Ch 4

4.5

解：由题可知：

$$\bar{x}_w = \frac{1}{\sum w_i} \sum w_i x_i \quad ①$$

$$\bar{y}_w = \frac{1}{\sum w_i} \sum w_i y_i \quad ②$$

$$Q_w(\beta_0, \beta_1) = \sum w_i (y_i - \beta_0 - \beta_1 x_i)^2$$

$$\Rightarrow \left.\frac{\partial Q}{\partial \beta_0}\right|_{\beta_0 = \hat{\beta}_{0w}} -2 \sum w_i (y_i - \hat{\beta}_{0w} - \hat{\beta}_1 x_i) = 0$$

且 $\left.\frac{\partial Q}{\partial \beta_1}\right|_{\beta_1 = \hat{\beta}_{1w}} -2 \sum w_i x_i (y_i - \hat{\beta}_{0w} - \hat{\beta}_1 x_i) = 0$ 时.

$$\Rightarrow \begin{cases} \hat{\beta}_{0w} \sum w_i + \hat{\beta}_{1w} \sum w_i x_i = \sum w_i y_i \\ \hat{\beta}_{0w} \sum w_i x_i + \hat{\beta}_{1w} \sum w_i x_i^2 = \sum w_i x_i y_i \end{cases}$$

故，将①②代入，可解得：

$$\hat{\beta}_{0w} = \bar{y}_w - \hat{\beta}_{1w} \bar{x}_w$$

$$\hat{\beta}_{1w} = \frac{\sum w_i (x_i - \bar{x}_w)(y_i - \bar{y}_w)}{\sum w_i (x_i - \bar{x}_w)^2}$$

4.9

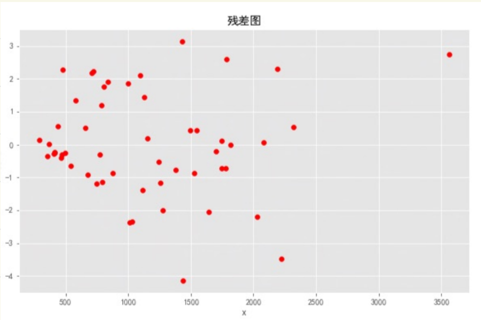
解：(1) 令 $y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$

$$\Rightarrow \hat{\beta}_1 = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2} = 0.0036$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x} = -0.7601$$

$$\Rightarrow \hat{y}_i = -0.7601 + 0.0036 x_i$$

残差图如下：



(2) 由(1)的残差图可以看出异方差，

于是考虑做 Breush-Pagan 和 White 检验

(不考虑使用戈德菲尔德-夸特检验因为观察到53的样本数太小)

结果如下：

```
name = ["Lagrange multiplier statistic", "p-value", "f-value", "f p-value"]
test = sms.het_breuschpagan(result.resid, result.model.exog)
lzip(name, test)
```

[32] ✓ 0.2s

```
[('Lagrange multiplier statistic', 6.334008062645265),
 ('p-value', 0.011844434081613621),
 ('f-value', 6.922266039658079),
 ('f p-value', 0.011230719809139865)]
```

```
sm.stats.diagnostic.het_white(result.resid, exog = result.model.exog)
```

[33] ✓ 0.1s

```
(6.355227644823807, 0.04168500436473889, 3.40618429672675, 0.04103719870974942)
```

发现两种检验的 p-Value 均小于 0.05，故可在 95% 的置信水平上拒绝原假设 $\alpha_1 = \alpha_2 = \cdots = \alpha_p = 0$，即存在异方差

(3) 不妨令 w_i 的权数为 -2, -1.5, -1, -0.5, 0, 0.5, 1, 1.5, 2

发现如下：

```
w0 = np.array(xx)

model = []
Log_likehood = {}
for i in range(-4,5):
    model.append(sm.WLS(y, x, weights=w0**(i*0.5)).fit())
    m = str(i*0.5)
    Log_likehood[m] = model[i+4].llf

Log_likehood
```

✓ 0.1s

```
{'-2.0': -95.83341494835537,
 '-1.5': -94.32170436040428,
 '-1.0': -94.33361361148387,
 '-0.5': -95.94305687070089,
 '0.0': -99.08042122802837,
 '0.5': -103.51157439626273,
 '1.0': -108.90346151039418,
 '1.5': -114.95608216430264,
 '2.0': -121.50001207422127}
```

故可以看出在 $w_i = (x_i)^{-1.5}$ 时，对数似然函数达到最大

$$\xRightarrow[m=-1.5]{WLS} \hat{\beta}_{1w} = 0.0035$$

$$\hat{\beta}_{0w} = -0.5510$$

$$\Rightarrow \hat{y} = -0.5510 + 0.0035 x_i$$

(4) $y' = \sqrt{y}$，对 y' 与 x 建立回归

$$\Rightarrow \hat{\beta}_1 = 0.0009$$

$$\hat{\beta}_0 = 0.6027$$

$$\hat{y}_i = 0.6027 + 0.0009 x_i$$

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