Systemic Risk and Stability in Financial Networks

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Introduction

Introduction

- The view that the architecture of the financial system plays a central role in shaping systemic risk has become conventional wisdom.
- There are two conflicting views on the relationship between the structure of the financial network and the extent of financial contagion.
    - A more interconnected architecture enhances the resilience of the system ti the insolvency of any individual bank. (Allen and Gale(2000), Freixas, Parigi, and Rochet(2000))
    - Dense interconnections may function as a destabilizing force, paving the way for systemic failures. (Vivier-Lirimont(2006), Blume te al.(2011, 2013))

Contribution

- This paper provides a framework for studying the network's role as a shock propagation and amplification mechanism.
- The Executive Director for Financial Stability at the Bank of England suggested that highly interconnected financial network may be **"robust-yet-fragile"**.
    - "within a certain range, connections serve as shock-absorbers and connectivity engenders robustness".
    - beyond the range, "the system flips to the wrong side of the knife-edge"

Main Results

- Regardless of the structure of the financial network, a payment equilibrium always exists and is generically unique.
- When the magnitude of negative shocks is below a certain threshold, a more diversified pattern of interbank liabilities leads to a less fragile financial network.
- As the magnitude or the number of negative shocks crosses certain thresholds, highly diversified lending patterns facilitate financial contagion and create a more fragile system.

Model

Model - Financial Institutions (1)

- $\mathcal{N} = \{1, \cdots, n\}$: risk-neutral banks
- The economy lasts for three periods, $t = 0, 1, 2$
- Each bank i is endowed with k_i units of capital that it can either hoard as cash, lend to other banks, or invest in a project
- Project yields a random return z_i at $t = 1$ and non-pledgeable long-term return of A at $t = 2$
- The bank can (partially) liquidate its project at $t = 1$, but can only recover a fraction $\zeta < 1$

Model - Financial Institutions (2)

- Interbank lending takes place through **standard debt contracts** signed at $t = 0$
- k_{ij} denote the amount of capital borrowed by bank j from bank i
- The face value of j's debt to i is equal to $y_{ij} = R_{ij}k_{ij}$ where R_{ij} is interest rate
- Each bank must meet an outside obligation of magnitude $v > 0$ at $t = 1$
- The sum of liabilities of bank i is thus equal to $y_i + v$, where $y_i = \sum_{j \neq i} y_{ji}$

Model - Financial Institutions (3)

- All debts have to be cleared at $t = 1$
- If bank j is unable to meet its $t = 1$ liabilities in full, it has to liquidate its project prematurely
- Assume that all junior creditors are of equal seniority
- If bank j can meet its senior liabilities, v, but defaults on its debt to the junior creditors, they are repaid in proportion to the face value of the contracts.

Model - The Financial Network

- Financial Network : the bilateral debt contracts in the economy as a weighted, directed graph on n vertices, where each vertex corresponds to a bank.
- The weight of the edge from i to j is equal to y_{ij}
- We denote a financial network with the collection of interbank liabilities $\{y_{ij}\}$
- A financial network is **symmetric** if $y_{ij} = y_{ji}$ for all pairs of banks i and j.
- A financial network is **regular** if $\sum_{j \neq i} y_{ij} = \sum_{j \neq i} y_{ji} = y$ for some y and all banks i

Example - The Financial Network

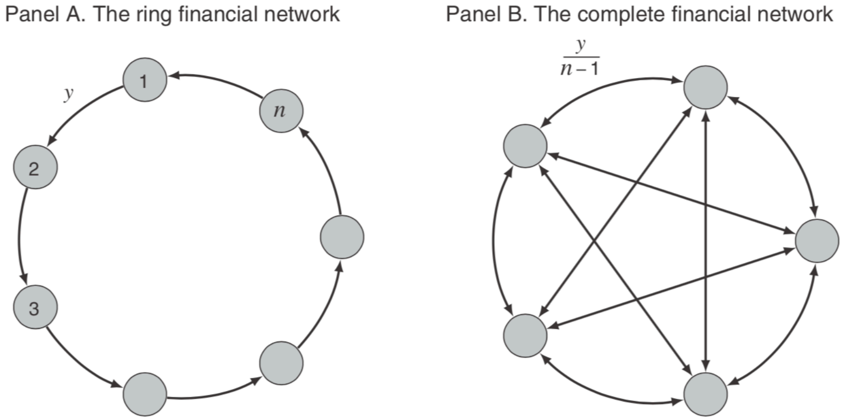

FIGURE 1. THE RING AND THE COMPLETE FINANCIAL NETWORKS

Payment Equilibrium

Liquidation

- x_{js} : the repayment by bank s on its debt to bank j at $t = 1$, $x_{js} \in [0, y_{js}]$
- When bank j does not liquidate its project, the total cash flow is $h_j = c_j + z_j + \sum_{s \neq j} x_{js}$
- If $h_j > v + y_j$, the bank can meet its liability in full, then $x_{ij} = y_{ij}$.
- If $h_j < v + y_j$, the bank needs to liquidate its project.
    - The bank liquidate its project up to the point where it can cover the shortfall $v + y_j - h_j$, or entirety to pay back as much as possible
- Mathematically, the bank's liquidation $l_j \in [0, A]$

$$l_j = \left[\min\{ \frac{1}{\zeta}(v + y_j - h_j), A \} \right]^+ \tag{1}$$

Repayment

- If the bank cannot pay its debts in full, it defaults and its creditors are repaid according to their seniority.
- If $h_j + \zeta A < v$, the bank defaults on its senior liabilities and its junior creditors receive nothing, $x_{ij} = 0$.
- If $h_j + \zeta A \in (v, v + y_j)$, senior liabilities are paid in full and the junior creditors are repaid in proportion to the face value of their contracts.
- Thus,

$$x_{ij} = \frac{y_{ij}}{y_j} \left[\min\{y_j, h_j + \zeta l_j - v\}\right]^+ \tag{2}$$

Payment Equilibrium

- **Definition 2** : For a given realization of the projects' short-term returns and the cash available to the banks, the collections $(\{x_{ij}\}, \{l_i\})$ of interbank debt repayments and liquidation decisions is a **payment equilibrium** of the financial network if (1) and (2) are satisfied for all i and j simultaneously.
- **Proposition 1** : For any given financial network, cash holdings, and realization of shocks, a payment equilibrium always exists and generically unique.

Social Surplus

- For any given financial network and the corresponding payment equilibrium, we define the social surplus in the economy as the sum of the returns to all agents; that is,

$$u = \sum_{i=1}^{n} (\pi_i + T_i)$$

where $T_i \leq v$ is the transfer from bank i to its senior creditors and π_i is the bank's profit

Financial Contagion

Assumptions

- We focus on the regular financial networks.
- We also assume that the short-term returns on the bank's investment are i.i.d. and only can have two values $z_i \in \{a, a - \epsilon\}$
  - $a > v$ is the return in the "business as usual" regime
  - $\epsilon \in (a - v + \zeta A, a)$ corresponding to the magnitude of a negative shock
- We assume that all banks hold the same amount of cash, which we normalize zero.
- We initially assume only one bank is hit with negative shock, $p = 1$ and the proceeds from liquidation are "trivial", $\zeta = 0$.

Financial Contagion

- **Proposition 2** : Conditional on the realization of p negative shocks, the social surplus in the economy is equal to

$$u = n(a + A) - p\epsilon - (1 - \zeta) \sum_{i=1}^{n} l_i$$

- **Definition 3** : Consider two regular financial networks $\{y_{ij}\}$ and $\{\tilde{y}_{ij}\}$. Conditional on the realization of p negative shocks,
    1. $\{y_{ij}\}$ is more **stable** than $\{\tilde{y}_{ij}\}$ if $E_p u \geq E_p \tilde{u}$, where E_p is the expectation conditional on the realization of p negative shocks.
    2. $\{y_{ij}\}$ is more **resilient** than $\{\tilde{y}_{ij}\}$ if $\min u \geq \min \tilde{u}$, where the minimum is taken over all positive realizations of p negative shocks.

Aggregate Interbank Liabilities

- **Proposition 3** : For a given regular financial network $\{y_{ij}\}$. let $\tilde{y}_{ij} = \beta y_{ij}$ for all $i \neq j$and some constant $\beta > 1$. Then, financial network $\{\tilde{y}_{ij}\}$ is less stable and resilient than $\{y_{ij}\}$
- Larger liabilities raise the exposure of each bank to the potential distress at its counterparties, hence facilitating contagion.

Small Shock Regime

- **Proposition 4** : Let $\epsilon^* = n(a - v)$ and suppose that $\epsilon < \epsilon^*$. Then, there exists y^* such that for $y > y^*$,
  1. The ring network is the least resilient and least stable financial network.
  2. The complex network is the most resilient and most stable financial network.
- Proposition 4 is thus in line with the observations made by Allen and Gale(2000) and Freixas, Parigi, and Rochet(2000).
- Intuition : a more diversified pattern of interbank liabilities implies that the burden of any potential losses is shared among more banks, creating a more robust financial system.

δ-connected Financial Network

- **Definition 5** : A regular financial network is δ-connected if there exists a collection of banks $\mathcal{S} \subset \mathcal{N}$ such that $\max\{y_{ij}, y_{ji}\} \leq \delta y$ for all $i \in \mathcal{S}$ and $j \notin \mathcal{S}$.
- In a δ-connected financial network, the fraction of liabilities of banks inside and outside of $\mathcal{S}$ to one another is no more than $\delta \in [0, 1]$.

Large Shock Regime

- **Proposition 6** : Suppose that $\epsilon > \epsilon^*$ and $y > y^*$. Then,
  1. The complete network and the ring networks are the least stable and least resilient financial networks.
  2. For small enough values of δ, any δ-connected financial network is strictly more stable and resilient than the ring and complete financial networks.
- When the magnitude of the negative shock crosses the critical threshold ϵ^*, the complete network exhibits a form of **phase transition**.
- Intuition : since all banks in the complete network are creditors of the distressed bank, the adverse effects of the negative shock are transmitted to them.

Shock Absorbers

- Excess liquidity of the non-distressed banks at $t = 1$
  - The impact of a shock is attenuated once it reaches banks with excess liquidity.
  - This mechanism is best utilized in dense financial network.
- The claim v of senior creditors of the distressed bank
  - The senior creditors can be forced to bear the losses, and hence limit the extent of contagion.
  - This mechanism is best utilized in weakly connected financial network.

Other Results

- Harmonic distance
  - Def :
$$m_{ij} = 1 + \sum_{k \neq j} \frac{y_{ik}}{y} m_{kj}$$
  - Distance between banks in a financial network, which takes into account the intensity of each connection.
  - If bank j is hit with negative shock, the banks whose harmonic distance is small defaults.
- Bottleneck parameter
  - Def :
$$\phi = \min_{\mathcal{S} \subset \mathcal{N}} \sum_{i \in \mathcal{S}} \sum_{j \notin \mathcal{S}} \frac{y_{ij}/y}{|\mathcal{S}||\mathcal{S}^c|}$$
  - How the financial network can be partitioned into two roughly equally-sized components.
  - We can see how many banks default by using this parameter.

Extensions

Extensions, Generalizations

- We can have same results even when some assumptions are relaxed.
    - Multiple shocks :multiple banks hit negative shocks
    - Non-trivial liquidation : $\zeta > 0$
    - Size heterogeneity : all assets and liabilities of bank *i* are scaled by a constant $\theta_i > 0$

Conclusion

Conclusion

- As long as the magnitude of negative shocks is below a certain threshold, a more diversified pattern of interbank relationship leads to less fragility.
- When negative shocks are larger than a certain threshold, weakly connected network is less prone to systemic failures.
- Policy implication
  - When regulating the extent and nature of interbank linkage, it must be based on the expected size of the negative shocks.
  - Efficiency of the network(working paper version)