3.8

解：$x_t = 10 + 0.5x_{t-1} + \varepsilon_t - 0.8\varepsilon_{t-2} + C\varepsilon_{t-3}$

$\Rightarrow x_t - 0.5x_{t-1} = 10 + \varepsilon_t - 0.8\varepsilon_{t-2} + C\varepsilon_{t-3}$

即：$(1-0.5B)x_t = 10 + (1-0.8B^2 + CB^3)\varepsilon_t$

$\Rightarrow x_t = 20 + \dfrac{1-0.8B^2+CB^3}{1-0.5B}\varepsilon_t$

当 $\dfrac{1-0.8B^2+CB^3}{1-0.5B}$ 能完整除出因子时，

$1-0.5B=0 \Rightarrow B=2$

故 $1-0.8\times4 + C\times8 = 0$

$\Rightarrow C = 0.275$

3.11

解：(4) $x_t = \varepsilon_t + 1.3\varepsilon_{t-1} - 0.4\varepsilon_{t-2}$

$\theta_1 = -1.3, \quad \theta_2 = 0.4$

$\theta(B) = 1 + 1.3B - 0.4B^2 = 0$ 时，

$B_1 = 3.89, \quad B_2 = -0.64$

$|B_2| < 1$

故不可逆

(5) $x_t = 0.7x_{t-1} + \varepsilon_t - 0.6\varepsilon_{t-1}$

$\theta(B) = 1 - 0.6B = 0$

$B = 1.67$

$|B| > 1$

故可逆

$\phi(B) = 1 - 0.7B = 0$

$B = 1.43$

$|B| > 1$

故平稳.

3.12

解：$x_t = 0.6x_{t-1} + \varepsilon_t - 0.3\varepsilon_{t-1} = \sum_{j=0}^{\infty} G_j\varepsilon_{t-j}$

由题得：

$$\begin{cases} G_0 = 1 \\ G_k = \sum_{j=1}^{k}\phi_j' G_{k-j} - \theta_k', & k\geq 1 \end{cases}$$

其中，$\phi_j' = \begin{cases} \phi_j, & 1\leq j\leq p \\ 0, & j>p \end{cases}$

$\theta_k' = \begin{cases} \theta_k, & 1\leq k\leq q \\ 0, & k>q \end{cases}$

故：$G_0 = 1$

$G_1 = \phi_1 G_0 - \theta_1 = 0.6 - 0.3 = 0.3$

$G_2 = \phi_1' G_1 + \phi_2' G_1 - \theta_2' = 0.3\times0.6 = 0.18$

$G_k = \phi_1 G_{k-1} = \phi_1^{k-1}G_1 = \frac{1}{2}\cdot0.6^k, \quad k\geq2$

$\Rightarrow x_t = \varepsilon_t + \frac{1}{2}\sum_{j=1}^{\infty}0.6^{j-1}\varepsilon_{t-j}$

3.14

证明：$x_t = 0.5x_{t-1} + \varepsilon_t - 0.25\varepsilon_{t-1}, \quad \varepsilon_t \sim WN(0,\sigma_\varepsilon^2)$

由题：

$G_0 = 1$

$G_1 = \phi_1 G_0 - \theta_1 = 0.5 - 0.25 = 0.25$

$G_k = \phi_1 G_{k-1} = \phi_1^{k-1}G_1 = 0.5^{k-1}\cdot0.25 = 0.5^{k+1}, \quad k\geq2$

$\rho_k = \dfrac{\gamma_k}{\gamma_0} = \dfrac{\sum_{j=0}^{\infty}G_jG_{j+k}}{\sum_{j=0}^{\infty}G_j^2}$

$\Rightarrow \rho_0 = 1$

$\rho_1 = \dfrac{\sum_{j=0}^{\infty}G_jG_{j+1}}{\sum_{j=0}^{\infty}G_j^2} = \dfrac{0.25+\sum_{j=1}^{\infty}0.5^{2j+3}}{1+\sum_{j=1}^{\infty}0.5^{2j+2}} = \dfrac{0.25+\frac{0.5^5}{1-0.5^2}}{1+\frac{0.5^4}{1-0.5^2}}$

$= 0.27$

$\rho_k = \dfrac{\sum_{j=0}^{\infty}G_jG_{j+k}}{\sum_{j=0}^{\infty}G_j^2} = \dfrac{0.5\sum_{j=0}^{\infty}G_jG_{j+k-1}}{\sum_{j=0}^{\infty}G_j^2} = 0.5\rho_{k-1}, \quad k\geq2$

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