2.8

证明：(1) 由题：

$$\frac{\hat\beta_1\sqrt{L_{xx}}}{\hat\sigma} = \frac{\hat\beta_1\sqrt{L_{xx}}}{\sqrt{\frac{1}{n-2}RSS}} = \frac{\hat\beta_1\sqrt{L_{xx}}\cdot\sqrt{n-2}}{\sqrt{RSS}}$$

$$\frac{\sqrt{n-2}\,r}{\sqrt{1-r^2}} = \frac{\sqrt{n-2}\,r}{\sqrt{1-ESS/TSS}} = \frac{\sqrt{n-2}\,r}{\sqrt{RSS/L_{yy}}} = \frac{\sqrt{n-2}\sqrt{\frac{ESS}{L_{yy}}}}{\sqrt{RSS/L_{yy}}}$$

$$= \frac{\sqrt{n-2}}{\sqrt{RSS}}\cdot\hat\beta_1\sqrt{L_{xx}}$$

$$= \frac{\hat\beta_1\sqrt{L_{xx}}}{\hat\sigma}$$ 得证

(2) 由题得：

$$SSR = ESS = \sum(\hat y_i-\bar y_i)^2 = \hat\beta_1^2\sum(x_i-\bar x)^2 = \hat\beta_1^2 L_{xx}$$

$$SSE = RSS = \sum(y_i-\hat y_i)^2 = (n-2)\hat\sigma^2$$

$$\Rightarrow F = \frac{\hat\beta_1^2 L_{xx}}{(n-2)\hat\sigma^2/(n-2)} = \frac{\hat\beta_1 L_{xx}}{\hat\sigma^2} = t^2$$

2.10

证明：由2.9可知：

$$Var(e_i) = [1-\frac{1}{n}-\frac{(x_i-\bar x)^2}{L_{xx}}]\sigma^2$$

$$\Rightarrow Ee_i^2-(Ee_i)^2 = Ee_i^2 = E((y_i-\hat y_i)^2)$$

$$= [1-\frac{1}{n}-\frac{(x_i-\bar x)^2}{L_{xx}}]\sigma^2$$

$$E(\hat\sigma^2) = E(\frac{1}{n-2}\sum_{i=1}^n(y_i-\hat y_i)^2)$$

$$= \frac{1}{n-2}\sum_{i=1}^n E((y_i-\hat y_i)^2)$$

$$= \frac{1}{n-2}\sum_{i=1}^n(1-\frac{1}{n}-\frac{(x_i-\bar x)^2}{L_{xx}})\sigma^2$$

$$= \frac{n-1}{n-2}\sigma^2-\frac{1}{n-2}\sum_{i=1}^n\frac{(x_i-\bar x)^2}{\sum_{j=1}^n(x_j-\bar x)^2}\sigma^2$$

$$= \frac{n-2}{n-2}\sigma^2$$

$$= \sigma^2$$

故无偏

2.12

解：假设原模型 $y_i=\beta_0+\beta_1x_i+\varepsilon_i$ 的回归方程如

$\hat y_i=\hat\beta_0+\hat\beta_1x_i$，其中，

$$\hat\beta_0 = \bar y-\hat\beta_1\bar x$$

$$\hat\beta_1 = \frac{\sum(x_i-\bar x)(y_i-\bar y)}{\sum(x_i-\bar x)^2}$$

① 自变量观测值都乘以2

即令 $z_i = 2x_i \Rightarrow \bar z = 2\bar x$

$$\hat\beta_1^{(1)} = \frac{\sum(z_i-\bar z)(y_i-\bar y)}{\sum(z_i-\bar z)^2}$$

$$= \frac{\sum(2x_i-2\bar x)(y_i-\bar y)}{\sum(2x_i-2\bar x)^2}$$

$$= \frac{2}{4}\hat\beta_1$$

$$= \frac{1}{2}\hat\beta_1$$

$$\hat\beta_0^{(1)} = \bar y-\hat\beta_1^{(1)}\bar z = \bar y-\frac{1}{2}\hat\beta_1\cdot2\bar x = \hat\beta_0$$

② 自变量的观测值都加上2

即令 $m_i = x_i+2 \Rightarrow \bar m = \bar x+2$

$$\hat\beta_1^{(2)} = \frac{\sum(m_i-\bar m)(y_i-\bar y)}{\sum(m_i-\bar m)^2}$$

$$= \frac{\sum(x_i+2-\bar x-2)(y_i-\bar y)}{\sum(x_i+2-\bar x-2)^2}$$

$$= \hat\beta_1$$

$$\hat\beta_0^{(2)} = \bar y-\hat\beta_1^{(2)}\bar m = \bar y-\hat\beta_1(\bar x+2) = \hat\beta_0-2\hat\beta_1$$