Problem Set 2. Econ 8107. Spring 2026

February 3, 2026

This problem set is due on Tuesday, Feb 17 by class time.
Please upload your electronic here:
https://www.dropbox.com/request/3K5csPBzoSo0UYlpUBr2

Everyone should write their own individual answers (including your own version of the code used, with your own comments) but you are encouraged to work in groups.

Note that there is an additional zip file containing the code for the last problem.

1. Continuing with the CARA Example

Consider the partial equilibrium CARA example with incomplete markets that we saw in class.

(a) Use your knowledge of the consumption function and the evolution of the cash in hands to derive an expression for the change in consumption. That is, solve for

$$c(x', s') - c(x, s)$$

where $x' = Ra(x, s) + y(s')$.

(b) Show that consumption is a random walk with drift. This is a slightly different version of the famous Hall (1978) result. What should be the relationship of changes in consumption with respect to predictable changes in future income?

(c) Show that in General Equilibrium, individual consumption is a random walk without any drift.

Consider now the same CARA environment, but where the individual endowment's process is as follows:

$$y(s') = w(s') + \eta(s')$$

where η is iid over time with $\eta(s') \sim N(0, \sigma_\eta^2)$ and the observed random variable $w(s)$ evolves according to an AR(1) process:

$$w(s') = \phi w(s) + (1 - \phi)\overline{w} + \epsilon(s')$$

where ϵ is iid over time with $\epsilon(s') \sim N(0, \sigma_\epsilon^2)$. That is, the endowment process is the sum of a (stochastic) permanent component and a (stochastic) transitory component.

(d) Similarly to what we did in class, guess that the log value function for the agent is linear in cash in hands and in $w(s)$. Solve for the consumption function and the evolution of the cash in hands.

(e) Derive a formula for the change in consumption (as in part a). Discuss how consumption responds to unexpected transitory income shocks as compared to unexpected persistent income shocks.

(f) Solve for the general equilibrium condition that the interest rate must satisfy in this case. Explain the difference with the equation shown in class.

2. Aiyagari and changes in the wage

Using the Aiyagari model presented in class, assume that utility is power:

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}$$

And suppose that the borrowing limit is $\phi = 0$. As usual, R is given. Suppose as well that the endowment is $y(s) = \omega \times l(s)$. Where ω is the wage and $l(s)$ is an agent's specific random variable, representing a random labor supply ($s \in S$).

(a) For a given R, show that if ω increases to $\lambda\omega$, policy functions that solve the consumer problem, satisfy

$$a(\lambda z|\lambda\omega, R) = \lambda a(z|\omega, R)$$
$$c(\lambda z|\lambda\omega, R) = \lambda c(z|\omega, R)$$

where z is cash in hands.

Hint: Conjecture and show that the value function is homogeneous of degree $(1-\gamma)$. That is $V(\lambda z|\lambda\omega, R) = \lambda^{1-\gamma}V(z|\omega, R)$. Alternatively, you can try to argue this from the sequence problem.

(b) Interpret the following condition for a stationary distribution of cash in hands given the wage and the interest rate, $F(z|\omega, R)$:

$$F(z|\omega, R) = \sum_{s \in S} \pi(s) \int_{\tilde{z}} \mathbf{1}(Ra(\tilde{z}|\omega, R) + y(s) \leq z) dF(\tilde{z}|\omega, R)$$

where $\mathbf{1}$ is an indicator function that returns one if the condition is satisfied and zero otherwise.

(c) What should happen to aggregate savings in the stationary distribution of the partial equilibrium problem (that is for fixed w and R), if the wage increases by λ (while maintaining R constant)?

Hint: Argue using (i) and (ii) that the new long run distribution satisfies: $F(\lambda z|\lambda\omega, R) = F(z|\omega, R)$*. Based on that figure out what should happen with assets, a, in the long run distribution*

(d) Would your answer change if ϕ was now the natural borrowing limit (instead of 0 as assumed above)?

3. A Ricardian equivalence in Aiyagari's model

Suppose that in the Aiyagari model presented in class we introduce a government. The government has an initial amount of debt given by D. Every period, the government raises lump-sum taxes from every agent in the economy, τ, to finance the interest rate payments on the debt. That is

$$rD = \tau$$

where $R = 1 + r$ is the equilibrium interest rate. The budget constraint of the individual is now

$$c + a' = y + Ra - \tau$$

where we take account of the taxes being paid. Suppose that the market imposes on individuals their natural debt limits.

(a) Show that the natural debt limit is

$$\frac{y_{min}}{r} - D$$

where y_{min} is the lowest shock. Then the borrowing constraint is $a' \geq -\left(\frac{y_{min}}{r} - D\right)$.

(b) Show that the solution to the consumer problem for a given interest rate R is independent of D in the following sense: if D increases, a increases one to one.

(c) Suppose that there is no capital, so that in equilibrium $\int adF(a) = D$. That is the sum of the agents asset holdings equals the amount of debt issued by the government, and where F is the stationary distribution of asset holdings. Argue that increases in D do not affect the equilibrium price, R, nor the properties of the economy. That is, D does not matter.

(d) Is it important for this neutrality result that the government tax policy affects the natural borrowing limit?

4. Incomplete Markets and Unemployment. A Numerical Analysis.

There is a unit of consumers in the economy. A consumer every period supplies l units of labor inelastically. The labor supply is either 1 (employed) or 0.5 (unemployed). The total labor income of the consumer is $w \times l$, where w is the wage.

Let $w = 1$ for now. Let P denote the transition matrix of the state of employment for an individual:

$$P = \begin{bmatrix} P_{00} & P_{01} \\ P_{10} & P_{11} \end{bmatrix}$$

where $P_{ij} = Prob(l_{t+1} = j | l_t = i)$.

Suppose the period is one year and that the borrowing limit is $\phi = 0$.

(a) What is the natural borrowing limit for the worker?

The consumer in this economy can save of a bond that has a annual gross return of $R = 1.04$. She has a standard power utility function, $u(c) = c^{1-\rho}/(1-\rho)$ with $\rho = 2$ and a (quarterly) discount factor of $\beta = 0.95$. **Use the code in the attached files** to answer the questions below.

(b) Describe the code. How is the program attaining the results? How is the optimization of the HH value function done? Explain in detail.

(c) The code will plot the transition mapping of "cash in hands". You will see two plots (one for each employment state). In addition, you should plot the optimal policy function for assets and consumption (both as a function of "cash in hands" and again, a plot per state). In which state (employed or unemployed) is the worker saving more for a given amount of cash in hands? Why is that?

We now proceed to characterize the stationary distribution of the model. The last part of the code computes the stationary distribution and makes an extra plot (the stationary distribution (p.d.f.) will look spiky, but not worry too much: this is expected for only 2 endowment realizations). Answer the following questions:

(d) What is the total asset holdings (across all employment states) in the stationary distribution?

(e) What fraction of people are borrowing constrained in the stationary distribution? Was that expected?

(f) How does the stationary distribution change if risk aversion is reduced, and $\rho = 1$ (log utility)?

Go back to $\rho = 2$.

(g) What is the total labor supply in the economy in the stationary distribution?

(h) Using that $F = Ak^{\alpha}l^{1-\alpha}$ where $\alpha = .36$, and that the annual depreciation rate of k equals $\delta = 0.1$, solve for the functions $W(R)$ and $K(R)$. Use the fact that $w(R) = 1$ for $R = 1.04$ to back out the value of A in the production function.

(i) Do we have a stationary general equilibrium for the interest rate R assumed?

(j) Create a grid for R from 1.04 to 1.05 (say with 20 points). Do Aiyagari's $\bar{A}(R) = K(R)$ plot. Identify the stationary general equilibrium.