Information Design and its Application to Finance

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2019/01/09

What is Information Design?

Mechanism Design and Information Design

- **Mechanism Design**
  - Fix an economic environment and information structure
  - Design the rules of the game to obtain desired outcomes
  - e.g., Auction, Matching
- **Information Design**
  - Fix an economic environment and the rules of the game
  - Design the information structure to obtain desired outcomes

Information Design

- (Loosely,) choosing information structure is choosing signal or additional information
- Kamenica and Gentzkow(2011) "Bayesian Persuasion" is an influential paper
  - prosecutor(sender) vs judge(receiver)
- Two types of information design
  - literal information design: there really is an information designer (most cases)
  - metaphorical information design: there is no information designer (e.g., Bergemann, Heumann, and Morris (2015))

Leading Cases

- Many players and uninformed information designer
  - Communication in Games : Mayerson (1991)
- One player and an informed information designer
  - "Bayesian Persuasion" : Kamenica Gentzkow (2011) and many literatures
- One player and an uninformed information designer
  - Very boring problem
- Many players and an informed information designer
  - Recent papers

General Setting

- Environment : players $1, \cdots, I$; payoff states Θ; prior $\psi \in \Delta(\Theta)$
- Game G : $(A_i, u_i)_{i=1}^I$ where $u_i : A \times \Theta \to \mathbb{R}$
- Information Structure S
  - $t_i \in T_i$ type of player i
  - $\pi : \Theta \to \Delta(T)$ type distribution

Timing

1. Information designer picks and commits to a rule for providing the players with extra messages
2. θ is realized
3. t_i's are privately realized
4. Players receive extra messages according to the information designer's rule
5. Players picks an action based on their prior information and the messages they receive
6. Payoffs are realized

Important Notions

- **Decision Rules**

$$\sigma : T \times \Theta \to \Delta(A)$$

- **Obedience**

  σ is obedient if, for all i, t_i, a_i and $a_i^{'}$,

$$\sum_{a_{-i}, t_{-i}, \theta} u_i((a_i, a_{-i}), \theta)\sigma(a|t, \theta)\pi(t|\theta)\psi(\theta)$$

$$\geq \sum_{a_{-i}, t_{-i}, \theta} u_i((a_i^{'}, a_{-i}), \theta)\sigma(a|t, \theta)\pi(t|\theta)\psi(\theta)$$

Basic Example

Investment Game

- One player "firm" and information designer
- action $a \in \{\text{invest}, \text{not invest}\}$
- state $\theta \in \{G, B\}$, $\psi(G) = \psi(B) = \frac{1}{2}$
- payoffs $u(a, \theta)$ are $(0 < x < 1)$:

$u(a,\theta)$	bad state (B)	good state (G)
invest	-1	x
not invest	0	0

- Information designer wants to maximize the probability of investment
- NB: firm **never** chooses "invest" without no additional information

Single Player without Prior Information

- Decision rule $\sigma : \Theta \to \Delta(A)$ is a pair (p_B, p_G)
- If they are obeyed, the ex ante distribution is:

$u(a\|\theta)\psi(\theta)$	bad state (B)	good state (G)
invest	$\frac{1}{2}p_B$	$\frac{1}{2}p_G$
not invest	$\frac{1}{2}(1-p_B)$	$\frac{1}{2}(1-p_G)$

- Obedience constraint

$$\frac{\frac{1}{2}p_B}{\frac{1}{2}p_B + \frac{1}{2}p_G}(-1) + \frac{\frac{1}{2}p_G}{\frac{1}{2}p_B + \frac{1}{2}p_G}x \geq 0$$

$$\Leftrightarrow p_G \geq \frac{p_B}{x}$$

Equilibrium Set

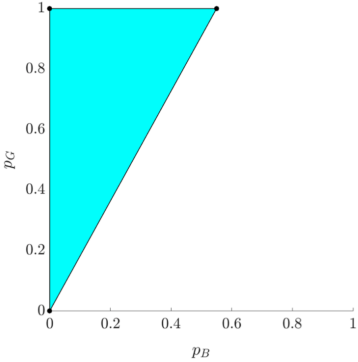

Figure 1: Investment Probabilities with Uninformed Player: x=55/100

- $(p_G, p_B) = (1, x)$ gives the highest probability of investment
- Information designer will obfuscate the state of the world

Single Player with Prior Information

- Observes its type $t \in \{b, g\}$ correctly with prob $q > \frac{1}{2}$

$\pi(t\|\theta)$	bad state (B)	good state (G)
bad signal b	q	$1-q$
good signal g	$1-q$	q

- Decision rule is now $(p_{Bg}, p_{Bg}, p_{Gb}, p_{Gg})$
- Obedience constraint

$$\begin{cases} (1-q)p_{Bg}(-1) + qp_{Gg}x \geq 0 \\ qp_{Bb}(-1) + (1-q)p_{Gb}x \geq 0 \end{cases}$$

- Joint distribution

$$\begin{cases} p_G = qp_{Gg} + (1-q)p_{Gb} \\ p_B = (1-q)p_{Bg} + qp_{Bb} \end{cases}$$

Equilibrium Set

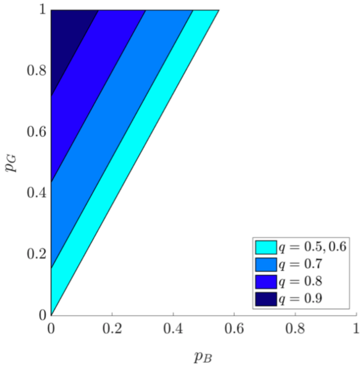

Figure 2: Investment Probability with Informed Player: x=55/100.

- More prior information shrinks the set of BCE
- Weak private information does not constraint information designer

Application to Finance

Persuation in Global Games
with Application to Stress Testing
Inostroza and Pavan, 2018, AER(R&R)

Motivation

- Damages to society of mix-coordination can be severe
  - Monte dei Paschi Siena
- A government's ability can be limited
- Persuasion : the instrument of last resort

Model

- Policy maker
- Agent $i \in [0, 1]$
- Action $a_i \in \{1 \text{ (attack)}, 0 \text{ (not attack)}\}$
- Regime outcome $r \in \{1 \text{ (change)}, 0 \text{ (not change)}\}$
- Regime rule

$$r = \begin{cases} 1 \text{ (if } R(\theta, A) \leq 0) \\ 0 \text{ (otherwise)} \end{cases}$$

- fundamentals $\theta \sim F$
- Private signal $x_i \sim p(\cdot|\theta)$

Payoffs

- PM's payoff

$$U^P(\theta, A) = \begin{cases} W(\theta, A) \text{ (if } r = 0) \\ L(\theta) \text{ (if } r = 1) \end{cases}$$

  with $W > L$
- Agent's payoff from attacking is zero
- Agent's payoff from not attacking is

$$u(\theta, A) = \begin{cases} g(\theta, A) \text{ (if } r = 0) \\ b(\theta, A) \text{ (if } r = 1) \end{cases}$$

  with $g > 0 > b$

Discrosure Policy

- Message function $m : [0,1] \to \mathcal{S}$
- $m_i \in \mathcal{S}$: information disclosed to i
- $M(\mathcal{S}) = \{\mathcal{S}^{[0,1]}\}$: the set of all possible message functions
- Disclosure policy $\Gamma = (\mathcal{S}, \pi)$ where $\pi : \Theta \to \Delta(M(\mathcal{S}))$

Timing

1. PM announces $\Gamma = (\mathcal{S}, \pi)$, and commits to it.
2. θ and $\boldsymbol{x}$ are realized.
3. m_i is disclosed to i according to diclosure policy.
4. Agents simulaneously choose their actions.
5. The regime outcome and payoffs are realized.

Important Concepts

- **Most Aggressive Rationlizable Profile**(MAMP)
  The strategy profile that minimizes the PM's ex-ante expected payoff over all profiles surviving IDISDS.
- **Perfect-coordination Property**(PCP)
  A disclosure policy has the perfect-coordination property if it induces all agents to take the same action, after any information it discloses.
- **Non-discriminatory Policy**
  PM disclose the same informaiton to all market participants, that is, $m_i = m_j$ for any $i, j \in [0, 1]$.

Perfect Coordination Property

Theorem

Given any Γ, there exists Γ^* satisfying PCP and yielding PM a payoff weakly higher than Γ.

- Policy Γ^* removes any **strategic uncertainty**.
- It preserves haterogeneity in **structual uncertainty**.
- Optimal stress test
  - need not be expected to generate consensus among market participants
  - but should be transparent enough to remove uncertainty about market response

Public Disclosures

Theorem

Given any non-discriminatory Γ, there exists a binary non-discriminatory $\Gamma^* = \{\{0, 1\}, \pi^*\}$ satisfying PCP and yielding PM a payoff weakly higher than Γ with an additional assumption.

- Optimal non-discriminatory policy: **stochastic pass/fail test**
- Optimality of such policies depends on agents' beliefs.

Monotone Test

Theorem

Given any non-discriminatory Γ, there exists a deterministic non-discriminatory $\Gamma^* = \{\{0,1\}, \pi^*\}$ satisfying PCP and yielding PM a payoff weakly higher than Γ under some conditions. The policy Γ^* is defined by θ^* such that, for all $\theta \leq \theta^*$, $\pi(1|\theta) = 1$, whereas, for all $\theta > \theta^*$, $\pi(0|\theta) = 1$

- With some assumptions, optimal non-discriminatory policy becomes a **monotone test**.
- Optimality of monotone policies may not be guaranteed even in canonical cases.

Discriminatory Discrosures

- In general, optimal stress test involves
  - public pass/fail announcement
  - discriminatory disclosures
- Benefits of discriminatory disclossure do not come from the possibility of tailoring the signals to prior beliefs.
- Rather, by enhancing the dispersion of posterior beliefs, PM maks it harder for agents to coordinate on a successful attack.
- However, with some assumptions, optimal policy is non-discriminatory.

Conclusion

- Optimal disclosure policy must satisfy PCP, that is, remove any strategic uncertainty.
- When PM is restricted to disclosing same information to the agents, optimal policy is simple pass/fail test.
- Optimal policy need not be monotone test.