Trading and Information Diffusion in Over-The-Counter Markets

Babus and Kondor, Econometrica, 2018

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Introduction

This paper

- This paper gives a novel approaching to modeling OTC market that reflects the features following:
    - bilateral transactions
    - dispersed price
    - persistent trading relationships
    - a few traders intermediate a large share of trading volume
- We focus on how decentralization and adverse selection jointly influence information diffusion, expected profits, trading costs, and welfare.

Outline

- Consider the **OTC game** and **conditional guessing game**.
- Derive the equilibrium in OTC game.
- Derive the equilibrium in the conditional guessing game.
- Show these equilibria are equivalent.
- Derive its properties.

Model

State and Signal

- n risk-neutral dealers who trade bilaterally a divisible risky asset.
- The value of the asset θ^i can be decompose to common value and individual value

$$\theta^i = \hat{\theta} + \eta^i$$

with $\theta^i \sim N(0, \sigma_{\hat{\theta}}^2)$ and $\eta^i \sim \text{i.i.d.} N(0, \sigma_{\eta}^2)$ and $Corr(\hat{\theta}, \eta^i) = 0$
- θ^i is normally distributed with mean 0 and variance $\sigma_{\theta}^2 = \sigma_{\hat{\theta}}^2 + \sigma_{\eta}^2$.
- The degree of interdependence can be written

$$\rho = \frac{\sigma_{\hat{\theta}}^2}{\sigma_{\theta}^2}$$

- Each dealer receives a private signal, s^i, such that

$$s^i = \theta^i + \epsilon^i$$

with $\epsilon^i \sim \text{i.i.d.} N(0, \sigma_{\epsilon}^2)$ and $Corr(\theta^j, \epsilon^i) = 0$ for all i and j.

Network

- Dealers are organized into a trading network, g.
- A link $ij \in g$ implies that i and j are potential trading partners.
- Let g^i denote the set of i's neighbors.
- $m^i \equiv |g^i|$ is the number of i's neighbors.
- Let q^i_{ij} denote the quality that dealer i trades over link ij.
- The price is denoted by p_{ij}.
- Links in the network are undirected, such that
    - if $ij \in g$, then $ji \in g$ also
    - $p_{ij} = p_{ji}$ and $q^i_{ij} = q^i_{ji}$

Strategy

- The strategy of dealer i is a map from the signal space to the space of **generalized demand functions**.
- For each dealer i with signal s^i, the strategy $\boldsymbol{Q}^i : R^{m^i} \to R^{m^i}$ that maps the vector of prices, $\boldsymbol{p}_{g^i} = (p_{ij})_{j \in g^i}$, into a vector of quantities she wishes to trade.

$$\boldsymbol{Q}^i(s^i; \boldsymbol{p}_{g^i}) = (Q^i_{ij}(s^i; \boldsymbol{p}_{g^i}))_{j \in g^i}$$

- When dealer i buys on the link ij, the quantity q^i_{ij} is positive. Conversely, when dealer i sells, q^i_{ij} is negative.
- Note that $Q^i_{ij}(s^i; \boldsymbol{p}_{g^i})$ depends only on $\boldsymbol{p}_{g^i}$, not on the full price vector.

Customers

- Each dealer also serves a price-sensitive customer base.
- Dealer i trades with the customers she associates to the link ij at the same price she trades with dealer j, p_{ij}, adjusted by an exogenous markup.
- For each transaction between i and j, the customer base generates a downward-sloping demand

$$D_{ij}(p_{ij}) = \beta_{ij} p_{ij}$$

with the constant $\beta_{ij} < 0$ is a summary statistic for dealer i and j's customers' preferences and the markup that the dealers charge.

Payoff

- The expected payoff for dealer i is

$$E\left[\sum_{j\in g^i} Q^i_{ij}(s^i;\boldsymbol{p}_{g^i})(\theta^i - p_{ij})|s^i,\boldsymbol{p}_{g^i}\right]$$

where p_{ij} is the elements of the price vector $\boldsymbol{p}$ defined by the smallest element of the set

$$\tilde{P}(\{\boldsymbol{Q}^i(s^i;\boldsymbol{p}_{g^i})\}_i,\boldsymbol{s}) \equiv \{\boldsymbol{p}|Q^i_{ij}(s^i;\boldsymbol{p}_{g^i}) + Q^j_{ij}(s^j;\boldsymbol{p}_{g^j}) + \beta_{ij}p_{ij} = 0, \forall ij \in g\}$$

Equilibrium

- **Definition 1** : A **Linear Bayesian Nash equilibrium** of the OTC game is a vector of linear generalized demand functions $\{\boldsymbol{Q}^1(s^1; \boldsymbol{p}_{g^1}), \cdots, \boldsymbol{Q}^n(s^n; \boldsymbol{p}_{g^n})\}$ such that $\boldsymbol{Q}^i(s^i; \boldsymbol{p}_{g^i})$ solves the problem

$$\max_{(Q^i_{ij})_{j \in g^i}} E\left\{ \left[\sum_{j \in g^i} Q^i_{ij}(s^i; \boldsymbol{p}_{g^i})(\theta^i - p_{ij}) \right] |s^i, \boldsymbol{p}_{g^i} \right\}$$

  for each dealer i, where $\boldsymbol{p} = P(\cdot, \boldsymbol{s})$.
- Solving this problem is equivalent to finding a fixed point in demand functions.

The Equilibrium

Derivation of Demand Functions

- We conjecture an equilibrium in linear demand functions

$$Q^i_{ij}(s^i; \boldsymbol{p}_{g^i}) = t^i_{ij}\left(y^i_{ij}s^i + \sum_{k\in g^i} z^i_{ij,ik}p_{ik} - p_{ij}\right)$$

- t^i_{ij} is the **trading intensity** of dealer i on the link ij
- y^i_{ij} and $z^i_{ij,ik}$ capture the effects specific to the dealer's private signal and the price p_{ik} on the quantity that dealer i demands on the link ij.

Equilibrium Price

- Market clearing condition is

$$Q^i_{ij}(s^i; \boldsymbol{p}_{g^i}) + Q^j_{ij}(s^j; \boldsymbol{p}_{g^j}) + \beta_{ij} p_{ij} = 0$$

- From the projection theorem, we can show $y^i_{ij} = y^i$ and $z^i_{ij,ik} = z^i_{ik}$ for all i, j and k.
- The equilibrium price between any pair of dealers i and j as a linear combination of the posterior beliefs of i and j:

$$p_{ij} = \frac{t^i_{ij} E(\theta^i | s^i, \boldsymbol{p}_{g^i}) + t^j_{ij} E(\theta^j | s^i, \boldsymbol{p}_{g^j})}{t^i_{ij} + t^j_{ij} - \beta_{ij}}$$

The Conditional Guessing Game

- Consider a set of n agents in the network g.
- Each agent i makes a guess e^i about her value of the asset θ^i.
- Dealer i chooses a guess function $\mathcal{E}^i(s^i; \boldsymbol{e}_{g^i})$.
- When the uncertainty is resolved, agent i recieves a payoff $-(\theta^i - e^i)^2$, where e^i is an element of $\boldsymbol{e}$ defined by the smallest element of the set

$$\Xi(\{\mathcal{E}^i(s^i; \boldsymbol{e}_{g^i})\}^i, \boldsymbol{s}) \equiv \{\boldsymbol{e} | e^i = \mathcal{E}^i(s^i; \boldsymbol{e}_{g^i}), \forall i\}$$

Deriving Posterior Beliefs

- **Definition 2** : An equilibrium of the conditional guessing game is given by a strategy profile $(\mathcal{E}^1, \cdots, \mathcal{E}^n)$ such that each agent i chooses strategy $\mathcal{E}^i : R \times R^{m^i} \to R$ to maximize her expected payoff
$$\max_{\mathcal{E}^i}\{-E((\theta^i - \mathcal{E}^i(s^i; \boldsymbol{e}_{q^i}))^2 | s^i, \boldsymbol{e}_{g^i})\}$$
where $\boldsymbol{e} = \Xi(\cdot, \boldsymbol{s})$.
- Agent i's optimal guess function is then given by
$$\mathcal{E}^i(s^i; \boldsymbol{e}_{g^i}) = E(\theta^i | s^i, \boldsymbol{e}_{g^i})$$

Equilibrium in Conditional Guessing Game

- **Proposition 1** : In the conditional guessing game, for any network g, there exists an equilibrium in linear guess functions such that
$$\mathcal{E}^i(s^i; \boldsymbol{e}_{g^i}) = \bar{y}^i s^i + \bar{\boldsymbol{z}}_{g^i} \boldsymbol{e}_{g^i}$$
for any i, where $\bar{y}^i$ is a scalar and $\bar{\boldsymbol{z}}_{g^i} = (\bar{z}^i_{ij})_{j \in g^i}$ is a row vector of length m^i.
- The guessing game has an equilibrium in any network.

Main Result

- **Proposition 2** : Let $\bar{y}^i$ and $\bar{\mathbf{z}}_{g^i} = (\bar{z}^i_{ij})_{j\in g^i}$ be the coefficients that support an equilibrium in the conditional game and let $e^i = E(\theta^i | s^i, \mathbf{e}_{g^i})$ be the corresponding equilibrium expectation of agent i. Then, there exists a Linear Bayesian Nash equilibrium in the OTC game whenever $\rho < 1$ and the following system has a solution $\{y^i, z^i_{ij}\}_{i=1,\cdots,n, j\in g^i}$ such that $z^i_{ij} \in (0,2)$:

$$\frac{y^i}{\left(1 - \sum_{k\in g^i} z^i_{ik} \frac{2-z^k_{ki}}{4-z^i_{ik}z^k_{ki}}\right)} = \bar{y}^i$$

$$z^i_{ij} \frac{\frac{2-z^i_{ij}}{4-z^i_{ij}z^j_{ji}}}{\left(1 - \sum_{k\in g^i} z^i_{ik} \frac{2-z^k_{ki}}{4-z^i_{ik}z^k_{ki}}\right)} = \bar{z}^i_{ij}, \forall j \in g^i$$

Proposition 2 (cont.)

- All z^i_{ij} are determined by ρ and the ratio $\sigma = \frac{\sigma^2_\epsilon}{\sigma^2_\theta}$ and independent of β_{ij}. The equilibrium demand functions are given by

$$Q^i_{ij}(s^i; \boldsymbol{p}_{g^i}) = t^i_{ij} \left(y^i_{ij} s^i + \sum_{k \in g^i} z^i_{ij,ik} p_{ik} - p_{ij} \right)$$

with

$$t^i_{ij} = -\beta_{ij} \frac{2 - z^j_{ij}}{z^i_{ij} + z^j_{ji} - z^i_{ij} z^j_{ji}}$$

The equilibrium beliefs are $E(\theta^i | s^i, \boldsymbol{p}_{g^i}) = y^i s^i + \sum_{j \in g^i} z^i_{ij} p_{ij}$, whereas the equilibrium prices and quantities are

$$p_{ij} = \frac{t^i_{ij} e^i + t^j_{ij} e^j}{t^i_{ij} + t^j_{ij} - \beta_{ij}}, \; q^i_{ij} = t^i_{ij}(e^i - p_{ij})$$

Interpretation

- Sufficient conditions under which we can construct an equilibrium of the OTC game building on an equilibrium of the conditional guessing game.
- By considering guessing game, we can solve a simpler fixed-point problem.
- Proposition 2 describes a simple numerical algorithm to find the OTC game for any network.
- Proposition 3 :
    1. In any network in the circulant family, the equilibrium of the OTC game exists.
    2. In a star network, the equilibrium of the OTC game exists.

Information Diffusion

Information Diffusion via Prices

- How the market structure affects the info. diffusion?
  - How strategic behaviors affect the diffusion?
  - How network interacts with prices as information aggregator?
- **Proposition 4** : In any Linear Bayesian Nash equilibrium of the OTC game, the vector with elements e^i defined as

$$e^i = E(\theta^i | s^i, \boldsymbol{p}_{g^i})$$

  is an equilibrium expectation vector in the conditional guessing game.
- The equivalence of beliefs on the two games implies that any feature of the beliefs in the OTC game must be unrelated in any way to price manipulation, market power, or other profit-related motives.

Network Structure and Diffusion

- **Proposition 5** : Suppose that there exists an equilibrium in the OTC game. Then, in any connected network g, each bilateral prices is a linear combination of all signals in the economy, with strictly positive weight on each signal.
- A decentralized trading structure can be surprisingly effective in transmitting information.
- Each price partially incorporates all the private signals in the economy.

Reveal by Price

- Dealers in the OTC market do not learn from prices all the relevant information in the economy.
- The equilibrium prices are **privately fully revealing** if, for each dealer i, $(s^i, \boldsymbol{p}_{g^i})$ is a sufficient statistic of the vector of signals $\boldsymbol{s}$, in the estimation of θ^i.
- In the complete network, prices are privately fully revealing.
  - Since dealer i observes $n-1$ prices and she knows her own signal, she can invert the prices to obtain the signals of the other dealers.

Planner's Problem

- We study efficiencies in how agents learn from prices.
- **Informational efficiency**:

$$U(\{\bar{y}^i, \bar{\boldsymbol{z}}_{g^i}\}_{i\in\{1,\cdots,n\}}) \equiv -E\left[\sum_i (\theta^i - \mathcal{E}^i(s^i; \boldsymbol{e}_{g^i}))^2 | \boldsymbol{s}\right]$$

  where $\mathcal{E}^i(\cdot)$ is the guess function of dealer i in the conditional guessing game.
- Planner's Problem: maximize $U(\{\bar{y}^i, \bar{\boldsymbol{z}}_{g^i}\})$ w.r.t. $\mathcal{E}$ subject to $\boldsymbol{e} = \Xi(\cdot, \boldsymbol{s})$ and $\Xi(\{\mathcal{E}^i(s^i; \boldsymbol{e}_{g^i})\}^i, \boldsymbol{s}) \equiv \{\boldsymbol{e} | e^i = \mathcal{E}^i(s^i; \boldsymbol{e}_{g^i}), \forall i\}$.

Informational Efficiency

- In general, beliefs are not constrained informationally efficient.
- This is a consequence of the learning externality arising from the interaction between the interdependent value environment and the network structure.

Simple Networks and Real-World OTC Markets

Welfare, Profits, Liquidity

- How the OTC market structure and adverse selection affect dealers' expected profit, welfare, and illiquidity?
- Expected Profit of dealer i:

$$E\left(\sum_{ij\in g^i} q^i_{ij}(\theta^i - p_{ij})\right) = \sum_{ij\in g^i} E((e^i - p_{ij})^2)$$

- Consumers' expected utility at link ij:

$$-\frac{\beta_{ij}}{2}E(p^2_{ij})$$

- Total welfare, the sum of profits and consumers' utility

$$\sum_{ij\in g}\left(\frac{\beta_{ij}}{2}E(p^2_{ij}) + E(\theta^i q^i_{ij}) + E(\theta^j q^j_{ij})\right)$$

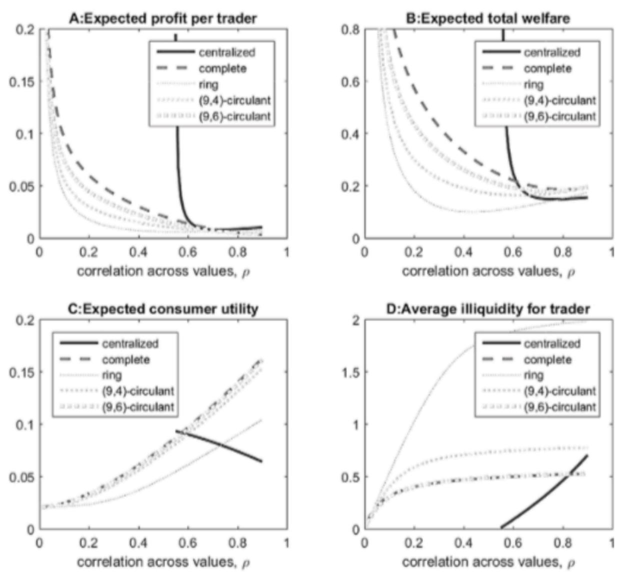
Illiquidity

- Equilibrium trading intensity:

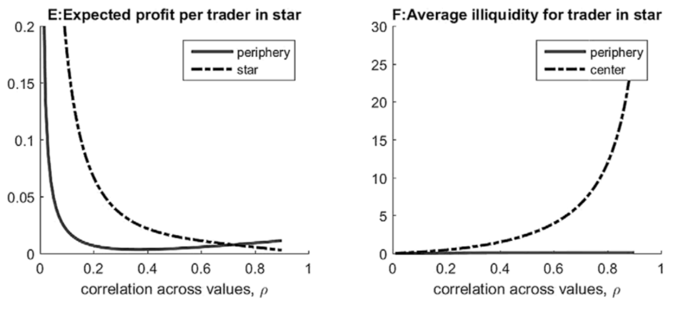
$$t_{ij}^{i} = t_{ij}^{j}(1 - z_{ij}^{j}) - \beta_{ij}$$

- z_{ij}^{j} is a natural measure of how much dealer j is concerned about adverse selection when trading with i.
- $\frac{1}{t_{ij}^{i}}$ is the price impact of trade i at link ij.
- Averaging $\frac{1}{t_{ij}^{i}}$ over the links of i provides a natural dealer-level illiquidity measure.

Comparative Statics



Star Network



- Central agents do not always earn higher expected profit than periphery agents.
- The market is less liquid for the central dealer than for the periphery dealer.

Conclusion

Conclusion

- Information diffusion through prices is unaffected by dealers' strategic motives.
- Each price partially incorporates the private information of all dealers.
- This paper identifies an informational externality that constrains the informativeness of prices.
- Decentralization can both increase or decrease welfare.
- The main determinant of a dealer's trading cost is not her centrality but rather the centrality of her counterparties.