the firm's demand curve. The market-demand curve is assumed to be known and has a downward slope (in price P). The market-demand function, *ceteris paribus*², is as assumed to be as follows:

$$Q = a_0^* - a_1^* \times P$$

The demand function is linear (in price P). It reflects decision-making by and preferences of consumers – and the monopolistic firm cannot change or manipulate this demand function. a_0^* = the demand for the good produced by the monopolist which is not sensitive to changes in the price P . a_1^* = the coefficient expressing the sensitivity of market demand to changes in P ; the sign is negative; hence a higher price will lower demand.

This means that it is impossible for the monopolistic firm to sell a large quantity Q and charge a high P . The firm faces a trade-off: if it wants to sell a large quantity Q , then it has to lower its P . Cake-ism is not possible.

² [Ceteris paribus](#) means: keeping all other factors constant/unchanged. Here we assume that income, tastes and other prices do not change.

We are interested in deriving marginal revenue. To do so, we must first define total revenue, which by definition is equal to: $TR = P \times Q$. The monopolist now has two options: it can use the price P as the (price-setting) instrument to maximize profits, or alternatively use quantity Q (quantity-setting) as the instrument for profit maximization. The monopolist cannot use both P and Q at the same – because it faces the independent demand function. If the firm sets P , consumers will demand a quantity Q based on the demand function; if the firm fixes Q , consumers will be willing to buy these goods at price P (following from the demand function). We will assume that the monopolist will use Q as its instrument. (Note that the model outcome will be the same if we use price instead of quantity).

Choosing Q as the instrument for profit maximization means that we have to express total revenue as a function of only Q . Put differently, we have to eliminate P from the equation $TR = P \times Q$. To do this, we begin by re-writing the market-demand function as the price-setting function of the monopolist. Market demand is:

$$Q = a_0^* - a_1^* \times P$$

Bringing P to the left-hand side and Q to the right-hand side, and re-arranging, gives:

$$P = \frac{a_0^*}{a_1^*} - \frac{1}{a_1^*} \times Q = a_0 - a_1 \times Q$$

where we define $a_0 = \frac{a_0^*}{a_1^*}$ and $a_1 = \frac{1}{a_1^*}$. In Figure 2, the price-setting curve is drawn. Note that the price-setting curve is also the market-demand curve.

Using the price-setting equation, we can define total revenue as follows:

$$TR = P \times Q = (a_0 - a_1 \times Q) \times Q = a_0 \times Q - a_1 \times Q^2$$

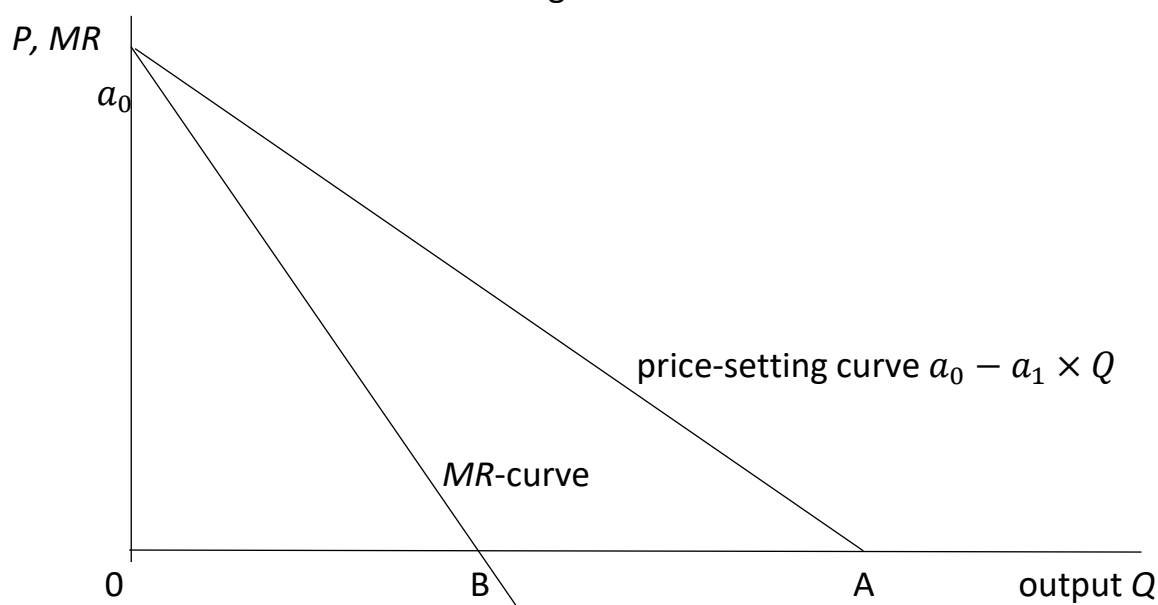
Marginal revenue of the monopolist then is:

$$MR = \frac{dTR}{dQ} = a_0 - 2 \times a_1 \times Q$$

For the monopolist, marginal revenue is not constant (and equal to P), but declining as Q increases. The reason is that if the firm steps up output, it can only

sell the higher output if it lowers the price. Accordingly, the extra revenue obtained from producing and selling one additional unit of output declines, the more the monopolistic firm produces. The declining MR -curve is shown in Figure 2. Note that the slope of the MR -curve ($= -2 \times a_1$) is twice as large in absolute terms than the slope of the price-setting curve ($-a_1$). As a result, the distance OB is half the distance OA in Figure 2.

Figure 2

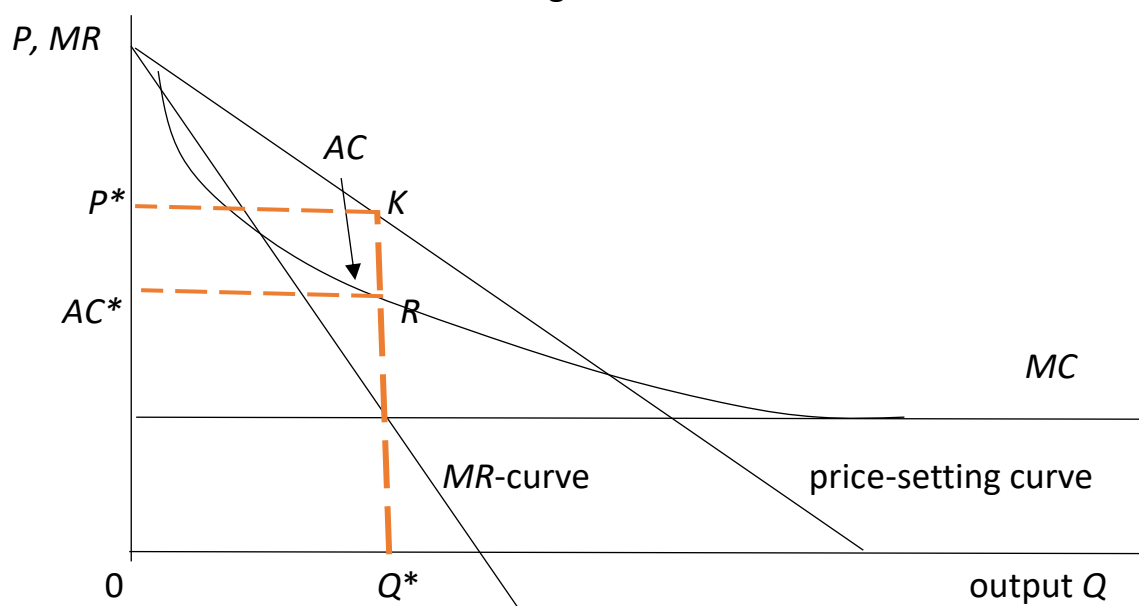


We can now put Figures 1 and 2 together – to identify the profit-maximizing decision to be made by the monopolist.

Profit maximization by the monopolist

Figure 3 integrates the marginal-cost side and the marginal-revenue side of the problem in one graph. The point of intersection between the MR -curve and the MC -curve defines the profit-maximizing level of output Q^* . The profit-maximizing price P^* (corresponding with Q^*) can be found by going up (from Q^*) to the price-setting curve – and identify P^* .

Figure 3



We can solve the monopoly model algebraically. We have seen that $MC = b_1$ and $MR = a_0 - 2 \times a_1 \times Q$. The condition for maximum profits is: $MR = MC$. Hence, we get:

$$\begin{aligned}
 b_1 &= a_0 - 2 \times a_1 \times Q \quad \rightarrow \\
 2 \times a_1 \times Q &= a_0 - b_1 \quad \rightarrow \\
 Q^* &= \frac{a_0 - b_1}{2 a_1} \quad \text{and} \quad P^* = \frac{1}{2} a_0 + \frac{1}{2} b_1
 \end{aligned}$$

Supernormal profits

Super-normal profits are defined as $\Pi = TR - TC$. In Figure 3, TR is equal to the rectangular area $OQ^*K P^*$ - because $TR = P^* \times Q^*$. TC is equal to rectangular area $OQ^*R A C^*$ - because $TC = AC^* \times Q^*$. This means that super-normal profits are equal to area $AC^*R K P^*$ in Figure 3.

These monopoly super-normal profits are the highest profits which can be extracted from this market. The margin $(P - AC)$ is largest in a monopoly, both in the short run and the long run (as long as the monopoly is not contested by potential entrants). This is the exact opposite of the long-run outcome in perfect competition, when $(P = AC)$ and only normal profits remain for firms. The equilibrium outcome is therefore the best-possible outcome for the

monopolistic firm. For consumers, in contrast, the monopoly outcome is the worst possible outcome in the short run: consumers pay for the super-normal profits and the supply of goods is restricted (and much lower than in perfect competition). Consumer welfare is compromised in the short run in a monopoly. But this needs not be the case in the long(-er) run.

Static versus dynamic efficiency

The monopoly model is essentially a static model. This is a limitation, because we do not know how the firm will use its super-normal profits. Let us suppose that the monopolist retains the profits as profit reserve (on its balance sheet), which it re-invests in expansion of productive capacity, R&D and innovation. If the innovation is successful, this may result in lower average and marginal costs of production. In Figure 3, the AC and MC curves will shift downwards – and as a result, profit-maximizing output Q^* will increase and the equilibrium price P^* will decline. This way, the monopolistic firm could be ‘dynamically efficient’, i.e. using profits to finance innovation to reduce costs and/or improve products and/or develop new products & services.

Technological progress requires funding – and internal (accumulated) profits can be a useful source of financing innovation. This could lead to progress: lower costs, cheaper goods, new goods – which is what we call dynamic efficiency. This is beneficial to consumers in the long run, if the monopolist is entrepreneurial.

Joseph Schumpeter pointed to the conflict between static efficiency and dynamic efficiency which exists in perfect competition and in monopoly.

- In perfect competition, $P = AC$ in long-run equilibrium. There are no super-normal profits. This benefits consumers (cheap goods) but it deprives firms from internal sources of finance to fund innovation. Perfect competition may achieve static efficiency (the lowest prices for consumers), but it fails to bring about dynamic efficiency (i.e. allow for profits to finance R&D, product development and innovation). Consumers may be best off in perfect competition in the short run, but not in the long run.

- In monopoly, $P > AC$. Consumers pay high prices (because of the presence of super-normal profits) and the supply of goods is restricted. Static efficiency for consumers is compromised. But if the profits are used to finance innovation, consumers will benefit in the longer run from better, new, and cheaper products & services. Schumpeter argued that dynamic efficiency is of more importance to improvements in living standards and welfare than static efficiency.

This Schumpeterian logic underpins the legal intervention of patents. A patent provides an innovating firm with a temporary monopoly (roughly for 15-18 years), in which this firm can make super-normal profits to recoup the cost of the earlier innovation and finance further innovations. The trade-off between static and dynamic efficiency is resolved by giving priority to innovation – and dynamic efficiency of firms.

Limitations of the Theory of Monopoly

The theory of monopoly assumes that the monopolist is strictly working under the impetus of profit maximization. This may or may not be the case – after all, the monopolist is sheltered from external competition and could also aim for other goals. A key issue, for dynamic efficiency, is whether or not the super-normal profits are re-invested into R&D, upgrading and innovation. There is nothing in the theory which tells us that the monopolistic firm will actually do this.

One could invoke shareholders' pressure on the monopolistic firm to retain and re-invest the profits in innovation and technological progress as a channel to ensure that the firm uses its profits to enhance its dynamic efficient (to the ultimate benefit of consumers). In reality, this is not what shareholders do, however. Empirical research for the 500 biggest U.S. corporations shows that in the past two decades shareholders have pressured these big firms to disburse all their profits (billions of dollars per year) to the shareholders themselves. Firms used around 40% of their profits to pay dividends to shareholders and more than 50% of their profits to buy back their own shares (in the stock market). Share-buybacks drive up the stock market prices of these share – which benefits the shareholders.

[William Lazonick and Matt Hopkins \(2020\)](#) document how S&P 500 biggest U.S. corporations channelled \$5.3 trillion to shareholders during the past decade (2010-2019) - rather than invest in technologies for the common good. The \$5.3 trillion amounts to 54% of cumulative profits. They single out Microsoft and write:

“Yet from fiscal 1996 through the third quarter of fiscal 2020, Microsoft repurchased \$244 billion worth of its own shares, equal to 65 percent of its profits, and distributed \$161 billion in dividends, representing 43 percent of profits.

On its website, Microsoft calls its buybacks and dividends “[Cash Returned to Shareholders](#).” Yet, as Gates must know, the only funds that Microsoft has ever raised from the public stock market amount to \$59 million: all the proceeds of its initial public offering in March 1986. So how can Microsoft “return” cash to parties who never invested in the company’s productive capabilities but simply bought and sold shares already outstanding on the stock market?”

This raises the crucial issue: how to control (monopoly) business power for the common good? Here we enter the terrain of market regulation.

Exercises and questions

Exercise 1

A monopoly faces market demand $Q = 30 - P$ and has a (total) cost function $C(Q) = \frac{1}{2} Q^2$. Find the profit maximizing price and quantity and the resulting (supernormal) profit of the monopolistic firm.

Exercise 2

A monopolist faces a demand for its product given by: $P = 15 - \frac{1}{2} Q$.

Its total cost takes the following form: $TC = 5 + Q^2$.

Find the monopolist's profit maximizing price and quantity. How much supernormal profit does the monopolist earn?

Exercise 3

Consider the following monopolistic market. Total demand for the commodity Q is given by the demand function: $Q = 120 - 4P$, where Q = quantity demanded and P = market price. The total cost function of the profit-maximizing firm is as follows: $TC = \frac{3}{4} Q^2 - 50Q + 1000$.

- (a) Derive the profit-maximizing level of output Q^* and the profit-maximizing price P^* . How big are the monopolist's supernormal profits?
- (b) What happens to the profit-maximizing level of output Q^* when the fixed costs of production increase by 50%? Explain your answer.

Exercise 4

Suppose that a monopolist faces a demand curve of $P = 100 - 2Q$. (The price is in euros).

The firm has costs $TC = 3Q^2$ (costs are expressed in euros).

- (a) What is the profit-maximizing level of output Q ?
- (b) How big are supernormal profits?
- (c) If the firm has to pay a fee of €150 to the government in order to start business, how will the optimal level of output change?

Exercise 5

Suppose a monopolist faces the following demand curve: $P = 4000 - 20Q$. The price is expressed in euros. The monopolistic firm incurs a fixed (annual) cost of €100000. Its long-run marginal cost of production is constant and equal to €40.

1. What is the monopolist's profit maximizing level of output?
2. What price will the profit maximizing monopolist charge?
3. How much profit will the monopolist make if she maximizes her profit?

Now suppose that the fixed (annual) cost increase to €150000.

4. What is the monopolist's profit maximizing level of output?
5. What price will the profit maximizing monopolist charge?
6. How much profit will the monopolist make if she maximizes her profit?
7. What is the impact of fixed cost on profit-maximisation?

Exercise 6

A monopolist faces the demand curve $P = 11 - Q$, where P is measured in euros per unit and Q in thousands of units. The monopolist has a constant average cost of €6 per unit.

1. Draw the average and marginal revenue curves and the average and marginal cost curves.
2. What are the monopolist's profit-maximizing price and quantity?
3. What is the resulting profit?

Exercise 7

Consider the monopolistic firm Macro-HardTM which produces computer-chips manufacturing machines. The demand for these machines is $Q = 10000/(P^2)$. P is measured in millions of euros. The firm's total cost is $TC = 520 + 5Q$ (in million euros).

1. What price should Macro-HardTM charge to maximize profit?
2. What quantity does it sell?
3. How much profit does it make?
4. Would it be better off shutting down in the short run?

Questions

1. What is the defining feature of a monopoly?
2. Explain four main causes of a monopoly and give an example of each.
3. The monopolist cannot simultaneously determine price and quantity. Why is this the case?
4. What is the difference between static and dynamic efficiency?
5. What is the rationale of giving a patent monopoly to an innovating firm?
6. How did the biggest U.S. corporations use their (super-normal) profits?
What is your view on these uses?

The answers to the questions can be found in this lecture note 😊.

Answers to the exercises

Exercise 1

The monopoly produces at the point where $MR = MC$.

In this question: $TR = P * Q = (30 - Q) * Q = 30Q - Q^2$.

Hence $MR = 30 - 2Q$.

$MC = Q$.

Equating MR and MC gives us $Q = 10$.

From the demand equation we can find $P = 30 - Q = 20$.

(Supernormal) Profit = $PQ - C(Q) = 20 \cdot 10 - 50 = 150$.

Exercise 2

The monopoly produces at the point where $MR = MC$.

In this question: $TR = P * Q = (15 - \frac{1}{2}Q) * Q = 15Q - \frac{1}{2}Q^2$.

Hence $MR = 15 - Q$.

$MC = 2Q$.

Equating MR and MC gives us $Q = 5$.

From the demand equation we can find $P = 15 - \frac{1}{2}Q = 12\frac{1}{2}$.

(Supernormal) Profit = $PQ - C(Q) = 12\frac{1}{2} \cdot 5 - 30 = 32\frac{1}{2}$.

Exercise 3

We first derive $TR = P*Q$. We know that $Q = 120 - 4P$; hence, $P = 30 - \frac{1}{4}Q$.

This means that $TR = (30 - \frac{1}{4}Q)*Q = 30Q - \frac{1}{4}Q^2 \rightarrow MR = dTR/dQ = 30 - \frac{1}{2}Q$

Marginal cost = $dTC/dQ = 1\frac{1}{2}Q - 50$.

The condition for maximum profits is: $MR = MC \rightarrow 30 - \frac{1}{2}Q = 1\frac{1}{2}Q - 50 \rightarrow Q = 40$; $P = 20$ euro. $TR = P*Q = 20*40 = 800$. $TC = 200$. Supernormal profits = 600.

- $Q^* = 40$; $P^* = 20$; supernormal profits = 600.
- Nothing. Fixed costs increase from 1000 to 1500. But this does not affect MC. Hence, $MR = MC$ remains unchanged, and $Q^* = 40$. However, supernormal profits decline from 600 to 100 (because of the increase in fixed costs by 500).

Exercise 4

We first derive the total revenue function $TR = P*Q = (100 - 2Q)*Q = 100Q - 2Q^2$

This gives as marginal revenue $MR = (dTR/dQ) = 100 - 4Q$.

Marginal cost = $(dTC/dQ) = 6Q$.

The condition for maximum profits is: $MR = MC \rightarrow 100 - 4Q = 6Q \rightarrow Q = 10$; $P = 80$.

$TR = 10*80 = \text{euro } 800$. $TC = 3*10^2 = \text{euro } 300$. Monopoly supernormal profits = euro 500.

- Profit-maximizing $Q = 10$.
- Monopoly supernormal profits = euro 500.
- The fee is a fixed cost item. TC now becomes: $TC = 150 + 3Q^2$. Clearly, marginal cost do NOT change. Hence the profit-maximizing Q stays the same (at 10). Supernormal profits, however, are reduced by euro 150 – to euro 350.

Exercise 5

We first derive total revenue of the monopolist:

$$TR = P * Q = 4000 * Q - 20 * Q^2$$

This gives as marginal revenue: $MR = 4000 - 40 * Q$.

Profit-maximisation requires that $MC = MR \rightarrow 40 = 4000 - 40 * Q \rightarrow Q = 99$; $P = €2020$.

$TR = P * Q = € 199980$. $TC = 100000 + 40Q = € 103960$ Max. Profits = € 96020.

1. $Q = 99$
2. $P = €2020$
3. Maximum profits = € 96020
4. $Q = 99$ (stays the same)
5. $P = €2020$ (stays the same)
6. Maximum profits = € 46020
7. The increase in fixed costs does not affect the decision concerning the profit-maximising level of output (and price), but it does reduce the size of maximum profits (one-for-one).

Exercise 6

We first derive total revenue of the monopolist:

$$TR = P * Q = (11 - Q) * Q = 11Q - Q^2$$

Average revenue $AR = TR/Q = 11 - Q$ is the demand function.

$$MR = dTR/dQ = 11 - 2Q$$

Constant average costs = 6 $\rightarrow TC = 6 * Q \rightarrow AC = TC/Q = 6$

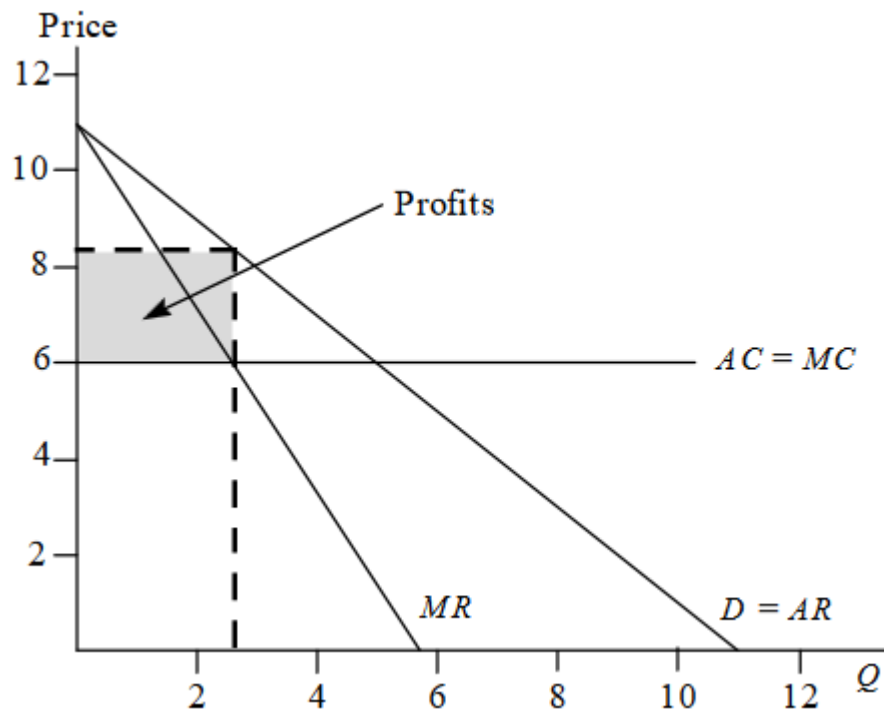
$$MC = dTC/dQ = 6$$

Profit maximization requires that $MC = MR \rightarrow 6 = 11 - 2Q \rightarrow Q = 2\frac{1}{2} \text{ (x 1000)}$

$P = 11 - 2\frac{1}{2} = 8\frac{1}{2}$; $TR = 8\frac{1}{2} * 2500 = € 21250$; $TC = 6 * 2500 = € 15000$

Maximum profits = $TR - TC = €6250$.

1. See figure below.
2. $Q = 2500$; $P = €8\frac{1}{2}$
3. Max. profits = €6250



Exercise 7

Total revenue $TR = P \cdot Q$.

We know that demand is: $Q = 10000/(P^2) \rightarrow P = 100/\sqrt{Q} \rightarrow$

Hence $TR = 100 \cdot \sqrt{Q} \rightarrow MR = 50/\sqrt{Q}$

$MC = dTC/dQ = 5$

Profit-maximisation requires that $MC = MR \rightarrow 5 = 50/\sqrt{Q} \rightarrow Q = 100 \rightarrow P = € 10 \text{ million}$.

$TR = € 1 \text{ billion}$; $TC = € 1020 \text{ million}$. Maximum profits = $TR - TC = -€ 20 \text{ million}$.

1. $P = € 10 \text{ million}$ (per machine).
2. $Q = 100 \text{ machines}$.
3. Maximum profits = $-€ 20 \text{ million}$
4. Although maximum profit is negative, price $€ 10 \text{ million}$ is above the average variable cost of $€ 5 \text{ million}$ and therefore, the firm should not shut down in the short run. Since most of the firm's costs are fixed, the firm loses $€ 520 \text{ million}$, if nothing is produced. If the profit-maximizing quantity is produced, the firm loses only $€ 20 \text{ million}$.