

MOT111A FINANCIAL MANAGEMENT:

LECTURE 5

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All lectures and workshops are mandatory and are held at 10:45-12:30 hours Coffee hours are informal and voluntary and are held at 10.45-11.45 hours

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Activity	Instructor*	Date	Location^	Topic	Material	
Lecture 1	ZK	Tue 5 Sept	Flux Hall A	Introduction and financial statement analysis	Book chapters 1 (all) & 2 (all)	
Workshop 1	AR	Thu 7 Sept	Flux Hall A	Exercises based on lecture 1	Lecture 1	
Lecture 2	ZK	Tue 12 Sept	Echo-Hall B1	Evaluating investment opportunities	Chapters 3(3.1-3.4), 4 (all), 5(all) & 7 (all)	
Coffee	ZK/AR/WK	Thu 14	TPM			
Hour		Sept	Canteen			
					T Introduction Week	
Lecture 3	AR	Tue 26 Sept	Echo-Hall B1	Long-term financing and financial markets Valuing bonds	Slides + Chapter 6 (all)	
Workshop 2	AR	Thu 28 Sept	Flux Hall A	Exercises based on lectures 2 & 3	Lectures 2 & 3	
Lecture 4	ZK	Tue 3 Oct	Echo-Hall B1	Valuing stocks Risk analysis and portfolio investment	<ul> <li>Chapter 9 (9.1-9.2)</li> <li>Chapters 10: 10.1-10.3, 10.6-10.8</li> <li>Chapter 11: 11.1-11.2, 11.4 (until Example 11.9, p.404), 11.7-11.8</li> </ul>	
Workshop 3	AR	Thu 5 Oct	Flux Hall A	Exercises based on lecture 4	Lectures 4	
Lecture 5	ZK	Tue 10 Oct	Echo-Hall B1	Cost of capital and Capital structure	Chapter 12: 12.1, 12.5-12.7     Chapters 14 (all) & 15 (all)	
Coffee	ZK/AR/WK	Thu 12	TPM			
Hour		Oct	Canteen	4		
Lecture 6	ZK	Tue 17 Oct	Echo-Hall B1	Financial distress and payout policy	Chapters 16 (16.1-16.4) & 17 (17.1-17.4)	
Workshop 4	AR	Thu 19 Oct	Flux Hall A	Exercises based on lectures 5 & 6	Lectures 5 & 6	
Guest Lecture	To be announced	Tue 24 Oct	Echo-Hall B1	To be announced	Slides	
Coffee Hour	ZK/AR/WK	Thu 26 Oct	TPM Canteen			
Lecture 7 (Final)	ZK & AR	Thu 31 Oct	Echo-Hall B1	Q&A + Practice Exam Ralcheva; WK = Wesl	All materials discussed in Lectures 1-6	

<sup>\*</sup> ZK = Zenlin Roosenboom-Kwee; AR = Aleksandrina Ralcheva; WK = Wesley Koo

<sup>^</sup> Locations can be viewed at: https://esviewer.tudelft.nl/



# Structure of Lecture 5



COST OF CAPITAL



CAPITAL STRUCTURE



**DEBT & TAXES** 

# Lecture 5: Learning Objectives

- Estimate project cost of capital (all-equity comparables vs. levered firms comparables)
- Understand and compute a firm's net debt
- Compute weighted average cost of capital (WACC)
- Discuss the implications of debt policy (MM Propositions I and II)
- Understand and estimate the notion of interest tax shield
- Estimate the value of (un-)levered firms
- Understand the implications of the optimal level of leverage from the tax saving perspective

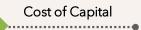




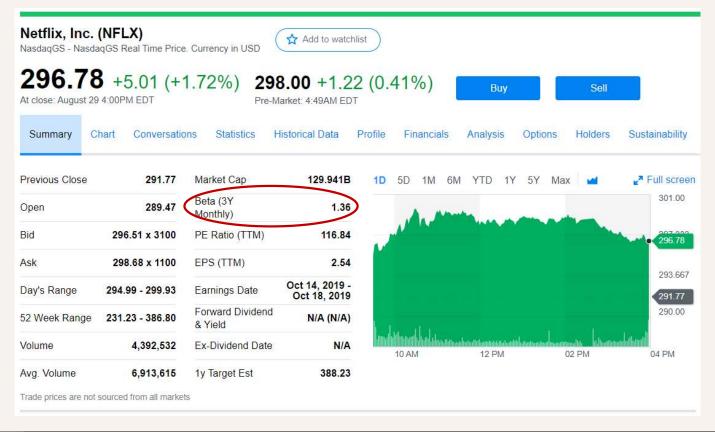
### Introduction to Cost of Capital (1 of 2)

- Costs of capital: Opportunity costs of creditors and owners have when investing in a business
  - For investors: the returns they could have earned on alternative, similar-risk investments
- Two alternatives of estimating a project's cost of capital:
  - 1. Alternative 1: All-equity comparables
  - Find an all-equity financed firm in a single line of business that is comparable to the project.
  - Use the comparable firm's equity beta (see <u>slide#6</u>) and cost of capital as estimates.
  - 2. Alternative 2: Levered firms comparables (see slide #7)
  - Estimate the cost of capital based on a portfolio of the comparable (i.e. levered) firm's debt and equity (see Figure 12.3)

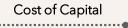




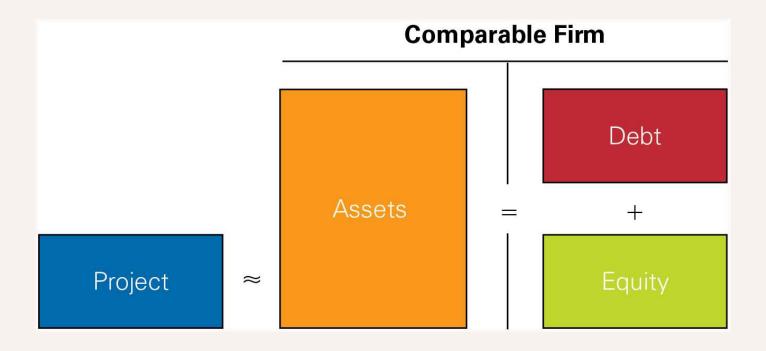
# Finding current/recent equity beta value







# Figure 12.3 Using a Levered Firm as a Comparable for a Project's Risk





### Introduction to Cost of Capital (2 of 2):

See Example 12.4

Cost of capital

See Example 12.5

Alt. 1: All-equity comparables

$$r_{project} = r_f + \beta_E (r_m - r_f)$$

Alt. 2: Levered firms as comparables

A. Compute  $r_E$  using CAPM and then compute  $r_{project}$  using asset or unlevered cost of capital

$$r_{U} = \frac{E}{E+D} r_{E} + \frac{D}{E+D} r_{D}$$

B. Or compute asset or unlevered beta then use CAPM to compute  $r_{project}$ 

$$\beta_{U} = \frac{E}{E+D} \beta_{E} + \frac{D}{E+D} \beta_{D}$$





## Textbook Example 12.4

#### Estimating the Beta of a Project from a Single-Product Firm

#### Problem

You have just graduated with an MBA and decide to pursue your dream of starting a line of designer clothes and accessories. You are working on your business plan, and believe your firm will face similar market risk to Lululemon (LULU). To develop your financial plan, estimate the cost of capital of this opportunity assuming a risk-free rate of 3% and a market risk premium of 5%.

#### Solution:

Check Yahoo! Finance, LULU has no debt  $\rightarrow$  LULU as an all-equity comparable firm. LULU's equity beta is estimated to be 0.80. Use CAPM formula to compute your project's cost of capital:

$$r_{project} = r_f + B_{LULU}(E[R_{Mkt}] - r) = 3\% + 0.80 \times 5\% = 7\%$$



## Textbook Example 12.5

#### Unlevering the Cost of Capital

#### **Problem**

Your firm is considering expanding its household products division. You identify Procter & Gamble (PG) as a firm with comparable investments. Suppose PG's equity has a market capitalization of \$144 billion and a beta of 0.55. PG also has \$37 billion of AA-rated debt outstanding, with an average yield of 3.1%. Estimate the cost of capital of your firm's investment given a risk-free rate of 3% and a market risk-premium of 5%.

Solution

A: Estimate PG's equity cost of capital using CAPM:  $r_E = 3\% + 0.55(5\%) = 5.75\%$ . Then based on PG as a levered firm comparable, estimate your firm's project cost of capital:

$$r_U = \frac{E}{E+D} r_E + \frac{D}{E+D} r_D \implies r_{project} = \frac{144}{144+37} 5.75\% + \frac{37}{144+37} 3.1\% = 5.21\%$$

B: Or estimate PG's unlevered beta by assuming PG's debt beta equals to zero (because of its high debt rating and the average yield is closed to the risk-free rate)

$$\beta_U = \frac{E}{E+D} \beta_E + \frac{D}{E+D} \beta_D \implies \beta_U = \frac{144}{144+37} 0.55 + \frac{37}{144+37} 0 = 0.438$$

Then use CAPM to compute your firm's project cost of capital:  $r_{project} = 3\% + 0.438(5\%) = 5.19\%$ 





### Cash and Net Debt

- Some firms maintain high cash balances
- Cash is a risk-free asset that reduces the average risk of the firm's assets
- Since the risk of the firm's enterprise value is what we're concerned with, leverage should be measured in terms of net debt (see Example 12.6)

Net Debt = Debt - Excess Cash and short-term investments





# Textbook Example 12.6: Cash & Beta

#### **Problem**

In early 2018, Microsoft Corporation had a market capitalization of \$716 billion, \$89 billion in debt, and \$133 billion in cash. If its estimated equity beta was 1.04, estimate the beta of Microsoft's underlying business enterprise.

#### Solution

Microsoft has net debt = (89-133) = -\$44 billion. Therefore, Microsoft's enterprise value is (716-44) = +672 billion, which is the total value of its underlying business on a debt-free basis and excluding cash. Assuming Microsoft's debt and cash investments are both risk-free, we can estimate the beta of this enterprise value as:

$$B_U = \frac{E}{E+D}B_E + \frac{D}{E+D}B_D = \frac{716}{716-44}1.04 + \frac{-44}{716-44}0 = 1.11$$

Note that in this case, Microsoft's equity is less risky than its underlying business activities due to its cash holdings.





## Financing and the Weighted Average Cost of Capital

- How might the project's cost of capital change if the firm uses leverage to finance the project?
- Perfect capital markets
  - In perfect capital markets, choice of financing does not affect cost of capital or project NPV
- Taxes A Big Imperfection
  - When interest payments on debt are tax deductible, the net cost to the firm is given by:
  - Effective after-tax interest rate = r(1-t)





# Example: Cost of Capital

	XYZ Liabilities and Owners' Equity	Opportunity Cost of Capital
Debt	€100	10%
Equity	€200	20%
	Creditors expect to earn at least 10% on their loans	Owners (shareholders) expect to earn at least 20% on their ownership of XYZ shares

Assuming a tax rate of 35%, to meet the expectations of creditors and owners:

- 1. How much money must XYZ earn annually on existing assets?
- 2. What rate of return must XYZ earn on existing assets?





### Solution Cost of Capital: Question 1

- ➤ How much money must XYZ earn annually on existing assets?
- Loan: interest payment of 10% which is subject to tax
  - Regarding Ioan, XYZ must earn = (1-tax rate) \* (Ioan interest) \* (debt) = (1-0.35)\*(0.1)\*(€100) = €6.50
- Equity: XYZ must earn = (ownership %) \* (share values) = (0.2) \* (200) = €40
- In total, XYZ must earn = €6.50 + €40 = €46.50 in order to meet the expectations of creditors and owners





### Solution Cost of Capital: Question 2

- > What rate of return must XYZ earn on existing assets?
- From the table, total investment at XYZ = €300
- With the €300 investment, XYZ must earn €46.50 → the required rate of return = €46.50/ €300 = 15.50%
- In formula:

$$r_W = \frac{(1-t)r_D D + r_E E}{D+E}$$

r<sub>W</sub>=cost of capital

 $r_D = cost of debt$  $r_F = cost of equity$ 

D = Debt

E = Equity

t = tax rate

r<sub>w</sub> is also known as weighted-average cost of capital (WACC), rewriting the formula:

$$WACC = \frac{D}{D+E}(1-t)r_D + \frac{E}{D+E}r_E$$

In our example, WACC is onethird the cost of debt plus twothirds the cost of equity





# The Weighted Average Cost of Capital

Weighted Average Cost of Capital (WACC)

$$r_{wacc} = \frac{E}{E+D} r_E + \frac{D}{E+D} r_D (1-\tau_C)$$

$$r_{\rm wacc} = r_{\!\! U} - \frac{D}{E + D} \tau_{\!\! C} r_{\!\! D}$$
 • Given a target leverage ratio:

- In a world with taxes:
  - $r_{ij}$  (unlevered cost of capital or pretax WACC)  $\rightarrow$  to evaluate an all-equity project with the same risk as the firm ( $r_U$  = the expected return of the firm's assets)
  - $r_{wacc}$   $\rightarrow$  to evaluate a project with the same risk and the same financing as the firm  $(r_{wacc} < \text{the expected return of the firm's assets})$





### Including risk in investment evaluation

- NPV: When estimating the present values of possible future cash in-/out-flows, we use discount rate
  - When including risk, the discount rate has to be risk-adjusted
  - Risk-adjusted discount rate = risk-free interest rate + risk premium rate
- IRR: compare IRR to a risk-adjusted discount rate

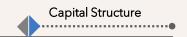




# Final Thoughts on the CAPM

- There is a number of assumptions made in the estimation of cost of capital using the CAPM.
- How reliable are the results?
  - Errors in cost of capital estimation are not likely to make a large difference in NPV estimates
  - CAPM is practical, easy to implement, and robust
  - CAPM imposes a disciplined approach to cost of capital estimation that is difficult to manipulate
  - CAPM requires managers to think about risk in the correct way

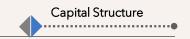




# Firm financing & Capital Structure

- Unlevered equity: equity in a firm with no debt
- Levered equity: equity in a firm that also has debt outstanding
- Capital structure: the relative proportions of debt, equity, and other securities that a firm has outstanding

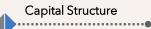




### Illustration: The effect of leverage on risk and return

- Suppose that as an entrepreneur, you have €1,000 and you are facing two possible ways to raise money in one year for a 50-50 chance return at either €900 (weak economy) or €1,400 (strong economy)
  - 1. Through common or preferred stock (equity financing, assume 100%) where you invest the whole €1,000
  - 2. Through debt financing of 80% (borrow €800) with a 1-year loan at 10%, invest €200



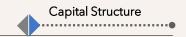


### Equity Financing vs. Debt Financing

1. 100% Equity Financing. You invest €1,000



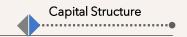




## In sum, the effect of leverage on risk and return:

- Leverage increases the risk of equity even when there is no risk that the firm will default
  - Unlevered equity: return of either -5% or 20%
  - Levered equity: return of either -45% or 80%
- The returns to equity holders are very different with and without leverage
  - $E(R_U) = 15\%$  and  $E(R_L) = 35\%$   $\rightarrow$  higher return to compensate for a higher risk
- Considering both sources of capital, the firm's average of cost of capital with leverage is 0.8(10%) + 0.2(35%) = 15% the same as for the unlevered firm





# Modigliani & Miller (MM) on capital structure

- Modigliani and Miller (1958) argued that with perfect capital markets, the total value of a firm should not depend on its capital structure
- A set of conditions for perfect capital markets:
  - Investors and firms can trade the same set of securities at competitive market prices equal to the present value of their future cash flows.
  - There are no taxes, transaction costs, or issuance costs associated with security trading.
  - A firm's financing decisions do not change the cash flows generated by its investments, nor do they reveal new information about them.



## MM proposition I & the law of one

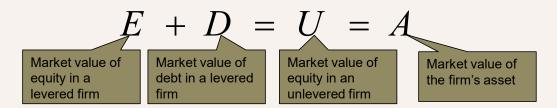
MM's argument: In the absence of taxes or other transaction costs:

total CF paid out to all of a firm's security holders = total CF generated by the firm's assets

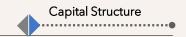
#### The law of one price:

the firm's securities and its assets must have the same total market value

• MM Proposition I: In a perfect capital market, the total value of a firm is equal to the market value of the total cash flows generated by its assets and is not affected by its choice of capital structure.



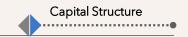




# Homemade Leverage

- MM demonstrated that if investors would prefer an alternative capital structure to the one the firm has chosen, investors can borrow or lend on their own and achieve the same result.
- Homemade Leverage
  - When investors use leverage in their own portfolios to adjust the leverage choice made by the firm (as long as they can borrow or lend at the same interest rate as the firm)
  - Investors can alter the leverage choice of the firm to suit their personal tastes either by adding more leverage or by reducing leverage.
  - With perfect capital markets, different choices of capital structure offer no benefit to investors and does not affect the value of the firm.





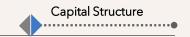
# Suppose you chose not to use debt, but an investor prefers to use debt (based on the illustration on slide #22 and slide #23)

An investor replicating levered equity using homemade leverage

		Year 0	Year 1: Cash flows	
		Initial cost	Strong economy	Weak economy
You	Unlevered equity	€1,000	€1,400	€900
Investor-	Loan (10% interest)	-800	-880	-880
	Levered equity	€200	€520	€20

Compare with slide #22





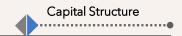
# Now, suppose you use debt but the investor prefers not to use any debt (based on the illustration on slide #22 and slide #23)

An investor replicating unlevered equity by holding debt & equity

		Year 0	Year 1: C	ash flows
		Initial cost	Strong economy	Weak economy
	Debt	€800	€880	€880
Investor	Levered equity	200	520	20
	Unlevered equity	€1,000	€1,400	€900

Compare with slide #22





# MM Proposition II

- MM Proposition I can be further used to derive an explicit relationship between leverage and the equity cost of capital:
  - The return on unlevered equity  $(R_U)$  is related to the returns of levered equity  $(R_F)$  and debt  $(R_D)$ :

$$\frac{E}{E+D}R_E + \frac{D}{E+D}R_D = R_U$$
Solving for  $R_E$ :  $R_E = \underbrace{R_U}_{\text{Risk without leverage}} + \underbrace{\frac{D}{E}(R_U - R_D)}_{\text{Additional risk due to leverage}}$ 

MM Proposition II: The cost of capital of levered equity increases with the firm's market value debt-equity ratio

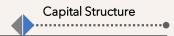




### MM Proposition II: Leverage, Risk, and the Cost of Capital

- Recall from slides #22 and #23:
  - If the firm is all-equity financed, the expected return on unlevered equity is 15%.
  - If the firm is financed with €800 of debt, the expected return of the debt is 10%.
  - Therefore, according to MM Proposition II, the expected return on equity for the levered firm is:
  - $-R_F = 15\% + 800/200 (15\% 10\%) = 35\%$





# MM Proposition II: Further implication

Leverage increases the risk and return to stockholders

Risk: 
$$R_E = R_U + \frac{D}{E}(R_U - R_D)$$

- Return: **ROE= net income / equity**; Net income = (EBIT-iD)(1-t)

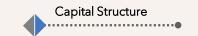
$$ROE = \frac{(EBIT - iD)(1 - t)}{E} = \frac{EBIT(1 - t)}{E} - \frac{iD(1 - t)}{E}$$

Substitute with:  $ROIC = \frac{EBIT(1-t)}{D+E}$ ; Note: Formula ROIC from lecture 1

$$ROE = ROIC \times \frac{D+E}{E} - i(1-t)\frac{D}{E}$$

If i'= i(1-t), then: 
$$ROE = ROIC + (ROIC - i')\frac{D}{E}$$





# Example Leverage, ROE, and EPS: Unlevered Firm (1)

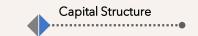
Unlevered firm, assuming no tax

Number of shares	1000			
Price per share	10			
value shares	10000			
	Possible Outcomes			
Operating income (EBIT)	500	1000	1500	2000
Earnings per share	0.50	1.00	1.50	2.00
Return on shares (%)	5	10	15	20

Since D= 0 then ROE = ROIC = 15%

Can this firm increase its ROE by taking advantage of the financial leverage (at i'=10% interest rate)?





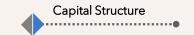
### Example Leverage, ROE, and EPS: Levered Firm (2)

Levered firm, assuming no tax

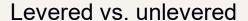
Number of shares (#)	500			
Price per share	10			
value shares	5000			
value debt	5000			
Interest at 10%	500			
		Possible (	Outcomes	
Operating income (A)	500	1000	1500	2000
Interest (B)	500	500	500	500
Equity earnings (C) = (A)-(B)	0	500	1000	1500
Earnings per share (D) = (C)/(#)	0	1.00	2.00	3.00
Return on shares (%)	0	10	20	30

ROIC = 
$$\frac{\text{EBIT}(1-t)}{D+E}$$
 = 1500/10000=15%  $ROE = ROIC + (ROIC - i')\frac{D}{E}$  = 15%  $+$  5% = 20%

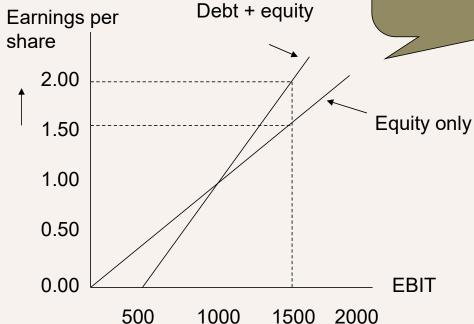




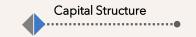
## Capital Structure Fallacy: Leverage & EPS



Financial leverage not only increases expected ROE, but also increases earnings per share (EPS)







### Does debt policy matter?

In short, Modigliani & Miller argue that:

I: Leverage does not increase a firm's market value

II: Leverage increases the risk and return to stockholders

>> How to reconcile both observations?

Leverage does not affect the required return on the *package* of debt and equity (cost of capital), but does affect the required return on the *individual* securities (see next slide)



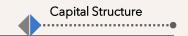
#### MOT111A: Financial Management

## Capital Structure

### Summarizing

			50% equity, 50% of 'cheaper'
	Unlevered	Levered	debt: Cost of capital = (50%*10%
Number of shares	1000	500	+ 50%*20%) = 15%
Price per share	10	10	MM Proposition I
Market value shares	10000	5000	I WIWI TO POSITION 1
value debt		5000	
Interest at 10%		500	
Operating income	1500	1500	
Interest cost		500	
Equity earnings		1000	
Earnings per share	1.50	2.00	MM Proposition II
Return on equity (%)	15	20	
Cost of equity of capital = 15°		Cost of	equity = 20%





# MM on the effect of leverage on a firm's cost of capital

With perfect capital markets, a firm's WACC is independent of its capital structure and is
equal to its equity cost of capital if it is unlevered, which matches the cost of capital of its
assets

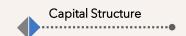
 $r_{wacc} = r_U = r_A$ 

• If the firm's capital structure is made up of multiple securities, then the WACC is calculated by computing the weighted average cost of capital of all of the firm's securities.

$$r_{wacc} = r_U = \frac{E}{E + D + W} r_E + \frac{D}{E + D + W} r_D + \frac{W}{E + D + W} r_W$$

See Example 14.6





# Textbook Example 14.6: WACC with multiple securities

#### **Problem**

Compute the WACCfor the entrepreneur's project with the capital structure described in Example 14.3.

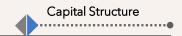
Solution:

From Example 14.3, the firm has three securities in its capital structure (debt, equity, and the warrant) with E = 440, D = 500, W = 60,  $r_E = 18.18\%$ ,  $r_D = 5\%$ ,  $r_W = 75\%$ . Given this capital structure, the project's WACC is computed as:

$$r_{wacc} = r_{U} = \frac{E}{E + D + W} r_{E} + \frac{D}{E + D + W} r_{D} + \frac{W}{E + D + W} r_{W}$$

$$r_{wacc} = \frac{\$440}{\$1000} (18.18\%) + \frac{\$500}{\$1000} (5\%) + \frac{\$60}{\$1000} (75\%) = 15\%$$





### Levered and Unlevered Betas

• The effect of leverage on the risk of a firm's securities can also be expressed in terms of beta:

$$\beta_U = \frac{E}{E + D} \beta_E + \frac{D}{E + D} \beta_D$$

Rearrange:

$$\beta_E = \beta_U + \frac{D}{E}(\beta_U - \beta_D)$$





# In the presence of taxes



In the absence of taxes, debt financing does not affect firm value (MM Proposition I)

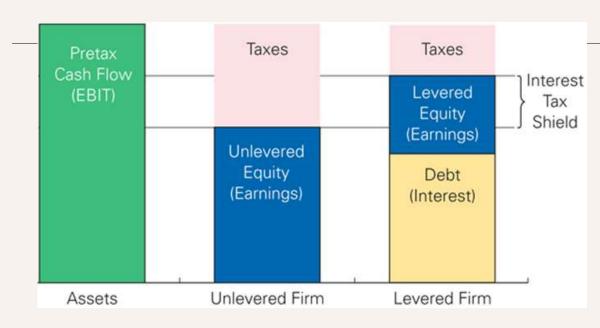


In the presence of taxes, prudent debt financing increases firm value





### The Interest Tax Shield and Firm Value



#### M&M Proposition I with taxes

The total value of the levered firm exceeds the value of the firm without leverage due to the present value of the tax savings from debt.

$$V^{L} = V^{U} + PV$$
(Interest Tax Shield)

$$\begin{pmatrix} Cash Flows to Investors \\ with Leverage \end{pmatrix} = \begin{pmatrix} Cash Flows to Investors \\ without Leverage \end{pmatrix} + (Interest Tax Shield)$$

Interest Tax Shield = Corporate Tax Rate × Interest Payments



# The Interest Tax Shield with Permanent Debt

• Suppose a firm borrows debt D and keeps the debt permanently. If the firm's marginal tax rate is  $\tau_c$ , and if the debt is riskless with a risk-free interest rate  $r_f$ , then the interest tax shield each year is  $\tau_c \times r_f \times D$ , and the tax shield can be valued as a perpetuity.

$$PV(\text{Interest Tax Shield}) = \frac{\tau_c \times \text{Interest}}{r_f} = \frac{\tau_c \times (r_f \times D)}{r_f}$$
  
=  $\tau_c \times D$ 



### WACC with Taxes

• With tax-deductible interest, the effective after-tax borrowing rate is  $r(1 - \tau_c)$  and the weighted average cost of capital becomes

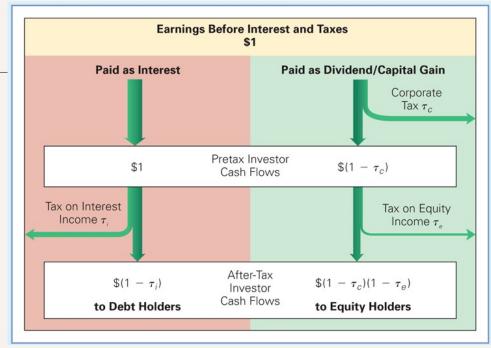
$$r_{wacc} = \frac{E}{E + D} r_E + \frac{D}{E + D} r_D (1 - \tau_c)$$

$$r_{wacc} = \underbrace{\frac{E}{E + D} r_E + \frac{D}{E + D} r_D}_{\text{Pretax WACC}} - \underbrace{\frac{D}{E + D} r_D r_D}_{\text{Reduction Due}}$$



to Interest Tax Shield

## Personal Taxes



$$V^L = V^U + \tau^* D$$

Effective tax advantage of debt =  $\tau^*$ 

$$\tau^* = \frac{(1 - \tau_i) - (1 - \tau_c)(1 - \tau_e)}{(1 - \tau_i)} = 1 - \frac{(1 - \tau_c)(1 - \tau_e)}{(1 - \tau_i)}$$





# Does a firm need to use 100% debt financing?

• The optimal level of leverage from a tax saving perspective is the level such that interest equals EBIT.

Interest = 
$$r_D \times \text{Debt} \leq \text{EBIT}$$
 or Debt  $\leq \text{EBIT} / r_D$ 

- The tax of disadvantage of excess leverage (where  $\tau_c = 0$ ):

$$\tau_{ex}^* = 1 - \frac{(1 - \tau_e)}{(1 - \tau_i)} = \frac{\tau_e - \tau_i}{(1 - \tau_i)} < 0$$

