Games 101 Course

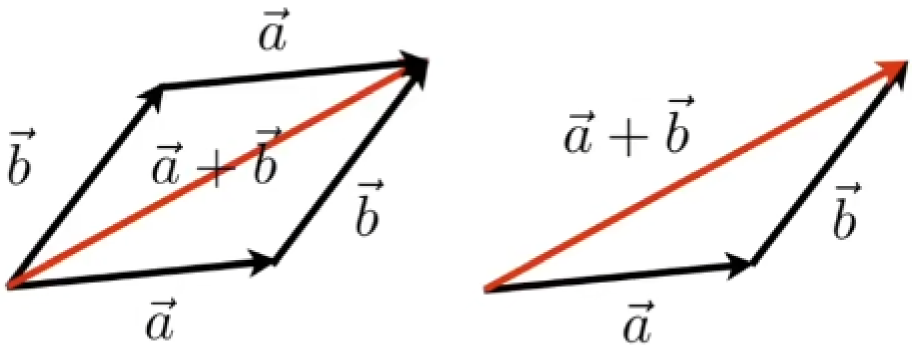
Linear Algebra Review

Vectors

- Usually written as $\vec{a}$ or in bold $\mathbf{a}$
- start and end point $\overrightarrow{AB} = B - A$
- Direction and length
- No absolute starting position

Vector Normalization

- Magnitude (length) of a vector written as $||\vec{a}||$
- Unit vector
    - A vector with magnitude of 1
    - Finding the unit vector of a vector (normalization): $\hat{a} = \vec{a}/||\vec{a}||$
    - used to represent directions



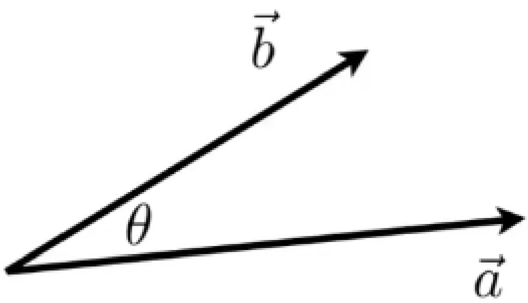
Vector Addition

- Geometrically: Parallelogram law & Triangle law
- Algebraically: Simply add coordinates

Cartesian Coordinates

- X and Y can be any (usually orthogonal unit) vectors
- $A = \binom{x}{y}$ A^T = (x ,y) | |A| | = $\sqrt{x^2 + y^2}$

Dot (scalar) product



$\vec{a} \cdot \vec{b} = ||\vec{a}|| ||\vec{b}|| cos\theta$

unit vector:

$cos\theta = \hat{a} \cdot \hat{b}$

get the angular θ

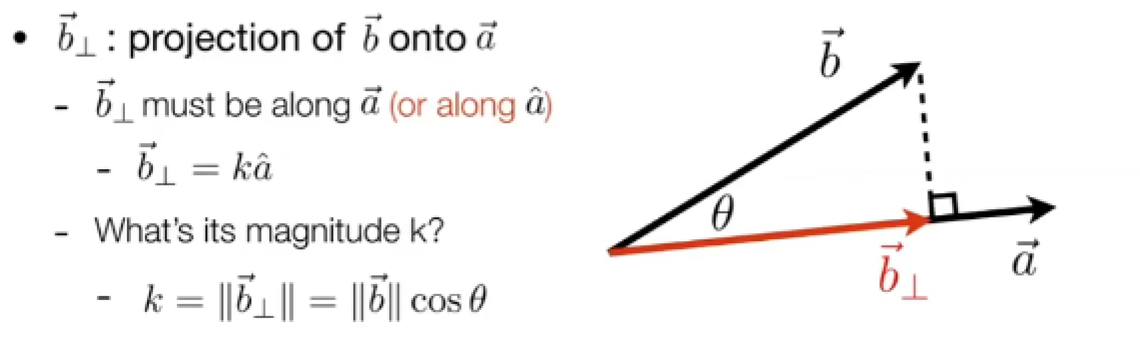
- In 2D

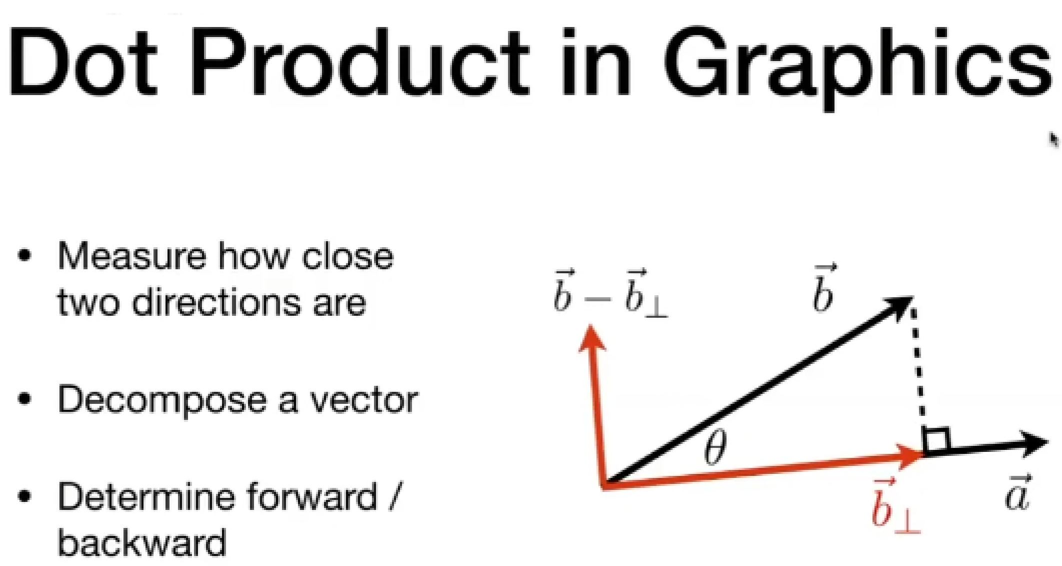
$\vec{a} \cdot \vec{b} = \binom{x_a}{y_a} \cdot \binom{x_b}{y_b} = x_a x_b + y_a y_b$

- In 3D

$$\vec{a} \cdot \vec{b} = \begin{pmatrix} x_a \\ y_a \\ z_c \end{pmatrix} \cdot \begin{pmatrix} x_b \\ y_b \\ z_b \end{pmatrix} = x_a x_b + y_a y_b + z_a + z_b$$

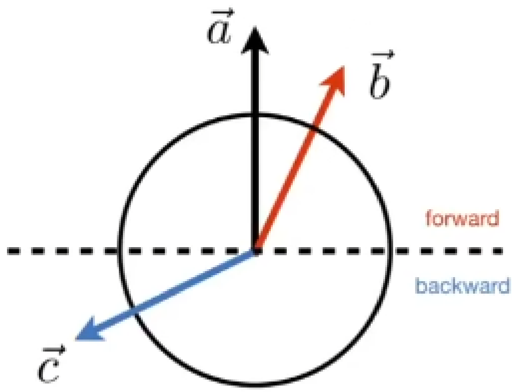
Dot Product for Projection





Dot Product in Graphics

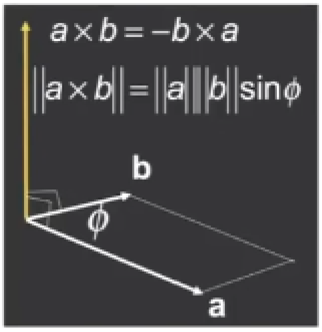
- Measure how close two directions are
- Decompose a vector
- Determine forward / backward



dot product > or < 0

Important --- the direction ---

Cross (vector) Product



- Cross product is orthogonal to two initial vectors
- Direction determined by right-hand rule
- Useful in constructing coordinate systems (later)

Cross Product: Cartesian Formula?

$$\vec{a} \times \vec{b} = \begin{pmatrix} y_a z_b - y_b z_a \\ z_a x_b - x_a z_b \\ x_a y_b - y_a x_b \end{pmatrix}$$

- Later in this lecture

$$\vec{a} \times \vec{b} = A^* b = \begin{pmatrix} 0 & -z_a & y_a \\ z_a & 0 & -x_a \\ -y_a & x_a & 0 \end{pmatrix} \begin{pmatrix} x_b \\ y_b \\ z_b \end{pmatrix}$$

- Determine left / right
- Determine inside / outside

Orthornormal Coordinate Frames

Orthonormal Coordinate Frames

- Any set of 3 vectors (in 3D) that

$$\|\vec{u}\| = \|\vec{v}\| = \|\vec{w}\| = 1$$

$$\vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{w} = \vec{u} \cdot \vec{w} = 0$$

$$\vec{w} = \vec{u} \times \vec{v} \quad \text{(right-handed)}$$

$$\vec{p} = (\vec{p} \cdot \vec{u})\vec{u} + (\vec{p} \cdot \vec{v})\vec{v} + (\vec{p} \cdot \vec{w})\vec{w}$$

(projection)

Matrix

Matrix-Matrix Multiplication

- # (number of) columns in A must = # rows in B
  (M x N) (N x P) = (M x P)

$$\begin{pmatrix} 1 & 3 \\ 5 & 2 \\ 0 & 4 \end{pmatrix} \begin{pmatrix} 3 & 6 & 9 & 4 \\ 2 & 7 & 8 & 3 \end{pmatrix} = \begin{pmatrix} 9 & ? & 33 & 13 \\ 19 & 44 & 61 & 26 \\ 8 & 28 & 32 & ? \end{pmatrix}$$

- Element (i, j) in the product is
  the dot product of row i from A and column j from B

Note: A*