Failures of Expectation Management: Learning from Counter-Cyclical Signalling Policy

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Abstract

I formalize the idea that the government can manage expectations through releasing policy-related signals, which can lead to rational waves of optimism and pessimism in the economy. I construct a model featuring incomplete information and housing, where the government attempts to counter-cyclically regulate the real estate market through signaling policy. Quantitative implications of my model indicate that such expectation management policies may fail due to the signaling channel: During a recession, stimulus policy signals trigger rational pessimism by informing people that the government views the economy as being in recession and in need of stimulus, which deepens the recession. Furthermore, by introducing people's misunderstanding of the signaling policy rule, I find a mitigated negative signaling effect, suggesting potential improvements for government expectation management policies.

Keywords: Expectation management, House prices, Imperfect information, Animal spirits

1 Introduction

Policy signals provide valuable information about changes in economic activity. Housing prices, output, and inflation exhibits comovement, but people often lack information about the current values of these variables and their future trends. Government policies can serve as signals, providing related information to people with incomplete information. In recent years, policy transparency and communication have become increasingly important. When governments implement counter-cyclical policies to regulate the economy, they also announce their current and future policy targets through explicit statements, issuing policy signals. These signals provide public information about the government's view of the economy's current state to market participants. I observe these signals, form expectations accordingly, and adjust my behavior, which in turn affects policy effectiveness. This paper refers to such a policy approach as **signalling policy** and treats it as an unconventional policy tool to study its effects.

Policy signals as a tool to manage expectations has become increasingly used by governments. After the 2008 global financial crisis, many central banks adopted unconventional monetary policies, known as forward guidance, which included explicit statements about the future path of interest rates and the implementation of quantitative easing, sending strong positive signals to the market to guide expectations and stimulate economic recovery. However, such stimulus policies in order to manage expectation often fail to achieve the expected effect. Del Negro et al. (2012) found that the stimulus effect of forward guidance predicted by rational expectations models far exceeded what was observed in real data, suggesting that some undetected mechanisms offset the expected effects of forward guidance. It leads the main contribution of my paper: it shows that policy-related signals can have a negative effect on expectation management once people realize that the government uses counter-cyclical policies to issue signals, they will rationally form pessimistic expectations if they observe a positive stimulus policy. Therefore, the potential signalling effect induces pessimism and hence reduces the effectiveness of stimulus policies. Moreover, this negative signalling effect changes as individuals gradually learn about the policy rule.

This paper proposes a new model of the signalling role of policy in generating and amplifying real estate market fluctuations. I embed my learning mechanism within a New Keynesian model with housing. In this model, a representative household has incomplete information about fundamental shocks, leading to incomplete information about key state variables, such as housing prices and nominal interest rates. However, they receive endogenous policy-related signals related to these variables each period. I assume that individuals fully understand the signalling policy rule, that is, the extent to which the government releases policy signals in response to the underlying economic states. In the model, the government uses both monetary policy and signalling policy to regulate the economy. Monetary policy follows a conventional Taylor rule, while signalling policy follows a linear signalling rule similar to the Taylor rule. Each period, the government observes endogenous variables like housing prices and interest rates to judge the current economic state and then issues positive or negative policy signals accordingly. This setup captures the policy communication between the government and market participants regarding monetary and housing policies.

My model features the signalling channels of such stimulus policy, creating a stimulus policy-pessimism feedback. Consider a scenario where the government observes a recession in the real estate market. In response, it releases a positive stimulus policy signal and regulates the economy through monetary policy. On one hand, there is a **direct policy effect**: as predicted by standard macroeconomic models, monetary policy boosts the real estate market. On the other hand, there is an **indirect signalling effect**: the positive policy signals might trigger rational pessimism if they inform people that the government views the market as being in recession needing stimulus, making the real estate market worse. Similarly, restrictive policies during periods of economic boom can also trigger rational optimism. Therefore, the presence of the signalling effect will make the economy even more prosperous in boom periods and more depressed in recession periods. The direct effect of monetary policy has been extensively studied in the literature, but the signalling effect has received less attention.

My model of learning from policy signals has several features that make it an appealing model for studying expectation management in the real estate market. First, the signal structure faced by households with incomplete information is fully microfounded and endogenous. This is crucial because the government must observe endogenous economic conditions and adjust its policies based on its assessment of prosperity or recession. Second, the form of the government's signalling policy rule is flexible, allowing my model to be extended to other government policies, such as fiscal policy. Third, the subjective perception of individuals regarding the parameters of the signalling policy rule can vary over time, enabling me to study how people learn about government policy.

Focus on impulse response functions (IRFs) I focus on impulse response functions to compare the effects of stimulus policies under different conditions. The impulse response simulation provides useful information and is easier to interpret in this context. I simulate the quantitative characteristics of the economy after the introduction of signalling policy and compare them with the results in a typical rational expectation model. The comparison shows that the signalling effect often results in a more pronounced boom during expansion and a deeper recession during downturns, weakening the effectiveness of monetary policy. This may help explain why stimulus policies often fall short of expectations, as the indirect signalling effect offsets the direct policy effect.

Finally, I enhance the model by incorporating an additional layer of incomplete information. I assume that households comprehend the linear form of the signalling policy rule but misunderstand its parameter values, reflecting a real-world scenario where households may be unaware of the government's countercyclical signalling intensity. Instead, households possess a subjective perception of the signalling policy rule, with prior beliefs about the policy parameters. As the simulation progresses, they receive data on government policies to update their subjective perception of the parameters, embodying a rational learning process regarding the unknown rule.

After introducing this rational learning process, I find that the negative impact of the signalling effect can be mitigated if individuals overestimate the government's responsiveness to economic conditions and amplified when they underestimate it. Therefore, if releasing policy signals is necessary and the signalling effect is unavoidable, this implies that the government should directly disclose its policy rules in cases of underestimation, as suggested in previous literature. This approach may represent a suboptimal solution because it can mitigate the signalling effect, but it is not always appropriate. In cases of overestimation, the government should perhaps refrain from disclosing policy rules, as this would amplify the signalling effect. In this regard, my recommendations on policy transparency diverge from those suggested by rational expectations models.

1.1 Literature review

This paper is closely related to the literature on managing expectations through policy signals. Morris & Shin (2002) and Morris & Shin (2008) constructed a model incorporating both private and public information, providing a basic framework for studying expectation management through monetary policy. They argued that economic agents rely on public information when forming expectations, and therefore, monetary policy transparency can influence market expectations by altering the public information set, ultimately affecting policy effectiveness. Furthermore, Morris & Shin (2008) emphasized that conveying information may be more important than directly using policy tools, asserting that managing and coordinating expectations is the core challenge of modern monetary policy. Ricco et al. (2014) used empirical data on expectations to demonstrate the signalling effects of fiscal policy, finding that fiscal stimulus effects are weaker during periods of high policy uncertainty. Their research highlights the importance of policy communication and transparency and points to policy signals as a potential policy tool. These papers support the positive aspects of using policy signals for managing expectations, whereas my model will demonstrate the negative endogenous effects of such signals.

Another study exploring the negative effects of policy signals is Baeriswyl & Cornand (2010), who investigated the signalling effects of monetary policy. They assumed that individuals update their expectations based on the central bank's interest rate decisions, which reveal the central bank's information about the economy. However, their model only considered explicitly formulated monetary policy rules, whereas my model incorporates subjective, time-varying learning of policy parameters, offering a more realistic depiction and richer analysis of signalling policy effects.

From a modeling perspective, this paper is related to the behavioral macroeconomics literature that assumes individuals hold incomplete information about current fundamental shocks and form expectations through related signals. Maçkowiak & Wiederholt (2015) assumed that firms and individuals simultaneously hold incomplete information and learn through active choices about signal precision, although their model used exogenous noise signals, which conflict with the endogenous policy forms I aim to study. Chahrour & Gaballo

(2021) modeled learning from endogenous market signals—specifically, housing prices—and examined the impact of sentiment-driven cycles on business fluctuations. I base my model on this framework, but instead, I focus on endogenous policy signals and introduce rational Bayesian learning by individuals about policy signals to study the effects of signalling policy.

This paper is also related to the literature on gradually learning about government policy functions through Bayesian updating. Dovis & Kirpalani (2021) studied reputation issues in sovereign debt default by constructing a sovereign debt model that categorizes governments as either commitment type (C) or strategic type (S). Individuals update their expectations about the government' s type by observing actions in each period, affecting the cost of borrowing based on perceived credibility. Pastor & Veronesi (2012) introduced government policy parameters in financial markets and assumed that individuals learn about these parameters over time. When a new policy replaces an old one, previous learning becomes obsolete, leading to heightened uncertainty and a drop in asset prices.

The paper is organized as follows. In Section 2, I describe the microfounded incomplete information model with housing, in which the government regulates the economy through signaling policy and monetary policy; In Section 3, I present the empirical results of the model's impulse response functions (IRFs) to illustrate the negative impact of policy signals on expectation management; In Section 4, I assume that agents do not fully understand the parameters of the signaling policy rule but have prior beliefs that are updated over time as data from impulse response simulations become available; In Section 5, I briefly describe the algorithm used in my model; In Section 6, I present my conclusions.

2 A Housing Model with learning from policy signals

In this section, I present an incomplete information model with housing, where agents with incomplete information observe endogenous policy signals and learn based on the correct signaling policy rule. This paper builds on the business cycle model with learning from house prices established by Chahrour & Gaballo (2021), extending it by introducing price stickiness and monetary policy, and assuming that households observe endogenous policy signals to quantitatively assess the signaling effect of expectation management based on these signals.

The economy consists of representative households who are price takers, consuming both housing and general consumption goods. Households supply labor for housing production and consumption goods production. Housing is produced by a perfectly competitive representative housing sector, while general consumption goods are composed of a series of differentiated goods produced by consumption goods firms, with monopolistic competition and Calvo price stickiness introduced in the consumption goods sector. Both the housing and consumption goods sectors are subject to sector-specific total factor productivity (TFP) shocks. The government uses countercyclical monetary policy and signaling policy to regulate the economy.

2.1 Household

The representative household chooses consumption, housing consumption, labor supply, money holdings, and risk-free bonds to maximize utility, which takes the form of money in utility (MIU):

$$\max_{C_t,\Delta_t,B_t,N_t^c,N_t^h,M_t} E_0 \sum_{t=0}^{\infty} \beta^t \left(\log \left(C_t^{\phi} \mathcal{H}_t^{1-\phi} \right) - v^c \frac{\left(N_t^c\right)^{1+\chi_c}}{1+\chi_c} - v^h \frac{\left(N_t^h\right)^{1+\chi_h}}{1+\chi_h} + v^m \frac{\left(\frac{M_t}{P_t}\right)^{1+\chi_m}}{1+\chi_m} \right)$$

In this utility function, C_t represents the total consumption goods purchased by the household from the consumption sector, and $\mathcal{H}_t$measures housing consumption composed of housing units purchased by the household each period. N_t^c is the labor supplied by the household to the consumption goods sector. Since consumption goods are differentiated, $N_t^c = \int_0^1 N_{it}^c \, di$ is the labor supplied to the housing sector. $M_t$represents the nominal money balance held by the household, and P_t is the price of total consumption goods, with $\frac{M_t}{P_t}$ representing the real money balance that generates utility.

Housing consumption consists of the previous period's old housing stock and newly acquired housing units in the current period. The old housing stock depreciates at a fixed rate. The law of motion for housing is:

$$\mathcal{H}_t = (1-\delta_h)\mathcal{H}_{t-1} + \Delta_t$$

The household's optimization is subject to the following budget constraint:

$$P_t C_t + Q_t \Delta_t + B_t + M_t \leq W_t^c N_t^c + W_t^h N_t^h + R_t B_{t-1} + M_{t-1} + \Pi_t^c + \Pi_{it}^h + T_t$$

The budget constraint holds for $t \in 0, 1, 2, \ldots with B_0 = 0$. P_t represents the price of total consumption goods C_t, and Q_t represents the price of newly acquired housing units Δ_t. W_t^c, W_t^h denote wages for labor supplied to different sectors, R_t is the nominal interest rate, and B_t represents the nominal risk-free bond held in the current period. Π_t^c, Π_{it}^h represent profits transferred from different sectors, and T_t is a one-time government transfer.

Dividing the budget constraint by the price of total consumption goods P_t yields the real quantities, denoted by lowercase letters:

$$C_t + q_t \Delta_t + b_t + m_t \leq w_t^c N_t^c + w_t^h N_t^h + \frac{R_t}{\Pi_t} b_{t-1} + m_{t-1} + \frac{\Pi_t^c + \Pi_{it}^h + T_t}{P_t}$$

Here, the MIU assumption is used to introduce monetary policy, but note that since I focus on monetary rules targeting interest rates, as long as money utility is separable, the actual quantity of money has no implications for the rest of the model and can therefore be ignored.[1]

2.2 Housing producers

Assuming the housing market is perfectly competitive, there is a representative housing producer who uses labor supplied by households to produce new housing units each period:

$$\Delta_t = e^{\xi_t^h} (N_t^h)^{\gamma_h}$$

The housing firm's profit maximization problem is:

$$\Pi_t^h = Q_t \Delta_t - W_t^h N_t^h$$

where W_t^h is the wage paid in the housing sector and Q_t is the price of housing.

The housing sector is subject to a sector-specific productivity shock ξ_t^h, which follows an AR(1) process:

$$\xi_t^h = \rho_h \xi_{t-1}^h + \varsigma_t^h$$

[1]Iacoviello (2005) has a extensive explanation about how MIU assumption makes the quantity of real money irrrelevant.

ς_t^h is an i.i.d. exogenous shock, distributed as $N(0, \sigma_{\varsigma^h}^2)$.

2.3 Consumption sector

Total consumption goods are composed of a series of differentiated intermediate goods and are sold by the consumption goods production firm. Intermediate goods are aggregated using a CES function:

$$Y_t = \left(\int_0^1 Y_{it}^{\frac{\epsilon-1}{\epsilon}} \, di \right)^{\frac{\epsilon}{\epsilon-1}}$$

ϵ represents the degree of product differentiation and is the source of monopoly power for intermediate goods firms. Each intermediate goods is produced by a specific intermediate goods firm using labor supplied by households. The production function is:

$$Y_{it} = e^{\xi_t^c} (N_{it}^c)^{\gamma_c}$$

ξ_t^c represents the sector-wide productivity shock in the consumption sector, which follows an AR(1) process:

$$\xi_t^c = \rho_c \xi_{t-1}^c + \varsigma_t^c$$

ς_t^c is an i.i.d. exogenous shock, distributed as $N(0, \sigma_{\varsigma^c}^2)$.

The consumption goods production firm maximizes profits by choosing the price and quantity of total consumption goods:

$$\Pi_t^c = P_t Y_t - \int W_{it}^c N_{it}^c di = P_t Y_t - W_t^c N_t^c$$

Next is the profit maximization problem for intermediate goods firms. Price stickiness is introduced through a standard Calvo-Yun mechanism. Intermediate goods are sold at a price of P_{it}, and there is a probability θ that the price cannot be adjusted each period, remaining at the previous period's level. The optimal price P_{it}^* in each period satisfies:

$$\sum_{k=0}^{\infty} (\theta\beta)^k E_t \left\{ \Lambda_{t,k} \left[\frac{P_{it}^*}{P_{it+k}} - \frac{\epsilon/(\epsilon-1)}{X_{t+k}} \right] Y_{it+k} \right\} = 0$$

X_{t+k} represents the mark-up for monopoly pricing, reflecting the marginal cost of producing intermediate goods, and $\Lambda_{t,k} = \frac{u'(C_{t+k})}{u'(C_t)}$ is the household's stochastic discounting factor.

The change in the total consumption goods price in each period is given by:

$$P_t = \left[\theta P_{t-1}^{1-\varepsilon} + (1-\theta)\left(P_t^*\right)^{1-\varepsilon}\right]^{1/(1-\varepsilon)}$$

2.4 Government policy

The government regulates the economy in a countercyclical manner using monetary policy and a signaling policy rule.

Monetary policy follows a standard Taylor rule:

$$R_t = (R_{t-1})^{\rho_R}\left(\Pi_t^{(1+\phi_\Pi)} R\right)^{1-\rho_R} \varepsilon_t^R$$

$0 < \rho_R < 1$ is the parameter associated with interest-rate inertia. $1 + \phi_\Pi$measures the sensitivity of interest rates to current ináation. ϵ_t^R is a i.i.d. shock process distributed as $N(0, \sigma_{\epsilon^R}^2)$. This rule captures the government's intent to regulate the economy countercyclically through monetary policy.

Similarly,I assume that the government releases policy signals in a countercyclical manner through a linear signaling policy rule, analogous to the Taylor rule:

$$S_t = \varphi_q Q_t + \varphi_r R_t$$

In this model, the government observes house prices and nominal interest rates each period and releases policy signals based on these variables. This captures the government's communication regarding real estate and monetary policy with the market. Given the countercyclical nature of the policy signals, I simply assume $\varphi_q < 0, \varphi_r < 0$.

However, it is important to explain that the sign of the coefficients does not effect individuals' expectation formation, as people learn signals through Kalman filter. Furthermore, the signs and magnitudes of these parameters are not crucial; my setting of $\varphi_q < 0$ and $\varphi_r < 0$ is solely intended to ensure that when the policy signal $S_t < 0$, it reflects an economic boom, where the government releases a negative policy signal for easier understanding. We will provide further explanation when discussing the mechanism of expectation formation through learning from policy signals in Section 5. [2]

2.5 Market clearing

Labor market clearing:

$$Y_t = C_t$$

Bond market clearing:

$$B_t = 0$$

2.6 Information structure

Based on Chahrour & Gaballo (2021), I introduce information friction into household's demand. I divide the household into an incomplete informed shopper, who decides the household's optimal consumption and housing, and a fully informed worker, who decides the labor supply, bonds, and money holdings. The rest of the production sectors and the government are fully informed.

I first define the full information set at time t is $\Omega_t = \left\{ \left\{\epsilon_k^R\right\}_0^t, \left\{\varsigma_k^c\right\}_0^t, \left\{\varsigma_k^h\right\}_0^t \right\}$, which includes the realized values of shocks from period 0 to period t. Thus, it also includes the realized values of all endogenous variables from period 0 to t. The incomplete information set at time t is $\{\Omega_{t-1}, S_t\}$. The shopper knows the full information set of the previous period, $t-1$, and thus the realized values of all endogenous variables from period 0 to $t-1$. However, due to not knowing the fundamental shocks realized at time t, the shopper does not know the

[2]When individuals are fully aware of the parameter values, they essentially serve only as a signaling channel for forming expectations. In simple terms, I assume that the government's signaling policy rule is $S_t = \varphi_q Q_t$, and current housing price $Q_t = 1$, then I have thest two scenarios:

1. The government issues a real estate policy signal with parameter $\varphi_q = -1$, thus $S_t = -1$
2. The government issues a real estate policy signal with parameter $\varphi_q = 2$, thus $S_t = 2$

Both of these scenarios inform individuals that the current housing price Q_t to be around 1 based on the received signals and the corresponding signaling policy rule.

specific values of the endogenous variables at time t. The shopper can observe the related policy signal S_t.

The worker and shopper maximize expected utility under different information sets, that is:

The shopper chooses consumption C_t and housing Δ_t to maximize utility based on the incomplete information set $\{S_t, \Omega_{t-1}\}$.

Thus, shoppers' optimization problem in each period is:

$$\max_{C_t,\Delta_t} \mathbb{E}_t \left[\sum_{i=t}^{\infty} \beta^i \left(\log\left(C_i^{\phi} \mathcal{H}_i^{1-\phi}\right) - v^c \frac{\left(N_i^{c*}\right)^{1+\chi_c}}{1+\chi_c} - v^h \frac{\left(N_i^{h*}\right)^{1+\chi_h}}{1+\chi_h} + v^m \frac{\left(\frac{M_i^*}{P_i}\right)^{1+\chi_m}}{1+\chi_m} \right) \mid S_t, \Omega_{t-1} \right]$$

subject to

$$\mathcal{H}_t = (1-\delta_h)\mathcal{H}_{t-1} + \Delta_t$$

$$P_t C_t + Q_t \Delta_t + B_t + M_t \leq W_t^c N_t^c + W_t^h N_t^h + R_t B_{t-1} + M_{t-1} + \Pi_t^c + \Pi_{it}^h + T_t$$

$$S_t = \varphi_q Q_t + \varphi_r R_t$$

The shoppers fully understand the form of the signalling policy rule and observe policy signals each period related to housing prices and nominal interest rates, allowing them to infer the underlying economic conditions. For example, when they observe a positive policy signal $S_t > 0$, given that $S_t = \varphi_q Q_t + \varphi_r R_t, \varphi_q < 0, \varphi_r < 0$, they update their expectations accordingly, anticipating either low housing prices ($Q_t < 0$) or low interest rates ($R_t < 0$), both of which lead to pessimism in the market.[3] The specific way in expectation formation will be discussed in Section 5.

The worker chooses labor supply N_t^h, N_t^c, bonds B_t, and money holdings M_t to maximize utility based on the full information set Ω_t.

$$N_t^c, N_t^h, B_t, M_t = \text{argmax}_{\left\{C_t^*, \Delta_t^*, N_t^c, N_t^h, B_t, M_t\right\}} E_t \left[U_t \mid \Omega_t\right]$$

The timing of each period in the model is as follows:

[3]This is because low housing prices ($Q_t < 0$) typically indicate a depressed real estate market, and due to the central bank's countercyclical monetary policy, low interest rates ($R_t < 0$) usually also reflect a current economic downturn.

1. Household splits into shoppers and worker-savers.

2. Shocks realize.

3. Government releases policy signals S_t based on the signaling policy rule.

4. The full information set in period t, Ω_t, is observed by firms and worker-savers. Shoppers observe the incomplete information set $\{S_t, \Omega_{t-1}\}$ and infer the realized fundamental shocks based on the signaling policy rule.

5. Production and trade take place. All players make their choices based on their respective information sets.

6. Market clearing.

7. Family members share information, revealing Ω_t to shoppers.

8. Shoppers update perceived signaling policy rules based on Bayesian learning.

Steps 3-6 are conducted together to find the fixed point. According to Chahrour, R., & Gaballo, G. (2021), dividing the household into shoppers and workers is not crucial, but it simplifies solving the model. What truly matters is that the shopper's lack of knowledge about shocks leads to incomplete information regarding current endogenous variables, requiring them to infer and form expectations based on government signals.

2.7 Equilibrium

Next, I define the equilibrium of the model:

Given initial conditions $\{B_{-1}, \mathcal{H}_{-1}, M_{-1}\}$, a rational expectations equilibrium is a set of prices $\left\{P_t, Q_t, W_t^c, W_t^h, R_t\right\}_{t=0}^{\infty}$ and quantities $\left\{B_t, N_t^c, N_t^h, C_t, \mathcal{H}_t, \Delta_t, Y_t\right\}_{t=0}^{\infty}$, which are contingent on the realization of the stochastic processes $\left\{\epsilon_t^R, \xi_t^c, \xi_t^h\right\}_{t=0}^{\infty}$, such that for each $t \geq 0$:

1. Shoppers and worker-savers optimize, i.e., $\left\{C_t, \Delta_t, N_t^c, N_t^h, B_t, M_t\right\}$ are solutions to $\max_{C_t, \Delta_t, N_t^c, N_t^h, B_t, M_t} E_t\left[U_t\right]$ given their respective information sets, and the budget constraint holds.

2. Local housing producers optimize, i.e., N_t^h, Δ_t are solutions to $\max_{N_t^h, \Delta_t} \Pi_t^h$.

3. Intermediate good producers optimize prices, i.e., P_{it}, N_{it}^c are solutions to $\max_{P_{it}, N_{it}^c} \Pi_{it}^c$.

4. Final consumption producers optimize, i.e., N_t^c is a solution to $\max_{N_t^c} \Pi_t^c$.

5. The government announces policy signals S_t based on the realization of the endogenous variables Q_t, R_t.

6. Markets clear.

2.8 FOCs of the model

I derive the first-order conditions of the model for better understanding and convert them into state-space equations via algorithms to facilitate my subsequent solution of the expectation formation problem. The specific algorithm is in Section 5. Lowercase letters represent real variables obtained by dividing the variables by the current total consumption goods price P_t. The first-order conditions of the model are as follows:

Shoppers make optimal decisions on consumption and housing consumption under incomplete information sets, resulting in the following first-order conditions:

$$[C_t]: \quad \phi \frac{1}{C_t} = \mathbb{E}_t \left[\lambda_t \mid S_t, \Omega_{t-1} \right]$$

$$[\mathcal{H}_t]: \quad (1-\phi) \frac{1}{\mathcal{H}_t} = \mathbb{E}_t \left[q_t \lambda_t \mid S_t, \Omega_{t-1} \right] + \mathbb{E}_t \left[q_{t+1} \lambda_{t+1} \mid S_t, \Omega_{t-1} \right]$$

λ_t is the shadow price of the household's budget constraint, representing the marginal utility the household can obtain from expanding the budget constraint. For the current shoppers, consumption and housing consumption are their own control variables, so they hold full information about them. However, since shoppers do not know the realization of the current fundamental shocks, they are unaware of current variables such as λ_t, q_t, etc. Therefore, their first-order conditions depend on the conditional expectations within their information set $\mathbb{E}_t \left[\cdot \mid S_t, \Omega_{t-1} \right]$. Taking the first-order condition of consumption C_t as an example, shoppers in period t receive relevant policy signals S_t, extract the signal to form the conditional expectation of the shadow price of the current budget constraint $\mathbb{E}_t \left[\lambda_t \mid S_t, \Omega_{t-1} \right]$, and decide the current consumption based on their expectations.

In period t of economic recession, the government engages in counter-cyclical economic adjustments and releases stimulus policy-related signals. Observing this, the shopper anticipates that the government' s actions align with a counter-cyclical economic stance. Consequently, the shopper develops a pessimistic outlook on the economy, raising their anticipated shadow price on budget constraints $\mathbb{E}_t [\lambda_t \mid S_t, \Omega_{t-1}]$, which I interpreted as a mechanism to increase savings in standard rational expectation model. [4] This pessimistic expectation leads shoppers to reduce current consumption C_t and housing consumption $\mathcal{H}_t$, thereby deepening the economic downturn.

The current workers, since they hold full information sets, are aware of the values of current variables such as λ_t, etc. At the same time, they infer the expected values of variables in period $t+1$, $\mathbb{E}_t [\lambda_{t+1} \mid \Omega_t]$, based on the law of motion obtained from solving the model. The full information and understanding of the law of motion make the workers' expectations rational, consistent with the results in rational expectation literature. Workers maximize household utility by choosing labor supply, bonds, and the amount of money held:

$$
\begin{aligned}
&[N_t^c]: \quad v^c (N_t^c)^{\chi_c} = \lambda_t w_t^c \\
&[N_t^h]: \quad v^h (N_t^h)^{\chi_h} = \lambda_t w_t^h \\
&[m_t]: \quad v^m (m_t)^{\chi_m} = \lambda_t - \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\pi_{t+1}} \mid \Omega_t \right] \\
&[b_t]: \quad \lambda_t = \mathbb{E}_t \left[\frac{R_t \lambda_{t+1}}{\pi_{t+1}} \mid \Omega_t \right]
\end{aligned}
$$

It can be seen that this household setup introduces a labor wedge into the model. When shoppers increase consumption due to optimistic expectations, workers have to increase the amount of labor supply, which makes the information friction have real effects.

[4] In a standard rational expectations model, when economy is in a recessionary scenario, households adjust their assessments of future uncertainty, thereby affecting their current decisions. Specifically, an economic downturn is typically accompanied by lower expected future income or increased uncertainty. When faced with heightened income uncertainty, households perceive their budget constraint as more pressing. At this point, the shadow price of the budget constraint λ_t rises. To smooth consumption, households tend to increase savings and reduce consumption in the present, a behavior commonly explained in the literature through mechanisms such as the substitution effect, income effect, or precautionary saving.

The first-order condition of the housing production sector is:

$$w_t^h = \gamma_h \frac{q_t \Delta_t}{N_t^h} e^{\xi_t^h}$$

Adding the production function and the law of motion for housing consumption:

$$\Delta_t = e^{\xi_t^h} (N_t^h)^{\gamma_h}$$
$$\mathcal{H}_t = (1 - \delta_h)\mathcal{H}_{t-1} + \Delta_t$$

Also, the first-order conditions of the consumption goods sector:

$$w_t^c = \gamma_c \frac{Y_t}{N_t^c} \frac{1}{X_t} e^{\xi_t^c}$$
$$X_{1t} = \frac{1}{C_t} \frac{X}{X_t} Y_t + \theta\beta\mathbb{E}_t \left[X_{1,t+1} \pi_{t+1}^{\xi} \mid \Omega_t \right]$$
$$X_{2t} = \frac{1}{C_t} Y_t + \theta\beta\mathbb{E}_t \left[X_{2,t+1} \pi_{t+1}^{\xi-1} \mid \Omega_t \right]$$
$$\pi_t = \left[\theta + (1-\theta)(\pi_t^*)^{1-\xi} \right]^{\frac{1}{1-\xi}}$$
$$\pi_t^* = \frac{X_{1t}}{X_{2t}} \pi_t$$

π_t is the current inflation level, calculated from the prices of consumption goods in two periods: $\pi_t = \frac{P_t}{P_{t-1}}$; π_t^* is the inflation level under optimal pricing: $\pi_t^* = \frac{P_t^*}{P_{t-1}}$; X_{1t}, X_{2t} are auxiliary variables used for pricing; X_t represents the monopoly pricing mark-up in the current period.

Adding the aggregate consumption goods production function:

$$Y_t = e^{\xi_t^c} (N_t^c)^{\gamma_c}$$

Finally, adding market-clearing conditions, the law of motion of shocks, and budget constraints, I obtain the dynamic equations of this economic system. After performing first-order linearization, it can be written in state-space form. The specific algorithm for solving the model is left for discussion in Section 5.

3 Dynamics

In this section, I aim to demonstrate the negative signaling effect of policy signals during the implementation of stimulus policies. Therefore, I primarily employ Impulse Response

Functions (IRFs) to simulate and compare the results after introducing the mechanism of learning from policy signals with those under the Full Information Rational Equilibrium (FIRE) scenario. My objective is to show that when the economy is subjected to exogenous shocks leading to periods of prosperity or recession, the government's attempt to regulate expectations through signaling policy tendencies can exacerbate the economic boom or bust.

As previously mentioned, the policy stimulus effects simulated under the rational model often significantly exceed the actual stimulus effects. Therefore, there must exist a potential channel that partially offsets the impact of stimulus policies. Accordingly, I define the simulation results under the FIRE scenario as the ideal outcomes. Subsequently, by introducing the mechanism of incomplete information and learning from policy signals, as described in Section 2, I define the simulation results of Section 2 as the scenario where the government attempts to manage expectations through policy signals. I define the difference between these two simulations as the signaling effect. Next, I present the results of the IRF simulations:

3.1 Parameter Values

Before conducting my simulations, I need to emphasize the specification of key model parameters. Since this paper is primarily a theoretical study, I do not undertake a full quantitative evaluation of the model. I aim only to demonstrate the changes in simulation results resulting from the introduction of my model's mechanisms. Almost all parameters are sourced from Chahrour & Gaballo (2021) and Iacoviello (2005). Below, I introduce the values of some key parameters:

According to Chahrour & Gaballo (2021), I set the discount factor $\beta = 0.99$, I set the weight on utility between consumption goods and housing consumption $\phi = 0.66$. Since the utility of money is additively separable, the parameters of monetary policy do not actually affect my simulation results. According to Iacoviello (2005), I set the probability that the intermediate goods sector cannot change prices $\theta = 0.75$. Regarding the Taylor rule for monetary policy, I set $\phi_\pi = 0.65$ to reflect the central bank's inclination towards controlling inflation.

Regarding the parameters in my model's mechanism, the response coefficients of the government's signaling policy rule to housing prices and nominal interest rates φ_q, φ_r, as I mentioned earlier, their magnitudes and signs do not affect the model's results. So I simply set $\varphi_q = -3, \varphi_r = -4$.

3.2 Impulse Responses of Monetary Shocks

A positive monetary policy shock represents the central bank's tightening of monetary policy by increasing nominal interest rates, thereby inducing a recession of economy. To effectively demonstrate the features of my baseline model, I first present the impulse responses of several key variables, which exhibit comovement:

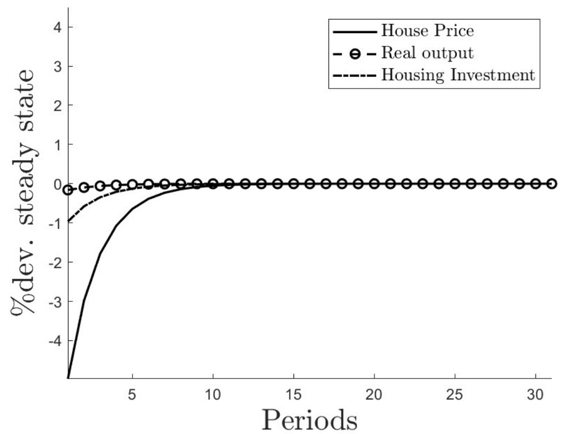

Figure 1: Comovements

It can be observed that a positive monetary policy shock, which increases the nominal interest rate R_t, leads households to increase their savings, resulting in a decline in the main economic variables. Consequently, the economy enters a recessionary period, during which housing prices Q_t, real output Y_t, and housing investment Δ_t exhibit comovement.

After the economy enters a recessionary period, the government releases positive policy signals in accordance with the signalling policy rule $S_t = \varphi_q Q_t + \varphi_r R_t$. Below, I present the dynamics of policy signals.

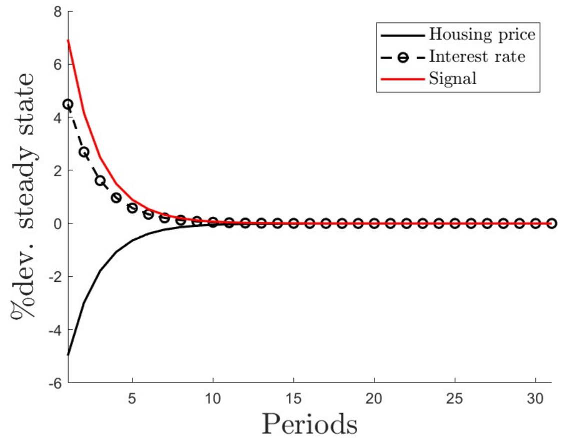

Figure 2: IRF of Policy Signals

It can be seen that the positive monetary policy shock causes a decline in housing prices Q_t, placing the real estate market in a recessionary state and the monetary policy in a tightening stance. According to my previous assumption, the government releases positive stimulus policy signals in the real estate sector to stimulate the economy. Therefore, the policy signal S_t is positive when the economy is in a recession, which triggers pessimistic expectations.

Next, I focus on the core variable, housing price Q_t, and compare the models from Section 2 with those under the Full Information Rational Equilibrium (FIRE) scenario:

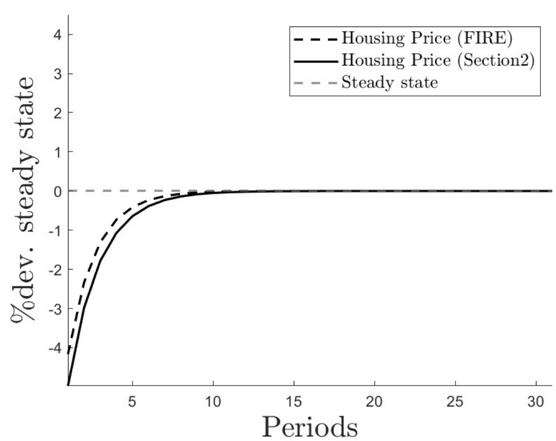

Figure 3: Housing Prices Comparasion (Bust Scenario)

As expected, in Figure 3, I observe that an increase in the interest rate has an immediate effect on housing prices Q_t. As stated earlier, there are several channels that can potentially explain the dynamics of house prices.

Firstly, through the **direct policy channel**, an increase in the interest rate causes housing prices, as a type of asset price, to decrease due to a substitution effect from holding money to other assets. Figure 3 illustrates that this is precisely what occurs; following the increase in the interest rate, house prices begin to fall and subsequently gradually return to the steady state.

Secondly, through the **indirect signalling channel**, the rise in nominal interest rates pushes the economy into a recessionary period. At this juncture, the government, adhering to the signalling policy, begins to release counter-cyclical positive policy signals. This action leads individuals to infer that the economy is currently in a recession. By comparing this with the FIRE scenario, I observe that the formation of pessimistic expectations caused by this indirect signalling effect leads individuals to maintain more pessimism, which in reality exacerbates the recessionary period.

Finally, I introduce a negative monetary policy shock to better illustrate the role of the signaling effect. A negative monetary policy shock signifies the central bank's expansionary monetary policy by decreasing nominal interest rates, thereby inducing a period of economic prosperity.

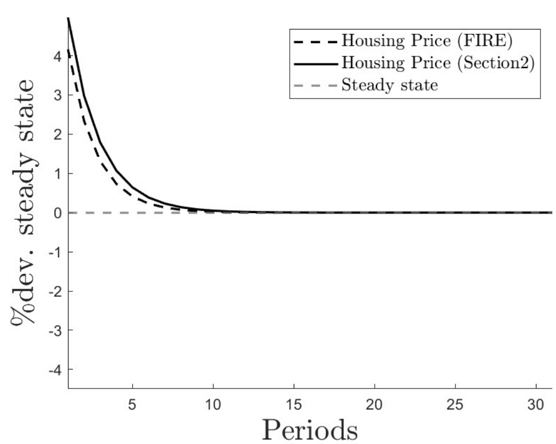

Figure 4: Housing Prices Comparasion (Boom Scenario)

Similarly, I compare the key variables of the FIRE scenario with Section 2 scenario.

Following the shock, housing prices increase in both models due to the direct policy effect. However, through the indirect signaling effect, my model exacerbates the economic prosperity. This occurs because individuals infer from the restrictive policy signals that the current economy is in a prosperous period, thereby further amplifying the prosperity during boom phases.

In summary, the presence of policy signals causes individuals with incomplete information in the economy to form pessimistic/optimistic expectations, thereby exacerbating economic recessions/booms. This offsets the direct policy effects of the government's stimulus/restrictive policies. I hope this mechanism can partially explain why traditional rational expectations models tend to overestimate the effects of stimulus policies. Moreover, policies where the government manages expectations through policy-related signals may prove to be negative.

4 Misunderstanding Signalling Policy Rule

4.1 learning policy rule by Bayesian updating

In real life, due to the opacity of the policy-making process, I often lack understanding of how the government sets policies. Even when the government enhances policy communication by releasing policy signals, individuals are unable to discern how these signals are generated. Therefore, I extend my model here by introducing an additional layer of incomplete information.

I assume that individuals are aware of the functional form of the signalling policy rule:

$$S_t = \varphi_q Q_t + \varphi_r R_t,$$

but they do not know the specific values of the parameters φ_q and φ_r. Accordingly, individuals possess their own subjective signalling policy rule:

$$S_t = \tilde{\varphi}_{q,t} Q_t + \tilde{\varphi}_{r,t} R_t,$$

where $\tilde{\varphi}_{q,t}$ and $\tilde{\varphi}_{r,t}$ represent the individual's perceived responsiveness of the policy signal to house prices Q_t and nominal interest rates R_t in period t, respectively. Typically, $\tilde{\varphi}_{q,t} \neq \varphi_q$

and $\tilde{\varphi}_{r,t} \neq \varphi_r$, reflecting the individual's misunderstanding of government policies. In reality, individuals are generally unaware of the specific processes involved in policy formulation, meaning they do not know the extent to which countercyclical policies respond to actual economic conditions. Each period, individuals receive a policy signal S_t and subsequently use their subjective signalling policy rule to infer the real economic conditions.

However, although individuals initially do not know the parameters of the government' s policy rule, at the end of each period, they learn both the realized signal S_t and the actual values of house prices Q_t and nominal interest rates R_t. Based on this data, individuals can update their understanding of the policy rule through rational learning, gradually learning the true policy rule. Below, I describe the process of individuals' rational learning:

Individuals update their beliefs using Bayesian learning. The parameters in their subjective rule at period 0 are $\tilde{\varphi}_{q,0}$ and $\tilde{\varphi}_{r,0}$, which follow a prior distribution:

$$\begin{bmatrix} \tilde{\varphi}_{q,0} \\ \tilde{\varphi}_{r,0} \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \varphi_q^0 \\ \varphi_r^0 \end{bmatrix}, \begin{bmatrix} \sigma_{\varphi_q}^2 & 0 \\ 0 & \sigma_{\varphi_r}^2 \end{bmatrix} \right).$$

The statistical learning model employed by individuals is:

$$S_t = \tilde{\varphi}_q Q_t + \tilde{\varphi}_r R_t + \varsigma_t^s,$$

where ς_t^s follows the distribution $\mathcal{N}(0, \sigma_{\varsigma^s}^2)$. This term represents an exogenous noisy shock introduced in the Bayesian learning process, which is only hypothesized in the individuals' subjective inference and does not exist in the actual model. According to my model's timing and information structure, at the end of period t, individuals obtain the information set $\Omega_t = \left\{ \left\{ \epsilon^R \right\}_0^t, \left\{ \varsigma^c \right\}_0^t, \left\{ \varsigma^h \right\}_0^t \right\}$, which is equivalent to$\Omega_t = \left\{ \left\{ S_t \right\}_0^t, \left\{ Q_t \right\}_0^t, \left\{ R_t \right\}_0^t \right\}$, providing the explanatory and dependent variables in the regression. Based on this, individuals perform Bayesian statistical learning to estimate the posterior distribution of the parameters in their subjective rule:

$$\begin{bmatrix} \tilde{\varphi}_{q,t} \\ \tilde{\varphi}_{r,t} \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \varphi_q^t \\ \varphi_r^t \end{bmatrix}, \Sigma_\varphi \right).$$

Shoppers use the posterior estimates of the means φ_q^t and φ_r^t as the parameters in their subjective policy rule to make inferences and form expectations.

Therefore, the timing of the extended model is as follows:

1. Households split into shoppers and worker-savers.

2. Shocks are realized.

3. The government releases policy signals S_t based on the signalling policy rule $S_t = \varphi_q Q_t + \varphi_r R_t$.

4. The full information set in period t, Ω_t, is observed by firms and worker-savers. Shoppers observe an incomplete information set $\{S_t, \Omega_{t-1}\}$ and infer the realized fundamental shocks based on the subjective signalling policy rule $S_t = \varphi_q^t Q_t + \varphi_r^t R_t$.

5. Production and trade take place. All players make their choices based on their respective information sets.

6. Markets clear.

7. Family members share information, revealing Ω_t to shoppers.

8. Shoppers update their perceived signalling policy rule based on Bayesian learning, obtaining the subjective signalling policy rule for period $t+1$: $S_{t+1} = \varphi_q^{t+1} Q_{t+1} + \varphi_r^{t+1} R_{t+1}$.

This is fundamentally consistent with the baseline model in Section 2. The only difference is that after the end of period t, individuals update their information sets, allowing them to revise their estimates of the parameters in the subjective policy rule. Once updated, individuals use the parameters estimated from period t's information set to make inferences and form expectations for period $t+1$.

Thus, shoppers' optimization problem in each period is:

$$\max_{C_t,\Delta_t} \mathbb{E}_t \left[\sum_{i=t}^{\infty} \beta^i \left(\log\left(C_i^{\phi} \mathcal{H}_i^{1-\phi} \right) - v^c \frac{\left(N_i^{c*}\right)^{1+\chi_c}}{1+\chi_c} - v^h \frac{\left(N_i^{h*}\right)^{1+\chi_h}}{1+\chi_h} + v^m \frac{\left(\frac{M_i^*}{P_i}\right)^{1+\chi_m}}{1+\chi_m} \right) \mid S_t, \Omega_{t-1} \right]$$

subject to

$$\mathcal{H}_t = (1-\delta_h)\mathcal{H}_{t-1} + \Delta_t$$

$$P_t C_t + Q_t \Delta_t + B_t + M_t \leq W_t^c N_t^c + W_t^h N_t^h + R_t B_{t-1} + M_{t-1} + \Pi_t^c + \Pi_{it}^h + T_t$$

$$S_t = \tilde{\varphi}_{q,t} Q_t + \tilde{\varphi}_{r,t} R_t$$

In the context of incomplete information, shoppers optimally choose consumption C_t and new housing purchases Δ_t. At this point, they form expectations based on their subjective policy rule to make optimal decisions.

4.2 Dynamics in Underestimate Scenario

Next, I utilize impulse response functions (IRFs) to demonstrate the characteristics of my model. I assume that in period 0, the economy experiences a unit positive monetary policy shock. Here, I suppose individuals underestimate the parameters, which means that the parameters of individuals' prior subjective policy rule, φ_q^0 and φ_r^0, have the same signs as the true signaling policy rule parameters φ_q and φ_r, but their magnitudes are smaller than the true values. Specifically:

$$\varphi_q^0 = -0.7, \quad \varphi_q = -3, \quad \text{with} \quad |\varphi_q^0| < |\varphi_q|$$
$$\varphi_r^0 = -1, \quad \varphi_r = -4, \quad \text{with} \quad |\varphi_r^0| < |\varphi_r|$$

This reflects that individuals initially underestimate the government's determination to regulate the economy through expectations management, as represented by the reaction coefficients, and subsequently learn the true parameters φ_q and φ_r gradually through Bayesian learning.

Below are the IRFs of the key variables in the model:

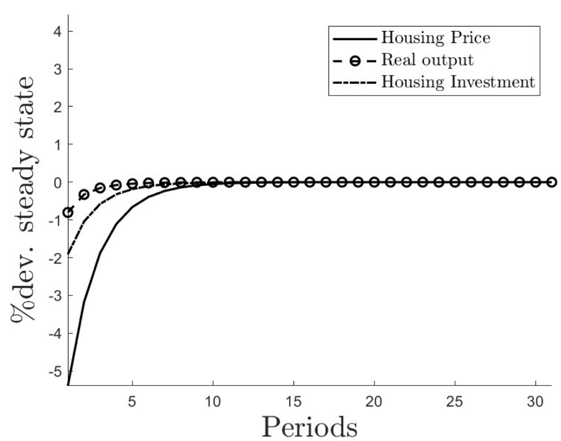

Figure 5: Comovements

It can be observed that the positive monetary policy shock leads to a decline in the main economic variables: house prices Q_t, real output Y_t, and housing investment Δ_t. This exhibits comovement similar to that in Section 2, reflecting the robustness of my model in describing comovement.

I then compare these results with those from Section 2 and under the Fully Informed Rational Expectations (FIRE) scenario. I select housing investment Δ_t as the variable to present (since the fluctuation in house prices Q_t is relatively small and does not display well, although the effect we discuss below still exists).

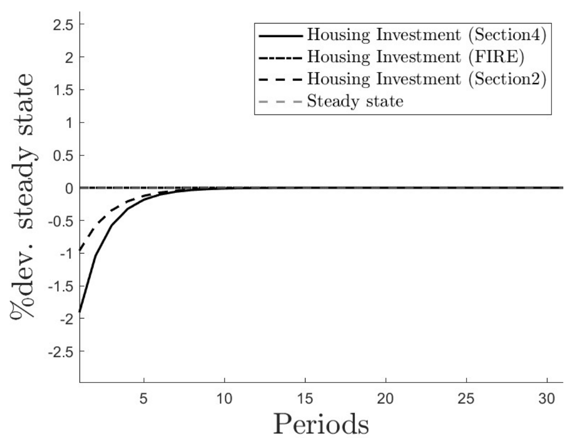

Figure 6: Housing Investments Comparasion (Underestimate Scenario)

It can be observed that housing investment Δ_t in Scenario 4 is more decreased than in Scenario 2, indicating that when individuals misunderstand the policy rule, the signaling effect—which exacerbates the recession—is further intensified. This occurs because, during a recession, the emergence of a positive policy signal of the same magnitude leads individuals who underestimate the reaction coefficients in the policy rule to anticipate a more severe economic downturn. Consequently, this triggers stronger pessimistic sentiments, amplifying the signaling effect beyond that observed in my baseline model and thereby deepening the recession.

As time progresses, individuals gradually learn the true policy reaction function, and housing investment converges toward that in my baseline model. The negative signaling effect diminishes accordingly. This indicates that the government needs to enhance communication regarding the policy reaction parameters in its policy communication.

Below is the process of the coefficients being gradually learned by individuals:

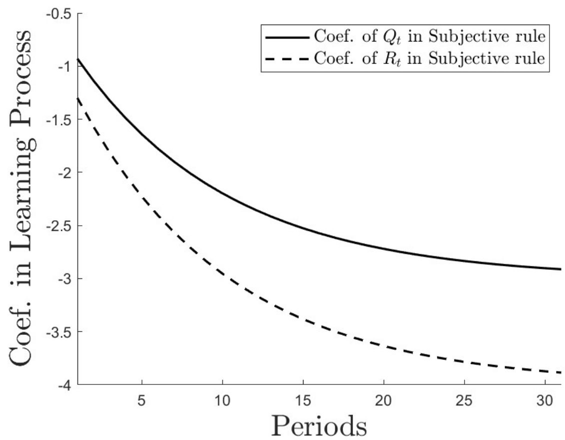

Figure 7: Learning process of Subjective policy rule

It can be observed that φ_q^t and φ_r^t gradually approach the true values as the simulation proceeds, reflecting the process of individuals incrementally learning the government's policy function.

4.3 Dynamics in Overestimate Scenario

To support my argument regarding the effect of misunderstanding the policy, I now assume individuals overestimate the parameters, which means individuals' prior subjective rule parameters φ_q^0 and φ_r^0 have the same signs as the true signaling policy rule parameters φ_q and φ_r, but their magnitudes are larger than the true values; specifically:

$$\varphi_q^0 = -5, \quad \varphi_q = -3, \quad \text{with} \quad |\varphi_q^0| > |\varphi_q|$$
$$\varphi_r^0 = -7, \quad \varphi_r = -4, \quad \text{with} \quad |\varphi_r^0| > |\varphi_r|$$

This reflects that individuals initially overestimate the government's determination to regulate the economy through expectations management and then gradually learn the true parameters φ_q and φ_r through Bayesian learning.

I compare the results with those from Section 2 and under the FIRE scenario, selecting housing investment Δ_t as the variable to present.

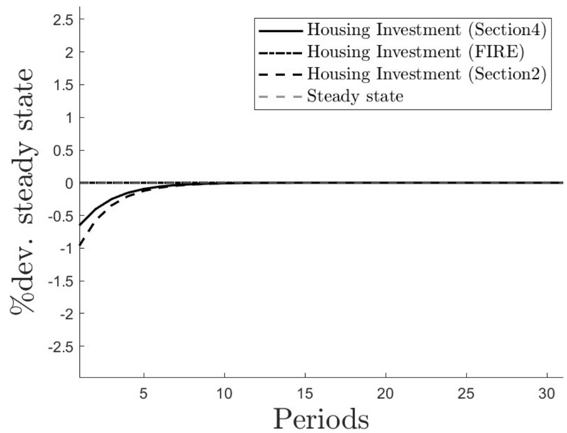

Figure 8: Housing Investments Comparasion (Overestimate Scenario)

It can be observed that housing investment Δ_t in Scenario 4 is more increased than in Scenario 2. When individuals overestimate the reaction degree of the policy rule, the signaling effect—which exacerbates the recession—is weakened. In this case, individuals tend to expect the economy to be in a less severe downturn, leading to less intense pessimistic sentiments and making the signaling effect smaller than in my baseline model. As time progresses, individuals gradually learn the true policy reaction function, and the negative signaling effect gradually increases.

In summary, this paper argues that if policy signals resulting from policy communication are unavoidable—that is, the signaling effect cannot be mitigated—the government should endeavor to adjust the parameters within the subjective signaling policy rule. Specifically, when individuals underestimate the policy rule, the government should openly disclose its policy rules, a recommendation strongly supported by traditional rational expectations models. While this approach constitutes a suboptimal solution, it is advisable under these circumstances. Conversely, when individuals overestimate the policy rule, the government should refrain from disclosing its policy rules, as doing so would amplify the negative signaling effect. Previous literature has suggested that conventional policy communication may be detrimental in such scenarios. Therefore, we posit that the government's strategy regarding expectation management and policy communication should adapt based on individuals' subjective perceptions of the policy rule. Disclosing the policy rule is not universally beneficial and should be contingent upon the prevailing subjective attitudes of the individuals involved.

5 Numerical algorithm

5.1 Solution in the Rational Expectation Model

This section introduces the solution of the baseline model in Section 2.

Before solving the model under incomplete information, I first solve the solution under Full Information Rational Expectations (FIRE). I adopt the algorithm from **?** to solve the FIRE model. I have a total of 20 equations and 20 variables. These variables are divided into state variables X_t and control variables Y_t. The state variables are $X_t = \left[\mathcal{H}_t, B_t, R_t, \xi_t^c, \xi_t^h\right]$, consisting of the current housing consumption $\mathcal{H}_t$, bonds held B_t, current nominal interest rate R_t, and current productivity shocks in the consumption sector and housing sector ξ_t^c, ξ_t^h. The control variables are $Y_t = [\pi_t, \pi_t^*, q_t, N_t^c, N_t^h, \ldots]$, consisting of a series of remaining variables, such as current inflation π_t, inflation under optimal pricing π_t^*, current housing price q_t, and labor supplied to the consumption goods sector and housing sector N_t^c, N_t^h. I can transform the first-order conditions into the following form:

$$f\left(\mathbb{E}_t[X_{t+1}], \mathbb{E}_t[Y_{t+1}], X_t, Y_t\right) = 0$$

Then, I perform log-linearization on the first-order conditions to obtain a 20x20 linear system, consisting of 20 equations and 20 variables. Lowercase letters represent their percentage deviations from the steady state. I have the linear system:

$$[fxp, fyp] \begin{bmatrix} \mathbb{E}_t[x_{t+1}] \\ \mathbb{E}_t[y_{t+1}] \end{bmatrix} = [fx, fy] \begin{bmatrix} x_t \\ y_t \end{bmatrix}$$

According to the algorithm of **?**, I can solve the FIRE model into the following form:

$$X_t = H \cdot X_{t-1} + \eta \cdot \epsilon_t$$
$$Y_t = G \cdot X_t$$

ϵ_t represents the exogenous shocks $\left\{\varsigma_t^c, \varsigma_t^h, \epsilon_t^R\right\}$. The state variable X_t evolves according to a law of motion; X_t in period t is determined by the state variable X_{t-1} in period $t-1$

and the exogenous shocks ϵ_t in period t. The control variable Y_t is determined by the current state variable X_t.

5.2 Expectation Formation Problem

Next, I solve the expectation formation problem.

I conjecture that the solution under incomplete information still has the following form:

$$X_t = \tilde{H} \cdot X_{t-1} + \eta \cdot \epsilon_t$$
$$Y_t = \tilde{G} \cdot X_t$$

According to Klein (2000), η is an exogenously given parameter related to the variance of shocks, which is independent of whether the information is complete or not. The solution of the model $\tilde{H}, \tilde{G}$ differs from the solution under full information H, G due to the setting of incomplete information.

I assume that workers, firms, and the government know the law of motion of the economic system and have full information in period t. The full information set is $\Omega_t = \left\{ \left\{ \epsilon_k^R \right\}_0^t, \left\{ \varsigma_k^c \right\}_0^t, \left\{ \varsigma_k^h \right\}_0^t \right\}$. Therefore, from period 0 onwards, they can deduce the realizations of all variables from period 0 to period t based on the law of motion, i.e., $\left\{ \left\{ X_k \right\}_0^t, \left\{ Y_k \right\}_0^t \right\}$.

I assume that shoppers know the law of motion of the economic system but are unaware of the realized fundamental shocks ϵ_t in the current period. Their incomplete information set in period t is Ω_{t-1}, S_t. Therefore, they can deduce the realizations of all variables from period 0 to period $t-1$ based on the law of motion, i.e., $\left\{ \left\{ X_k \right\}_0^{t-1}, \left\{ Y_k \right\}_0^{t-1} \right\}$, but cannot obtain the realizations of variables X_t, Y_t in period t.

Correspondingly, individuals in period t receive policy signals S_t related to these variables and use them to infer and form expectations:

$$S_t = \tilde{P} M [X_t, Y_t]$$

$\tilde{P}$ is the parameter of the signalling policy rule, and M is the selection matrix, selecting variables of interest to the government. In this model, I set it as Q_t, R_t to capture government policy communication on real estate policy and monetary policy. However, this setting is flexible and does not alter the main conclusions of this paper. Since I assume that individuals know the law of motion of economic variables but are unaware of the realized fundamental shocks ϵ_t in the current period, I have:

$$S_t = \tilde{P}M[X_t, Y_t] = \tilde{P}M[I, \tilde{G}]X_t = \tilde{Z}X_t$$

This expectation formation problem can be solved using the Kalman filter, treating the law of motion of the state variables as the transition equation and the signalling policy rule as the observation equation. I have:

$$\begin{aligned} \text{Transition Equation:} \quad & X_t = \tilde{H} \cdot X_{t-1} + \eta \cdot \epsilon_t \\ \text{Observation Equation:} \quad & S_t = \tilde{Z}X_t \end{aligned}$$

Applying the results of the Kalman filter, I can obtain:

$$\mathbb{E}_t[X_t \mid \Omega_{t-1}, S_t] = \mathbb{E}_{t-1}[X_t \mid \Omega_{t-2}, S_{t-1}] + \tilde{K}[S_t - \tilde{Z}\mathbb{E}_{t-1}[X_t \mid \Omega_{t-2}, S_{t-1}]]$$

The expectation in period t, $\mathbb{E}_t[X_t \mid \Omega_{t-1}, S_t]$, is based on the expectation in period $t-1$, $\mathbb{E}_{t-1}[X_t \mid \Omega_{t-2}, S_{t-1}]$. It makes a linear adjustment according to the difference between the signal received in the current period and the expectation in period $t-1$, with the adjustment magnitude determined by the Kalman gain matrix $\tilde{K}$.

Since shoppers know the law of motion of the economic system, I can obtain:

$$\begin{aligned} \mathbb{E}_t[Y_t \mid \Omega_{t-1}, S_t] &= \tilde{G}\mathbb{E}_t[X_t \mid \Omega_{t-1}, S_t] \\ \mathbb{E}_t[X_{t+1} \mid \Omega_{t-1}, S_t] &= \tilde{H}\mathbb{E}_t[X_t \mid \Omega_{t-1}, S_t] \\ \mathbb{E}_t[Y_{t+1} \mid \Omega_{t-1}, S_t] &= \tilde{G}\tilde{H}\mathbb{E}_t[X_t \mid \Omega_{t-1}, S_t] \end{aligned}$$

6 Conclusion

I investigate the effect of government expectations management through policy signals by introducing incomplete information. The quantitative results of the baseline model demonstrate the signaling effect: it influences individuals' expectations, making the economy more prosperous during booms and more depressed during recessions. Based on this channel, I believe that this kind of policy-related signal in the government's expectations management—that is, releasing policy-related signals to manage expectations—may be inappropriate, and may even have the opposite effect, reducing the effectiveness of stimulus/restrictive policies and often failing to achieve the expected impact.

By introducing an additional layer of uncertainty regarding individuals' misunderstandings of government policy, I find that the negative signaling effect increases when individuals underestimate the government's responsiveness to economic conditions and decreases when they overestimate it. Therefore, if releasing policy signals is necessary and the signaling effect is unavoidable, this implies that the government directly disclosing its policy rules, as suggested in previous literature, may constitute a suboptimal solution. However, this approach depends on individuals' subjective perceptions. In scenarios where the policy rule is underestimated, the government should disclose its policy rules, whereas in cases of overestimation, it should refrain from doing so.

Another possible solution in expectations management is to adopt target-based communication rather than policy-raleted communication, by announcing policy targets. This is a more powerful tool than policy signals because it demonstrates the government's subjective initiative as a strategic player in the game when the economy is excessively depressed or prosperous. I do not cover this aspect, and further research is awaiting to model this issue.

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