スラブ復号と格子基底縮小を用いた過負荷MIMO信号検出に関する一検討

早川　諒[†]　林　和則[†]　金子めぐみ[†]

† 京都大学大学院情報学研究科 〒 606-8501 京都市左京区吉田本町
E-mail: †rhayakawa@sys.i.kyoto-u.ac.jp, {kazunori,meg}@i.kyoto-u.ac.jp

あらまし　受信アンテナが送信ストリーム数よりも少ない過負荷 MIMO 伝送において，スラブ復号と格子基底縮小 (Lattice Reduction, LR) を用いた低演算量な信号検出法である Slab-LR-MMSE (Minimum Mean Square Error)-SIC (Successive Interference Cancellation) が提案されている．本稿では，Slab-LR-MMSE-SIC におけるいくつかのパラメータの決定法を提案する．また，Slab-LR-MMSE-SIC の誤り率の上界を導出し，Slab-LR-MMSE-SIC が最尤推定法と同じダイバーシチオーダーを達成可能であることを示す．

キーワード　過負荷 MIMO，格子基底縮小，スラブ復号

A Study on Overloaded MIMO Signal Detection Using Slab Decoding and Lattice Reduction

Ryo HAYAKAWA[†], Kazunori HAYASHI[†], and Megumi KANEKO[†]

† Graduate School of Informatics, Kyoto University, Yoshida-Honmachi, Sakyo-ku, Kyoto, 606-8501 Japan
E-mail: †rhayakawa@sys.i.kyoto-u.ac.jp, {kazunori,meg}@i.kyoto-u.ac.jp

Abstract　As a low complexity signal detection scheme for overloaded multiple-input multiple-output (MIMO) systems, Slab-LR (Lattice Reduction)-MMSE (Minimum Mean Square Error)- SIC (Successive Interference Cancellation) has been proposed. In this paper, we propose new selection criteria of some parameters in Slab-LR-MMSE-SIC. In addition, we derive an upper bound of the error probability of Slab-LR-MMSE-SIC and show that it can achieve the same diversity order as the optimal maximum likelihood (ML) detection.

Key words　overloaded MIMO, lattice reduction, slab decoding

1. Introduction

Many research efforts are being increasingly towards the design of future multiple-input multiple-output (MIMO) communication systems. In common MIMO systems, the number of receive antennas are set to be greater than that of transmitted streams. In some cases, however, sufficient number of receive antennas may not be available at the receiver due to limitations in size, weight, cost, and/or power consumption. Such MIMO systems, where the number of receive antennas are less than that of transmitted streams, are called overloaded (underdetermined) MIMO systems [1], [2].

For overloaded MIMO systems, various signal detection schemes have been proposed [1]–[8]. Maximum likeli-

hood (ML) detection can achieve the best bit error rate (BER) performance [9], however, its computational complexity increases exponentially with the number of transmitted streams because it searches over all possible candidates of transmitted signals. In order to reduce the complexity, slab-sphere decoding (SSD), which is based on the idea of sphere decoding [10], [11] for non-overloaded MIMO systems, has been proposed in [1]. Although SSD can achieve comparable performance as ML detection, its complexity is still high, especially in the worst case, where the number of possible candidates could not be reduced. On the other hand, to apply the lattice reduction (LR) [12], [13]-aided detectors [14]–[16] to overloaded MIMO systems, pre-voting cancellation (PVC) has been proposed [3]. Firstly, it divides the transmitted signals into two parts, the post-voting vector and pre-voting

vector. The former contains the same number of signal elements as that of receive antennas, and the latter contains the remaining elements. Next, it obtains the estimate of the post-voting vectors for each pre-voting vector candidate by using LR-aided MMSE-SIC detection. Finally, the estimate of the original signal is determined by maximum likelihood among all the possible candidates. Since this scheme searches all pre-voting vectors, its complexity increases exponentially with the difference between the number of the transmitted streams and that of receive antennas. To reduce the computational complexity, Slab-LR-MMSE-SIC, which uses both the idea of slab decoding [1] and LR-aided MMSE-SIC with PVC, has been proposed [17]. It uses slab decoding to reduce the candidates of the pre-voting vector and determines the post-voting vectors for each pre-voting vector candidate by LR-aided MMSE-SIC detection. By reducing the candidates of pre-voting vector, it can also reduce the computational complexity.

In this paper, we propose a selection criterion for the range of search in slab decoding. With the proposed criterion, we can determine the range of search by setting an acceptable error probability of slab decoding. We also propose a simple method for selecting the indexes of pre-voting vector and post-voting vector in order to reduce the number of candidates of pre-voting vector. In addition, we derive an upper bound for the error probability of Slab-LR-MMSE-SIC and show that Slab-LR-MMSE-SIC can achieve the full diversity order. Simulation results show that Slab-LR-MMSE-SIC can achieve almost the same BER performance as the conventional schemes while significantly reducing the number of candidates of pre-voting vector and hence the required computational complexity.

In the rest of the paper, we will use the following notations. Superscript $^\mathrm{T}$ and $^\mathrm{H}$ denote transpose and Hermitian transpose, respectively. $\boldsymbol{I}_n$ represents an $n \times n$ identity matrix and $\boldsymbol{0}_n := [0 \cdots 0]^\mathrm{T} \in \mathbb{R}^n$. For a vector $\boldsymbol{x} = [x_1 \cdots x_n]^\mathrm{T} \in \mathbb{R}^n$, $\|\boldsymbol{x}\| := \sqrt{\sum_{i=1}^n x_i^2}$ denotes ℓ_2-norm of $\boldsymbol{x}$. For a complex matrix $\boldsymbol{A}$, $\mathrm{Re}\{\boldsymbol{A}\}$ and $\mathrm{Im}\{\boldsymbol{A}\}$ represent the real and imaginary parts of $\boldsymbol{A}$, respectively. For a set $\mathcal{V}$, $|\mathcal{V}|$ denotes the cardinality of $\mathcal{V}$. $\mathrm{E}[\cdot]$ stands for expectation operator.

2. System Model

We consider an overloaded MIMO system with n transmit antennas and m receive antennas $(n > m)$. For simplicity, precoding is not considered and the number of transmitted streams is assumed to be equal to that of transmit antennas. In the transmitter, information bits are mapped to n symbols, converted by the serial-parallel converter, and sent from the transmit antennas. Here, $\tilde{s}_j$ $(j = 1, \ldots, n)$

represents the symbol sent from the j-th transmit antenna and $\tilde{\boldsymbol{s}} = [\tilde{s}_1 \cdots \tilde{s}_n]^\mathrm{T} \in \tilde{\mathcal{S}}^n$ is the transmitted signal vector, where $\tilde{\mathcal{S}}$ denotes the alphabet of the transmitted symbol, with $\mathrm{E}[\tilde{\boldsymbol{s}}] = \boldsymbol{0}_n$ and $\mathrm{E}[\tilde{\boldsymbol{s}}\tilde{\boldsymbol{s}}^\mathrm{H}] = \sigma_s^2 \boldsymbol{I}_n$. The received signal vector $\tilde{\boldsymbol{y}} = [\tilde{y}_1 \cdots \tilde{y}_m]^\mathrm{T} \in \mathbb{C}^m$, where $\tilde{y}_i$ $(i = 1, \ldots, m)$ is the received signal at the i-th receive antenna, is given by

$$\tilde{\boldsymbol{y}} = \tilde{\boldsymbol{H}}\tilde{\boldsymbol{s}} + \tilde{\boldsymbol{v}}, \tag{1}$$

where

$$\tilde{\boldsymbol{H}} = \begin{bmatrix} \tilde{h}_{1,1} & \cdots & \tilde{h}_{1,n} \\ \vdots & \ddots & \vdots \\ \tilde{h}_{m,1} & \cdots & \tilde{h}_{m,n} \end{bmatrix} \in \mathbb{C}^{m \times n} \tag{2}$$

represents the flat fading channel matrix and $\tilde{\boldsymbol{v}} = [\tilde{v}_1 \cdots \tilde{v}_m]^\mathrm{T} \in \mathbb{C}^m$ is a zero mean white complex Gaussian noise vector with covariance matrix of $\sigma_v^2 \boldsymbol{I}_m$.

3. Slab-LR-MMSE-SIC [17]

For overloaded MIMO systems, the conventional LR-aided detection [14]–[16] is not directly applicable given that the channel matrix $\tilde{\boldsymbol{H}}$ is fat. To apply LR-aided detection to overloaded MIMO systems, LR-aided MMSE-SIC detection with PVC has been proposed [3]. To reduce the required computational complexity of the detection with PVC, Slab-LR-MMSE-SIC has been proposed [17].

In the same way as the detection with PVC, Slab-LR-MMSE-SIC firstly divides the index set of the transmit antennas $\{1, \ldots, n\}$ into

$$\mathcal{A} = \{p_1, \ldots, p_{n-m}\} \subset \{1, \ldots, n\}, \tag{3}$$
$$\mathcal{B} = \{q_1, \ldots, q_m\} = \{1, \ldots, n\} \setminus \mathcal{A}. \tag{4}$$

In addition, we divide the elements of the transmitted signal vector $\tilde{\boldsymbol{s}}$ into two vectors as

$$\tilde{\boldsymbol{s}}_\mathcal{A} := [\tilde{s}_{p_1} \cdots \tilde{s}_{p_{n-m}}]^T \ (\text{pre-voting vector}), \tag{5}$$
$$\tilde{\boldsymbol{s}}_\mathcal{B} := [\tilde{s}_{q_1} \cdots \tilde{s}_{q_m}]^T \ (\text{post-voting vector}). \tag{6}$$

Similarly, the columns of the channel matrix $\tilde{\boldsymbol{H}} = [\tilde{\boldsymbol{h}}_1 \cdots \tilde{\boldsymbol{h}}_n]$ are divided into two matrices as

$$\tilde{\boldsymbol{H}}_\mathcal{A} := [\tilde{\boldsymbol{h}}_{p_1} \cdots \tilde{\boldsymbol{h}}_{p_{n-m}}], \tag{7}$$
$$\tilde{\boldsymbol{H}}_\mathcal{B} := [\tilde{\boldsymbol{h}}_{q_1} \cdots \tilde{\boldsymbol{h}}_{q_m}]. \tag{8}$$

By the above splitting, (1) can be rewritten as

$$\tilde{\boldsymbol{y}} = \tilde{\boldsymbol{H}}_\mathcal{A}\tilde{\boldsymbol{s}}_\mathcal{A} + \tilde{\boldsymbol{H}}_\mathcal{B}\tilde{\boldsymbol{s}}_\mathcal{B} + \tilde{\boldsymbol{v}}. \tag{9}$$

The conventional scheme with PVC considers all candidates of pre-voting vector $\tilde{\boldsymbol{s}}_\mathcal{A}$ and then estimate the post-voting vector $\tilde{\boldsymbol{s}}_\mathcal{B}$ corresponding to each candidate of $\tilde{\boldsymbol{s}}_\mathcal{A}$ by using LR-aided MMSE-SIC. Thus, the problem is to find a candidate $\tilde{\boldsymbol{u}}_\mathcal{A} \in \tilde{\mathcal{S}}^{n-m}$ and $\tilde{\boldsymbol{u}}_\mathcal{B} \in \tilde{\mathcal{S}}^m$ satisfying

$$\tilde{\boldsymbol{y}} = \tilde{\boldsymbol{H}}_{\mathcal{A}}\tilde{\boldsymbol{u}}_{\mathcal{A}} + \tilde{\boldsymbol{H}}_{\mathcal{B}}\tilde{\boldsymbol{u}}_{\mathcal{B}} + \tilde{\boldsymbol{v}}. \tag{10}$$

In Slab-LR-MMSE-SIC, however, we reduce the candidates of $\tilde{\boldsymbol{s}}_{\mathcal{A}}$ by using slab decoding[1] in order to decrease the computational complexity. First, we consider the real model equivalent to (9) as

$$\boldsymbol{y} = \boldsymbol{H}_{\mathcal{A}}\boldsymbol{s}_{\mathcal{A}} + \boldsymbol{H}_{\mathcal{B}}\boldsymbol{s}_{\mathcal{B}} + \boldsymbol{v}, \tag{11}$$

where

$$\boldsymbol{H}_{\mathcal{A}} := \begin{bmatrix} \mathrm{Re}\{\tilde{\boldsymbol{H}}_{\mathcal{A}}\} & -\mathrm{Im}\{\tilde{\boldsymbol{H}}_{\mathcal{A}}\} \\ \mathrm{Im}\{\tilde{\boldsymbol{H}}_{\mathcal{A}}\} & \mathrm{Re}\{\tilde{\boldsymbol{H}}_{\mathcal{A}}\} \end{bmatrix}, \; \boldsymbol{s}_{\mathcal{A}} := \begin{bmatrix} \mathrm{Re}\{\tilde{\boldsymbol{s}}_{\mathcal{A}}\} \\ \mathrm{Im}\{\tilde{\boldsymbol{s}}_{\mathcal{A}}\} \end{bmatrix},$$

$$\boldsymbol{H}_{\mathcal{B}} := \begin{bmatrix} \mathrm{Re}\{\tilde{\boldsymbol{H}}_{\mathcal{B}}\} & -\mathrm{Im}\{\tilde{\boldsymbol{H}}_{\mathcal{B}}\} \\ \mathrm{Im}\{\tilde{\boldsymbol{H}}_{\mathcal{B}}\} & \mathrm{Re}\{\tilde{\boldsymbol{H}}_{\mathcal{B}}\} \end{bmatrix}, \; \boldsymbol{s}_{\mathcal{B}} := \begin{bmatrix} \mathrm{Re}\{\tilde{\boldsymbol{s}}_{\mathcal{B}}\} \\ \mathrm{Im}\{\tilde{\boldsymbol{s}}_{\mathcal{B}}\} \end{bmatrix}. \tag{12}$$

Moreover, we transform (11) into

$$\boldsymbol{y} = \bar{\boldsymbol{H}}\bar{\boldsymbol{s}} + \boldsymbol{v}, \tag{13}$$

to implement slab decoding algorithm[1], where

$$\bar{\boldsymbol{H}} := \begin{bmatrix} \boldsymbol{H}_{\mathcal{B}} & \boldsymbol{H}_{\mathcal{A}} \end{bmatrix}, \tag{14}$$

$$\bar{\boldsymbol{s}} = \begin{bmatrix} \bar{s}_1 \cdots \bar{s}_{2n} \end{bmatrix}^{\mathrm{T}} := \begin{bmatrix} \boldsymbol{s}_{\mathcal{B}}^{\mathrm{T}} & \boldsymbol{s}_{\mathcal{A}}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}. \tag{15}$$

By using the QR decomposition of $\bar{\boldsymbol{H}} = \bar{\boldsymbol{Q}}\bar{\boldsymbol{R}}$, we can rewrite (13) as

$$\bar{\boldsymbol{z}} = \bar{\boldsymbol{R}}\bar{\boldsymbol{s}} + \bar{\boldsymbol{\eta}}, \tag{16}$$

where $\bar{\boldsymbol{z}} := \bar{\boldsymbol{Q}}^T\boldsymbol{y}$ and $\bar{\boldsymbol{\eta}} := \bar{\boldsymbol{Q}}^{\mathrm{T}}\boldsymbol{v}$. The equation of the $2m$-th row of (16) is given by

$$\bar{z}_{2m} = \bar{r}_{2m,2m}\bar{s}_{2m} + \cdots + \bar{r}_{2m,2n}\bar{s}_{2n} + \bar{\eta}_{2m}. \tag{17}$$

Thus, by using slab decoding algorithm, we find all $\bar{u}_{2m}, \ldots, \bar{u}_{2n}$ satisfying

$$-C_{\mathrm{SLAB}} \leqq \bar{z}_{2m} - (\bar{r}_{2m,2m}\bar{u}_{2m} + \cdots + \bar{r}_{2m,2n}\bar{u}_{2n}) \leqq C_{\mathrm{SLAB}}, \tag{18}$$

where C_{SLAB} is a constant whose selection will be discussed in Sect. 4.1. It should be noted here that slab decoding gives the candidates for not only $\boldsymbol{s}_{\mathcal{A}}$ but also $\bar{s}_{2m}$ because $\boldsymbol{s}_{\mathcal{A}} = [\bar{s}_{2m+1} \cdots \bar{s}_{2n}]^{\mathrm{T}}$. However, we utilize the candidates of $\boldsymbol{s}_{\mathcal{A}}$ only, and $\bar{s}_{2m}$ will be estimated as one of the elements of the post-voting vector using LR-aided MMSE-SIC later. From the candidates of $\boldsymbol{s}_{\mathcal{A}}$, we can also obtain the candidates of $\tilde{\boldsymbol{s}}_{\mathcal{A}}$. Let L denote the number of candidates of $\tilde{\boldsymbol{s}}_{\mathcal{A}}$ obtained by using slab decoding algorithm, and we represent the L candidates as $\tilde{\boldsymbol{u}}_{\mathcal{A}}^1, \ldots, \tilde{\boldsymbol{u}}_{\mathcal{A}}^L$.

Next, we obtain candidates of post-voting vectors $\tilde{\boldsymbol{u}}_{\mathcal{B}}^1, \ldots, \tilde{\boldsymbol{u}}_{\mathcal{B}}^L$ corresponding to $\tilde{\boldsymbol{u}}_{\mathcal{A}}^1, \ldots, \tilde{\boldsymbol{u}}_{\mathcal{A}}^L$ by LR-aided MMSE-SIC. Assuming $\tilde{\boldsymbol{u}}_{\mathcal{A}} = \tilde{\boldsymbol{u}}_{\mathcal{A}}^\ell$ ($\ell = 1, \ldots, L$), (10) can

be rewritten as

$$\boldsymbol{r}^\ell := \tilde{\boldsymbol{y}} - \tilde{\boldsymbol{H}}_{\mathcal{A}}\tilde{\boldsymbol{u}}_{\mathcal{A}}^\ell = \tilde{\boldsymbol{H}}_{\mathcal{B}}\tilde{\boldsymbol{u}}_{\mathcal{B}} + \tilde{\boldsymbol{v}}. \tag{19}$$

Since (19) can be regarded as the model of $m \times m$ MIMO system, we can obtain $\tilde{\boldsymbol{u}}_{\mathcal{B}}^\ell$, i.e., the estimate of $\tilde{\boldsymbol{u}}_{\mathcal{B}}$ corresponding to $\tilde{\boldsymbol{u}}_{\mathcal{A}}^\ell$, by using the conventional LR-aided MMSE-SIC.

Finally, we choose the best candidate, which maximizes the likelihood, as the detected signal. Specifically, we obtain

$$\hat{\ell} = \arg\min_{\ell \in \{1, \ldots, L\}} \left\| \tilde{\boldsymbol{y}} - \tilde{\boldsymbol{H}}_{\mathcal{A}}\tilde{\boldsymbol{u}}_{\mathcal{A}}^\ell - \tilde{\boldsymbol{H}}_{\mathcal{B}}\tilde{\boldsymbol{u}}_{\mathcal{B}}^\ell \right\|^2 \tag{20}$$

and select $\tilde{\boldsymbol{u}}_{\mathcal{A}}^{\hat{\ell}}$ and $\tilde{\boldsymbol{u}}_{\mathcal{B}}^{\hat{\ell}}$ as the estimates of $\tilde{\boldsymbol{s}}_{\mathcal{A}}$ and $\tilde{\boldsymbol{s}}_{\mathcal{B}}$, respectively.

4. Proposed Selection Criteria of the Parameters

4.1 Selection of C_{SLAB}

In Slab-LR-MMSE-SIC, the selection of C_{SLAB} is one of the key issues, since it has a direct impact on the computational complexity and performance. Here, we discuss how to set C_{SLAB}, such that the complexity is reduced as much as possible while keeping the performance loss to an acceptable level.

Let $P_{e,\mathrm{SLAB}}$ be the probability that the true transmit signals $\bar{s}_{2m}, \ldots, \bar{s}_{2n}$ is not included in the set of candidates obtained with slab decoding. For true $\bar{s}_{2m}, \ldots, \bar{s}_{2n}$, we have

$$\bar{z}_{2m} - (\bar{r}_{2m,2m}\bar{s}_{2m} + \cdots + \bar{r}_{2m,2n}\bar{s}_{2n}) = \bar{\eta}_{2m} \tag{21}$$

from (17), thus

$$P_{e,\mathrm{SLAB}} = 1 - \Pr(-C_{\mathrm{SLAB}} \leqq \bar{\eta}_{2m} \leqq C_{\mathrm{SLAB}}). \tag{22}$$

Since $\bar{\boldsymbol{\eta}} = \bar{\boldsymbol{Q}}^{\mathrm{T}}\boldsymbol{v}$, $\bar{\eta}_{2m}$ is written as

$$\bar{\eta}_{2m} = \bar{q}_{1,2m}v_1 + \bar{q}_{2,2m}v_2 + \cdots + \bar{q}_{2m,2m}v_{2m}, \tag{23}$$

where $\bar{q}_{i,2m}$ ($i = 1, \ldots, 2m$) represents the $(i, 2m)$ element of $\bar{\boldsymbol{Q}}$. Moreover, since v_i ($i = 1, \ldots, 2m$) are Gaussian random variables with zero mean and variance $\sigma_v^2/2$ and $\bar{\boldsymbol{Q}}$ is an orthogonal matrix, $\bar{\eta}_{2m}$ is also a Gaussian random variable with zero mean and variance $\sigma_v^2/2$. Therefore, $P_{e,\mathrm{SLAB}}$ can be calculated as

$$P_{e,\mathrm{SLAB}} = 2 \int_{C_{\mathrm{SLAB}}}^{+\infty} \frac{1}{\sqrt{2\pi(\sigma_v^2/2)}} \exp\left(-\frac{x^2}{2(\sigma_v^2/2)}\right) \mathrm{d}x$$
$$= \mathrm{erfc}\left(C_{\mathrm{SLAB}}/\sigma_v\right), \tag{24}$$

where

$$\mathrm{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{+\infty} \exp\left(-t^2\right) \mathrm{d}t \tag{25}$$

is the complementary error function. From (24), C_{SLAB} can be given as

$$C_{\text{SLAB}} = \sigma_v \text{erfc}^{-1}\left(P_{e,\text{SLAB}}\right), \tag{26}$$

thus we can determine C_{SLAB} by setting the acceptable probability $P_{e,\text{SLAB}}$ that slab decoding fails to obtain the true $\bar{s}_{2m}, \ldots, \bar{s}_{2n}$.

4.2 Selection of $\mathcal{A}$ and $\mathcal{B}$

The selection of $\mathcal{A}$ and $\mathcal{B}$ also has an impact on the computational complexity, because the number of candidates of pre-voting vectors obtained by slab decoding depends on the selection. Thus, we consider how to select $\mathcal{A}$ and $\mathcal{B}$ here, and propose a simple but effective selection criterion named maximum variance (MV) criterion.

Let $X := z_{2m} - \left(\bar{r}_{2m,2m}\bar{S}_{2m} + \cdots + \bar{r}_{2m,2n}\bar{S}_{2n}\right)$, where $\bar{S}_{2m}, \ldots, \bar{S}_{2n}$ are i.i.d. random variables distributed uniformly on $\mathcal{S}$. Then, the probability that a realization of $(\bar{u}_{2m}, \ldots, \bar{u}_{2n}) = \left(\bar{S}_{2m}, \ldots, \bar{S}_{2n}\right)$ satisfies (18) is $\text{Pr}\left(-C_{\text{SLAB}} \leqq X \leqq C_{\text{SLAB}}\right)$, which should be minimized to reduce the number of candidates of pre-voting vectors obtained by slab decoding. In our approach, therefore, we select $\mathcal{A}$ and $\mathcal{B}$ so that the variance of X is as large as possible. Since

$$\mu_x := E[X] = z_{2m}, \tag{27}$$
$$\sigma_x^2 := E[(X - \mu_x)^2] = \left(\bar{r}_{2m,2m}^2 + \cdots + \bar{r}_{2m,2n}^2\right)\sigma_s^2/2, \tag{28}$$

we obtain

$$\left[\hat{\mathcal{A}}, \hat{\mathcal{B}}\right] = \arg\max_{[\mathcal{A},\mathcal{B}]}\left\{\bar{r}_{2m,2m}^2 + \cdots + \bar{r}_{2m,2n}^2\right\}, \tag{29}$$

where $\bar{r}_{2m,i}$ $(i = 2m, \ldots, 2n)$ denotes the $(2m, i)$ element of the upper triangular matrix obtained by QR decomposition of $\bar{H} = [H_{\mathcal{B}}\ H_{\mathcal{A}}]$. Note that MV criterion does not require LR for the evaluation, thus it has much lower complexity than max-min diagonal (MD) criterion in [18], which needs LR for each candidates of $\mathcal{A}$ and $\mathcal{B}$.

5. Performance Analysis

In this section, we evaluate the error probability of Slab-LR-MMSE-SIC taking a similar approach as in [3]. Let $\mathcal{U}_{\mathcal{A}} = \{\tilde{u}_{\mathcal{A}}^1, \ldots, \tilde{u}_{\mathcal{A}}^L\}$ and $\mathcal{U} = \{\tilde{u}^1, \ldots, \tilde{u}^L\}$ be the set of the candidates of $\tilde{s}_{\mathcal{A}}$ obtained by using slab decoding and that of $\tilde{s}$ after LR-aided MMSE-SIC, respectively. In addition, we denote $\hat{s}$ as the final estimate of the true transmitted signal vector $\tilde{s}$ obtained with Slab-LR-MMSE-SIC. The error probability of Slab-LR-MMSE-SIC P_e is written as

$$P_e = 1 - \text{Pr}(\hat{s} = \tilde{s} \mid \tilde{s} \in \mathcal{U})\,\text{Pr}(\tilde{s} \in \mathcal{U})$$
$$= 1 - (1 - \text{Pr}(\hat{s} \neq \tilde{s} \mid \tilde{s} \in \mathcal{U}))(1 - \text{Pr}(\tilde{s} \notin \mathcal{U}))$$
$$= 1 - (1 - P_{e,\text{SEL}})(1 - P_{e,\text{PV}}), \tag{30}$$

where $P_{e,\text{SEL}} = \text{Pr}(\hat{s} \neq \tilde{s} \mid \tilde{s} \in \mathcal{U})$ is the probability that

the final estimate of $\tilde{s}$ is not equal to $\tilde{s}$ though $\mathcal{U}$ contains $\tilde{s}$, and $P_{e,\text{PV}} = \text{Pr}(\tilde{s} \notin \mathcal{U})$ is the probability that $\mathcal{U}$ does not contain the true transmitted signal vector. As mentioned in Sect. 3., the candidates of $\bar{s}_{2m}$ are not used in order to reduce the candidates of $\tilde{s}_{\mathcal{A}} = [\bar{s}_{2m+1} \cdots \bar{s}_{2n}]^{\text{T}}$ though they were obtained by slab decoding. Thus, the probability that the set of candidates of the pre-voting vector $\mathcal{U}_{\mathcal{A}}$ includes the true pre-voting vector $\tilde{s}_{\mathcal{A}}$ is greater than the success probability of slab decoding, namely $\text{Pr}(\tilde{s}_{\mathcal{A}} \in \mathcal{U}_{\mathcal{A}}) \geqq 1 - P_{e,\text{SLAB}}$. Similarly, we also define $\tilde{s}_{\mathcal{B}}$ as the true post-voting vector. Thus, $P_{e,\text{PV}}$ can be bounded as

$$P_{e,\text{PV}}$$
$$= 1 - \text{Pr}(\tilde{u}_{\mathcal{B}}^\ell = \tilde{s}_{\mathcal{B}} \mid \tilde{u}_{\mathcal{A}}^\ell = \tilde{s}_{\mathcal{A}})\,\text{Pr}(\tilde{s}_{\mathcal{A}} \in \mathcal{U}_{\mathcal{A}})$$
$$\leqq 1 - \text{Pr}(\tilde{u}_{\mathcal{B}}^\ell = \tilde{s}_{\mathcal{B}} \mid \tilde{u}_{\mathcal{A}}^\ell = \tilde{s}_{\mathcal{A}})(1 - P_{e,\text{SLAB}})$$
$$= (1 - \text{Pr}(\tilde{u}_{\mathcal{B}}^\ell = \tilde{s}_{\mathcal{B}} \mid \tilde{u}_{\mathcal{A}}^\ell = \tilde{s}_{\mathcal{A}}))(1 - P_{e,\text{SLAB}}) + P_{e,\text{SLAB}}$$
$$\leqq (1 - \text{Pr}(\tilde{u}_{\mathcal{B}}^\ell = \tilde{s}_{\mathcal{B}} \mid \tilde{u}_{\mathcal{A}}^\ell = \tilde{s}_{\mathcal{A}})) + P_{e,\text{SLAB}}, \tag{31}$$

where $1 - \text{Pr}(\tilde{u}_{\mathcal{B}}^\ell = \tilde{s}_{\mathcal{B}} \mid \tilde{u}_{\mathcal{A}}^\ell = \tilde{s}_{\mathcal{A}})$ is the probability that the post-voting vector $\tilde{s}_{\mathcal{B}}$ is not correctly estimated in (19) for the true pre-voting vector $\tilde{s}_{\mathcal{A}}$. As shown in [3], [19], we have

$$1 - \text{Pr}(\tilde{u}_{\mathcal{B}}^\ell = \tilde{s}_{\mathcal{B}} \mid \tilde{u}_{\mathcal{A}}^\ell = \tilde{s}_{\mathcal{A}})$$
$$\leqq c_{mm}\left(\frac{2}{c_\delta}\right)^m \frac{(2m-1)!}{(m-1)!}\left(\frac{1}{\sigma_v^2}\right)^{-m}, \tag{32}$$

where c_{mm} is a constant depending on m, $1/2 < \delta < 1$ is a parameter in complex LLL algorithm, and

$$c_\delta = 2^{\frac{m}{2}}\left(\frac{2}{2\delta - 1}\right)^{-m(m+1)/4}. \tag{33}$$

If $\tilde{s} \in \mathcal{U}$ and the transmitted signal vector $\tilde{s}$ can be correctly obtained with ML detection among all possible candidates, Slab-LR-MMSE-SIC can also select $\tilde{s}$ among candidates in $\mathcal{U}$ by using (20), which obtains the candidate having maximum likelihood among candidates in $\mathcal{U}$. Thus, we have $P_{e,\text{SEL}} \leqq P_{e,\text{ML}}$, where $P_{e,\text{ML}}$ is the error probability of ML detection, which can be bounded as $P_{e,\text{ML}} \leqq c_{\text{ML}}\left(1/\sigma_v^2\right)^{-m}$, where c_{ML} is a constant independent of σ_v^2 [20]. Hence, the error probability of Slab-LR-MMSE-SIC P_e is bounded as

$$P_e = P_{e,\text{PV}} + P_{e,\text{SEL}} - P_{e,\text{PV}}P_{e,\text{SEL}}$$
$$\leqq P_{e,\text{PV}} + P_{e,\text{ML}}$$
$$\leqq c_{mm}\left(\frac{2}{c_\delta}\right)^m \frac{(2m-1)!}{(m-1)!}\left(\frac{1}{\sigma_v^2}\right)^{-m}$$
$$+ P_{e,\text{SLAB}} + c_{\text{ML}}\left(\frac{1}{\sigma_v^2}\right)^{-m}. \tag{34}$$

It should be noted that we can control $P_{e,\text{SLAB}}$ as the parameter of the acceptable error probability of slab decoding. Specifically, by choosing $P_{e,\text{SLAB}}$ as $P_{e,\text{SLAB}} \propto \left(1/\sigma_v^2\right)^{-m}$, P_e can be bounded as
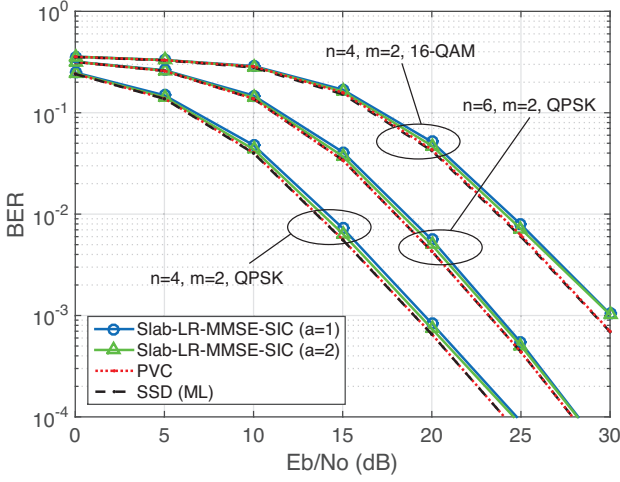
Fig. 1 BER performance

Fig. 2 Ratio of the number of pre-voting vector candidates $p = L/\left|\tilde{\mathcal{S}}\right|^{n-m}$

$$P_e \leqq c \left(\frac{1}{\sigma_v^2}\right)^{-m}, \qquad (35)$$

where c is a constant which is independent of σ_v^2, meaning that Slab-LR-MMSE-SIC can achieve a diversity order of m. Since the maximum diversity order for $n \times m$ overloaded MIMO is known to be m [3], Slab-LR-MMSE-SIC can achieve the full diversity order.

6. Simulation Results

In this section, we demonstrate the performance of Slab-LR-MMSE-SIC using computer simulations. In the simulations, $\tilde{\boldsymbol{H}}$ is assumed to be time-invariant and is composed of i.i.d. complex Gaussian random variables with zero mean and unit variance. All the results are obtained by averaging the performance for 1,000 realizations of $\tilde{\boldsymbol{H}}$. The acceptable error probability of slab decoding in Slab-LR-MMSE-SIC is set to be $P_{e,\mathrm{SLAB}} = BER_{\mathrm{ML}}/a$ at each E_b/N_0, where $a > 0$ is a constant and BER_{ML} is the BER achieved by ML detection, which is the same as that of SSD. The parameter δ in the complex LLL algorithm is set as $\delta = 3/4$.

6.1 BER performance

First, we evaluate the BER performance of Slab-LR-MMSE-SIC and conventional signal detection schemes. Figure 1 shows the BER performance of Slab-LR-MMSE-SIC for $n = 4$, $m = 2$ with QPSK modulation, for $n = 6$, $m = 2$ with QPSK modulation, and for $n = 4$, $m = 2$ with 16-QAM. The BERs of slab sphere decoding (SSD) and the conventional LR-aided MMSE-SIC detection (PVC), are also plotted in the same figures. In all the figures, we can see that the conventional LR-aided MMSE-SIC detection can achieve almost the same BER performance as the optimal ML detection. In addition, Slab-LR-MMSE-SIC can also achieve similar performance as the conventional schemes. It should be noted here that Slab-LR-MMSE-SIC with the proper parameter
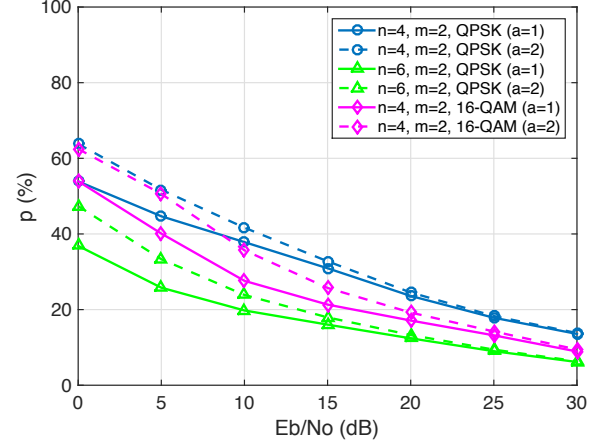
choice can achieve the same diversity order as ML detection, which is confirmed by the simulation results.

6.2 Number of candidates of pre-voting vectors

Next, we evaluate the number of candidates obtained by slab decoding in Slab-LR-MMSE-SIC. Figure 2 shows the ratio of the number of candidates $p = L/\left|\tilde{\mathcal{S}}\right|^{n-m}$ in percentage, where L denotes the number of candidates of $\tilde{\boldsymbol{s}}_{\mathcal{A}}$ obtained by slab decoding in Slab-LR-MMSE-SIC, and $\left|\tilde{\mathcal{S}}\right|^{n-m}$ represents that in the conventional scheme with PVC. Since different values of C_{SLAB} are used for each E_b/N_0, the amount p for higher E_b/N_0 is less than that for lower E_b/N_0. We can also see that smaller a, which means larger $P_{e,\mathrm{SLAB}}$, results in larger reduction of the number of candidates though it entails a slight performance degradation as shown in Fig. 1. In addition, we observe a larger reduction of the number of candidates for greater values of $n - m$ and higher modulation levels. Note that, by reducing the number of candidates for $\tilde{\boldsymbol{s}}_{\mathcal{A}}$, the required number of LR-aided MMSE-SIC detections for $\tilde{\boldsymbol{s}}_{\mathcal{B}}$ is also reduced. Therefore, Slab-LR-MMSE-SIC is able to reduce the computational complexity as compared to the conventional schemes, while achieving a near-optimal performance.

6.3 Comparison of the selection criteria of $\mathcal{A}$ and $\mathcal{B}$

We demonstrate the effectiveness of the proposed MV criterion against the conventional MD criterion for $n = 4, m = 2$ with QPSK modulation. Figure 3 shows the BER performances of Slab-LR-MMSE-SIC with MV criterion and MD criterion. We can observe that both MV criterion and MD criterion provide almost the same BER performance as the optimal ML detection, while MD criterion requires higher computational complexity than MV criterion. Figure 4 shows the ratio of the number of candidates of pre-voting vector obtained with each criterion. We can see that Slab-
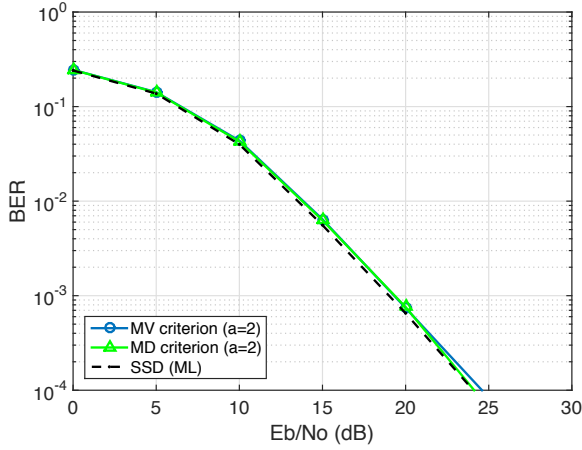
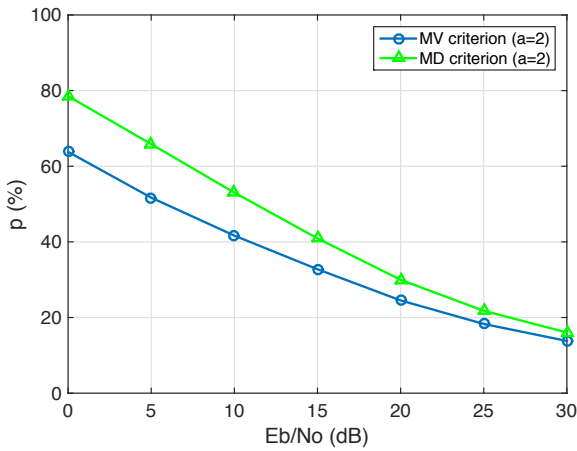Fig. 3   BER performance ($n = 4, m = 2$, QPSK)

Fig. 4   Ratio of the number of pre-voting vector candidates
$p = L / \left| \tilde{\mathcal{S}} \right|^{n-m} (n = 4, m = 2$, QPSK)

LR-MMSE-SIC with MV criterion has less candidates than that with MD criterion, and hence the proposed MV criterion enables further reduction of the required computational complexity.

7. Conclusion

In this paper, we have proposed the selection criteria for the parameters of the overloaded MIMO signal detection scheme with slab decoding and LR, referred as Slab-LR-MMSE-SIC. With the proposed parameter choice in slab decoding and the selection of the indexes for the pre-voting vector, Slab-LR-MMSE-SIC can significantly reduce the candidates of pre-voting vector compared to the conventional scheme, closely approaching the BER performance of optimal ML detection. In addition, we have analyzed the error probability of Slab-LR-MMSE-SIC and have clarified that it can achieve full diversity order with a proper parameter choice in slab decoding.

References

[1] K. K. Wong *et al.*, "Efficient high-performance decoding for overloaded MIMO antenna systems," IEEE Trans. Wireless Commun., vol. 6, no. 5, pp. 1833–1843, May 2007.

[2] Z. Yang *et al.*, "A new approach for fast generalized sphere decoding in MIMO systems," IEEE Signal Process. Lett., vol. 12, no. 1, pp. 41–44, Jan. 2005.

[3] L. Bai *et al.*, "Lattice reduction aided detection for underdetermined MIMO systems: a pre-voting cancellation approach," Proc. IEEE VTC2010-Spring, Taipei, Taiwan, pp. 1–5, May 2010.

[4] P. Wang *et al.*, "A low-complexity generalized sphere decoding approach for underdetermined MIMO systems," Proc. IEEE ICC 2006, Istanbul, Turkey, vol. 9, pp. 4266–4271, June 2006.

[5] N. Bakinde *et al.*, "Improved signal detection approach using genetic algorithm for overloaded MIMO systems," Proc. IEEE WiCOM 2008, Dalian, China, pp.1–4, Oct. 2008.

[6] S. Denno *et al.*, "A virtual layered space time receiver with maximum likelihood channel detection," Proc. IEEE VTC2009-Spring, Barcelona, Spain, pp. 1–5, April 2009.

[7] K. Obaidullah *et al.*, "Efficient algorithm with lognormal distributions for overloaded MIMO wireless system." Proc. APSIPA ASC 2012, California, USA, pp. 1–4, Dec. 2012.

[8] Y. Sanada, "Performance of joint maximum-likelihood decoding for block coded signal streams in overloaded MIMO-OFDM system," Proc. IEEE ISPACS 2013, Naha, Japan, pp. 775–780, Nov. 2013.

[9] A. van Zelst *et al.*, "Space division multiplexing (SDM) for OFDM systems," Proc. IEEE VTC2000-Spring, Tokyo, Japan, pp. 1070-1074, May 2000.

[10] B. Hassibi *et al.*, "On the Sphere-Decoding Algorithm I: expected complexity," IEEE Trans. Signal Process., vol. 53, no. 8, pp. 2806-2818, Aug. 2005.

[11] M. Pohst, "On the computation of lattice vectors of minimal length, successive minima and reduced basis with applications," ACM SIGSAM Bulletin, vol. 15, pp. 37–44, Feb. 1981.

[12] Y. H. Gan *et al.*, "Complex lattice reduction algorithms for low-complexity MIMO detection," Proc. IEEE GLOBECOM 2005, St. Louis, USA, vol. 5, pp. 2953–2957, Nov. 28-Dec. 2, 2005.

[13] A. K. Lenstra *et al.*, "Factoring polynomials with rational coefficients," Math. Ann., vol. 261, pp. 515–534, July 1982.

[14] D. Wübben *et al.*, "Near-maximum-likelihood detection of MIMO systems using MMSE-based lattice reduction," Proc. IEEE ICC2004, Paris, France, vol. 2, pp. 798–802, June 2004.

[15] H. Yao *et al.*, "Lattice-reduction-aided detectors for MIMO communication systems," in Proc. IEEE GLOBECOM 2002, Taipei, Taiwan, pp. 424-428, Nov. 2002.

[16] C. Windpassinger *et al.*, "Low-complexity near-maximum-likelihood detection and precoding for MIMO systems using lattice reduction," Proc. IEEE ITW 2003, Paris, France, pp. 345–348, Mar. 2003.

[17] R. Hayakawa *et al.*, "An overloaded MIMO signal detection scheme with slab decoding and lattice reduction," Proc. APCC2015, Kyoto, Japan, pp. 42–46 , Oct. 2015.

[18] J. Choi *et al.*, "User selection criteria for multiuser systems with optimal and suboptimal LR based detectors," IEEE Trans. Signal Process., vol. 58, no. 10, pp. 5463–5468, June 2010.

[19] X. Ma *et al.*, "Performance analysis for MIMO systems with lattice-reduction aided linear equalization," IEEE Trans. Commun., vol. 56, pp. 309–318, Feb. 2008.

[20] A. Paulraj *et al.*, Introduction to space-time wireless communications, Cambridge university press, 2003.