APS 502: Financial Engineering

Project Assignment

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Q1 P1:

From the spots rates given, we can derive the forward rates:
$F(1,2) = 0.02$
$F(2,3) = 0.03$
$F(3,4) = 0.04$
$F(4,5) = 0.05$
$F(5,6) = 0.06$
We can then use the forward rates to carry the cash left of each period forward.

The problem can be formulated as follows:
minimize

$108x_1 + 94x_2 + 99x_3 + 92.7x_4 + 96.6x_5 + 95.9x_6 + 92.9x_7 + 110x_8 + 104x_9 + 101x_{10} + 107x_{11} + 102x_{12} + 95.2x_{13}$

subject to:

$10x_1 + 7x_2 + 8x_3 + 6x_4 + 7x_5 + 6x_6 + 5x_7 + 10x_8 + 8x_9 +6x_{10} +10x_{11} + 7x_{12} + 100x_{13} - z_1 \geq 500;$

$10x_1 + 7x_2 + 8x_3 + 6x_4 + 7x_5 + 6x_6 + 5x_7 + 10x_8 + 8x_9 +6x_{10} +110x_{11} + 107x_{12} + 1.02z_1 - z_2 \geq 200;$

$10x_1 + 7x_2 + 8x_3 + 6x_4 + 7x_5 + 6x_6 + 5x_7 + 110x_8 +108x_9 +106x_{10} + 1.03z_2 - z_3 \geq 800;$

$10x_1 + 7x_2 + 8x_3 + 6x_4 + 7x_5 + 106x_6 + 105x_7 + 1.04z_3 - z_4 \geq 400;$

$10x_1 + 7x_2 + 8x_3 + 106x_4 + 107x_5 + 1.05z_4 - z_5 \geq 700;$

$110x_1 + 107x_2 + 108x_3 + 1.06z_5 \geq 900;$

Where $x_i \geq 0$; $x_j \geq 0$ ($i = 1..13$; $j = 1..5$)

Q1 P2,P3:

For these 2 parts, we just need one more constraint each to complete the linear programming problem.

P2:

$108x_1 + 94x_2 + 99x_3 + 92.7x_4 + 96.6x_5 + 95.9x_6 \leq 0.5(108x_1 + 94x_2 + 99x_3 + 92.7x_4 + 96.6x_5 + 95.9x_6 + 92.9x_7 + 110x_8 + 104x_9 + 101x_{10} + 107x_{11} + 102x_{12} + 95.2x_{13})$;

P3:

$108x_1 + 94x_2 + 99x_3 + 92.7x_4 + 96.6x_5 + 95.9x_6 \leq 0.25(108x_1 + 94x_2 + 99x_3 + 92.7x_4 + 96.6x_5 + 95.9x_6 + 92.9x_7 + 110x_8 + 104x_9 + 101x_{10} + 107x_{11} + 102x_{12} + 95.2x_{13})$;

Q1 Results and code:

P1:

```
c = [108, 94, 99, 92.7,96.6,95.9,92.9,110,104,101,
      107,102,95.2,0,0,0,0,0]
A = -[10,7,8,6,7,6,5,10,8,6,10,7,100,-1,0,0,0,0;
      10,7,8,6,7,6,5,10,8,6,110,107,0,1.02,-1,0,0,0;
      10,7,8,6,7,6,5,110,108,106,0,0,0,0,1.03,-1,0,0;
      10,7,8,6,7,106,105,0,0,0,0,0,0,0,0,1.04,-1,0;
      10,7,8,106,107,0,0,0,0,0,0,0,0,0,0,0,1.05,-1;
      110,107,108,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1.06]
b = -[500,200,800,400,700,900]
Aeq = []
beq = []
lb = [0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0]
ub =
[inf,inf,inf,inf,inf,inf,inf,inf,inf,inf,inf,inf,inf,inf,inf,inf,inf,
inf]
[x,fval] = linprog(c,A,b,Aeq,beq,lb,ub)
```

x = [8.1818, 0 ,0 ,0 , 5.7774 ,2.6202 ,0 , 0 , 6.1298 ,0 ,0.1180 ,0 , 3.1180]

z=[0,0,0,0,0]

fval =2.6400e+03

P2:

```
c = [108, 94,99,92.7,96.6,95.9,92.9,110,104,101,
      107,102,95.2,0,0,0,0,0]
A = -[10,7,8,6,7,6,5,10,8,6,10,7,100,-1,0,0,0,0;
      10,7,8,6,7,6,5,10,8,6,110,107,0,1.02,-1,0,0,0;
      10,7,8,6,7,6,5,110,108,106,0,0,0,0,1.03,-1,0,0;
      10,7,8,6,7,106,105,0,0,0,0,0,0,0,0,1.04,-1,0;
      10,7,8,106,107,0,0,0,0,0,0,0,0,0,0,0,1.05,-1;
      110,107,108,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1.06;
      -108,-94,-99,-92.7,-96.6,-95.9,92.9,110,104,101,107,102,
      95.2,0,0,0,0,0]
      %-108*0.5,-94*0.5,-99*0.5,-92.7*0.5,-96.6*0.5,-
      %95.9*0.5,92.9*0.5,110*0.5,104*0.5,101*
      %0.5,107*0.5,102*0.5,95.2*0.5,0,0,0,0,0]
b = -[500,200,800,400,700,900,0]
Aeq = []
beq = []
lb = [0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0]
ub =
[inf,inf,inf,inf,inf,inf,inf,inf,inf,inf,inf,inf,inf,inf,inf,inf,inf,
inf]
[x,fval] = linprog(c,A,b,Aeq,beq,lb,ub)
```

x = [0,8.4112 ,0 ,0,5.5027 ,0,3.3566 ,0 ,6.3502 ,0 ,0.3183 ,0 ,3.3183]

z = [0 ,0,0 ,49.8396 ,0]

fval =2.6444e+03

P3:

```
c = [108, 94, 99, 92.7,96.6,95.9,92.9,110,104,101,107,
      102,95.2,0,0,0,0,0];
A = -[10,7,8,6,7,6,5,10,8,6,10,7,100,-1,0,0,0,0;
      10,7,8,6,7,6,5,10,8,6,110,107,0,1.02,-1,0,0,0;
      10,7,8,6,7,6,5,110,108,106,0,0,0,0,1.03,-1,0,0;
      10,7,8,6,7,106,105,0,0,0,0,0,0,0,0,1.04,-1,0;
      10,7,8,106,107,0,0,0,0,0,0,0,0,0,0,0,1.05,-1;
      110,107,108,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1.06;
      -108*0.75,-94*0.75,-99*0.75,-92.7*0.75,-96.6*0.75,-
95.9*0.75,92.9*0.25,110*0.25,104*0.25,101*0.25,107*0.25,102*0.25,95.2
*0.25,0,0,0,0,0] ;
b = -[500,200,800,400,700,900,0];
Aeq = [];
beq = [];
lb = [0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0];
ub =
[inf,inf,inf,inf,inf,inf,inf,inf,inf,inf,inf,inf,inf,inf,inf,inf,inf,
inf];
[x,fval] = linprog(c,A,b,Aeq,beq,lb,ub)
```

x =[0,7.1271 ,0 ,0 ,0 ,0,10.4068,0,6.4637 ,0 ,0.4215 ,0,3.4215]

z = [0 ,0 ,0 ,742.6027,129.6225]

fval =2.6798e+03

Interpretation:

Based on the fval calculated, we can see that portfolio 3 costs the most, portfolio 2 costs the second most, and portfolio 1 costs the least. We may conclude that the less you are allowed to put into the B rating bonds, the more the portfolio costs.

Q2 P1(a):

The basic statistics for adjusted closing prices of SPY, GOVT, and EEMV from Jan 2014 to end of Jan 2022:

Stocks/Assets	SPY	GOVT	EEMV
Arithmetic Average	0.011577	0.002009	0.003444
Geometric Average	0.010779	0.001948	0.002782
Standard Deviation	0.040117	0.011159	0.036398

The covariance matrix:

covariance	SPY	GOVT	EEMV
SPY	0.001609	-0.000114	0.001014
GOVT	-0.000114	0.000125	-0.000057
EEMV	0.001014	-0.000057	0.001325

Q2 P1(b):

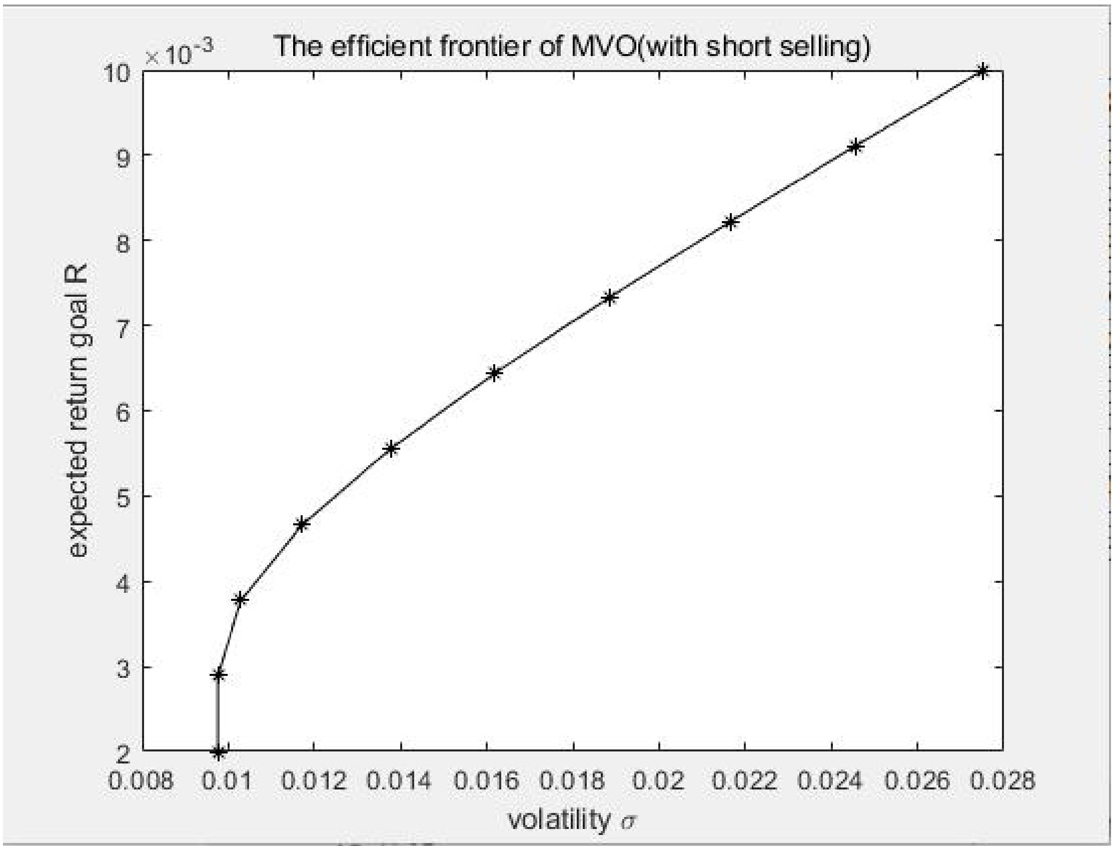
Results and code:

```
Q = [0.001609,  -0.000114,  0.001014;
    -0.000114,  0.000125,   -0.000057;
    0.001014,   -0.000057,  0.001325;];
n = 3;
c = zeros(n,1);
A = -[0.010779,0.001948,0.002782]
%b = -[];
Aeq = [ones(1,n)]%[1,1,1];
beq = [1];
ub = [inf;inf;inf;];
%for short selling
lb = [-inf;-inf;-inf;]
%without short selling:
%lb = [0;0;0;]

goal_R = linspace(0.002,0.01,10)
%goal_R=[0.003:0.000856:0.0107]; % expected return goals
x = zeros(length(goal_R),n);
std_devi = zeros(length(goal_R),1)
variance = zeros(length(goal_R),1)

for a=1:length(goal_R)
    b = -goal_R(a)
    [x(a,:), fval(a,1)] = quadprog(Q, c, A,b, Aeq,beq, lb,ub);
    std_devi(a,1)=(x(a,:)*Q*x(a,:)')^.5;
    % standard deviation = (x'*Q*x)^.5
    variance(a,1)=x(a,:)*Q*x(a,:)';
end
plot(std_devi, goal_R, '-k*')
xlabel('volatility \sigma')
ylabel('expected return goal R')
title('The efficient frontier of MVO')
```

If we allow short selling(w can be negative):
(W_1:SPY, W_2:GOVT, W_3:EEMV)

Target return	w_1	w_2	w_3	variance
0.002	0.1001	0.8674	0.0325	0.00009516
0.0028889	0.1046	0.8664	0.0290	0.00009517
0.0037778	0.2124	0.8428	-0.0552	0.00010613
0.0046667	0.3211	0.8190	-0.1400	0.00013763
0.0055556	0.4298	0.7951	-0.2249	0.00018969
0.0064444	0.5384	0.7713	-0.3098	0.00026227
0.0073333	0.6471	0.7475	-0.3946	0.0003554
0.0082222	0.7558	0.7237	-0.4795	0.00046907
0.0091111	0.8644	0.6999	-0.5643	0.0006033
0.01	0.9731	0.6761	-0.6492	0.00075805

The corresponding efficient frontier:

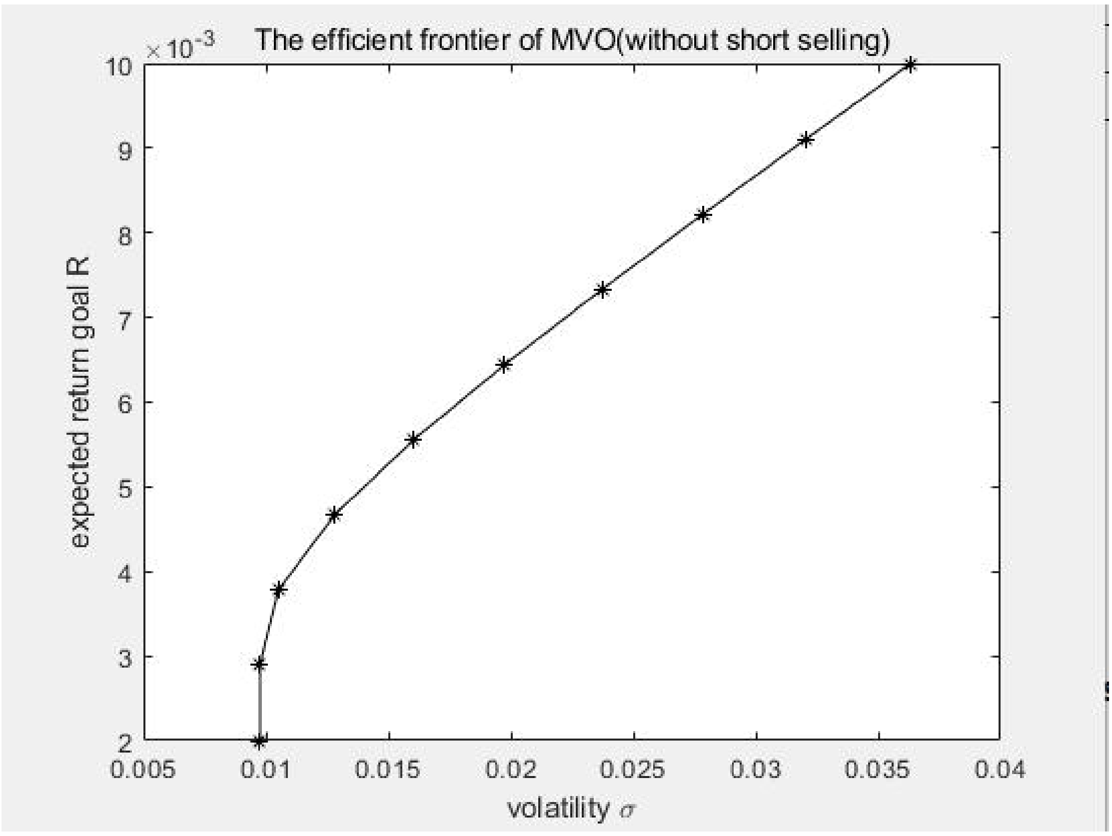


If we do not allow short selling($w \geq 0$):

(w_3 does have some weights for the last 7 portfolio, but they are too small that we cannot view it if we let the result remain 4 decimal places. One way to show their weights is to add "format long" or "format shortg" at the beginning of the matlab file. But this can make the result messy(around 10 decimal places))

Target return	w_1	w_2	w_3	variance
0.002	0.1000	0.8673	0.0326	0.00009516
0.0028889	0.1044	0.8664	0.0291	0.00009517
0.0037778	0.2072	0.7927	0.0001	0.00011021
0.0046667	0.3079	0.6921	0.0000	0.00016379
0.0055556	0.4085	0.5915	0.0000	0.00025717
0.0064444	0.5092	0.4908	0.0000	0.00039027
0.0073333	0.6098	0.3902	0.0000	0.00056314
0.0082222	0.7105	0.2895	0.0000	0.00077577
0.0091111	0.8111	0.1889	0.0000	0.0010282
0.01	0.9118	0.0882	0.0000	0.0013203

The corresponding efficient frontier:



Q2 P1(c):
The monthly returns of Feb 2022 for the 3 assets are as follows:
SPY:-0.02952
GOVT:-0.00772
EEMV:-0.00112

	P1:Minimum variance portfolio with short selling(10.01% SPY, 86.74% GOVT, 3.25% EEMV)	P2:Minimum variance portfolio without short selling(10% SPY, 86.73% GOVT,3.26% EEMV)	P3:Portfolio with 70% SPY, 20% GOVT, 10% EEMV	P4:Equally weighted portfolio (all 1/3)
return	-0.009688	-0.009684	-0.02232	-0.012787

Interpretation: The returns for all 3 assets in February are negative. So all portfolios have negative returns but some helps to minimize the loss. If we rank the returns of the 4 portfolio, we have p2>p1>p4>p3. We can clearly see that SPY has much larger negative returns than the other 2 assets, so if a portfolio put a larger weight on SPY, this portfolio must have a lower return. And this is actually true here for the 4 portfolio returns we calculated. The standard deviations(risk,volatility) for the 3 assets are 0.04(SPY), 0.01(GOVT) and 0.036(EEMV). So it is not hard to see that a person holding a portfolio with more weights on SPY and EEMV will bear much higher risk(eg.p3) and holding portfolio with more weight on GOVT will bear much lower risk(ag. p1,p2). If one wants higher returns, he has to bear higher risks. This is true especially in our situation here when all the returns of the assets are negative and we want to hedge the risk. If a person is risk averse, he will choose p2 so that he will have a small loss. But if he is a risk seeker, he may choose p3 and he will suffer comparatively bigger losses.

Q2 P2:

Similar to P1(a), we calculate the following statistics:

Stocks/assets	ACN	BR	CBOE	CME	ICE
Arithmetic Average	0.018421	0.017721	0.011821	0.015792	0.013322
Geometric Average	0.016701	0.016015	0.009665	0.014357	0.011825
Standard Deviation	0.059091	0.059232	0.065333	0.054294	0.055712

The covariance matrix:

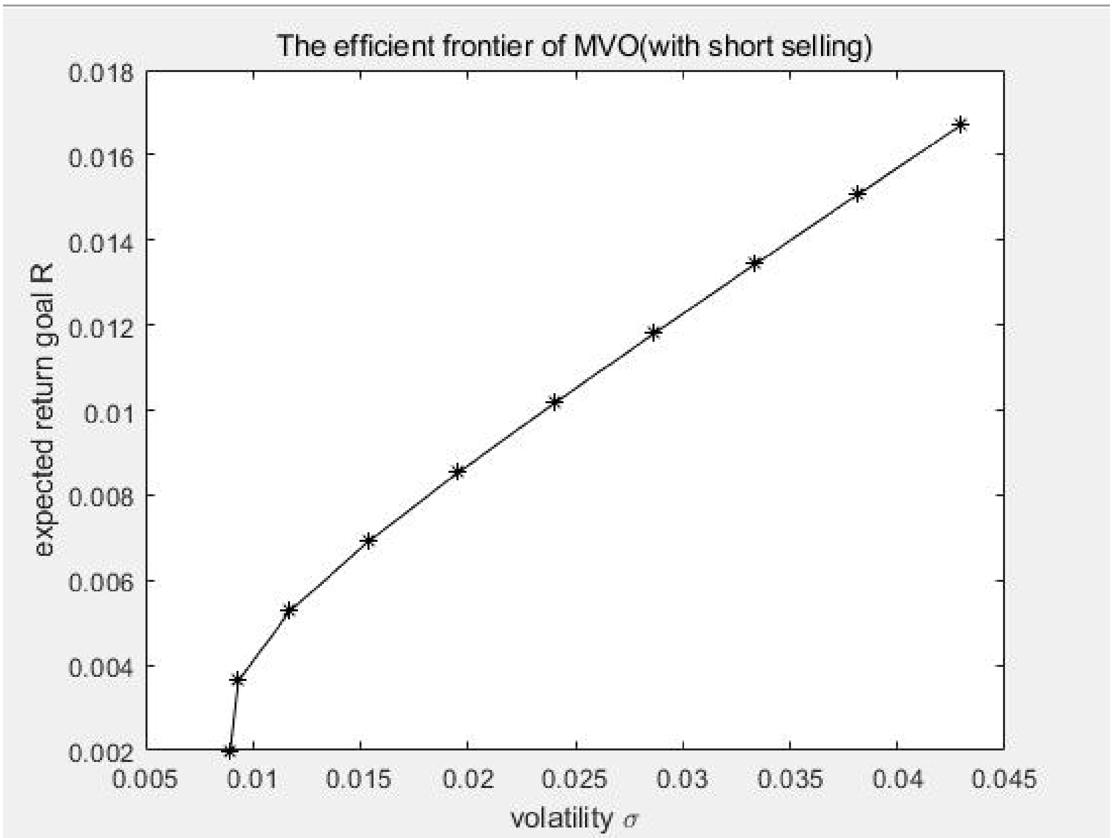
	SPY	GOVT	EEMV	ACN	BR	CBOE	CME	ICE
SPY	0.001609	-0.000114	0.001014	0.001893	0.001482	0.000940	0.000816	0.001274
GOVT	-0.000114	0.000125	-0.000057	-0.000061	0.000033	-0.000017	-0.000121	-0.000086
EEMV	0.001014	-0.000057	0.001325	0.000926	0.000891	0.000323	0.000192	0.000392
ACN	0.001893	-0.000061	0.000926	0.003492	0.002259	0.001281	0.001069	0.001869
BR	0.001482	0.000033	0.000891	0.002259	0.003508	0.001046	0.001112	0.001446
CBOE	0.000940	-0.000017	0.000323	0.001281	0.001046	0.004268	0.001924	0.001670
CME	0.000816	-0.000121	0.000192	0.001069	0.001112	0.001924	0.002948	0.001837
ICE	0.001274	-0.000086	0.000392	0.001869	0.001446	0.001670	0.001837	0.003104

The corresponding code is as follows:

```
Q = [0.001609,  -0.000114,  0.001014,   0.001893,   0.001482,
0.000940,   0.000816,   0.001274;
-0.000114,  0.000125,   -0.000057,  -0.000061,  0.000033,   -0.000017,
-0.000121,  -0.000086;
0.001014,   -0.000057,  0.001325,   0.000926,   0.000891,   0.000323,
0.000192,   0.000392;
0.001893,   -0.000061,  0.000926,   0.003492,   0.002259,   0.001281,
0.001069,   0.001869;
0.001482,   0.000033,   0.000891,   0.002259,   0.003508,   0.001046,
0.001112,   0.001446;
0.000940,   -0.000017,  0.000323,   0.001281,   0.001046,   0.004268,
0.001924,   0.001670;
0.000816,   -0.000121,  0.000192,   0.001069,   0.001112,   0.001924,
0.002948,   0.001837;
0.001274,   -0.000086,  0.000392,   0.001869,   0.001446,   0.001670,
0.001837,   0.003104;];

n = 8;
c = zeros(n,1);
A = -
[0.010779,0.001948,0.002782,0.016701,0.016015,0.009665,0.014357,0.011
825];
%b = -[];
Aeq = [ones(1,n)]%[1,1,1];
beq = [1];
ub = [inf;inf;inf;inf;inf;inf;inf;inf;];
%for short selling
lb = [-inf;-inf;-inf;-inf;-inf;-inf;-inf;-inf;]
%without short selling:
%lb = [0;0;0;0;0;0;0;0]

goal_R=linspace(0.002,0.0167,10);
%goal_R=[0.002:0.00147:0.0167]; % expected return goals

x = zeros(length(goal_R),n);
std_devi = zeros(length(goal_R),1)
variance = zeros(length(goal_R),1)

for a=1:length(goal_R)
    b = -goal_R(a)
    [x(a,:), fval(a,1)] = quadprog(Q, c, A,b, Aeq,beq, lb,ub);
    std_devi(a,1)=(x(a,:)*Q*x(a,:)')^.5;
    % standard deviation = (x'*Q*x)^.5
    variance(a,1)=x(a,:)*Q*x(a,:)';
end
plot(std_devi, goal_R, '-k*')
xlabel('volatility \sigma')
ylabel('expected return goal R')
title('The efficient frontier of MVO')
```

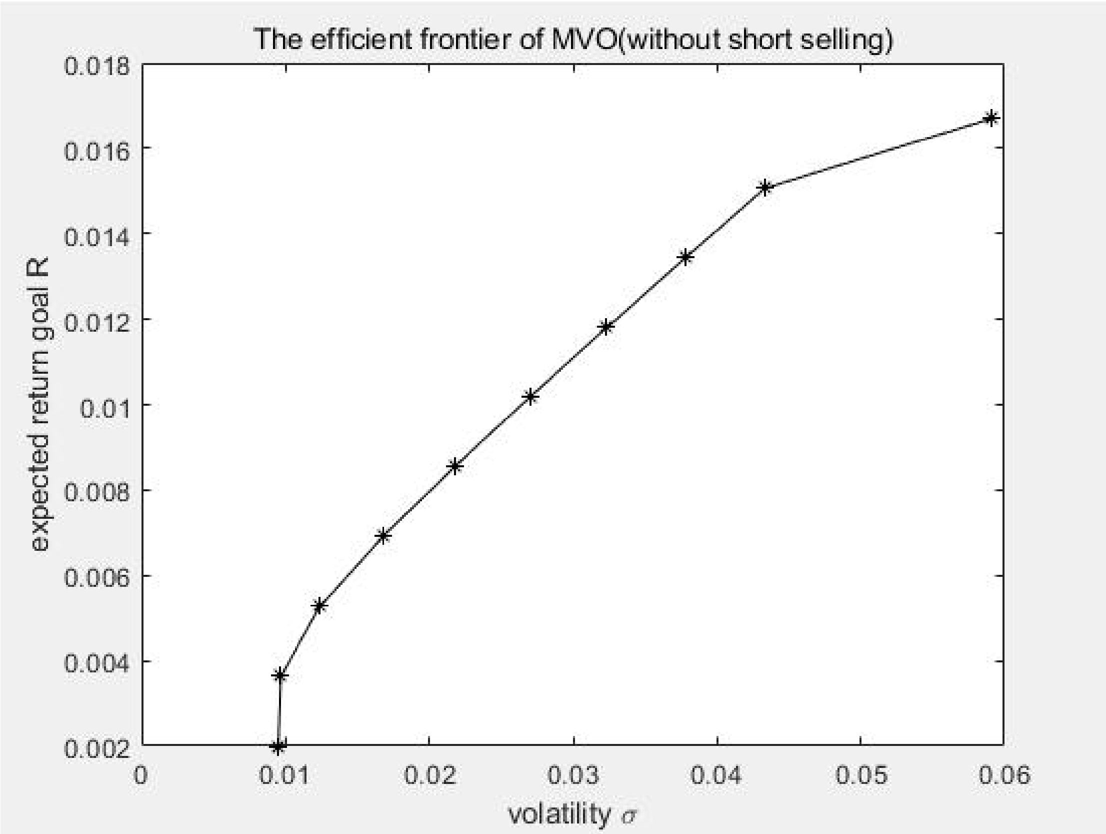
If we allow short selling(w can be negative):
(W_1:SPY, W_2:GOVT, W_3:EEMV, W4:ACN, W5:BR, W6:CBOE, W7:CME, W8:ICE)

Target return	W1	W2	W3	W4	W5	W6	W7	W8	variance
0.002	0.1643	0.8554	0.0276	-0.0360	-0.0514	-0.0197	0.0612	-0.0014	0.000080168
0.0036333	0.2041	0.8316	-0.0233	-0.0208	-0.0344	-0.0253	0.0852	-0.0171	0.000087072
0.0052667	0.2788	0.7870	-0.1187	0.0078	-0.0026	-0.0358	0.1301	-0.0466	0.00013715
0.0069	0.3535	0.7424	-0.2141	0.0363	0.0292	-0.0463	0.1750	-0.0760	0.00023568
0.0085333	0.4282	0.6979	-0.3094	0.0648	0.0609	-0.0568	0.2199	-0.1055	0.00038263
0.010167	0.5029	0.6533	-0.4048	0.0934	0.0927	-0.0673	0.2648	-0.1350	0.00057803
0.0118	0.5776	0.6087	-0.5002	0.1219	0.1245	-0.0778	0.3097	-0.1644	0.00082188
0.013433	0.6523	0.5642	-0.5956	0.1504	0.1563	-0.0883	0.3546	-0.1939	0.0011141
0.015067	0.7270	0.5196	-0.6910	0.1789	0.1881	-0.0988	0.3995	-0.2233	0.0014549
0.0167	0.8017	0.4750	-0.7864	0.2075	0.2198	-0.1093	0.4444	-0.2528	0.001844



If we do not allow short selling(w ≥ 0):
(W_1:SPY, W_2:GOVT, W_3:EEMV, W4:ACN, W5:BR, W6:CBOE, W7:CME, W8:ICE)

Target return	W1	W2	W3	W4	W5	W6	W7	W8	variance
0.002	0.0604	0.8430	0.0515	0.0000	0.0000	0.0000	0.0451	0.0000	9.0099e-05
0.0036333	0.1087	0.8231	0.0105	0.0000	0.0000	0.0000	0.0578	0.0000	9.3573e-05
0.0052667	0.1324	0.7029	0.0000	0.0387	0.0088	0.0000	0.1172	0.0000	0.00015362
0.0069	0.0961	0.5974	0.0000	0.0945	0.0476	0.0000	0.1643	0.0000	0.00028104
0.0085333	0.0598	0.4920	0.0000	0.1504	0.0863	0.0000	0.2115	0.0000	0.0004721
0.010167	0.0236	0.3865	0.0000	0.2061	0.1251	0.0000	0.2586	0.0000	0.00072682
0.0118	0.0000	0.2762	0.0000	0.2563	0.1628	0.0000	0.3047	0.0000	0.0010453
0.013433	0.0000	0.1568	0.0000	0.2959	0.1984	0.0000	0.3488	0.0000	0.0014287
0.015067	0.0000	0.0374	0.0000	0.3355	0.2340	0.0000	0.3929	0.0000	0.0018774
0.0167	0.0000	0.0000	0.0000	0.9985	0.0015	0.0000	0.0000	0.0000	0.0034884



Interpretation:
With more asset choices, we are able to create portfolios that can bring us more portfolio return but also at a relatively higher risk level. With short selling, we are able to obtain the same expected return(as the system without short selling) but at a lower risk level.