Social Scientific Values

MOT1442 Q2

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Overview

- Last week we looked at an overview of philosophy of science
- Reasoning is core to science *and* philosophy
- This week, we're looking at reasoning in formal argumentation

Arguments

- When we talk about reasoning, we’re talking about *arguments*
- Arguments are used to extend our knowledge
- They are the *bridge* between beliefs already justified and novel ones
- How do we know when they’re good or bad?
- This is our core question, today

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Structure of the lecture

- First half:
  - Informal Fallacies
  - Formal logic 1
- Second half:
  - Formal logic 2
  - Formal Fallacies

- Note: these lecture slides are quite detailed, you might want to download them from Brightspace to follow along!

Problems with Arguments

- When arguments are bad, it's usually because they employ some form of *fallacy*.
- Fallacies can be either *informal* or *formal*.
- **Informal fallacies** are characterised by an error in the content of an argument.
- **Formal fallacies** are characterised by an error in the structure of an argument.

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Informal Fallacies

- These are the most common types of errors.
- The error is *contentful*
- For example, *ad hominem* is an informal fallacy.
- Latin for '*against the person*'
- Here's an example of an *ad hominem* argument:

  P1. Fred says the bar is open

  P2. But Fred likes to drink, so he would say that

  C. The bar isn't really open

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Informal Fallacies – Ad Hominem

P1. Fred says the bar is open

P2. But Fred likes to drink, so he would say that

C. The bar isn't really open

- The proposition ('the bar is open') isn't the target
- Instead, Fred is the target.
- P2's function: Fred is unreliable, therefore what Fred says is false.
- This, however, isn't a reliable inference (unreliable people say true things from time to time!)
- When the target of an argument is the speaker and not the content, the ad hominem fallacy is usually being employed

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Informal Fallacies – Other Examples

- More examples of informal fallacies:
  - Appeal to nature (covered last week)
  - Naturalistic fallacy
  - *Post hoc ergo propter hoc*
  - Appeal to the law
  - Appeal to ignorance
  - Begging the question

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Informal Fallacies – Other Examples

- Naturalistic Fallacy
  - Important to note: Different from appeal to nature
  - Concerns the descriptive/normative distinction
  - Moving from descriptive claims, to normative claims
  - We can't infer what *should* be the case, from what *is* the case (or vice versa)
  - Example:
    - **Premise:** People need to drink 5 glasses of water a day, and eat plenty of vegetables to be healthy
    - **Conclusion:** I should drink 5 glasses of water a day, and eat plenty of vegetables

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Informal Fallacies – Other Examples

- *Post hoc ergo propter hoc*
  - Latin: ‘after this, therefore because of this’
  - ‘Because *x* followed *y, x* must have been *caused* by *y*’
  - Correlation does not imply causation
    - Example:
      - **Premise:** I felt sick after I ate all that food last night
      - **Conclusion:** The food must have made me feel ill

      - **Premise:** Every time I go to the doctor, I get a bad diagnosis
      - **Conclusion:** I should stop going to the doctor

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Informal Fallacies – Other Examples

- *Appeal to the law*
  - Conflates illegality with immorality
  - Just because an act is illegal, doesn’t imply necessarily that that act is immoral (or vice versa)
  - Examples:
    - The court says it’s legal to not pay your workers, therefore employers have no moral duty to look after them.
    - Murder is illegal. Taking drugs is illegal. Murder is morally wrong, and therefore so is taking drugs.

Blow to gig economy workers after UK supreme court rules against collective bargaining rights

Top court says riders were self-employed contractors and do not have a right to collective negotiations on pay and conditions

The court upheld past judgments that UK riders were independent self-employed contractors. Photograph: Nathan Stirk/Getty Images

Deliveroo riders do not have the right to collective negotiations on pay and conditions, the UK’s top court has ruled, in a blow to gig economy

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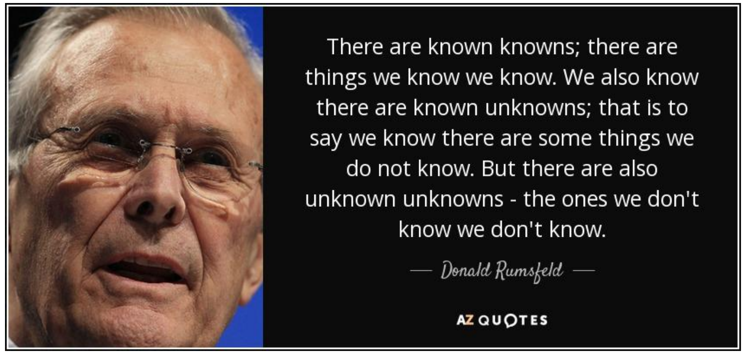
Informal Fallacies – Other Examples

- *Appeal to ignorance*
  - Concluding that a proposition is true because there is no evidence against it
  - We have no evidence that aliens don’t exist, therefore, they do
  - We have no long-term evidence that vaccines are safe, therefore they are dangerous
  - US defense secretary Donald Rumsfeld, lead up to the Iraq War, in reference to WMDs (2002):
    - ‘Simply because you do not have evidence that something exists does not mean that you have evidence that it doesn't exist’



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Informal Fallacies – Other Examples

- *Begging the question*
  - Technical term – different from common usage
  - To assume the truth of your conclusion in the premises
  - Examples:
    - ‘I’m in charge because what I say goes!’
    - ‘Opium induces sleep because it has soporific qualities’
    - ‘Multiculturalism will not work because different cultures cannot coexist’


soporific 1 of 2 adjective

sop·o·rif·ic ˌsä-pə-ˈri-fik

Synonyms of *soporific* ›

1 a : causing or tending to cause sleep

soporific drugs

b : tending to dull awareness or alertness

2 : of, relating to, or marked by sleepiness or lethargy

Informal Fallacies

- There are numerous informal fallacies, and countless examples of them
- Depend on the *content* of the argument
- Require you to look at the argument content to detect their presence

Formal Fallacies – first thing's first

- There is another way of detecting faults with arguments, however
- This method only requires we look at the *structure* of an argument
- Before we look at these fallacies, we first need a methodology for analysing this structure

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Arguments - Reminder

- Take our example from last week, once more:
  - Premise 1. If today is Thursday, then tomorrow it will be Friday.
  - Premise 2. Today is Thursday.
  - Conclusion. Tomorrow, it will be Friday

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Arguments - Reminder

- This argument is *valid*. An argument is valid *if and only if* the truth of the premises entail the truth of the conclusion. What that means is, *if* the premises are true, the conclusion *must* be true.
  - Premise 1. If today is Thursday, then tomorrow it will be Friday.
  - Premise 2. Today is Thursday.
  - Conclusion. Tomorrow, it will be Friday

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Arguments - Validity

- An argument ***can still be valid***, even if the premises are ***actually false***.
- As long as it's the case that, *if they are true*, the conclusion would *have to be true*, then the argument is valid.
  - Premise 1. If today is Wednesday, then tomorrow it will be Thursday.
  - Premise 2. Today is Wednesday.
  - Conclusion. Tomorrow, it will be Thursday.

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Arguments - Validity

- When an argument is *valid*, ***and*** the premises are *true*, the argument is *sound.*
  - Premise 1: If today is Thursday, then tomorrow is Friday.
  - Premise 2: Today is Thursday.
  - Conclusion: Tomorrow is Friday.

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Arguments – Translating sentences

- First of all, we translate the sentences of an argument into symbolic form. We'll let the letters *p* and *q* stand for whole sentences.
- So, to take our earlier example:
  - P1. If today is Thursday, then tomorrow is Friday
  - P2. Today is Thursday
  - C. Tomorrow is Friday
- We'll translate this to:
  - P1. If *p*, then *q*
  - P2. *p*
  - *C. q*

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Arguments – Logical Connectives

P1. If *p*, then *q*
P2. *p*
C. q

- There were some extra parts we didn't translate (namely, the *if… then…* part).
- We then have to translate the *connectives* (i.e., the parts that connect sentences in arguments).
- There are 5 logical connectives in propositional logic. These translate as follows:

English connective	Logical connective
P or Q	$P \lor Q$
P and Q	$P \land Q$
If P then Q	$P \rightarrow Q$
P, if and only if Q	$P \longleftrightarrow Q$
Not-P	$\neg P$

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Arguments – Logical Connectives

- So, to translate our earlier example:
  - P1. If p, then q
  - P2. p
  - C. q
- This becomes:
  - P1. $p \rightarrow q$
  - P2. p
  - C. q

English connective	Logical connective
P or Q	$P \vee Q$
P and Q	$P \wedge Q$
If P then Q	$P \rightarrow Q$
P, if and only if Q	$P \longleftrightarrow Q$
Not-P	$\neg P$

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Arguments – Logical Connectives

- So, to translate our earlier example:
  - P1. If p, then q
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- This becomes:
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Not-P	$\neg P$

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Arguments – Logical Connectives

- Let's translate another, more complicated example:
  - P1. If it’s raining, then I’m not happy.
  - P2. It’s raining, and I’m wearing a raincoat.
  - C. I’m not happy.

English connective	Logical connective
P or Q	$P \vee Q$
P and Q	$P \wedge Q$
If P then Q	$P \rightarrow Q$
P, if and only if Q	$P \longleftrightarrow Q$
Not-P	$\neg P$

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Arguments – Logical Connectives

- First of all, substitute sentences for letters:
  - P1. If it's raining, then I'm not happy.
  - P2. It's raining, and I'm wearing a raincoat.
  - C. I'm not happy.
- Becomes:
  - P1. If p, then not-q
  - P2. p and r
  - C. not-q

English connective	Logical connective
P or Q	$P \vee Q$
P and Q	$P \wedge Q$
If P then Q	$P \rightarrow Q$
P, if and only if Q	$P \longleftrightarrow Q$
Not-P	$\neg P$

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Arguments – Logical Connectives

- And now, substitute the English connectives for logical operators:
  - P1. If p, then not-q
  - P2. p and r
  - C. not-q
- Becomes:
  - P1. $p \rightarrow \neg q$
  - P2. $p \wedge r$
  - C. $\neg q$
- Bonus: is this valid or invalid?

English connective	Logical connective
P or Q	$P \vee Q$
P and Q	$P \wedge Q$
If P then Q	$P \rightarrow Q$
P, if and only if Q	$P \longleftrightarrow Q$
Not-P	$\neg P$

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Arguments - Validity

- We have to give *meaning* to the logical connectives.
- We give them meaning, in propositional logic, by giving their *truth conditions.*

English connective	Logical connective
P or Q	$P \vee Q$
P and Q	$P \wedge Q$
If P then Q	$P \rightarrow Q$
P, if and only if Q	$P \longleftrightarrow Q$
Not-P	$\neg P$

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Arguments - Validity

- So, let's take ‘and’, as an example. Under what conditions is the sentence ‘**p and q**’ (‘p $\wedge$ q’) true?
- If I said, ‘it’s raining and it’s cold’, the sentence is only true when it’s both raining, and it’s cold.
- The sentence ‘it’s raining and it’s cold’ isn’t true when only <u>one</u> of the two *conjuncts* (i.e., p, q) are true.
- Equally, ‘it’s raining and it’s cold’ isn’t true when <u>neither</u> of them are true.

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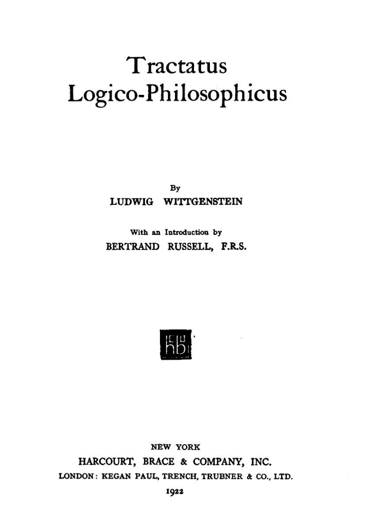

Tractatus
Logico-Philosophicus

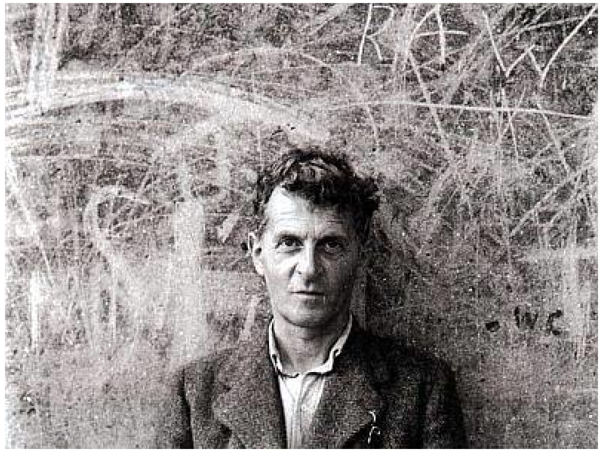
By
LUDWIG WITTGENSTEIN

With an Introduction by
BERTRAND RUSSELL, F.R.S.

NEW YORK
HARCOURT, BRACE & COMPANY, INC.
LONDON: KEGAN PAUL, TRENCH, TRUBNER & CO., LTD.
1922

Arguments – Truth Tables

- We can do this more rigorously using a *truth table.*
- A truth table exhaustively describes the conditions under which the sentence could be true or false.



Arguments – Truth Tables

- Once again, take our example: *'it's raining and it's cold'.*
- Substitute 'it's raining' for P, and 'it's cold' for 'Q'
- There are a total of four combinations of how the world might be here, given either the truth or falsity of both P and Q.
- It could be that:
  - P is true, and Q is true (it's raining, and it's cold')
  - P is true, and Q is false (it's raining but it's **not** cold)
  - P is false, and Q is true (it's **not** raining but it's cold)
  - P is false, and Q is false (it's **not** raining and it's **not** cold)

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Arguments – Truth Tables

- We'll symbolise true as 1, and false as 0.
  - P is true, and Q is true
  - P is true, and Q is false
  - P is false, and Q is true
  - P is false, and Q is false

- Putting our earlier possible combinations in a table, we get this:

P (*'it's raining'*)	Q (*'it's cold'*)
1	1
1	0
0	1
0	0

Arguments – Truth Tables (Conjunction)

- Now, we can define our connective '*and*' (also known as *conjunction*) by saying when the sentence 'P and Q' is true or false in each of these possible combinations:

P ('*it's raining*')	**Q ('*it's cold*')**	**P ∧ Q (*it's raining and it's cold'*)**
1	1	1
1	0	0
0	1	0
0	0	0

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Arguments – Truth Tables (Conjunction)

- Notice, it's only when P and Q are both true, that the sentence 'P ∧ Q' is true.
- This is exactly how we want it. As we said, it's only when both 'P' is true *and* 'Q' is true, that 'P ∧ Q' was true.

P	Q	P ∧ Q
1	1	1 (true)
1	0	0 (false)
0	1	0 (false)
0	0	0 (false)

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Arguments – Truth Tables (Disjunction)

- And we can expand this for the other 4 operators. So, let's do the truth table for 'or' (also known as *disjunction*):

P	Q	P ∧ Q	**P ∨ Q**
1	1	1	1
1	0	0	1
0	1	0	1
0	0	0	0

Arguments – Truth Tables (Disjunction)

- This one is a little trickier. Usually, in English, we use the *exclusive ‘or’.*
- For example, if I say, ‘I have an apple, **or** I have a banana’, we don’t think the sentence is true if I have **both** an apple *and* a banana.
- In propositional logic, however, we treat ‘or’ as *inclusive,* so if I said ‘I have an apple, or I have a banana’, and I have both an apple and a banana, then the sentence is true, when translated.

P ‘I have an apple’	Q ‘I have a banana’	P ∧ Q ‘I have an apple *and* a banana’	**P ∨ Q ‘I have an apple *or* a banana’**
1	1	1	1
1	0	0	1
0	1	0	1
0	0	0	0

This is the circumstance in which I have both an apple and a banana.

Arguments – Truth Tables (Disjunction)

- Nonetheless, the other three results are quite straightforward. If I have either an apple, or I have a banana, then P v Q is true. It's only in circumstances where I have neither that the sentence is false (i.e., the bottom line).

P	Q	P ∧ Q	P ∨ Q
1	1	1	1
1	0	0	1
0	1	0	1
0	0	0	0

This is the circumstance in which I have neither an apple nor a banana.

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Arguments – Truth Tables (Material Conditional)

- There's three more connectives to go. First of all, there's the 'if… then…' connective (also known as *the material conditional*). The truth table for the material conditional is as follows:

P	Q	$P \wedge Q$	$P \vee Q$	$P \rightarrow Q$
1	1	1	1	1
1	0	0	1	0
0	1	0	1	1
0	0	0	0	1

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Arguments – Truth Tables (Material Conditional)

- The material conditional turns out as true in all circumstances *except* those in which the **antecedent** (the 'P' in 'P → Q') is true, but the **consequent** (the 'Q' in 'P → Q') is false.

P	Q	P ∧ Q	P ∨ Q	P → Q
1	1	1	1	1
1	0	0	1	0
0	1	0	1	1
0	0	0	0	1

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Arguments – Truth Tables (Material Conditional)

- This is potentially the most counterintuitive truth table you'll meet.
- If we swap P and Q for actual sentences, it makes it easier to test whether the sentence aligns with our intuition.
- Let **P** stand for **'it rains',** and **Q** stand for **'the floor gets wet'.**
- *(Situation 1)* The first line makes intuitive sense. 'If it rains then the floor gets wet' is true when it's true that it's raining, and the floor is indeed wet.
- *(Situation 2)* Similarly, it's intuitively sensible that 'If it rains then the floor gets wet' is false when it rains, but the floor isn't wet.

	P	Q	P ∧ Q	P ∨ Q	**P → Q**
Situation 1	1	1	1	1	1
Situation 2	1	0	0	1	0
	0	1	0	1	1
	0	0	0	0	1

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Arguments – Truth Tables (Material Conditional)

- However, this is where things start to make less sense.
- According to our truth table, the sentence ‘if it rains, then the floor gets wet’ is true when it doesn’t rain, and the floor is indeed wet. (i.e., *situation 3*)
- Similarly, according to the truth table, ‘if it rains, then the floor gets wet’ is true when it doesn’t rain, and the floor isn’t wet. (i.e., situation 4)

	P	Q	P ∧ Q	P ∨ Q	P → Q
	1	1	1	1	1
	1	0	0	1	0
Situation 3	0	1	0	1	1
Situation 4	0	0	0	0	1

Arguments – Truth Tables (Material Conditional)

- We'll go into why this is in more detail in the tutorials.
- For now, the best way to think of it is this: the only circumstance in which 'if it rains, then the floor gets wet' is *definitely* false, is when it rains, and the floor isn't wet (*i.e., situation* 2).
- In a circumstance in which it doesn't rain, we just don't know whether the conditional 'if it rains, then the floor gets wet' is true; we haven't *tested it,* so to speak.
- Out logic is *two-valued* – things can be either true, or false. And if we can't be sure it's false, then it can only be the other option – true!

	P	Q	P ∧ Q	P ∨ Q	**P → Q**
	1	1	1	1	1
Situation 2	1	0	0	1	0
	0	1	0	1	1
	0	0	0	0	1

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Arguments – Truth Tables (Biconditional)

- Our penultimate operator is the *biconditional,* or the ‘if and only if’ (sometimes written ‘iff’).
- Sentences with a biconditional are true when either **1)** both P and Q are true (*i.e., situation 1*), or **2)** when both P and Q are false (*i.e., situation 2*).

	P	Q	P ∧ Q	P ∨ Q	P → Q	**P⟵Q**
Situation 1	1	1	1	1	1	1
	1	0	0	1	0	0
	0	1	0	1	1	0
Situation 2	0	0	0	0	1	1

Arguments – Truth Tables (Biconditional)

- This makes intuitive sense, which we can see if we translate a biconditional into English.
- So, take the sentence, ‘I’ll go to the party, if and only if you go too’.
- In this circumstance, the sentence is true only 1) when both of us go (i.e., P and Q are true), or 2) when neither of us go (i.e., P and Q are false).
- If only one of us goes to the party, then the sentence is false, and this is what the truth table says.

P	Q	$P \wedge Q$	$P \vee Q$	$P \rightarrow Q$	$P \longleftarrow Q$
1	1	1	1	1	1
1	0	0	1	0	0
0	1	0	1	1	0
0	0	0	0	1	1

Arguments – Truth Tables (Negation)

- Our final operator, the ‘not’ operator (also known as *negation*) only operates on one letter, so it’s truth table is slightly different.
- It’s also very simple; when P is false, the sentence ‘not-P’ is true. Similarly, when P is true, then the sentence ‘not-P’ is false.
- Again, this is intuitive. The sentence ‘it’s not raining’ is false when it’s raining, and true when it is!

P	¬P
1	0
0	1

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Arguments – Truth Tables

- Altogether then, our truth table looks like this.
- These are the definitions of the logical operators; they fully define their meaning.

P	Q	$P \land Q$	$P \lor Q$	$P \rightarrow Q$	$P \longleftarrow Q$	$\neg P$
1	1	1	1	1	1	0
1	0	0	1	0	0	0
0	1	0	1	1	0	1
0	0	0	0	1	1	1

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Arguments – Truth Tables

- We can do truth tables for complex compound sentences with multiple connectives within them.
- I’ll quickly demonstrate this now, but this is something we’ll do together in more detail in the tutorials as well.

Arguments – Truth Tables

- So, for example, lets build the truth table for the sentence ‘$P \land (P \rightarrow Q)$’

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Arguments – Truth Tables

- So, for example, lets build the truth table for the sentence:
  - ‘P ∧ (P → Q)’
- We always start by the full configuration of the possible combinations of the sentences that appear in the compound sentence (i.e., P and Q).
- These never change. They only extend if you add in more atomic sentences

P	Q		
1	1		
1	0		
0	1		
0	0		

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Arguments – Truth Tables

- Next, we'll break the sentence down into its constituent parts. ***Original sentence:** "P ∧ (P → Q)'*
- Sentences in brackets are always *well-formed formulae* (i.e., they can't be broken down further)
- Therefore, we know we'll need a truth table for '(P → Q)'
- This is the same as it appears on a normal truth table

P	**Q**		**P → Q**
1	1		1
1	0		0
0	1		1
0	0		1

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Arguments – Truth Tables

- Next, we have to add in the ‘P ∧ …’ portion of our compound sentence.
- But what values do we enter here?

P	Q	P ∧	P → Q
1	1		1
1	0		0
0	1		1
0	0		1

Arguments – Truth Tables

- On our original truth table, we had the truth value for ‘P ∧ Q’, but now we have different values for the second *conjunct*.

Original Truth Table for Conjunction

P	Q	P ∧ Q
1	1	1
1	0	0
0	1	0
0	0	0

Initial values for the second conjunct

P	Q	P ∧	P → Q
1	1		1
1	0		0
0	1		1
0	0		1

Our new values, as we’re looking for the truth value of ‘P ∧ (P → Q)’, rather than ‘P ∧ Q’.

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Arguments – Truth Tables

- What our initial truth table told us was essentially a formula for working out what the value of ‘P $\wedge$ *anything’* is.
- It tells us all the possible circumstances we could meet.
- It says that, if our *first conjunct* is 1, and our *second conjunct* is 1, then the sentence, as a whole, is true (i.e., is a 1) (*situation 1*).
- If our antecedent is 1, and our consequent is 0, the sentence is false, and we have a 0 (*situation 2*).
- Et cetera.

	P	Q	P $\wedge$ Q
Situation 1	1	1	1
Situation 2	1	0	0
	0	1	0
	0	0	0

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Arguments – Truth Tables

- Our second conjunct here is ‘P → Q’, and we’ve already got the truth values for this sentence (we just worked it out!).
- Looking at our original truth table, as we saw, when we have a situation in which the first conjunct is 1, and the second conjunct is 1, the sentence, as a whole, has a value of 1 (i.e., *situation 1*).
- Therefore, we know the sentence ‘P∧(P→Q)’ is true when ‘P’ is true, and ‘P→Q’ is true. We can put a value of ‘1’ in the top line. This value is the value for the sentence, as a whole (as the conjunction is the *main operator*).

	P	Q	P ∧	P → Q
Situation 1	1	1	1	1
	1	0		0
	0	1		1
	0	0		1

Truth values of ‘P → Q’

Original Truth Table for Conjunction

P	Q	P ∧ Q
1	1	1
1	0	0
0	1	0
0	0	0

Arguments – Truth Tables

- We can then go on and complete the rest of the table.
- In **situation 2**, we have a 1 and a 0. Therefore, according to our original truth table for conjunction, we have a 0.
- In situation 3, we have a 0 and a 1. Therefore, according to our original truth table for conjunction, we have a 0.
- In situation 4, we have a 0 and a 1. Therefore, according to our original truth table for conjunction, we have a 0.

	P	Q	P ∧	P → Q
	1	1	1	1
Situation 2	1	0	0	0
Situation 3	0	1		1
Situation 4	0	0		1

Original Truth Table for Conjunction

P	Q	P ∧ Q
1	1	1
1	0	0
0	1	0
0	0	0

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Arguments – Truth Tables

- We can then go on and complete the rest of the table.
- In situation 2, we have a 1 and a 0. Therefore, according to our original truth table for conjunction, we have a 0.
- In **situation 3**, we have a 0 and a 1. Therefore, according to our original truth table for conjunction, we have a 0.
- In situation 4, we have a 0 and a 1. Therefore, according to our original truth table for conjunction, we have a 0.

	P	Q	P $\wedge$	P $\rightarrow$ Q
	1	1	1	1
Situation 2	1	0	0	0
Situation 3	0	1	0	1
Situation 4	0	0		1

Original Truth Table for Conjunction

P	Q	P $\wedge$ Q
1	1	1
1	0	0
0	1	0
0	0	0

Arguments – Truth Tables

- We can then go on and complete the rest of the table.
- In situation 2, we have a 1 and a 0. Therefore, according to our original truth table for conjunction, we have a 0.
- In situation 3, we have a 0 and a 1. Therefore, according to our original truth table for conjunction, we have a 0.
- In **situation 4**, we have a 0 and a 1. Therefore, according to our original truth table for conjunction, we have a 0.

	P	Q	P ∧	P → Q
	1	1	1	1
Situation 2	1	0	0	0
Situation 3	0	1	0	1
Situation 4	0	0	0	1

Original Truth Table for Conjunction

P	Q	P ∧ Q
1	1	1
1	0	0
0	1	0
0	0	0

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Arguments – Truth Tables

- And there we have it. This is the truth value of the sentence ‘$P \wedge (P \rightarrow Q)$’ (circled in red).
- We have a mechanical system for calculating whether or not a sentence is true or false, given whatever configuration of the truth of the sentences that compose it.

P	Q	P ∧	P → Q
1	1	1	1
1	0	0	0
0	1	0	1
0	0	0	1

Note: if asked to give a truthtable for a sentence, the red ringed part is the ‘answer’, so to speak.

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Arguments – Truth Tables

- With this truth table, we can construct truth tables for any sentence, as long as we can translate it in the method described.
- We're only limited by computational power.

P	Q	P ∧ Q	P ∨ Q	P → Q	P⟷Q	¬P
1	1	1	1	1	1	0
1	0	0	1	0	0	0
0	1	0	1	1	0	1
0	0	0	0	1	1	1

Arguments – Truth Tables

- There's no other method by which to memorise the truth tables for the connectives, other than to learn them – they're definitions, and as such, they can't be derived from anything else.
- That said, learning how they correspond (and differ) from their natural language equivalents can help you remember.
- We'll go through this in the tutorials, so don't worry if you're struggling with this.

P	Q	$P \wedge Q$	$P \vee Q$	$P \rightarrow Q$	$P \leftrightarrow Q$	$\neg P$
1	1	1	1	1	1	0
1	0	0	1	0	0	0
0	1	0	1	1	0	1
0	0	0	0	1	1	1

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Arguments – Truth Tables

- How do we make truth tables for sentences with more than 2 atomic sentences?
- How would we make a truth table for:
  - ‘$P \rightarrow (Q \vee R)$’
- Our current truth table only mentions has space two of the variables

P	Q	$P \rightarrow (Q \vee R)$
1	1	
1	0	
0	1	
0	0	

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Arguments – Truth Tables

- We need to extend the truth table such that it includes R.
- We do this like so:

P	Q	$P \rightarrow (Q \lor R)$
1	1	
1	0	
0	1	
0	0	

R	P	Q	$(P \rightarrow (Q \lor R)$
1	1	1	
1	1	0	
1	0	1	
1	0	0	
0	1	1	
0	1	0	
0	0	1	
0	0	0	

Arguments – Truth Tables

- Notice here, our old truth table appears in the new one. It's just been extended.

P	Q	$P \rightarrow (Q \vee R)$
1	1	
1	0	
0	1	
0	0	

R	P	Q	$(P \rightarrow (Q \vee R)$
1	1	1	
1	1	0	
1	0	1	
1	0	0	
0	1	1	
0	1	0	
0	0	1	
0	0	0	

Arguments – Truth Tables

- We extend it because now we have to consider 1) all the old situations, when R is true in those, **and** 2) all the old situations, in which R is false in those.
- Note: these are the *only* novel combinations that are possible. This exhausts them.
- We double the number of rows, and add an additional column, if we have a fourth variable.
- R, P, and Q could remain unchanged

P	Q	P→ (Q v R)
1	1	
1	0	
0	1	
0	0	

R	P	Q	(P→ (Q v R)
1	1	1	
1	1	0	
1	0	1	
1	0	0	
0	1	1	
0	1	0	
0	0	1	
0	0	0	

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Arguments – Truth Tables

- To do complete the truth table, we follow the exact same rules.

P	Q	P→ (Q v R)
1	1	
1	0	
0	1	
0	0	

R	P	Q	(P→ (Q v R)
1	1	1	
1	1	0	
1	0	1	
1	0	0	
0	1	1	
0	1	0	
0	0	1	
0	0	0	

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Arguments – Truth Tables

- Split the sentences into its atomic elements.

R	P	Q	(P→	(Q v R)
1	1	1		
1	1	0		
1	0	1		
1	0	0		
0	1	1		
0	1	0		
0	0	1		
0	0	0		

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Arguments – Truth Tables

Note here: we're not considering 'P', because 'P' isn't in the sentence (Q v R)

- Apply the rules as we learnt them before
- For *disjunction* ('or', the *v* symbol), we're looking for *at least* one of the disjuncts, here 'Q' and 'R', to be true
- Start with the ones with *no* true value for either Q or R

R	P	Q	(P→	(Q v R)
1	1	1		
1	1	0		
1	0	1		
1	0	0		
0	1	1		
0	1	0		0
0	0	1		
0	0	0		0

Original rule for 'or'

P	Q	P V Q
1	1	1
1	0	1
0	1	1
0	0	0

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Arguments – Truth Tables

Note here: we're not considering 'P', because 'P' isn't in the sentence (Q v R)

- All the other values, are true, since they're not false!

R	P	Q	(P→	(Q v R)
1	1	1		1
1	1	0		1
1	0	1		1
1	0	0		1
0	1	1		1
0	1	0		0
0	0	1		1
0	0	0		0

Original rule for 'or'

P	Q	P V Q
1	1	1
1	0	1
0	1	1
0	0	0

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Arguments – Truth Tables

Now we're concerned with P, as it is the *antecedent*

- Now we turn to the *conditional* (the '→' part).
- Our rule was, true in all but those situations in which the first letter is true, and the second is false.
- Here, there's only one situation which is like that.

R	P	Q	(P→	(Q v R)
1	1	1		1
1	1	0		1
1	0	1		1
1	0	0		1
0	1	1		1
0	1	0	0	0
0	0	1		1
0	0	0		0

Original rule for '→'

P	Q	P → Q
1	1	1
1	0	0
0	1	1
0	0	1

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Arguments – Truth Tables

- The rest of the column is 1
- Add in all the other values, and we have our truth table for the sentence (P → (Q v R)

R	P	Q	(P→	(Q v R)
1	1	1	1	1
1	1	0	1	1
1	0	1	1	1
1	0	0	1	1
0	1	1	1	1
0	1	0	0	0
0	0	1	1	1
0	0	0	1	0

Original rule for '→'

P	Q	P → Q
1	1	1
1	0	0
0	1	1
0	0	1

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Arguments – Truth Tables

- With our new understanding of formal logic in hand, we’re now in a position to understand *formal fallacies*

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Arguments – Truth Tables

- To remind ourselves, formal fallacies concern problems with an arguments structure
- The problem we’re looking for is *invalidity,* and we can find it using truth tables
- How do we test if an argument is valid, using truth tables?

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Arguments – Truth Tables

- Take the following argument:
  - P1. If the moon is closer to the earth than the sun, then it’s gravitational pull on the earth is weaker
  - P2. The moon’s gravitational pull on the earth is weaker than the Sun’s
  - C. The moon is closer to the earth than the sun

Arguments – Truth Tables

- Take the following argument:
  - P1. If the moon is closer to the earth than the sun, then it’s gravitational pull on the earth is weaker
  - P2. The moon’s gravitational pull on the earth is weaker than the Sun’s
  - C. The moon is closer to the earth than the sun

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Arguments – Truth Tables

- Replace with variables:
  - P1. If A, then B
  - P2. A
  - C. B

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Arguments – Truth Tables

- Replace the operators:
  - P1. $A \rightarrow B$
  - P2. A
  - C. B

Arguments – Truth Tables

- Draw the truth table for the premises, and the conclusion:

A	B	$A \rightarrow B$	B	$\models$ A
1	1			
1	0			
0	1			
0	0			

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Arguments – Truth Tables

- Draw the truth table for the premises, and the conclusion:

A	B	A → B	B	⊨ A
1	1	1	1	1
1	0	0	0	1
0	1	1	1	0
0	0	1	0	0

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Arguments – Truth Tables

- How do we test for validity?:

A	B	$A \rightarrow B$	B	$\vDash$ A
1	1	1	1	1
1	0	0	0	1
0	1	1	1	0
0	0	1	0	0

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Arguments – Truth Tables

- **Definition of validity**: an argument is valid **iff** when the premises are true, the conclusion must be true as well.
- So, we look at each instance of when the premises are all true, and see if the conclusion is true as well.

A	B	$A \rightarrow B$	B	$\vDash$ A
1	1	1	1	1
1	0	0	0	1
0	1	1	1	0
0	0	1	0	0

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Arguments – Truth Tables

- We have two such instances
- When we look, however, we can see that it's possible for the premises to be true, and the conclusion to be false
- We know, therefore, that the argument is **invalid**

A	B	$A \rightarrow B$	B	$\models$ A
1	1	1	1	1
1	0	0	0	1
0	1	1	1	**0**
0	0	1	0	0

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Arguments – Truth Tables

- We have two such instances
- When we look, however, we can see that it's possible for the premises to be true, and the conclusion to be false
- We know, therefore, that the argument is **invalid**

A	B	$A \rightarrow B$	B	$\models$ A
1	1	1	1	1
1	0	0	0	1
0	1	1	1	**0**
0	0	1	0	0

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Arguments – Truth Tables

- And notice, unlike the informal fallacies, as long as the argument has this form, we can know it's invalid.
- We can swap out A and B for *anything,* and it would still be invalid
- We have a general methodology for assessing validity of arguments

A	B	$A \rightarrow B$	B	$\models$ A
1	1	1	1	1
1	0	0	0	1
0	1	1	1	**0**
0	0	1	0	0

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Arguments – Truth Tables

- This is an example of the formal fallacy that is called *affirming the consequent*
- It's quite common to see examples of such reasoning (indeed, I don't think the example was obviously invalid)
- Formal and informal fallacies are not mutually exclusive – can you think of any examples that combine both *affirming the consequent* <u>and</u> *post hoc ergo propter hoc*?

A	B	$A \rightarrow B$	B	$\models$ A
1	1	1	1	1
1	0	0	0	1
0	1	1	1	**0**
0	0	1	0	0

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Questions

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If we have time – 3 variable argument, and test for validity