3.16

解： $x_t-\mu = \phi_1(x_{t-1}-\mu)+\varepsilon_t$

$\hat{\mu}=10,\quad \hat{\phi}_1=0.3,\quad \hat{\sigma}_\varepsilon^2=9$

(1) 不妨令 $y_t = x_t-\mu$

则： $y_t = \phi_1 y_{t-1}+\varepsilon_t$

$\hat{y}_{t+3} = 0.3^3 y_t = 0.027(x_t-\hat{\mu}) = 0.027x_t - 0.$

$\hat{x}_{t+3} = 0.027x_t - 0.27 + \hat{\mu} = 0.027x_t + 9.73$

其由 Green函数递推：

$G_0 = 1$

$G_1 = \phi_1 G_0 = 0.3$

$G_2 = \phi_1 G_1 + \phi_2 G_0 = 0.09$

$Var(\hat{e}_{3y}) = \hat{\sigma}_\varepsilon^2 \sum_{i=0}^{2} G_i^2 = 9\cdot(1+0.3+0.09) = 12.51$

$\xRightarrow{y_t=x_t+\mu} Var(\hat{e}_{2x}) = Var(\hat{e}_{3y}) = 12.51$

故 x_{t+3} 的 95% 置信区间：

$(\hat{x}_{t+3}-1.96\sqrt{12.51},\ \hat{x}_{t+3}+1.96\sqrt{12.51})$

即： $(0.027x_t+2.798,\ 0.27x_t+16.662)$

(2) $Var(\hat{e}_y) = \hat{\sigma}_\varepsilon^2 \sum_{i=0}^{1} G_i^2 = 9\cdot(1+0.3) = 11.7$

$\Rightarrow Var(\hat{e}_{2x}) = Var(\hat{e}_{2y}) = 11.7$

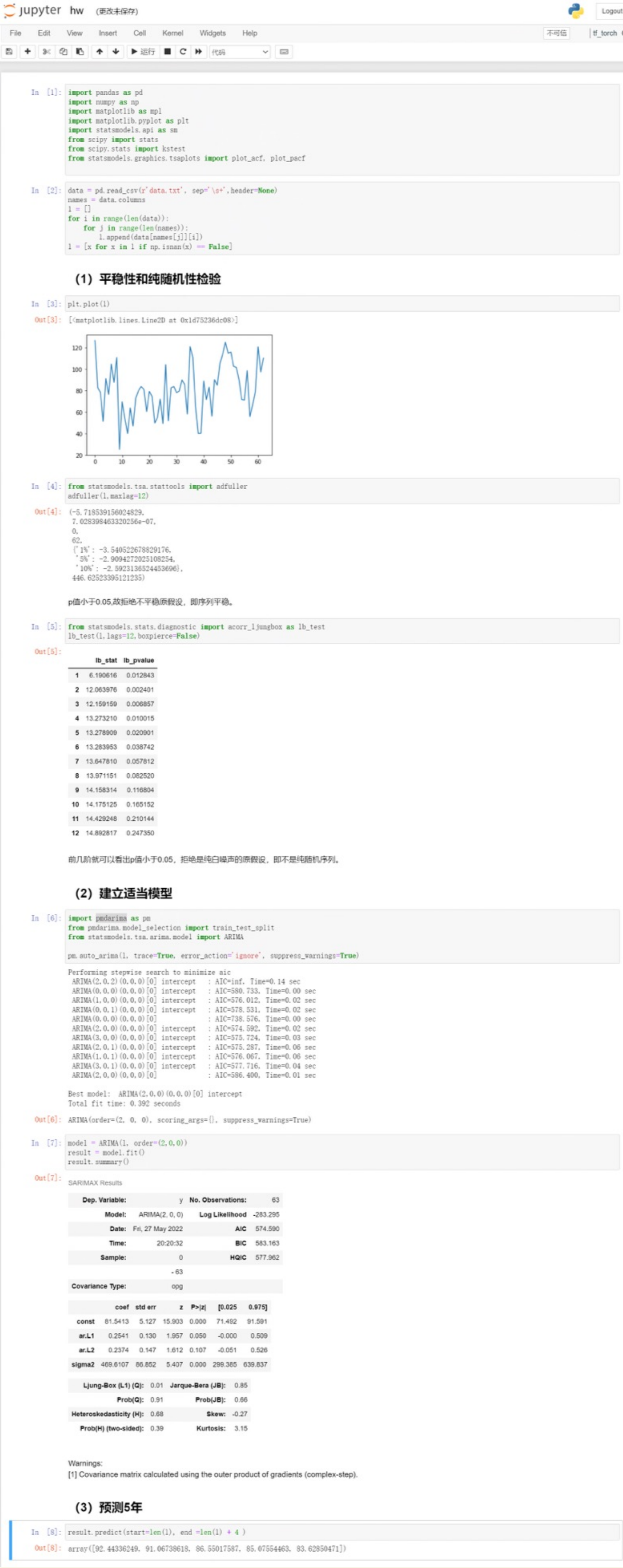
$x_{t+1}-\mu = \phi_1(x_t-\mu)+\varepsilon_{t+1}$

代入，

$(\frac{0.027}{0.3}(x_{t+1}+0.7\hat{\mu})+3.026,\ \frac{0.027}{0.3}(x_{t+1}+0.7\hat{\mu})+16.43$

即： $(4.601,\ 18.009)$

3.17



```
In [1]: import pandas as pd
        import numpy as np
        import matplotlib as mpl
        import matplotlib.pyplot as plt
        import statsmodels.api as sm
        from scipy import stats
        from scipy.stats import kstest
        from statsmodels.graphics.tsaplots import plot_acf, plot_pacf
```

```
In [2]: data = pd.read_csv(r'data.txt', sep='\s+',header=None)
        names = data.columns
        l = []
        for i in range(len(data)):
            for j in range(len(names)):
                l.append(data[names[j]][i])
        l = [x for x in l if np.isnan(x) == False]
```

（1）平稳性和纯随机性检验

```
In [3]: plt.plot(l)
Out[3]: [<matplotlib.lines.Line2D at 0x1d75236dc08>]
```

```
In [4]: from statsmodels.tsa.stattools import adfuller
        adfuller(l,maxlag=12)
Out[4]: (-5.718539156024829,
         7.028398463320256e-07,
         0,
         62,
         {'1%': -3.540522678829176,
          '5%': -2.9094272025108254,
          '10%': -2.5923136524453696},
         446.62523395121235)
```

p值小于0.05,故拒绝不平稳原假设，即序列平稳。

```
In [5]: from statsmodels.stats.diagnostic import acorr_ljungbox as lb_test
        lb_test(l,lags=12,boxpierce=False)
Out[5]:
```

	lb_stat	lb_pvalue
1	6.190616	0.012843
2	12.063976	0.002401
3	12.159159	0.006857
4	13.273210	0.010015
5	13.278909	0.020901
6	13.283953	0.038742
7	13.647810	0.057812
8	13.971151	0.082520
9	14.158314	0.116804
10	14.175125	0.165152
11	14.429248	0.210144
12	14.892817	0.247350

前几阶就可以看出p值小于0.05，拒绝是纯白噪声的原假设，即不是纯随机序列。

（2）建立适当模型

```
In [6]: import pmdarima as pm
        from pmdarima.model_selection import train_test_split
        from statsmodels.tsa.arima.model import ARIMA

        pm.auto_arima(l, trace=True, error_action='ignore', suppress_warnings=True)

        Performing stepwise search to minimize aic
         ARIMA(2,0,2)(0,0,0)[0] intercept   : AIC=inf, Time=0.14 sec
         ARIMA(0,0,0)(0,0,0)[0] intercept   : AIC=580.733, Time=0.00 sec
         ARIMA(1,0,0)(0,0,0)[0] intercept   : AIC=576.012, Time=0.02 sec
         ARIMA(0,0,1)(0,0,0)[0] intercept   : AIC=578.531, Time=0.02 sec
         ARIMA(0,0,0)(0,0,0)[0]             : AIC=738.576, Time=0.00 sec
         ARIMA(2,0,0)(0,0,0)[0] intercept   : AIC=574.592, Time=0.02 sec
         ARIMA(3,0,0)(0,0,0)[0] intercept   : AIC=575.724, Time=0.03 sec
         ARIMA(2,0,1)(0,0,0)[0] intercept   : AIC=575.287, Time=0.06 sec
         ARIMA(1,0,1)(0,0,0)[0] intercept   : AIC=576.067, Time=0.06 sec
         ARIMA(3,0,1)(0,0,0)[0] intercept   : AIC=577.716, Time=0.04 sec
         ARIMA(2,0,0)(0,0,0)[0]             : AIC=586.400, Time=0.01 sec

        Best model:  ARIMA(2,0,0)(0,0,0)[0] intercept
        Total fit time: 0.392 seconds
Out[6]: ARIMA(order=(2, 0, 0), scoring_args={}, suppress_warnings=True)
```

```
In [7]: model = ARIMA(l, order=(2,0,0))
        result = model.fit()
        result.summary()
Out[7]:
```

SARIMAX Results

Dep. Variable:	y	No. Observations:	63
Model:	ARIMA(2, 0, 0)	Log Likelihood	-283.295
Date:	Fri, 27 May 2022	AIC	574.590
Time:	20:20:32	BIC	583.163
Sample:	0	HQIC	577.962
	- 63		
Covariance Type:	opg		

	coef	std err	z	P>|z|	[0.025	0.975]
const	81.5413	5.127	15.903	0.000	71.492	91.591
ar.L1	0.2541	0.130	1.957	0.050	-0.000	0.509
ar.L2	0.2374	0.147	1.612	0.107	-0.051	0.526
sigma2	469.6107	86.852	5.407	0.000	299.385	639.837

Ljung-Box (L1) (Q):	0.01	Jarque-Bera (JB):	0.85
Prob(Q):	0.91	Prob(JB):	0.66
Heteroskedasticity (H):	0.68	Skew:	-0.27
Prob(H) (two-sided):	0.39	Kurtosis:	3.15

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-step).

（3）预测5年

```
In [8]: result.predict(start=len(l), end =len(l) + 4 )
Out[8]: array([92.44336249, 91.06738618, 86.55017587, 85.07554463, 83.62850471])
```

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马