

Week Five

EPA143A

THE IS-LM MODEL

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ANSWERS

EXERCISE W-5.1

1. Derive the IS-curve.

$$y = c + g + i = 200 + 0.25(y - 200) + 250 + 150 + 0.25y - 1000 \times r$$

$$y = 550 + 0.5y - 1000 \times r$$

$$y = 1100 - 2000 \times r \quad \text{This is the IS-curve.}$$

2. Derive the LM-curve.

$$\left(\frac{M^S}{p}\right) = 1600 = \left(\frac{M^D}{p}\right) = 2 \times y - 8000 \times r$$

$$y = 800 + 4000 \times r \quad \text{This is the LM-curve.}$$

3. Solve for equilibrium real GDP y^* and the equilibrium real interest rate r^* . Calculate c^* and i^* .

$$y = 1100 - 2000 \times r = 800 + 4000 \times r \rightarrow r^* = \frac{300}{6000} = 0.05 \text{ or } 5\% \text{ and } y^* = 1000$$

4. Let $\left(\frac{M^S}{p}\right) = 1840$. Repeat questions (1)-(3). Explain the direction of change in y^* and r^* .

$$\text{The new LM-curve becomes: } y = 920 + 4000 \times r. \quad r^* = \frac{180}{6000} = 0.03 \text{ or } 3\% \text{ and } y^* = 1040.$$

The central bank decided to increase money supply by 40 units. In response to the increased availability of money, banks lowered the (real) interest rate; this, in turn, stimulated business investment and as a result, real GDP increased from 1000 units to 1040 units. Monetary

stimulus did lead to an increase in economic activity and higher real GDP – by ‘crowding in’ business investment (through the lower interest rate r^*).

5. Let $\left(\frac{M^S}{p}\right) = 1600$ and let $g = 400$. Repeat questions (1)-(3). Explain the direction of change in y^* and r^* . Explain why there is crowding out of private expenditure by public expenditure.

The new IS-curve becomes: $y = 1400 - 2000 \times r$. The LM-curve remains:

$$y = 800 + 4000 \times r. \text{ Hence, we get: } r^* = \frac{600}{6000} = 0.1 \text{ or } 10\% \text{ and } y^* = 1200.$$

The government increase spending by 150 units. The multiplier (in the IS-curve) is equal to 2. Real GDP would have increased by $2 \times 150 = 300$ units. But higher real GDP raises the transactions demand for money (equation (5)); money supply is fixed at 1600. Hence, the (real) interest rate has to increase – which depresses business investment and lowers money demand. In equilibrium, real GDP y^* has increased by 20%, not 30%, because the interest rate has risen from 5% to 10%. The difference of 100 units represents the decline in private business investment, caused by the real interest rate increase. 100 units of private spending have been crowded out by the higher interest rate, caused by the fiscal stimulus.

EXERCISE W-5.2

1. Derive the IS-curve.

$$\begin{aligned} y &= c + g + i + ne \\ &= 2000 + 0.6(y - 300) + 300 + 300 - 3000 \times r + 400 - 200 \times e \\ y &= 2820 + 0.6y - 3000 \times r - 200 \times e \\ y &= 7050 - 7500 \times r - 500 \times e \quad \text{IS-curve.} \end{aligned}$$

2. Derive the LM-curve.

$$\begin{aligned} \left(\frac{M^D}{p}\right) &= 0.2 \times y - 1000 \times r = \left(\frac{M^S}{p}\right) = 500 \\ y &= 2500 + 5000 \times r \quad \text{LM-curve.} \end{aligned}$$

3. Derive the BT-curve.

$$ne = 400 - 200 \times er = 0$$

$$er = 2 \quad \text{BT-curve}$$

4. We know that $er = 2$ (in equilibrium). This reduces the IS-curve to:

$$y = 6050 - 7500 \times r. \quad \text{The LM-curve is: } y = 2500 + 5000 \times r$$

$$y = 2500 + 5000 \times r = 6050 - 7500 \times r \rightarrow r^* = (3550/12500) = 0.284 \text{ or } 28.4\%.$$

$$y^* = 2500 + 5000 \times r = 3920.$$

5. Now $ne = 500 - 200 \times er$. The LM-curve stays the same: $y = 2500 + 5000 \times r$.

The new BT-curve: $er = 2.5$.

$$y = c + g + i + ne$$

$$= 2000 + 0.6(y - 300) + 300 + 300 - 3000 \times r + 500 - 200 \times e$$

$$y = 2920 + 0.6y - 3000 \times r - 200 \times er$$

$$y = 7300 - 7500 \times r - 500 \times er \quad \text{The new IS-curve.}$$

Since $er = 2.5$, this gives: $y = 6050 - 7500 \times r$

This gives: $r^* = (3550/12500) = 0.284$ or 28.4%; and $y^* = 3920$. There is no change in real GDP or the real interest rate. Explanation:

We assume that the economy's balance of trade $ne = e - m = 0$ is (always) in equilibrium, because the exchange rate will adjust to bring about equilibrium. This means that the equation $y = c + g + i + e - m$ reduces to $y = c + g + i$. Exports and imports do not matter for macro-economic equilibrium (under this assumption).

EXERCISE W-5.3

Let us solve the IS-LM model first.

The IS-curve (we solve for r):

$$y = c + g + i$$

$$y = c_0 + (1 - \sigma) \times (y - t_0) + g_0 + i_0 - \alpha \times r$$

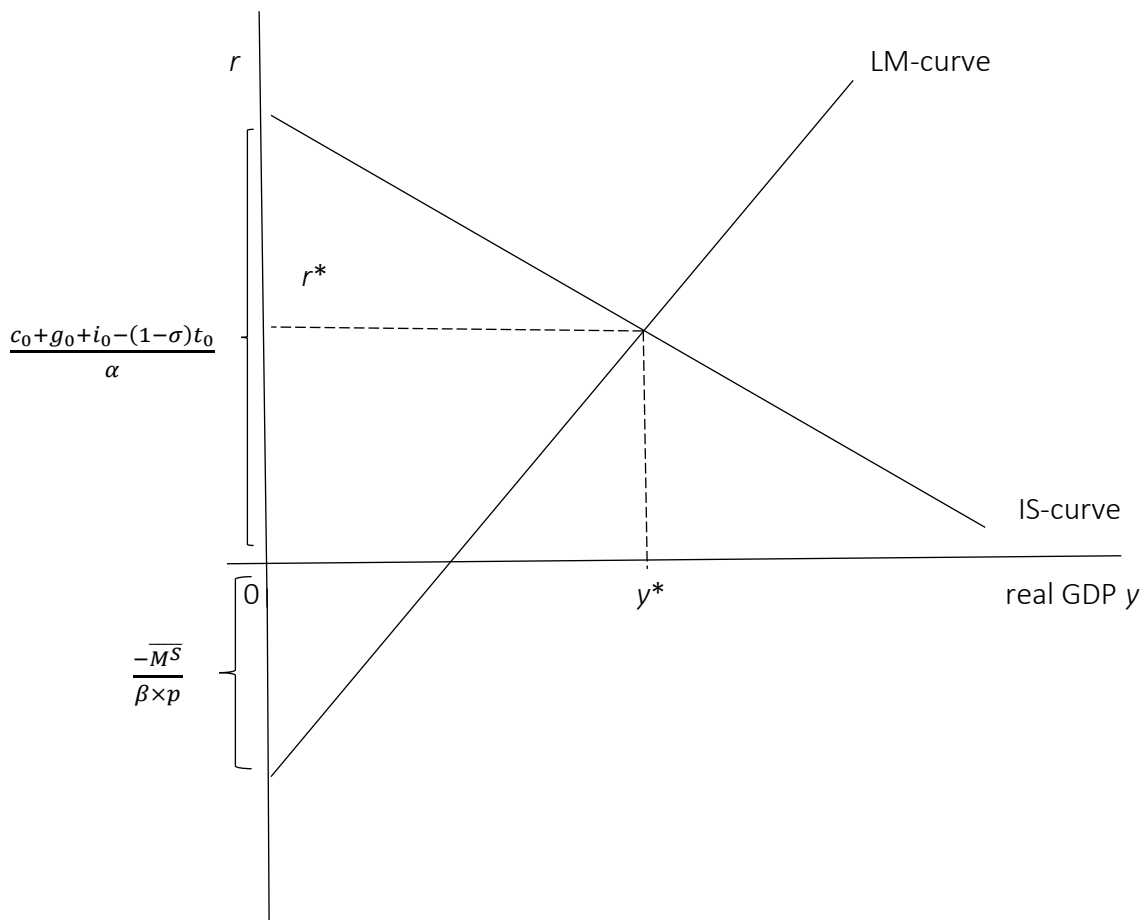
$$\alpha \times r = c_0 + (1 - \sigma) \times y - y - (1 - \sigma)t_0 + g_0 + i_0$$

$$r = \frac{c_0 + g_0 + i_0 - (1 - \sigma)t_0}{\alpha} - \frac{\sigma \times y}{\alpha} \quad \text{the IS-curve}$$

The LM-curve (we solve for r):

$$\left(\frac{M^S}{p}\right) = \left[\frac{\overline{M^S}}{p}\right] = \left(\frac{M^D}{p}\right) = v \times y - \beta \times r$$

$$r = \frac{v \times p \times y - \overline{M^S}}{\beta \times p} \quad \text{the LM-curve}$$



1. In response to
 - (a) a decline in the average propensity to save (σ), the IS-curve will shift to the LEFT (down) and because the (negative) slope coefficient ($\frac{\sigma}{\alpha}$) will decline, the IS-curve will decline less steeply than in the figure.
 - (b) an increase in autonomous private investment, the IS-curve will shift to the RIGHT (since i_0 will rise). Higher investment has an unconstrained multiplier impact on real GDP, as long as the interest rate is unchanged.
 - (c) an increase in income taxation (t_0) the IS-curve will shift to the LEFT (or down). Autonomous demand goes down, which reduces real GDP – at the unchanged interest rate.
 - (d) good news about the economy, the IS-curve will shift to the RIGHT (i_0 will rise).
2. In response to
 - (a) a decline in nominal money supply M^S , the LM-curve will shift to LEFT (or up). The central bank reduces money supply; for the same rate of interest, the level of economic activity has to go down.
 - (b) a decline in the general price level p , the LM-curve will shift to the RIGHT (or down). With a lower general price level, the real value of money supply increases, which allows a greater number of “transactions”; there is (in real terms) more money available, and the rate of interest can go down.

EXERCISE W-5.4

1. Goods market equilibrium:

$$y = c + i = 100 + 0.8 \times y + 120 - 3 \times r = 0.8 \times y + 220 - 3 \times r$$

$$y = 5 \times (220 - 3 \times r) = 1100 - 15 \times r \quad \text{The IS-curve}$$
 Money market equilibrium:

$$120 = 0.2 \times y - 5 \times r \rightarrow y = 600 + 25 \times r \quad \text{The LM-curve}$$
2. Equilibrium:

$$1100 - 15r = 600 + 25r \rightarrow 40r = 500 \rightarrow r^* = 12.5\%; y^* = 912.5$$
3. Goods market equilibrium:

$$y = c + i = 100 + 0.8 \times y + 160 - 3 \times r = 0.8 \times y + 260 - 3 \times r$$

$$y = 5 \times (260 - 3 \times r) = 1300 - 15 \times r \quad \text{IS-curve}$$

Money market equilibrium:

$$120 = 0.2 \times y - 5 \times r \rightarrow y = 600 + 25 \times r \quad \text{The LM-curve}$$

Equilibrium:

$$1300 - 15r = 600 + 25r \rightarrow 40r = 700 \rightarrow r^* = 17.5\%; y^* = 1037.5$$

4. Public investment increased by 40. The multiplier is 5. Hence, y^* would rise by 200 units (to 1112.5 units), if the real interest rate remains unchanged. But r^* rises from 12.5% to 17.5% and this reduces y^* by 75 units (15 x change in $r^* = 5$) to 1037.5 units. Private consumption and private investment therefore decline by 75 units (= the extent of crowding out).
5. To determine the accommodating monetary policy, the central bank will look at the equation for money market equilibrium: $\frac{M^s}{p} = 0.2 \times y - 5 \times r$. We already know that $y^* = 1112.5$ (without crowding out) and that r^* must remain unchanged at 12.5%. Hence, we can write: $\frac{M^s}{1} = 0.2 \times 1112.5 - 5 \times 12.5 = 160$. Accommodating monetary policy means that the central bank should raise money supply by 40 units. Doing so, the rate of interest can stay unchanged, as the government raises public investment by 40. The multiplier process works without restrictions or negative side-effects, and y^* increases by 200 units. There is – in this case – no crowding out.