Perspective HW4 Moving Beyond Linearity

```
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```

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gss_test

```
set.seed(214)

#Use Uchicago Remote Desktop, address bit tricky
#Manually improt the data
gss_train
```

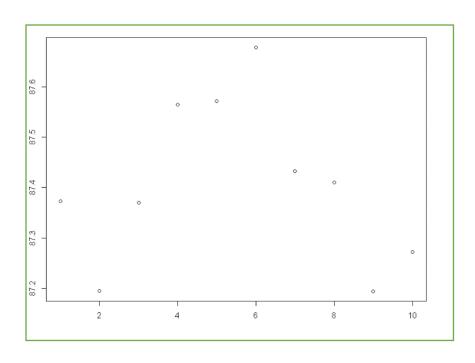
Egalitarianism and income

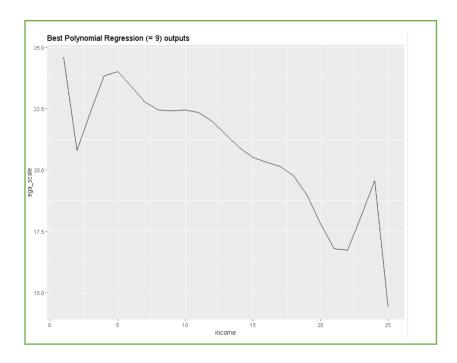
1. Polynomial Regression

```
#Polynomial Regression
k <- 10 #10 Cross Validation
fold <- sample(k, nrow(gss_train), replace = T)</pre>
mse <- numeric(k)</pre>
s \leftarrow seq(1,10, by = 1)
cv<- numeric(length(s))</pre>
for (j in seq_along(s))
  for (i in seq_len(k))
    set <- fold == i
    fold1 <- gss_train[set,]</pre>
    fold2 <- gss_train[!set,]</pre>
    f <- lm(egalit_scale ~ poly(income06, span[j]), data = fold2)</pre>
    pre <- predict(f, fold1)</pre>
    mse[i] <- mean((pre - fold1$egalit_scale)^2, na.rm = T)</pre>
  cv[j] <- mean(mse)</pre>
}
print(cv)
print(min(cv))
plot(s, cv)
> print(cv)
[1] 87.37284 87.19469 87.37046 87.56479 87.57150 87.67797 87.43290 87.40981 87.19437 87.27274
> print(min(cv))
```

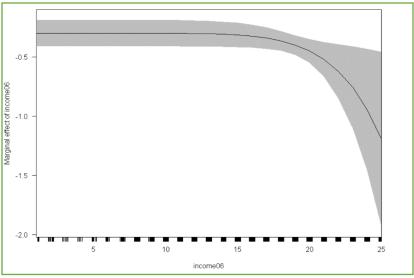
[Reference - Polynomial CV]

[1] 87.19437





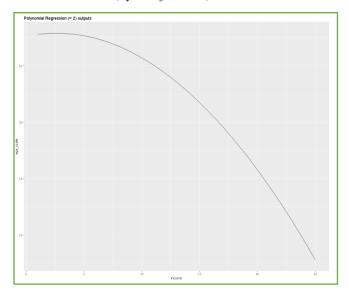
```
#AME plot
pl_use <- lm(egalit_scale ~ income06 + I(income06^9), data = gss_train)
margins(pl_use)
ame_graph <- cplot(pl_use, "income06", what = "effect")</pre>
```



[Reference - AME]

When deciding which polynomial number to use, my cross-validation found that when poly num equals to 2 or 9, the MSE becomes the lowest, and when poly = 9, the MSE is slightly lower than poly = 2. Normally speaking, when the poly number grows larger, the risk of overfitting will be higher. Therefore, in most cases, I will choose poly = 2 over poly = 9. But here, I will stick with poly = 9. And when we plot the model when poly = 9, we can see a relatively rough curve, going up sometimes and going down sometimes, this shows the risk of overfitting. If the poly = 2, it will yield a smooth downward curve (as shown in the graph below).

```
pl <- lm(egalit_scale ~ poly(income06, 2), data = gss_train, raw = T)
pl %>% prediction %>% ggplot(aes(x = income06)) +
   geom_line(aes(y = fitted)) +
   labs(title = "Polynomial Regression (= 2) outputs",
        x = "income", y = "ega_scale")
```



ME is a downward curve, decrease with the income become larger. The AME is -0.4796.

2. Step Function

```
#Step Function
step_error <- rep (0, 24)

for (i in 2:25) {

    gss_train$income_cut <- cut(gss_train$income06, i)

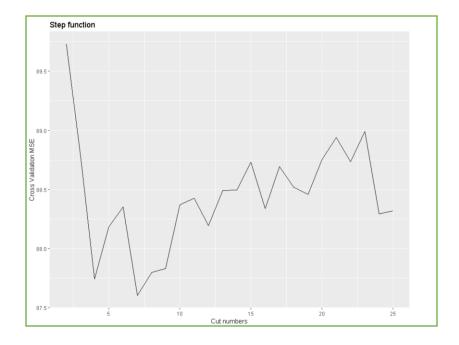
    step_fit = glm(egalit_scale ~ income_cut, data = gss_train)

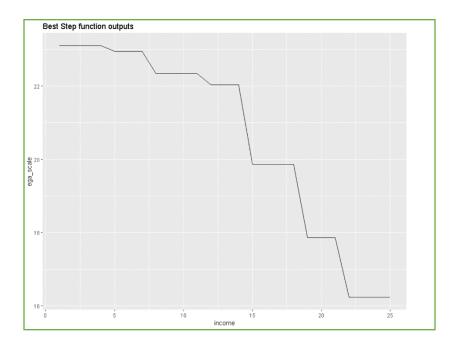
    step_error[i - 1] <- cv.glm(gss_train, step_fit, K = 10) $ delta[1]
}

step_error

[1] 89.72870 88.77890 87.74390 88.18452 88.35354 87.60359 87.79998 87.82956 88.37060 88.42418 88.19459 88.49154 88.49383 [14] 88.73069 88.33865 88.69247 88.51801 88.46033 88.75236 88.93881 88.73486 88.99052 88.29318 88.32065
```

[Reference – Step Function CV]

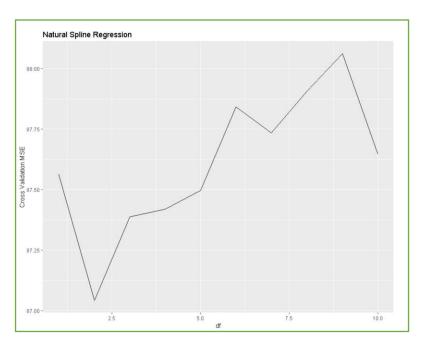


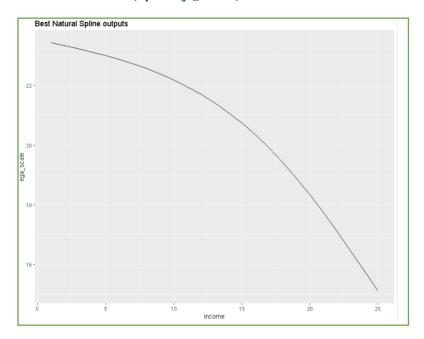


When deciding which polynomial number to use, my cross-validation found that when cut num equals to 4 or 7, the MSE becomes the lowest, and when cut = 7, the MSE is slightly lower than cut = 4. The situation here is kind of similar to the polynomial. Because as the cut number goes higher, the risk of overfitting also goes higher. Therefore, like what I have mentioned in the previous part, I may choose cut num = 4 over cut num = 7 when facing a real life problem. And as we can see from the graph, as the income goes up, the egalit scale goes down.

3. Natural Spline

```
#Natural spline regression
n_{error} \leftarrow rep(0,10)
for (i in 1:10) {
  n.fit = glm(egalit_scale ~
                   ns(income06, df = i), data = gss_train)
  n_error[i] <- cv.glm(gss_train, n.fit, K = 10) $ delta[1]</pre>
}
n_error
 n_error
 [1] 87.56360 87.04151 87.38626 87.41819 87.49702 87.84182 87.73431 87.90574 88.06180 87.64580
tibble(df = 1:10, mse = n_error) %>%
  ggplot(aes(df, mse))+
  geom_line() +
  labs(title = "Natural Spline Regression"
      x = "df", y = "Cross Validation MSE")
\#As we can see from the step_error output and the graph, when the df = 2
#The MSE is the lowest. Thus, df = 2 is our optimal choice
```





For the Natural Spline Regression, the cross validation found that when df equals to 2, the MSE is lowest, this is a relatively reasonable result, as the risk of overfitting will not be as high as my best polynomial regression and step functions. And as we can see from the output, it is a downward curve, as the incomes goes higher, the eaglit_scale goes down.

Egalitarianism and everything

gss_train

```
gss_test
[Reference – K-fold Cross Validation]
4A. Linear Regression
#Linear Regression
trControl = train.control)
print(lr)
> print(lr)
Linear Regression
1481 samples
 45 predictor
Pre-processing: centered (126), scaled (126)
Resampling: Cross-Validated (10 fold)
Summary of sample sizes: 1332, 1333, 1333, 1333, 1334, ...
Resampling results:
         Rsquared MAE
 8.001996 0.3199094 6.331663
```

4B. Elastic net regression

Tuning parameter 'intercept' was held constant at a value of TRUE

```
> print(En)
almnet
1481 samples
 45 predictor
Pre-processing: centered (126), scaled (126)
Resampling: Cross-Validated (10 fold)
Summary of sample sizes: 1332, 1333, 1333, 1333, 1332, 1333, ...
Resampling results across tuning parameters:
  alpha.
              lambda.
                            RMSE
                                       Rsquared
                                                   MAF
  0.1000000
              0.001421607
                            7.963711
                                                   6.269071
                                      0.3242854
  0.1000000
              0.002204224
                            7.963711
                                       0.3242854
                                                   6.269071
              0.003417684
  0.1000000
                            7 963711
                                       0.3242854
                                                   6 269071
  0.1000000
              0.005299173
                              963559
                                       0.3243071
                                                   6.269018
  0.1000000
              0.008216451
                              963015
                                       0.3243839
                                                   6.268832
                              .961940
.957485
  0.1000000
              0.012739737
                                       0.3245193
                                                   6.268293
              0.019753164
                                       0.3250398
                                                   6.265283
  0.1000000
  0.1000000
              0.030627593
                              951415
                                       0.3257290
                                                   6.261255
  0.1000000
              0.047488566
                              943037
                                       0.3266628
                                                   6.256380
              0.073631772
                              932404
                                       0.3278057
                                                   6.249692
  0.1000000
              0.114167225
                              917928
                                       0.3293580
  0.1000000
                                                   6.239786
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                                       0.3346589
                                                   6.208345
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                              841887
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                                                   6.189970
  0.1000000
              0.659851520
                              814628
                                       0.3417186
                                                   6.177966
  0.1000000
              1.023110202
                              791134
                                       0.3456947
                                                   6.178948
              1.586348526
                                       0.3496402
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                              814383
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              3.813739259
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                                       0.3483725
                                                   6.393509
              5.913262949
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                                       0.3384461
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                                                   6.269190
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                                       0.3448791
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                                       0.3494406
                                                   6.183292
                                       0.3515554
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                                                   6.231070
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                                                   6.331925
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              5.913262949
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                                       0.3242790
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                              963904
                                                   6.269063
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              0.002204224
                              963904
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                                                   6.269063
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                                       0.3243282
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                                       0.3244038
                                                   6.268771
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                              961557
                                       0.3245871
                                                   6.267958
  0.1947368
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                              956861
                                       0.3251499
                                                   6.264829
              0.019753164
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  0.1947368
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                                                   6.260560
  0.1947368
              0.030627593
                              941417
                                       0.3269425
                                                   6.255302
  0.1947368
              0.047488566
                              930201
                                       0.3281914
                                                   6.248020
  0.1947368
              0.073631772
                              915037
                                       0.3298710
                                                   6.236997
  0.1947368
              0.114167225
                              892752
                                       0.3324714
                                                   6.221198
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              0.177018085
                              863969
                                       0.3358855
                                                   6.201521
  0.1947368
              0.274469337
                              832782
                                       0.3397228
                                                   6.181717
  0.1947368
              0.425569042
                              806302
                                       0.3432216
                                                   6.170065
  0.1947368
              0.659851520
                               781173
                                       0.3474982
                                                   6.173222
  0.1947368
              1.023110202
                              770486
                                       0.3515530
                                                   6.195878
                              .808493
  0.1947368
              1.586348526
                                       0.3508527
                                                   6.271423
              2.459658442
                                                   6.403292
  0.1947368
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                                       0.3449486
  0.1947368
              3.813739259
                            8.124452
                                       0.3247850
                                                   6.637337
              5.913262949
  0.1947368
                            8.433606
                                       0.2961576
                                                   6.951740
```

0.194736	8 5.913262949	8.433606	0.2961576	6.951740
0.242105	3 0.001421607	7.963884	0.3242822	6.269055
0.242105	3 0.002204224	7.963846	0.3242877	6.269042
0.242105	3 0.003417684	7.963374	0.3243550	6.268880
0.242105	3 0.005299173	7.962758	0.3244416	6.268685
0.242105	3 0.008216451	7.960087	0.3247682	6.266917
0.242105		7.954533	0.3254375	6.263297
0.242105	3 0.019753164	7.946826	0.3263496	6.258486
0.242105		7.937246	0.3274381	6.252604
0.242105	3 0.047488566	7.924575	0.3288459	6.243920
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0.242105		7.906534	0.3309142	6.230743
0.242105	3 0.114167225	7.881153	0.3339223	6.212645
0.242105	3 0.177018085	7.849795	0.3377218	6.190992
0.242105	3 0.274469337	7.819718	0.3415131	6.174354
0.242105	3 0.425569042	7.794907	0.3450135	6.167378
0.242105	3 0.659851520	7.770635	0.3497916	6.174575
0.242105	3 1.023110202	7.777079	0.3520031	6.216956
0.242105	3 1.586348526	7.834415	0.3497159	6.309537
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		8.248143	0.3079777	6.764343
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0.289473	7 0.001421607	7.963977	0.3242843	6.269194
0.289473		7.963860	0.3243009	6.269154
0.289473		7.963334	0.3243758	6.268983
0.289473		7.962321	0.3245084	6.268468
0.289473	7 0.008216451	7.958348	0.3249856	6.265758
0.289473		7.952232	0.3257168	6.261718
0.289473	7 0.019753164	7.943571	0.3267435	6.256606
0.289473		7.933214	0.3279113	6.249929
0.289473	7 0.047488566	7.919144	0.3294869	6.239737
				5.233737
0.289473	7 0.073631772	7.898255	0.3319413	6.224668
0.289473	7 0.114167225	7.870198	0.3352986	6.204872
0.289473	7 0.177018085	7.838371	0.3392139	6.183276
0.289473		7.810725	0.3427599	6.169758
0.289473	7 0.425569042	7.784332	0.3468305	6.167622
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0.289473	7 5.913262949	8.703908	0.2590147	7.220674
0.336842		7.964140	0.3242641	6.269243
0.336842	1 0.002204224	7.963869	0.3243027	6.269150
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0.336842		7.961520	0.3246060	6.267926
0.336842	1 0.008216451	7.956657	0.3251961	6.264653
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0.336842		7.776518	0.3483006	6.168905
0.336842	1 0.659851520	7.767323	0.3519148	6.192886
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0.336842	1 1.586348526	7.922591	0.3400323	6.411794
0.336842		8.153480	0.3129608	6.662662
0.336842	1 3.813739259	8.440455	0.2840032	6.955954
0.336842	1 5.913262949	8.809903	0.2440736	7.324316
0.384210		7.964104	0.3242720	6.269233
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0.384210			0.3244014	6.268942
0.384210	5 0.005299173	7.960568	0.3247222	6.267210
0.384210			0.3253860	6.263641
0.384210	5 0.012739737	7.947522	0.3262994	6.258855
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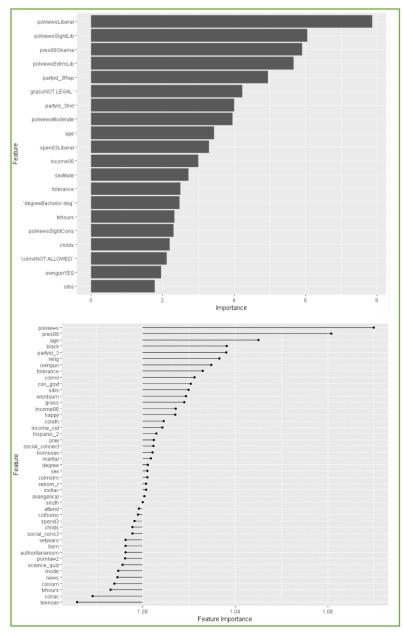
```
0.3842105
             0.073631772
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                                                6.324968
  0.4315789
             1.586348526
                           8.033254
                                     0.3248674
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             2.459658442
                           8.290758
                                     0.2945868
                                                6.806374
             3.813739259
                                     0.2579105
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                           8.615261
                                                 7.129818
  0.4315789
             5.913262949
                           8.958597
                                     0.2406458
                                                 7.470594
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                                                6.269269
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                           7.963581
                                     0.3243535
                                                6.269098
                             962539
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  0.4789474
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                             868564
                                     0.3356384
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             0.114167225
                           7 836318
                                     0.3396465
                                                6.180681
  0.4789474
             0.177018085
                                                 6.167479
                           7.809214
                                     0.3431129
  0.4789474
             0.274469337
                             782343
                                     0.3471982
                                                 6.165246
  0.4789474
             0.425569042
                           7.764511
                                     0.3514334
                                                 6.178520
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                           7 788771
                                     0.3512017
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             1.023110202
                           7.880683
                                     0.3429491
                                                6.360068
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             1.586348526
                           8.088944
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                                     0.2871131
                                                 6.864022
  0.4789474
             3.813739259
                           8.690031
                                     0.2462273
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                           9.043297
  0.4789474
             5.913262949
                                     0.2364955
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             0.001421607
                           7 963930
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             0.002204224
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                                                6.269009
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                             961996
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                                                 6.268202
  0.5263158
             0.005299173
                             957282
                                     0.3251302
                                                 6.265021
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                           7 950597
                                     0.3259394
                                                6.260674
                                     0.3270023
                                                 6.255417
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             0.012739737
                           7.941658
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             0.019753164
                             930583
                                     0.3282638
                                                 6.248075
  0.5263158
             0.030627593
                           7.915251
                                     0.3300210
                                                6.236735
                           7.892257
             0.047488566
                                                 6.219776
  0.5263158
                                     0.3327858
             0.073631772
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                             862025
                                     0.3364648
  0.5263158
             0.114167225
                           7.829978
                                     0.3404786
                                                6.176710
             0.177018085
                                     0.3437393
  0.5263158
                           7.804669
                                                 6.165435
  0.5263158
             0.274469337
                             777269
                                     0.3481194
                                                6.165850
  0.5263158
             0.425569042
                           7.764922
                                     0.3518515
                                                 6.185927
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             0.659851520
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             1.023110202
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                                     0.3385934
  0.5263158
             1.586348526
                           8.141189
                                     0.3090075
                                                 6.645934
                                                6.919285
7.257954
  0.5263158
             2.459658442
                           8.406377
                                     0.2792138
                           8.741730
                                     0.2420381
             3.813739259
  0.5263158
             5.913262949
                           9.128851
                                     0.2316765
  0.5263158
 [ reached getOption("max.print") -- omitted 200 rows ]
RMSE was used to select the optimal model using the smallest value.
The final values used for the model were alpha = 0.7631579 and lambda = 0.2744693.
```

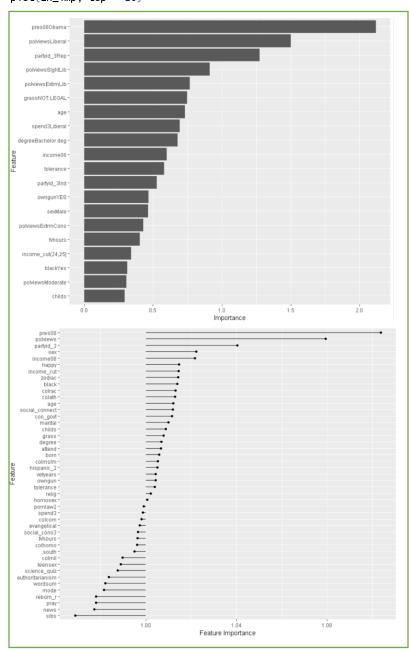
4C. Principal component regression

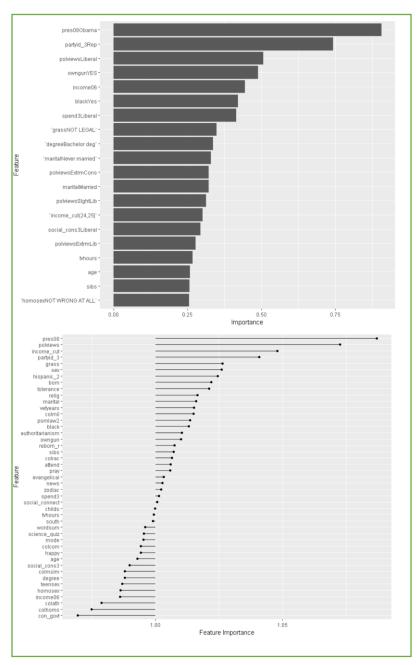
```
> print(PCR)
Principal Component Analysis
1481 samples
  45 predictor
Pre-processing: centered (126), scaled (126)
Resampling: Cross-Validated (10 fold)
Summary of sample sizes: 1333, 1333, 1333, 1333, 1333, ...
Resampling results across tuning parameters:
  ncomp RMSE
                   Rsquared
         9.538930 0.02917644 7.942473
  1
         8.506431 0.22241439 6.970061
   2
         8.509472 0.22177232 6.972125
   3
         8.470417 0.22858791 6.902642
   4
         8.318597 0.25518726 6.725908
         8.243409 0.26796198 6.656584
   6
   7
         8.197885 0.27666653 6.598401
         8
   a
         8.156923 0.28357205 6.552702
  10
        8.097610 0.29339570 6.511722
8.091704 0.29410077 6.501094
  11
  12
         8.072141 0.29708435 6.479742
  13
  14
         8.064798 0.29866450 6.472188
         8.069661 0.29780218 6.475315
8.071798 0.29732416 6.475514
  15
  16
  17
         8.066098 0.29844331 6.472919
         8.062285 0.29893324 6.465727
8.058146 0.29955950 6.457077
 18
  19
         8.037543 0.30313151 6.439954
  20
RMSE was used to select the optimal model using the smallest value.
The final value used for the model was ncomp = 20.
```

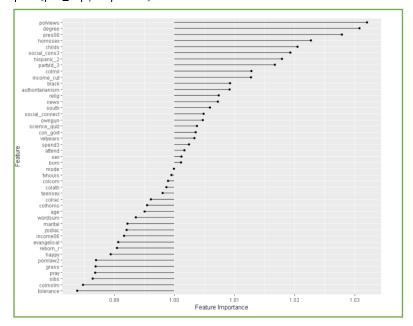
4D. Partial least squares regression

5. feature importance









From the four graphs above, we can see very clearly that for different models, the feature importance varies. Some features are more important in one model, but not so important in another. But still, we can find that some features such as pres08 and polviews rank high is almost every model. I am not an expert in this, but if eglit_scale is indeed highly correlated with politics, then this makes sense. Also, I decided the importance of the features based on their MAE. If the importance is decided based on other standard for example MSE, the result may also be very different. Therefore, when facing a real problem, which model to choose and which loss to take should be important.

Note:

Green Square: Output of R codes.

Reference:

Polynomial CV: https://stackoverflow.com/questions/43686539/using-cross-validation-to-select-the-optimal-polynomial-degree-in-r

AME: https://cran.r-project.org/web/packages/margins/vignettes/Introduction.html

Step Function CV: https://stackoverflow.com/questions/42190337/cross-validating-step-functions-in-r

K-fold Cross Validation: http://www.sthda.com/english/articles/38-regression-model-validation/157-cross-

validation-essentials-in-r/

Variance Importance: https://www.r-bloggers.com/variable-importance-plot-and-variable-selection/
https://www.rbocumentation.org/packages/iml/versions/0.9.0/topics/FeatureImp