

# Ultrasound Image Reconstruction with Denoising Diffusion Restoration Models

## DGM4MICCAI - 2023

Yuxin Zhang

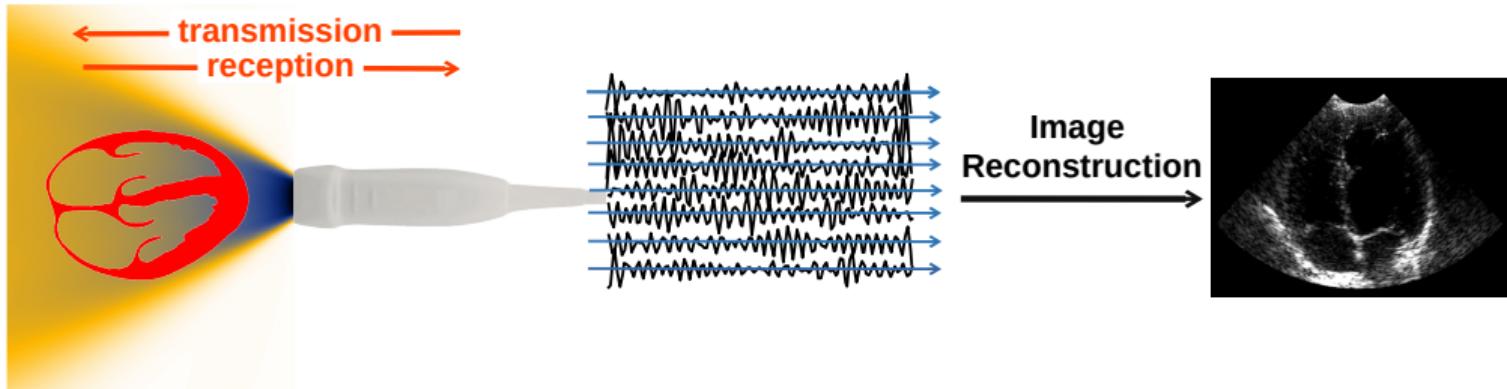
**Supervisors :** Clément Huneau, Jérôme Idier, Diana Mateus

Nantes Université, École Centrale Nantes, LS2N,  
CNRS, UMR 6004, F-44000 Nantes, France

8 - October - 2023

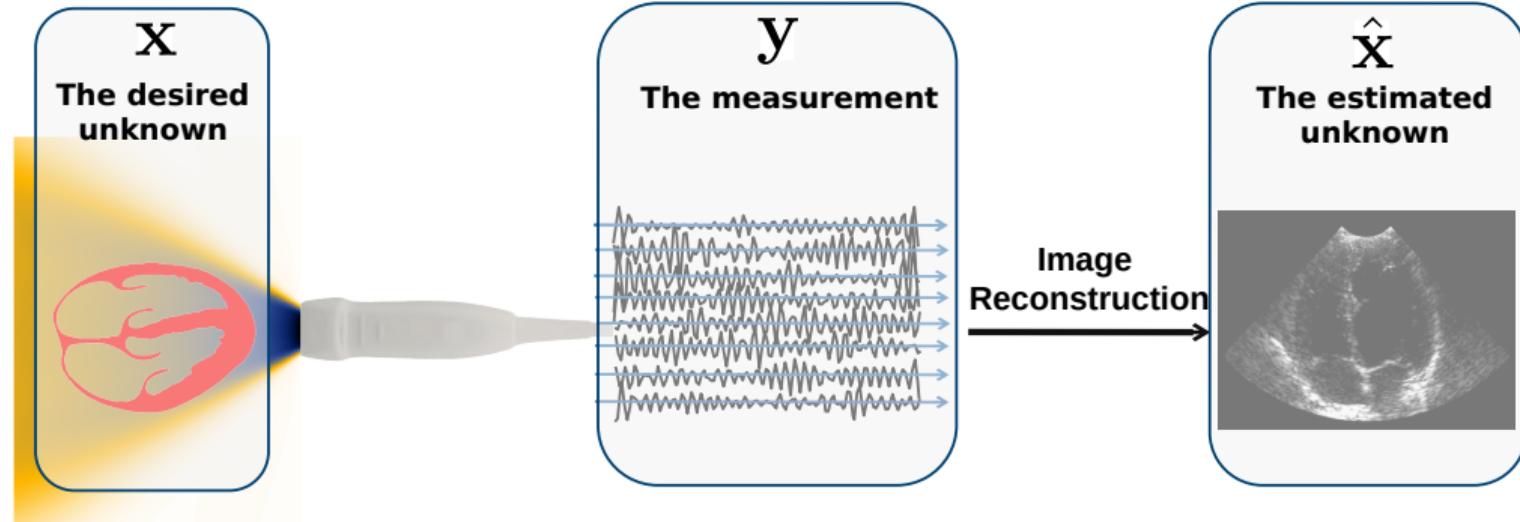


# Ultrasound Imaging



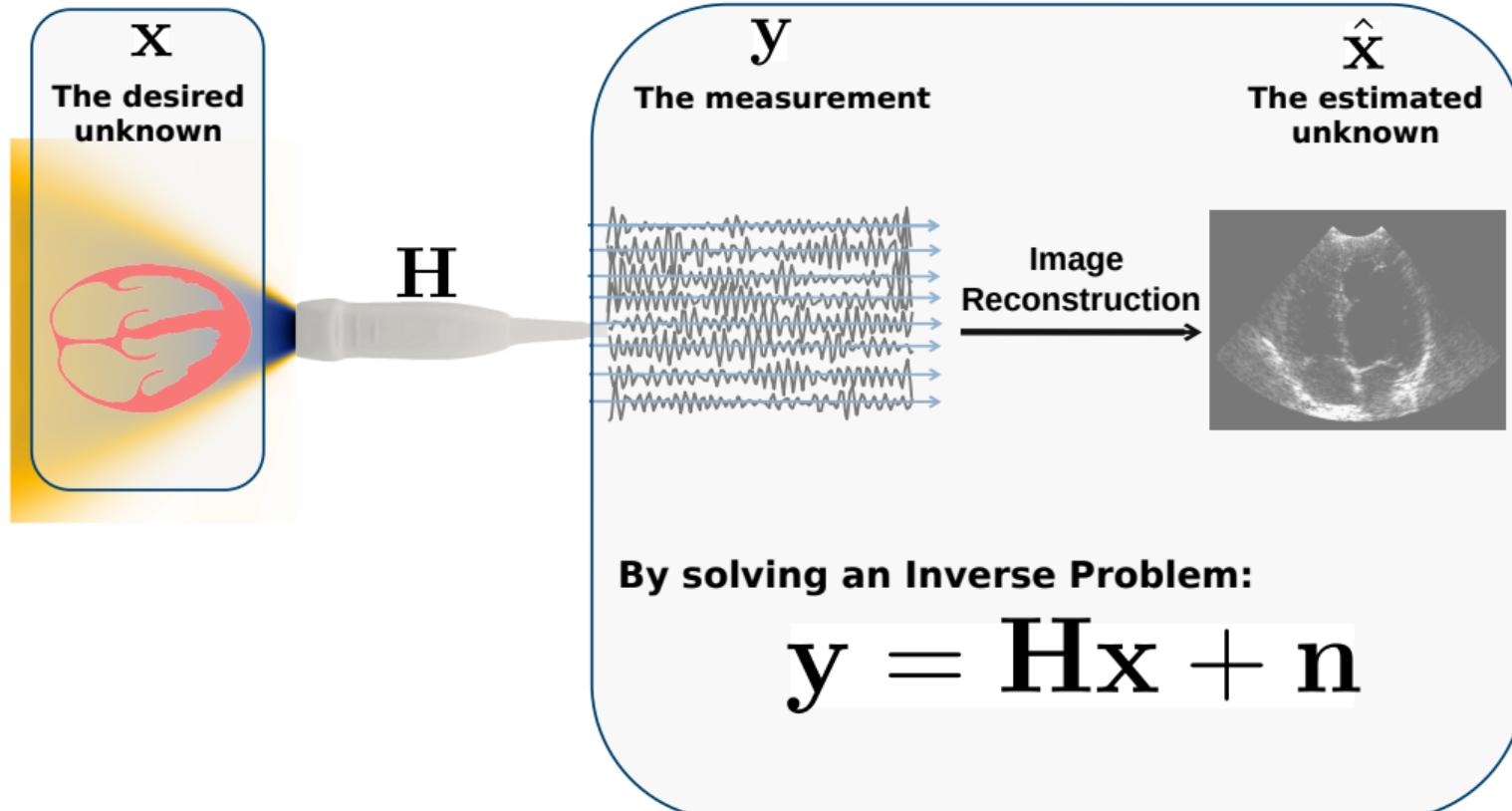
Source : [https://www.biomedcardio.com/files/Tracking\\_motions\\_in\\_the\\_body.pdf](https://www.biomedcardio.com/files/Tracking_motions_in_the_body.pdf)

# Ultrasound Imaging



Source : [https://www.biomedcardio.com/files/Tracking\\_motions\\_in\\_the\\_body.pdf](https://www.biomedcardio.com/files/Tracking_motions_in_the_body.pdf)

# Image Reconstruction → an Inverse Problem



Source : [https://www.biomecardio.com/files/Tracking\\_motions\\_in\\_the\\_body.pdf](https://www.biomecardio.com/files/Tracking_motions_in_the_body.pdf)



## Model-based

$$\hat{\mathbf{x}} = \arg \min_{\mathbf{x}} \frac{1}{2} \|\mathbf{y} - \mathbf{Hx}\|_2^2 + \phi_{\text{reg}}$$

Ozkan et al. IEEE Trans. Ultrason. Ferroelectr. Freq. Control. 2018

Goudarzi et al. IEEE Trans. Ultrason. Ferroelectr. Freq. Control. 2022

# Inverse Problem Solving



## Model-based

$$\hat{\mathbf{x}} = \arg \min_{\mathbf{x}} \frac{1}{2} \|\mathbf{y} - \mathbf{Hx}\|_2^2 + \phi_{\text{reg}}$$

Ozkan et al. IEEE Trans. Ultrason. Ferroelectr. Freq. Control. 2018  
Goudarzi et al. IEEE Trans. Ultrason. Ferroelectr. Freq. Control. 2022



## Learning-based



Perdios et al. IEEE Trans. Ultrason. Ferroelectr. Freq. Control. (accepted)

# Inverse Problem Solving



## Model-based

$$\hat{\mathbf{x}} = \arg \min_{\mathbf{x}} \frac{1}{2} \|\mathbf{y} - \mathbf{Hx}\|_2^2 + \phi_{\text{reg}}$$

Ozkan et al. IEEE Trans. Ultrason. Ferroelectr. Freq. Control. 2018  
Goudarzi et al. IEEE Trans. Ultrason. Ferroelectr. Freq. Control. 2022



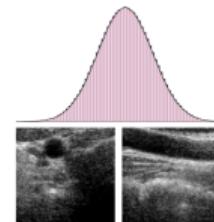
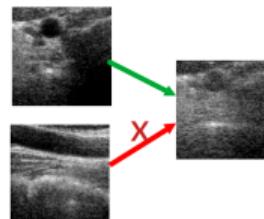
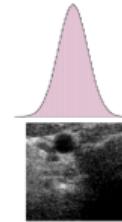
## Learning-based



Perdios et al. IEEE Trans. Ultrason. Ferroelectr. Freq. Control. (accepted)



Hybrid



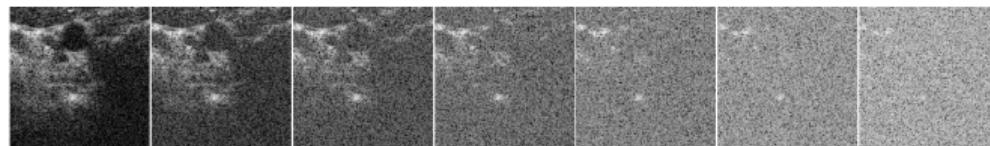
Posterior  $\leftarrow$  Likelihood (Inverse Problem)  $\times$  Prior (Diffusion Model)

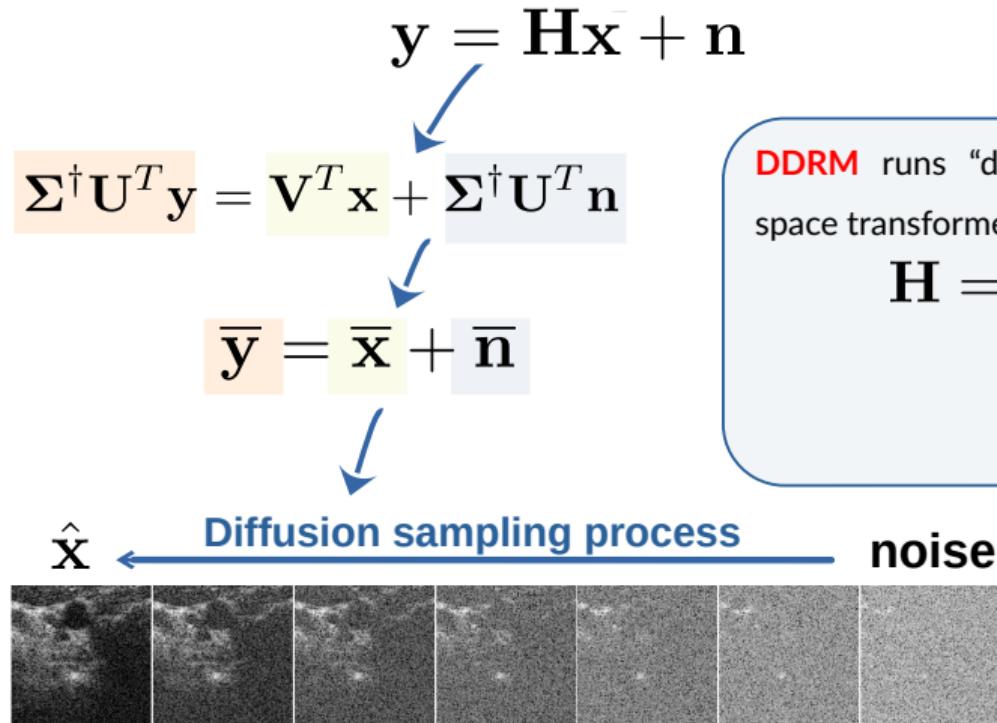
# Conditional Diffusion Sampling

$$\mathbf{y} = \mathbf{Hx} + \mathbf{n}$$

- Song Y et al. Solving inverse problems in medical imaging with score-based generative models. ICLR, 2022
- Song J et al. Pseudoinverse-guided diffusion models for inverse problems. ICLR, 2023
- Chung H et al. Score-based diffusion models for accelerated MRI. Med Image Anal. 2022
- Chung H et al. Diffusion posterior sampling for general noisy inverse problems. ICLR, 2023
- Kawar B et al. Denoising diffusion restoration models. NeurIPS. 2022 **(DDRM)**

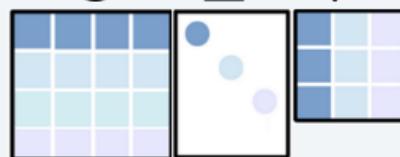
$\hat{\mathbf{x}}$  ← Diffusion sampling process noise





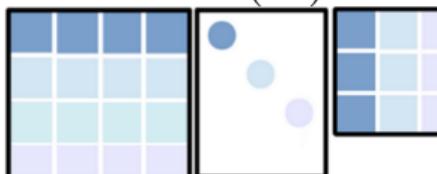
**DDRM** runs “denoising and/or inpainting” in the space transformed by **Singular Value Decomposition**

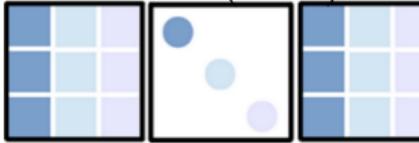
$$\mathbf{H} = \mathbf{U} \Sigma \mathbf{V}^T$$

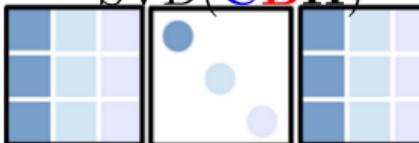


**DDRM** (Kawar et al. NeurIPS 2022)  
initially for **natural images**

## Data Compressing & Noise Whitening

$$\mathbf{y} = \text{SVD}(\mathbf{H}) \quad \mathbf{x} + \text{white } \mathbf{n}$$


$$\mathbf{B}\mathbf{y} = \text{SVD}(\mathbf{BH}) \quad \mathbf{x} + \text{colored } \mathbf{Bn}$$


$$\mathbf{CB}\mathbf{y} = \text{SVD}(\mathbf{CBH}) \quad \mathbf{x} + \text{white } \mathbf{CBn}$$




## Data Compressing & Noise Whitening

$$\mathbf{y} = \text{SVD}(\mathbf{H}) \mathbf{x} + \text{white } \mathbf{n}$$

$$\mathbf{B}\mathbf{y} = \text{SVD}(\mathbf{BH}) \mathbf{x} + \text{colored } \mathbf{Bn}$$

$$\mathbf{CB}\mathbf{y} = \text{SVD}(\mathbf{CBH}) \mathbf{x} + \text{white } \mathbf{CBn}$$



# Datasets

Natural Images

VS

Ultrasound Images (**SIGNED**)

Pre-trained on :



Figure – the ImageNet dataset (1,281,167 images) [\(?\)](#)

Fine-tuned on :

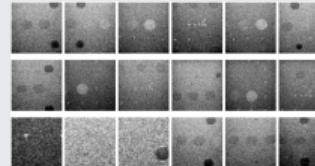


Figure – Examples of the self-acquired dataset (800 images)

Test set : PICMUS dataset [\(?\)](#) gives the observation  $y$ .

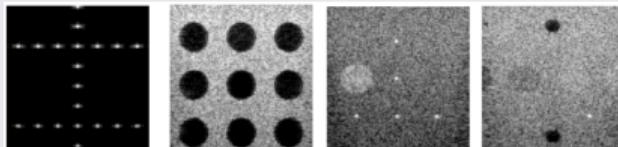
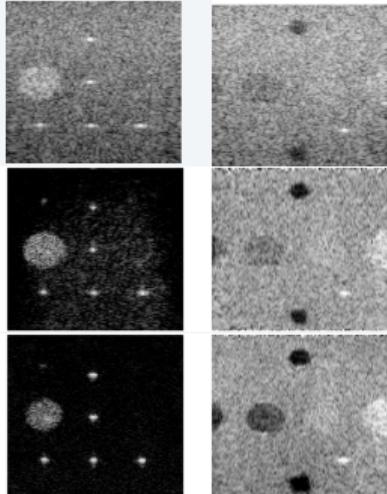


Figure – Examples of PICMUS reconstructed ultrasound images

# Results

## 1 transmission (Fast acquisition)

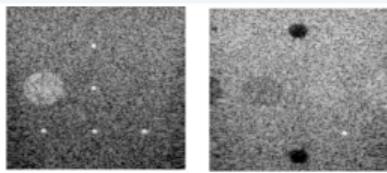


Baseline (DAS1)

**DRUS (ours)**  
( $\mathbf{B}_y = \mathbf{B}_{Hx} + \mathbf{B}_n$ )

**WDRUS (ours)**  
( $\mathbf{CB}_y = \mathbf{CB}_{Hx} + \mathbf{CB}_n$ )

## 75 transmissions (Slow acquisition)



Golden standard  
(DAS75)

	Resolution (FWHM [mm]↓)		Contrast (CNR[dB] ↑)
	Axial	Lateral	
Baseline	0.51	1.21	8.15
<b>DRUS</b>	0.26	0.69	<b>12.9</b>
<b>WDRUS</b>	<b>0.25</b>	0.62	11.95
Golden standard	0.49	0.59	12.05



## Diffusion Inverse Problem Solver

Model-based



Learning-based



## Ultrasound Inverse Problem Model

noise whitened

data compressed

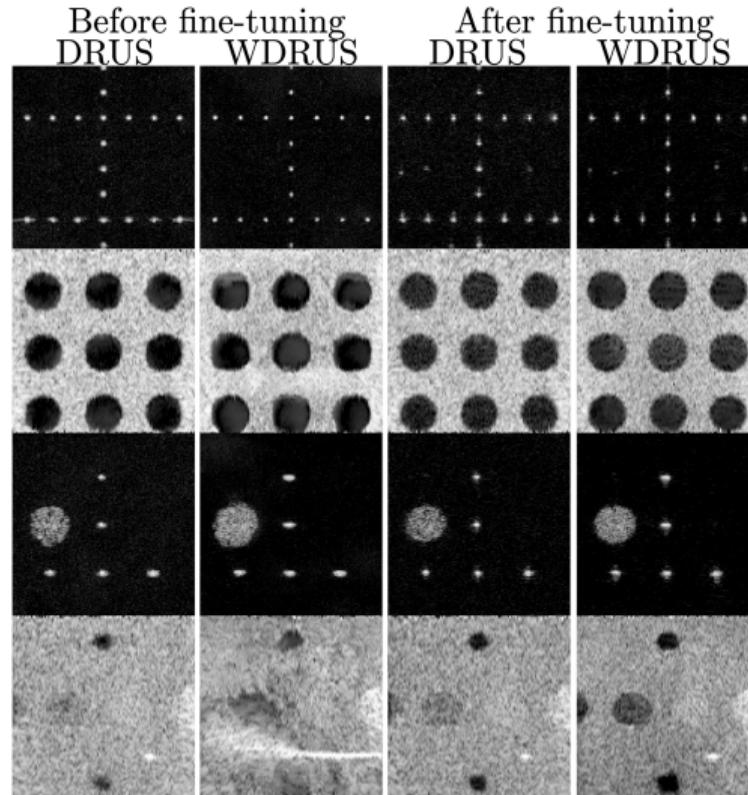
original



## Fine-Tuning from a Natural-Image Diffusion Model

Thank you !

## Fine-tuning



## B and C in a simple case

$$* \mathbf{B} = \mathbf{H}^t$$

$$* \mathbf{C} = \Lambda^{-\frac{1}{2}} \mathbf{V}^t, \text{ where } \text{eig}(\mathbf{B}\mathbf{B}^t) = \mathbf{V}\Lambda\mathbf{V}^t$$

$$\begin{aligned} \text{Cov}(\mathbf{CBn}) &= E[\mathbf{CBnn}^t \mathbf{B}^t \mathbf{C}^t] = \gamma^2 \mathbf{CBB}^t \mathbf{C}^t = \\ &\gamma^2 \mathbf{CV}\Lambda\mathbf{V}^t \mathbf{C}^t = \gamma^2 \mathbf{I}_M \end{aligned}$$

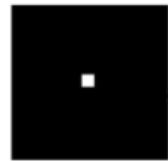
In summary

$$\mathbf{y} = \mathbf{Hx} + \mathbf{n}$$

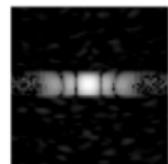
$$\underline{\underline{\mathbf{By}}} = \mathbf{BHx} + \mathbf{Bn} \text{ (DRUS)}$$

$$\underline{\underline{\mathbf{CBy}}} = \mathbf{CBHx} + \mathbf{CBn} \text{ (WDRUS)}$$

ground truth (x) measurement (y)



By



CBy

