

# Ultrasound Image Reconstruction with Deep Learning

## ED seminar - 2023

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30 - June - 2023



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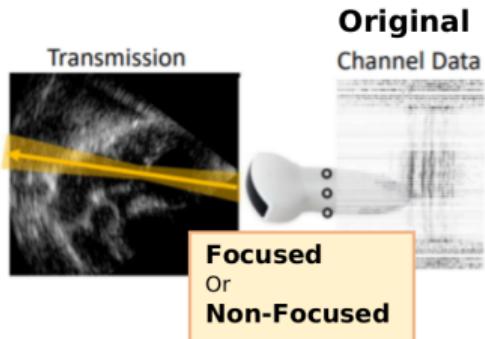
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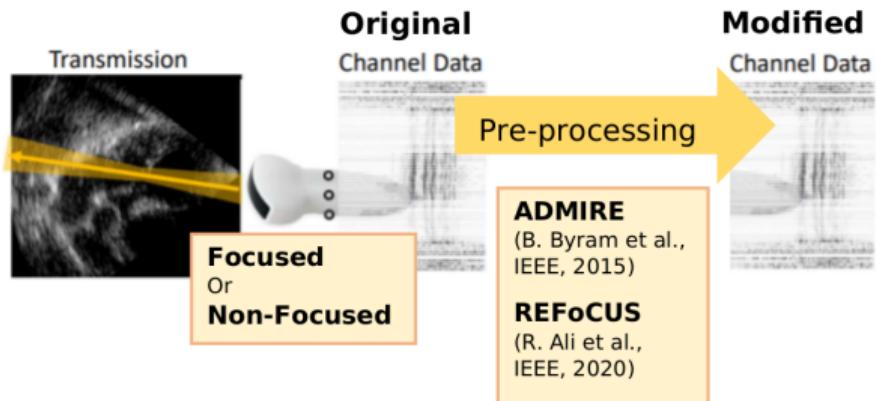
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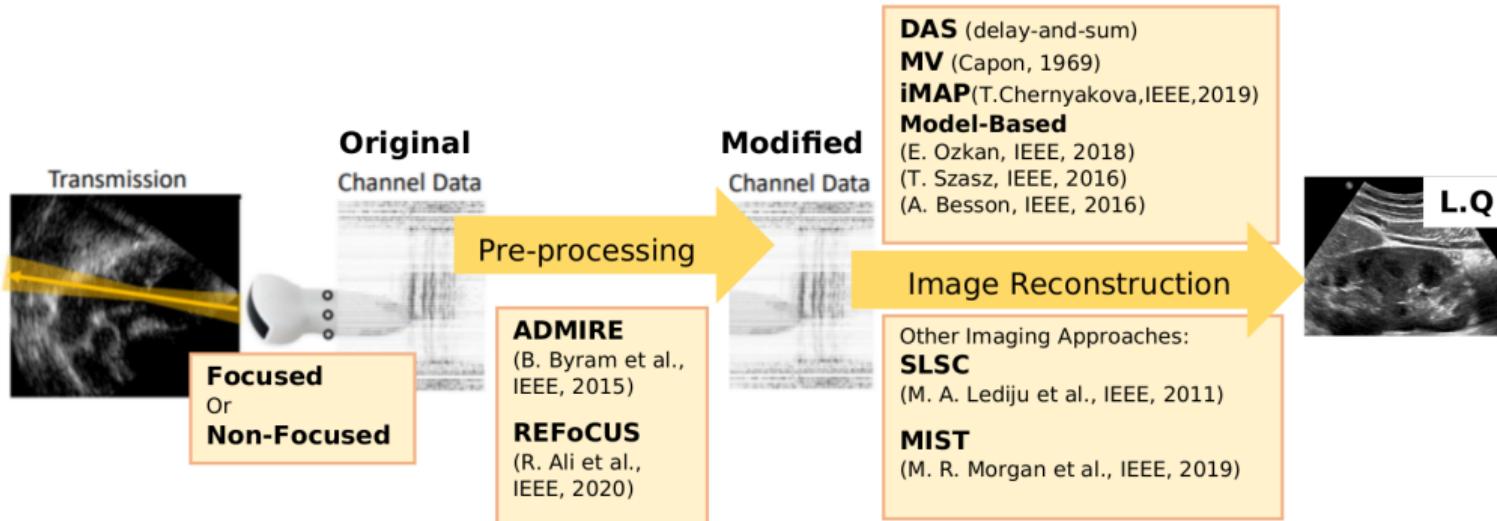
# Medical Ultrasound Image Reconstruction Workflow



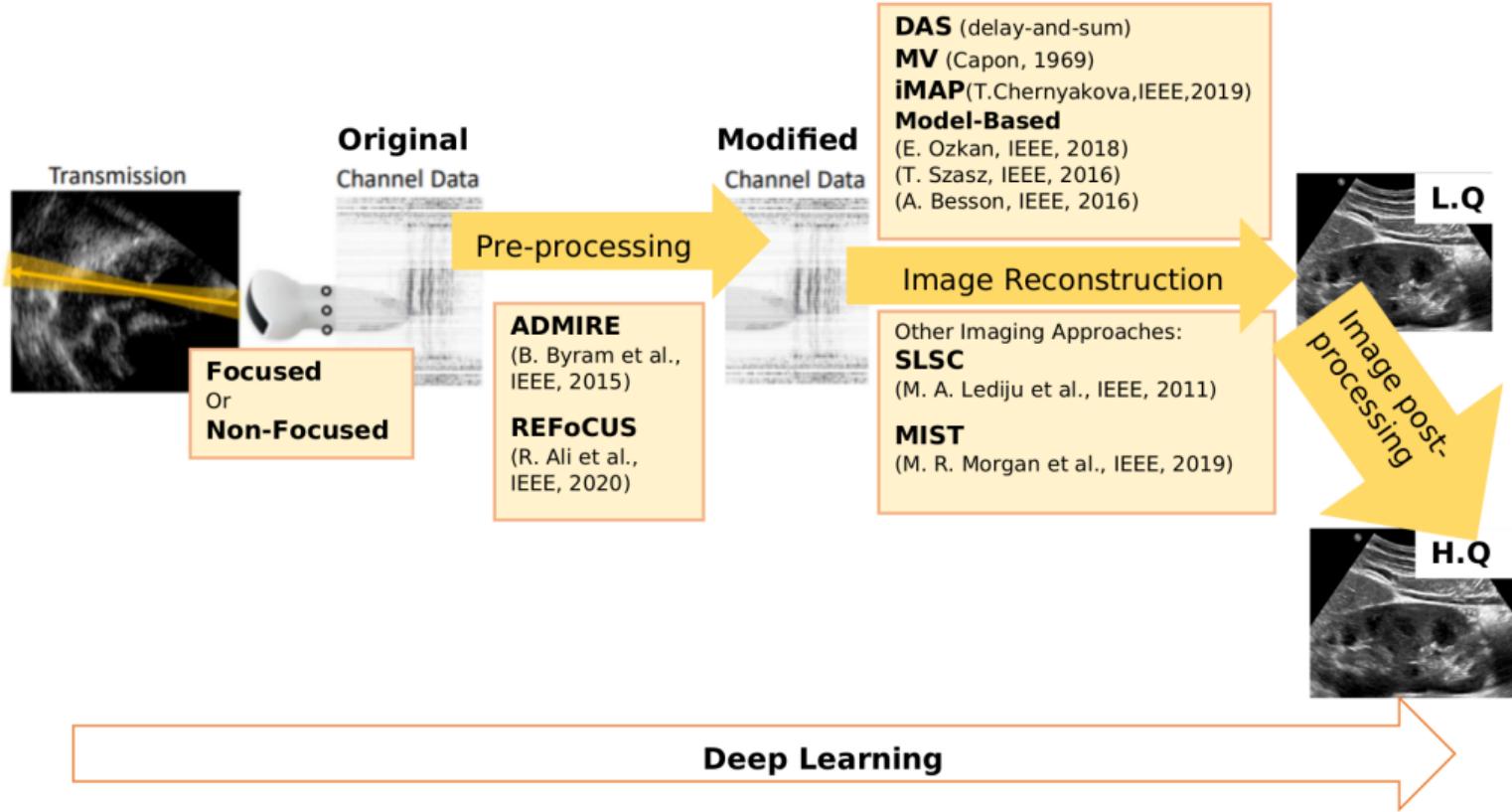
# Medical Ultrasound Image Reconstruction Workflow



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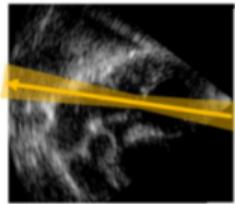
# Medical Ultrasound Image Reconstruction Workflow



# Inverse Problem of Ultrasound Image Reconstruction

$$y = Hx + n$$

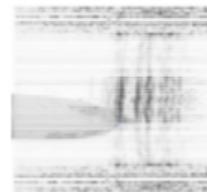
**x : reflectivity map**



**H : model matrix**

with the info. of  
**-time delay**  
**-pulse-echo response**

**y : channel data**



# Solving the Inverse Problem of Ultrasound Image Reconstruction

$$\text{Solving } \mathbf{y} = \mathbf{Hx} + \mathbf{n} \quad \text{by} \quad \hat{\mathbf{x}} = \arg \min_{\hat{\mathbf{x}}} \frac{1}{2} \|\mathbf{y} - \mathbf{Hx}\|_2^2 + \phi_{reg}$$

State-of-the-art :  $\phi_{reg}$  based on the prior assumptions [1-4] / data-adaptive [5]

## (1) Smoothness in frequency domain

$$\frac{1}{2} \|\mathbf{W}_f \mathbf{D}_1 \text{abs}(\mathbf{Fx})\|_2^2 + \frac{1}{2} \|\mathbf{W}_f \mathbf{D}_2 \text{abs}(\mathbf{Fx})\|_2^2, \text{ where } \mathbf{F} = \text{DCT}$$

(Ozkan et al. [2018])

## (2) Smoothness in spatial domain

$$\|\mathbf{D}_1 \text{Env}(\mathbf{x})\|_1 + \|\mathbf{D}_2 \text{Env}(\mathbf{x})\|_1$$

(Zhang et al. [2021])

$$\|\nabla \mathbf{x}\|_2^2$$

(Bodnariuc et al. [2016])

## (3) Sparsity in wavelet domain

$$\left\| \psi^\dagger \mathbf{x} \right\|_1, \text{ where } \psi = \frac{1}{\sqrt{8}} [\psi_1, \psi_2, \dots, \psi_8]$$

(Zhang et al. [2021]/Carrillo et al. [2015]/Carrillo et al. [2013])

## (4) Sparsity in spatial domain

$$\text{Tikhonov} - \|\mathbf{x}\|_1 \text{ and } \|\mathbf{x}\|_2^2$$

(Szasz et al. [2016])

$$\text{use envelope} - \|\text{Env}(\mathbf{x})\|_1$$

(Zhang et al. [2021])

## (5) Data-adaptive

$$\frac{1}{2} \mathbf{x}^T (\mathbf{x} - \text{Denoi}(\mathbf{x}))$$

[Regularization by Denoising (RED)]  
under the Plug-and-Play (PnP) framework

(Goudarzi et al. [2022])

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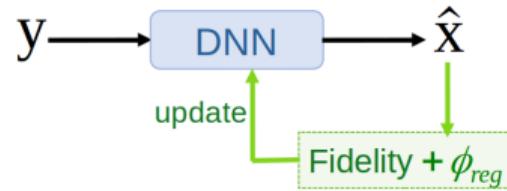
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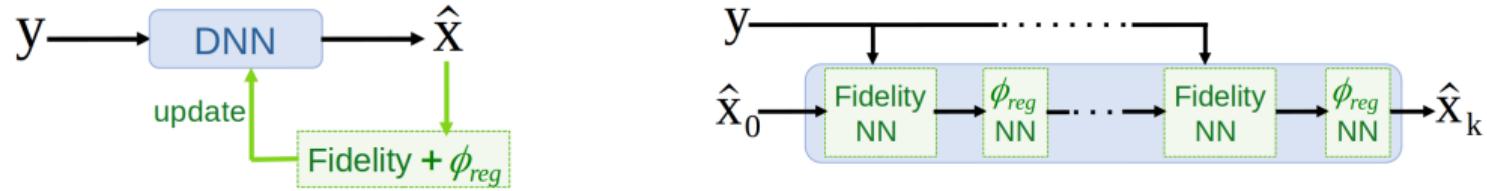
(Goudarzi et al. [2022])

Solve with Deep Neural Networks ?

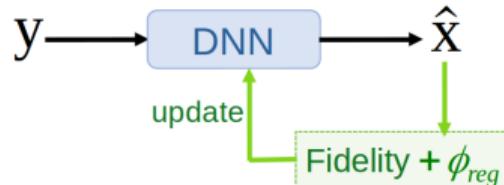
# Solving the Inverse Problem of Ultrasound Image Reconstruction



# Solving the Inverse Problem of Ultrasound Image Reconstruction



# Solving the Inverse Problem of Ultrasound Image Reconstruction



- **Self-Supervised** (Zhang et al. [2021])

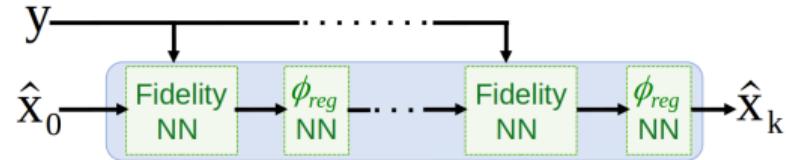
Fidelity  $[H\hat{x}, y]$      $\phi_{reg}$  based on the prior assumptions

inaccurate prior knowledge

- **Fully Supervised** (Perdios et al. [2022])

Fidelity  $[\hat{x}, x]$     Not leverage  $\phi_{reg}$

→ requires a lot of  $[L.Q, H.Q]$  data pairs



- **Supervised Unrolling** (Luijten et al. [2023])

Fidelity  $[\hat{x}, x]$     Learned  $\phi_{reg}$

→ requires a lot of  $[L.Q, H.Q]$  data pairs

Common drawback :

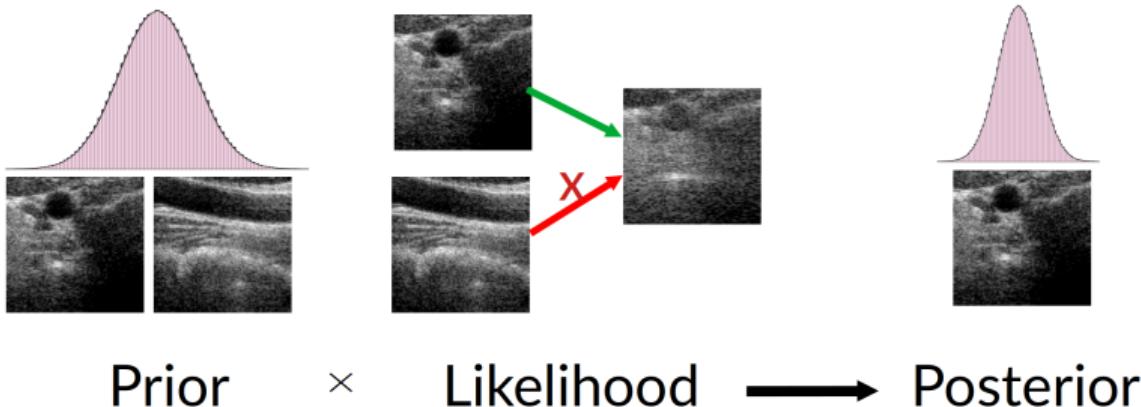
1 Trained DNN  $\longleftrightarrow$  1 Inverse Problem Model

[Med Image Anal] Ultrasound Image Reconstruction from Plane Wave Radio-Frequency Data by Self-Supervised Deep Neural Network (Zhang et al. [2021])

[IEEE TUFFC] CNN-based Image Reconstruction Method for Ultrafast Ultrasound Imaging (Perdios et al. [2022])

[IEEE ICASSP] Neural Maximum-a-Posteriori Beamforming for Ultrasound Imaging (Luijten et al. [2023])

# Solving the Inverse Problem of Ultrasound Image Reconstruction



- **Leverage the Generative Priors**
  - + One Trained Generative model  $\longleftrightarrow$  One Unlimited Inverse Problem Models
  - + assumed learned prior
  - +  $\{\text{LQ}, \text{HQ}\}$  HQ required for training

The solving methods can be adapted to other inverse problems

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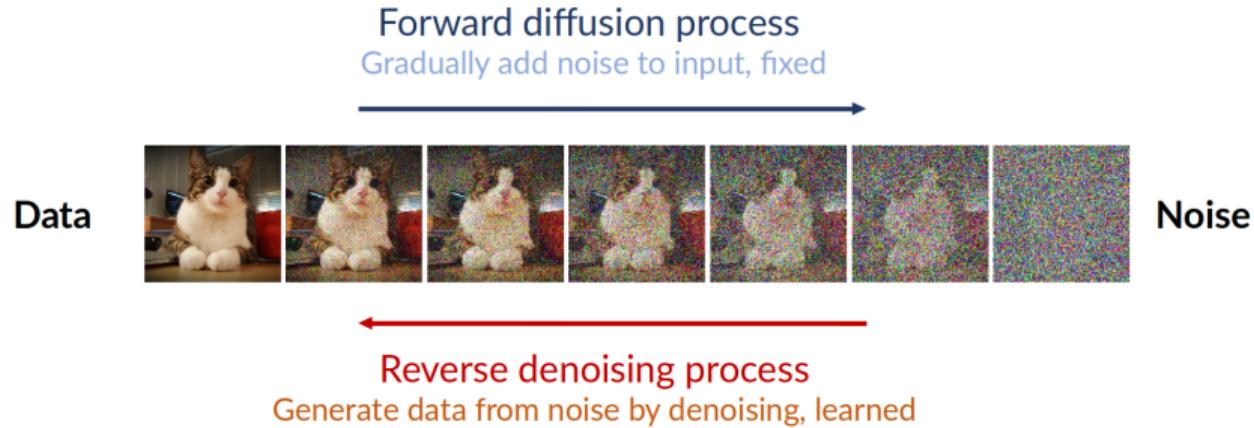
Forward Models of Ultrasound Image Reconstruction

Results

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# Diffusion Models

\* Generative Models : **Diffusion Models**



# Diffusion Models

Unconditional sampling :

Data



Noise

Reverse denoising process

Generate data from noise by denoising, learned

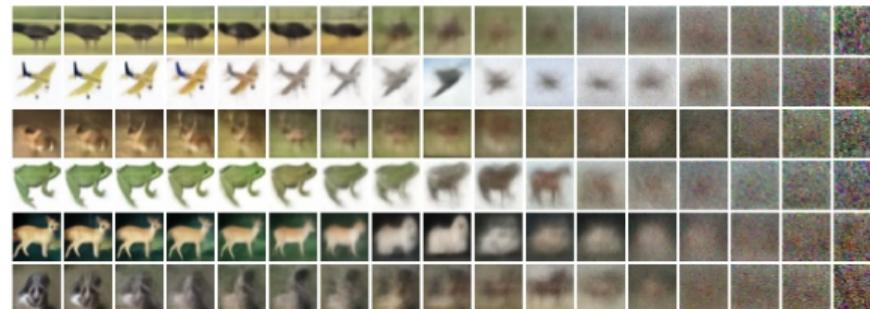


Figure – Unconditional CIFAR10 progressive generation ([Ho et al. \[2020\]](#)).

# Diffusion Models

Unconditional sampling :

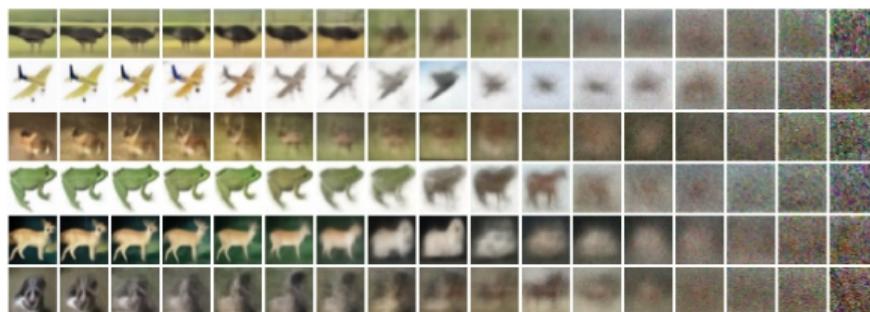
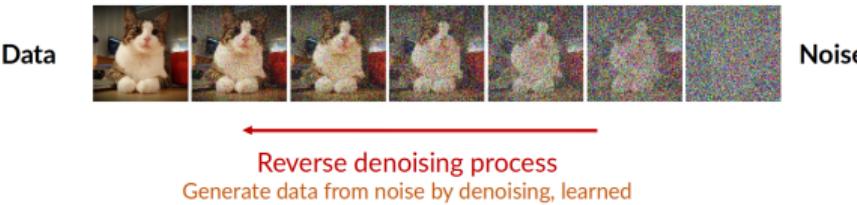


Figure – Unconditional CIFAR10 progressive generation ([Ho et al. \[2020\]](#)).

Conditional sampling :

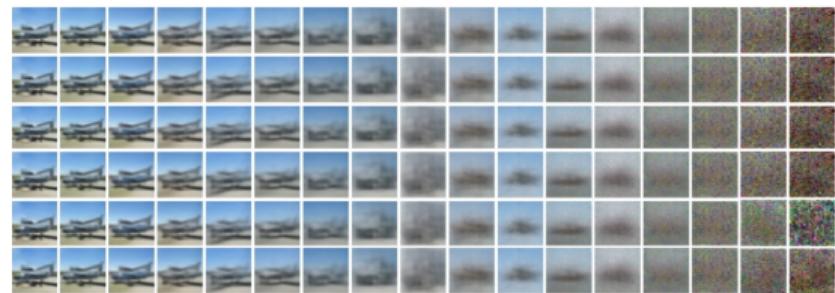
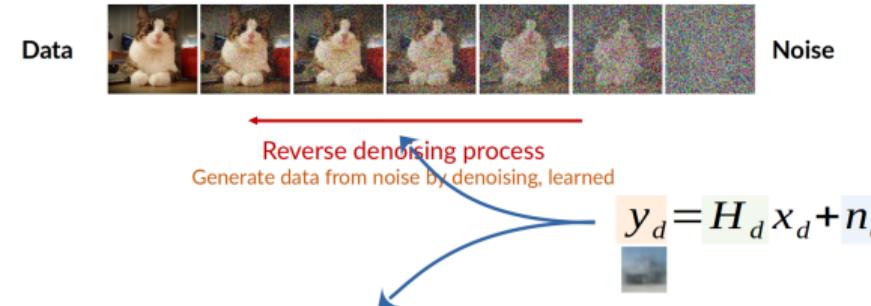


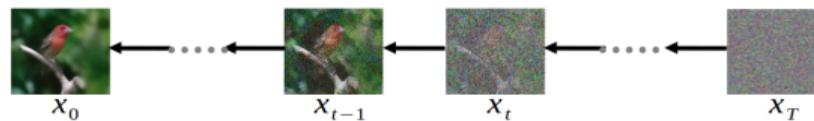
Figure – Generation process of a conditioned generator.

# Denoising Diffusion Restoration Models

Start with a simple case :

$$y_d = H_d x_d + n_d$$

[Diagonal]  
[Noisy observation]                                    [Noise, i.i.d Gaussian, known variance]



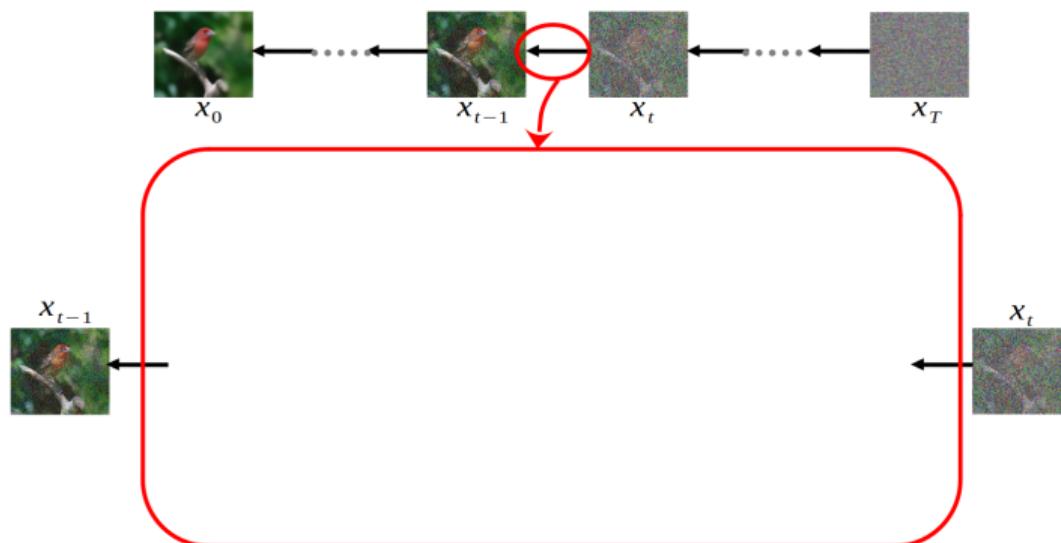
[NeurIPS] Denoising Diffusion Restoration Models ([Kawar et al. \[2022\]](#))

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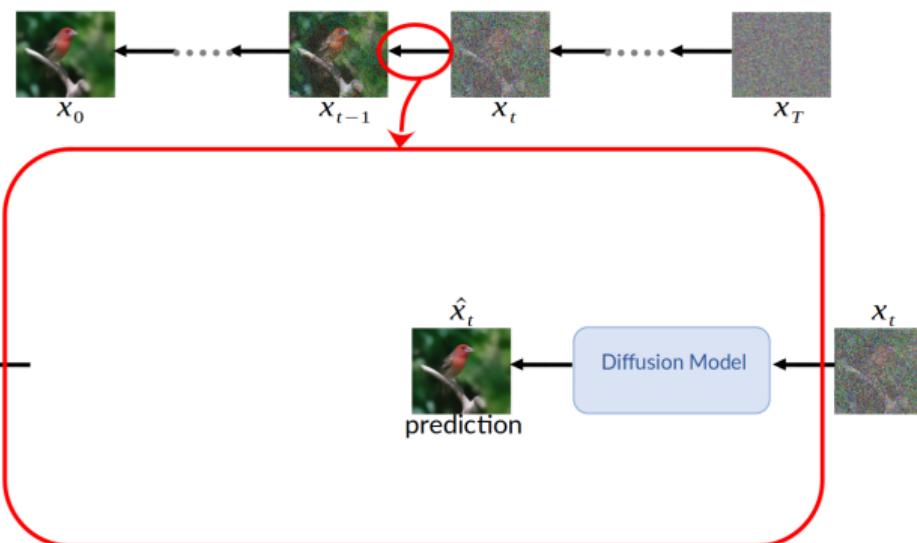
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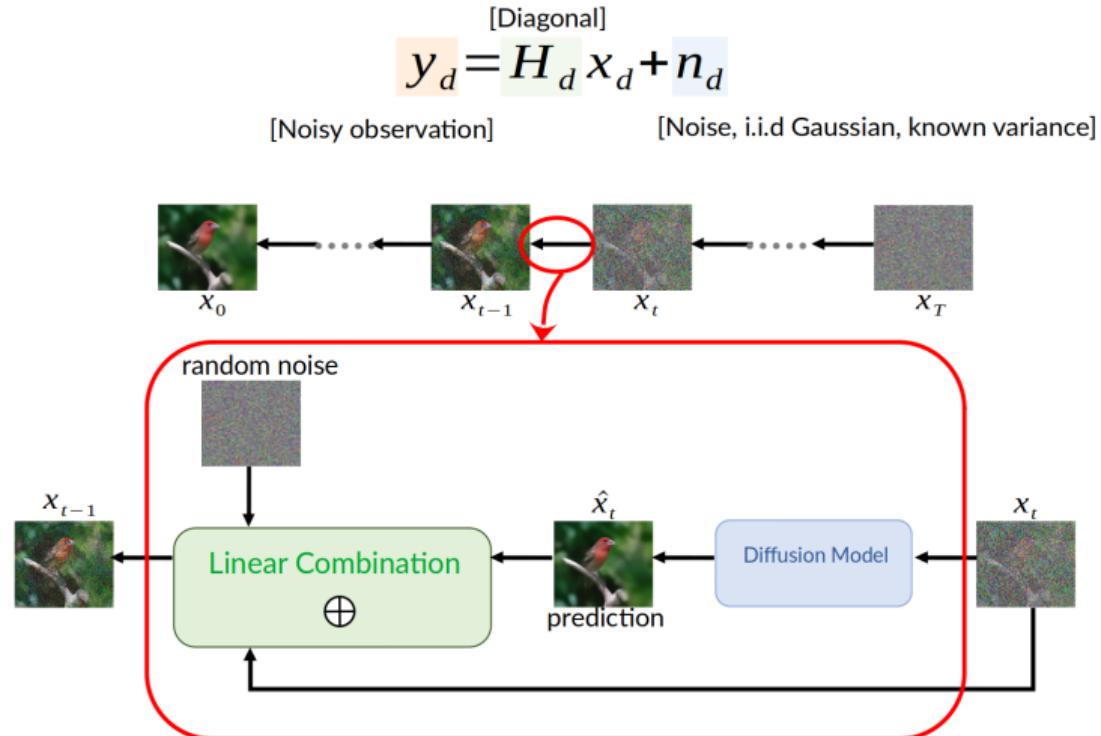
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# Denoising Diffusion Restoration Models

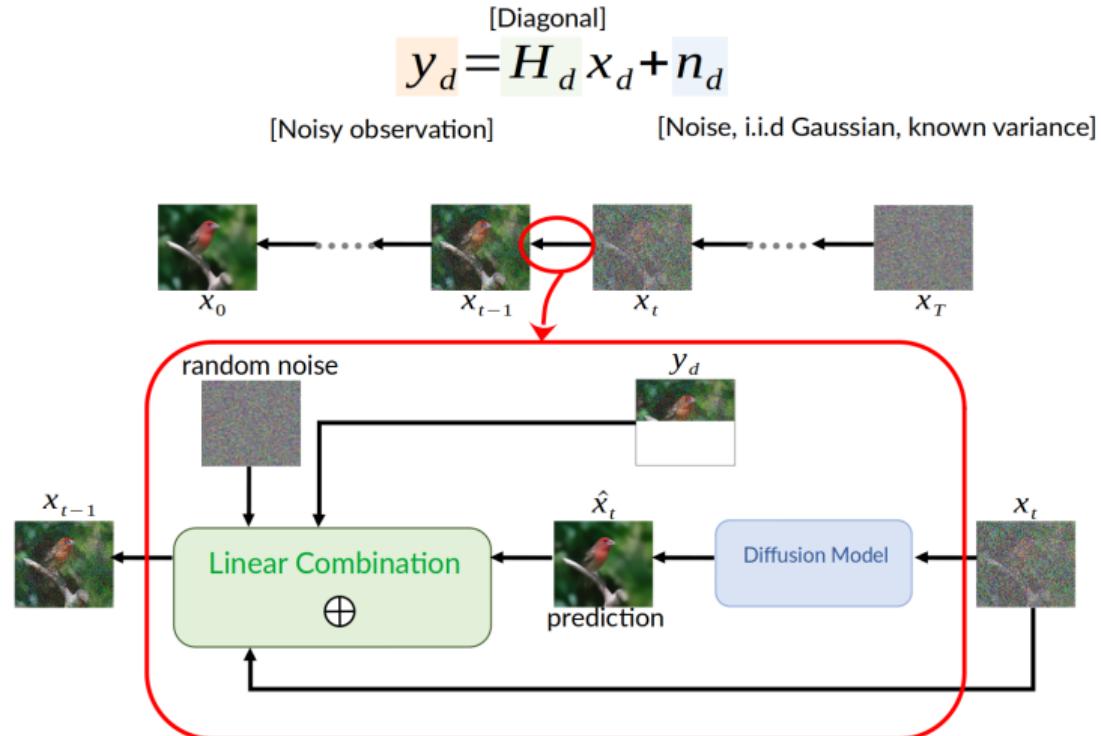
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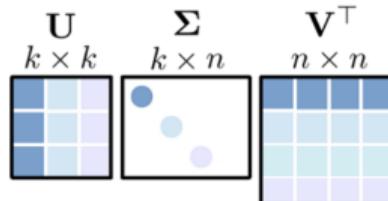


[NeurIPS] Denoising Diffusion Restoration Models ([Kawar et al. \[2022\]](#))

# Denoising Diffusion Restoration Models

Most general case : **any linear inverse problem**

$$H_d = U \Sigma V^T$$



H is “diagonal” in transformed space from SVD

$$\begin{aligned} \Sigma^\dagger U^T y_d &= V^T x_d + \Sigma^\dagger U^T n_d \\ \text{Curved arrow pointing right} \quad \overline{y_d} &= \overline{x_d} + \overline{n_d} \end{aligned}$$

**DDRM:** run “denoising and/or inpainting”, but in the space transformed by SVD

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From  $y = Hx + n$  to  $By = BHx + Bn$

$$(H \in \mathbb{R}^{KL \times N}) \quad (x \in \mathbb{R}^{N \times 1}) \quad (n \in \mathbb{R}^{KL \times 1}) \quad (y \in \mathbb{R}^{KL \times 1})$$
$$\begin{matrix} & \bullet \\ N_z & \end{matrix} \cdot \begin{matrix} x \\ N_x \end{matrix} + n = \begin{matrix} y \\ K \\ L \end{matrix}$$

$x$ : reflectivity map  
 $n$ : white noise  
 $y$ : channel data

$K$ : number of time samples  
 $L$ : number of channels  
 $N = N_x \times N_z$ : number of pixels

Figure – Forward model of ultrasound image reconstruction

From  $y = Hx + n$  to  $By = BHx + Bn$

$$(H \in \mathbb{R}^{KL \times N}) \quad (x \in \mathbb{R}^{N \times 1}) \quad (n \in \mathbb{R}^{KL \times 1}) \quad (y \in \mathbb{R}^{KL \times 1})$$

$\bullet$   $N_z$        $+n$       =

$N_x$

$L$

$K$

$x$ : reflectivity map  
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Problem : TOO MUCH data to control !

Solution :

COMPRESS the data by applying an operator  $B \approx H^t$

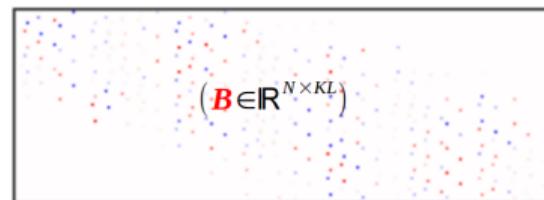


Figure – Matrix  $B$

Figure – Forward model of ultrasound image reconstruction

From  $y = Hx + n$  to  $By = BHx + Bn$

$$(H \in \mathbb{R}^{KL \times N}) \quad (x \in \mathbb{R}^{N \times 1}) \quad (n \in \mathbb{R}^{KL \times 1}) \quad (y \in \mathbb{R}^{KL \times 1})$$
$$x : \text{reflectivity map}$$
$$n : \text{white noise}$$
$$y : \text{channel data}$$
$$N = N_x \times N_z : \text{number of pixels}$$

Figure – Forward model of ultrasound image reconstruction

Then :

$$(BH \in \mathbb{R}^{N \times N}) \quad (x \in \mathbb{R}^{N \times 1}) \quad (Bn \in \mathbb{R}^{N \times 1}) \quad (By \in \mathbb{R}^{N \times 1})$$
$$BH : \text{forward model with compression}$$
$$Bn : \text{noise with compression}$$
$$By : \text{compressed channel data}$$

Figure – Data-compressed forward model

From  $y = Hx + n$  to  $By = BHx + Bn$

$$(H \in \mathbb{R}^{KL \times N}) \quad (x \in \mathbb{R}^{N \times 1}) \quad (n \in \mathbb{R}^{KL \times 1}) \quad (y \in \mathbb{R}^{KL \times 1})$$

$x$ : reflectivity map  
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Figure – Forward model of ultrasound image reconstruction

Then :

$$(BH \in \mathbb{R}^{N \times N}) \quad (x \in \mathbb{R}^{N \times 1}) \quad (Bn \in \mathbb{R}^{N \times 1}) \quad (By \in \mathbb{R}^{N \times 1})$$

Figure – Data-compressed forward model

Conflict :

colored noise  $Bn$  does not meet the assumption of DDRM

Solution : Apply a whitening operator  $C$

From  $\mathbf{By} = \mathbf{BHx} + \mathbf{Bn}$  to  $\mathbf{CBy} = \mathbf{CBHx} + \mathbf{CBn}$

$$(\mathbf{H} \in \mathbb{R}^{KL \times N}) \quad (\mathbf{x} \in \mathbb{R}^{N \times 1}) \quad (\mathbf{n} \in \mathbb{R}^{KL \times 1}) \quad (\mathbf{y} \in \mathbb{R}^{KL \times 1})$$

$$\begin{matrix} \mathbf{x} \\ \mathbf{n} \end{matrix} \cdot \begin{matrix} N_z \\ N_x \end{matrix} + \mathbf{n} = \begin{matrix} \mathbf{y} \\ K \\ L \end{matrix}$$

*x*: reflectivity map  
*n*: white noise  
*y*: channel data  
*K*: number of time samples  
*L*: number of channels  
*N* =  $N_x \times N_z$ : number of pixels

Figure – Forward model of ultrasound image reconstruction

$$(\mathbf{B} \mathbf{H} \in \mathbb{R}^{N \times N}) \quad (\mathbf{x} \in \mathbb{R}^{N \times 1}) \quad (\mathbf{B} \mathbf{n} \in \mathbb{R}^{N \times 1}) \quad (\mathbf{B} \mathbf{y} \in \mathbb{R}^{N \times 1})$$

$$\mathbf{B} \mathbf{H} \cdot \mathbf{x} + \mathbf{B} \mathbf{n} = \mathbf{B} \mathbf{y}$$

Figure – Data-compressed forward model

$$(\mathbf{C} \mathbf{B} \mathbf{H} \in \mathbb{R}^{M \times N}) \quad (\mathbf{x} \in \mathbb{R}^{N \times 1}) \quad (\mathbf{C} \mathbf{B} \mathbf{n} \in \mathbb{R}^{M \times 1}) \quad (\mathbf{C} \mathbf{B} \mathbf{y} \in \mathbb{R}^{M \times 1})$$

$$\mathbf{C} \mathbf{B} \mathbf{H} \cdot \mathbf{x} + \mathbf{C} \mathbf{B} \mathbf{n} = \mathbf{C} \mathbf{B} \mathbf{y}$$

*M* ( $\leq N$ )

Figure – Noise-whitened and data-compressed forward model

# Solve the Inverse Problem of Ultrasound Image Reconstruction with DDMR

Inverse Problem Models :

Use  $\begin{cases} \mathbf{By} = \mathbf{B}\mathbf{Hx} + \mathbf{Bn} & (\text{DRUS}) \\ \mathbf{CBy} = \mathbf{CBHx} + \mathbf{CBn} & (\text{WDRUS}) \end{cases}$  to recover  $\mathbf{x}$  with the given  $\mathbf{y}$ .

Test set : PICMUS dataset ([Liebgott et al. \[2016\]](#)) gives the observation  $\mathbf{y}$ .

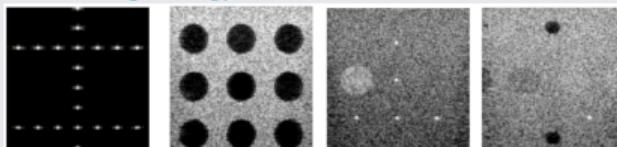


Figure – Examples of PICMUS reconstructed ultrasound images

Diffusion Model :

Fine-tune the public-available one which was trained on the ImageNet dataset (1,281,167 images) ([Russakovsky et al. \[2015\]](#))

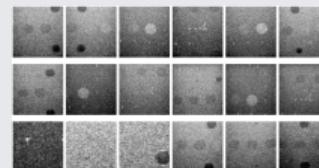
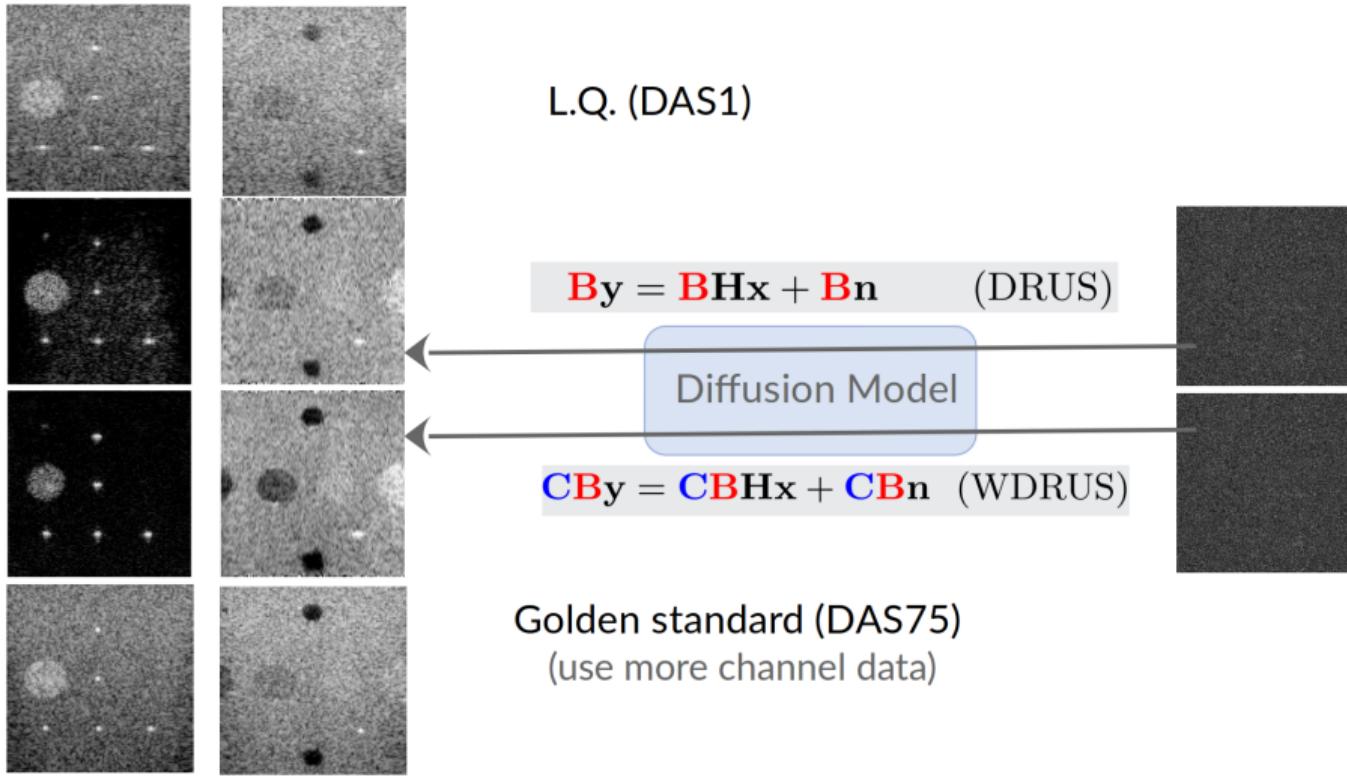
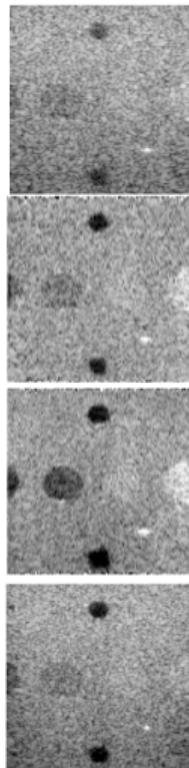
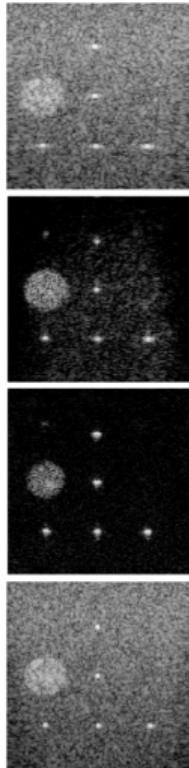


Figure – Examples of the fine-tune set (800 images)

## Results : compare against the reference



## Results : compare against the reference



L.Q. (DAS1)

DRUS

WDRUS

Golden standard  
(DAS75)

	Resolution (FWHM [mm]↓)		Contrast (CNR[dB] ↑)
	Axial	Lateral	
DAS1	0.51	1.21	8.15
DRUS	0.26	0.69	<b>12.9</b>
WDRUS	<b>0.25</b>	0.62	11.95
DAS75	0.49	0.59	12.05

## Conclusion and Future work

### Ultrasound Image Reconstruction with Denoising Diffusion Restoration Models

- + 1 pre-trained Diffusion Model → different Inverse Problem Models
- + ~~training from scratch~~ Fine-tuning with [LQ , HQ] image pairs

- Artifacts
- Requiring the SVD of the model matrix

DGM4MICCAI workshop at MICCAI 2023 (submit)

Future work :

- Enlarging the train/test dataset
- Removing the dependency on SVD

Thank you !

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