APS1022 PROJECT ASSIGNMENT

Yuyang Shao

1002961636

Part1.estimation of model parameters

In this assignment, expected market return μ, risk aversion coefficient λ, covariance matrix Q and the risk free rate(rf) are the major parameters that need to be estimated. I collected the adjusted close price data from Dec 2004 - Nov 2008 from Yahoo Finance. The parameters are estimated using the data from Dec 2004 - Sep 2008 and the data from Oct 2008 - Nov 2008 are for testing purposes. The risk free rate(rf) is obtained by taking average of the 10 year treasury bond rate from 2005-2008. The monthly risk free rate is calculated as 1/12 of the yearly rate. μ is calculated as the mean returns of the monthly adjusted close prices. The covariance matrix Q is computed from the correlations between the monthly stock returns. λ is estimated according to the suggestion from Idzorek.

Part2.models and general performances

2.1 Mean Variance Optimization

$max\ \ \mu^T x - \lambda x^T Q x\ \ w.r.t\ x$

$subject\ to\ \ 1^T x = 1$

This classic model basically tries to maximize return and minimize the risk. With the risk aversion coefficient λ, one can have different portfolio weights according to how much risk he can bear. Besides its simplicity, this model is very sensitive to return/price changes. Even a small change can cause significant weights shifts. This nature causes MVO's unstable performance from time to time. So, short selling with MVO is extremely risky. In practice, it is a good idea to set an upper bound and lower bound for the weights(x) to prevent too much shift in weights and reduce risk. The performance on the return data from Dec 2004 - Sep 2008 and related quantities for optimal MVO portfolio are as follows.

```
MVO result:
MVO Portfolio return = 0.080629
MVO Portfolio variance = 0.009606
MVO Standard deviation = 0.098012
MVO Sharpe ratio = 0.785700
```

2.2 Robust Mean Variance Optimization

$max\ \ \mu^T x - \lambda x^T Q x\ \ w.r.t\ x$

$subject\ to\ \ \mu^T x - \delta^T |x| \geq R$

$1^T x = 1$

The Robust MVO model is very similar to the classic MVO model but it has one more constraint. It introduces an uncertainty set around the estimated expected returns and estimate around it so that it becomes more stable and less sensitive. It generally provides better estimations than classic MVO. I used the box uncertainty set here. I also need λ in the objective function for efficient frontier generation later, so I formulate the problem similar to MVO with just one more return constraint with the penalty term. The performance on the return data from Dec 2004 - Sep 2008 and related quantities for optimal Robust MVO portfolio are as follows.

```
Robust MVO with 90% CL result:
Robust MVO with 90% CL Portfolio return = 0.108295
Robust MVO with 90% CL Portfolio variance = 0.018436
Robust MVO with 90% CL Standard deviation = 0.135778
Robust MVO with 90% CL Sharpe ratio = 0.770914
```

```
Robust MVO with 95% CL result:
Robust MVO with 95% CL Portfolio return = 0.110653
Robust MVO with 95% CL Portfolio variance = 0.019814
Robust MVO with 95% CL Standard deviation = 0.140761
Robust MVO with 95% CL Sharpe ratio = 0.760378
```

2.3 Risk Parity Optimization(no short selling)

$$\min_{\boldsymbol{x}} \quad \sum_{i=1}^{n}\sum_{j=1}^{n} \left(x_i(\boldsymbol{Qx})_i - x_j(\boldsymbol{Qx})_j\right)^2$$

$$\text{s.t.} \quad \mathbf{1}^T\boldsymbol{x} = 1,$$

$$\boldsymbol{x} \geq 0.$$

The risk parity optimization technique allocates weights in a way that the risk contributions of all assets are the same(or the difference between the individual risk

contribution is minimized). It can be seen very clearly by scrutinizing the objective function alone. This method is highly resistant to the sudden change in the stock price or return of a single stock. It is a relatively safer method compared to MVO and robust MVO(portfolio variance much lower than MVO and robust MVO). Conversely, it is hard to earn higher profit as well with such low portfolio variance. For this method, the short selling is not allowed, so when a stock crush happens, it is highly likely that it will get a low negative return. The performance on the return data from Dec 2004 - Sep 2008 and related quantities for optimal risk parity optimization portfolio are as follows.

```
RPO result:
RPO Portfolio return = 0.006314
RPO Portfolio variance = 0.000476
RPO Standard deviation = 0.021826
RPO Sharpe ratio = 0.123360
```

2.4 Market Capitalization Portfolio

This portfolio does not require solving any convex optimization problem. The portfolio weights are calculated as (Each company's market cap/total market cap of 20 chosen companies). Each company's market capitalization is calculated as its average market capitalization from 2005 - 2008. This technique can significantly reduce risk if the portfolio is diversified enough. Its weakness is obvious, too. As you can see from the result comparison of the 4 methods, market cap portfolio is relatively less risky compared to the MVO and robust MVO methods. Although it can hedge risk well, it is unlikely to bring high return as well since the portfolio variance is low. Using the statistics shown below, the risk averse coefficient λ can be obtained, which is calculated as suggested by Idzorek : λ = (expected return of the market - rf rate)/variance of the market. λ is used in MVO and Robust MVO as well. The performance on the return data from Dec 2004 - Sep 2008 and related quantities for market cap portfolio are as follows.

```
Market Cap result:
Market Cap return = 0.007225
Market Cap variance = 0.000906
Market Cap Standard deviation = 0.030101
Market Cap Sharpe ratio = 0.119720
```

Part3. answer to question A(Oct 2008)

	return	variance	standard_deviation	sharpe_ratio
MVO	0.185880	0.009606	0.098012	1.859565
Robust_MVO_90	0.305243	0.018436	0.135778	2.221430
Robust_MVO_95	0.303770	0.019814	0.140761	2.132322
Risk Parity Optimization	-0.145989	0.000476	0.021826	-6.854690
Market Capitalization	-0.116334	0.000906	0.030101	-3.985125

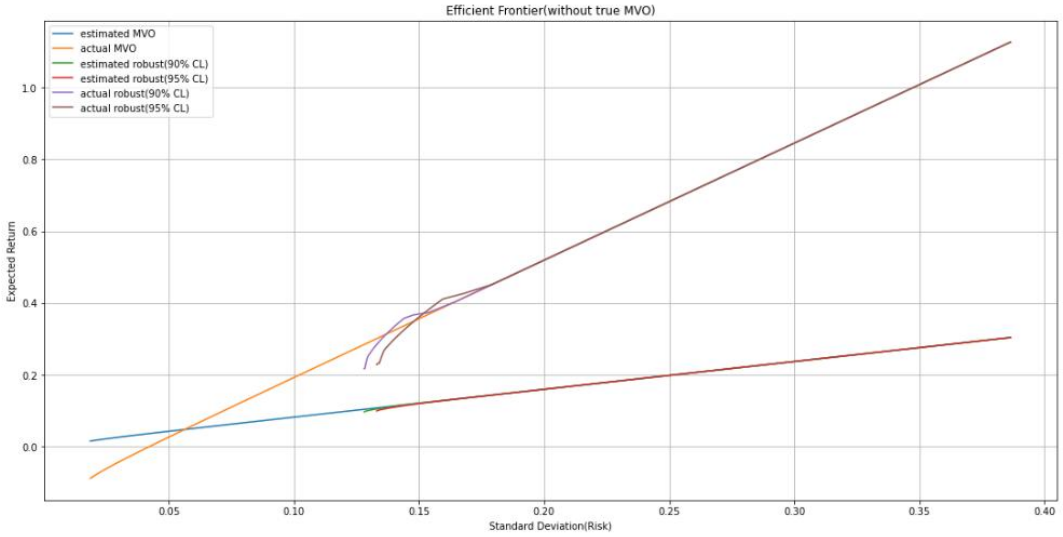
From the table, we can clearly see that the mean variance related approaches outperform the risk parity approach and the market cap approach. The robust MVO portfolios have the highest returns and Sharpe ratio. The stock crash at that time have severe effect on the stock prices. It is surprising that all stocks have a negative return except only ED in Oct 2008. This can explain the negative return for risk parity optimization(RPO) portfolio and market cap portfolio. We do not allow short selling for RPO. So when 19 out of 20 stocks are going down, there is no way that RPO can generate positive return without short selling. The portfolio weights for market cap approach should be positive since we allocate weights according to each company's market capitalization. So it is nearly impossible for it to generate positive return in this situation as well. For MVO and Robust MVO, the short selling is allowed. The results are impressive, they all bring large profit even during the stock crush period. With a good understanding of mean return and covariance matrix, the MVOs utilizes short sells to efficiently hedge the risk. The robust MVO portfolios outperform classic MVO portfolio because of the extra return constraint with the penalty. The variance of the RPO and market cap portfolios are much smaller than the MVO portfolios.

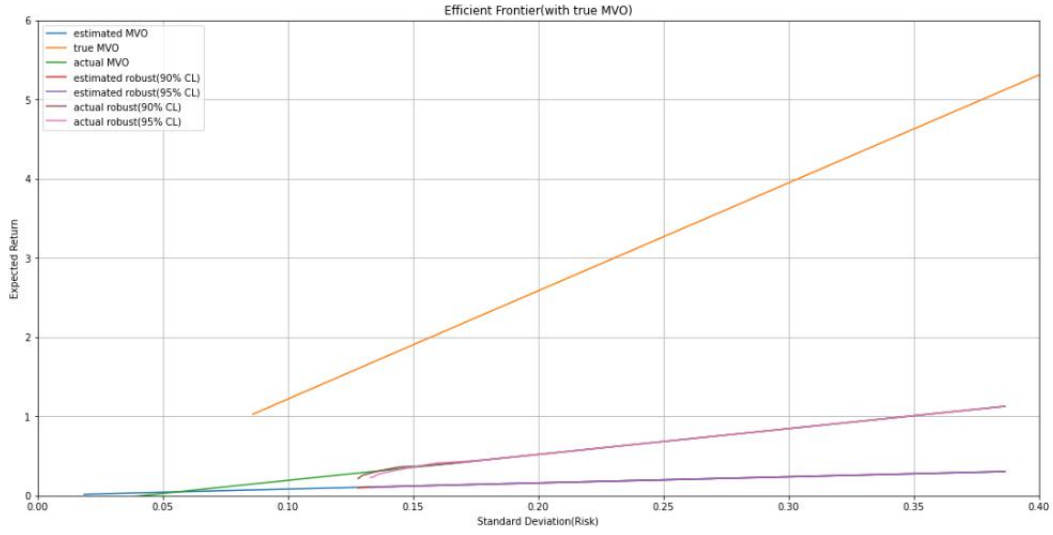
Part4. answer to question B(Nov 2008)

	return	variance	standard_deviation	sharpe_ratio
MVO	0.264199	0.009606	0.098012	2.658637
Robust_MVO_90	0.158755	0.018436	0.135778	1.142555
Robust_MVO_95	0.095052	0.019814	0.140761	0.649542
Risk Parity Optimization	-0.008364	0.000476	0.021826	-0.549125
Market Capitalization	-0.039984	0.000906	0.030101	-1.448652

Compared to returns of October, Around 10 companies have positive returns in November. Like in October 2008, RPO and market cap portfolio have negative returns because of the same reasons. But they perform better in November(less negative return) since more companies start to have positive returns in November 2008. The statistics mimic the one in October except that MVO portfolio has the highest return and Sharpe ratio this month. Again, the portfolios which allow short selling(MVO portfolios) have higher returns because only short selling can help portfolios gain reasonable returns when most of the companies have negative stock returns in the portfolio.

Part6. answer to question C





The efficient frontier for true MVO has much higher return for all levels of risk compared to other efficient frontiers(EF), To see clearly the relationships of the rest of the EFs, the first figure does not contain the true MVO EF. The second figure contains all the seven EFs. Each EF is constructed using 120 points(expected return, volatility pairs). Normally we would expected that for the same volatility, the ranking of EFs by profit from high to low is :1.estimated EF, 2.true EF, 3.actual EF. But the actual ranking I get is 1.true EF, 2.actual EF, 3.estimated EF. Given the situation in Oct 2008, it is not hard to understand the reason. The stock crush has a severe influence on the stock price. In Oct 2008, Only 1 out of 20 chosen stocks has a positive return. This means that short selling is the only way for a portfolio to profit. The expectation of the relationship between the 3 EFs is based on normal situation(eg.10 positive returns and 10 negative returns). It cannot simply be applied to the extreme situation like Oct 2008. It is reasonable to expect some change in the relationships. The estimated frontiers tend to exaggerate certain errors in the input parameters, resulting in biased estimates of portfolio performance. Those estimates do not capture much of the potential opportunities for short selling in this harsh period. The relationships between assets may be significantly different from the historical relationships. The reason for the true MVO EF to have such high return is that it uses the true parameters instead of estimated value. Through MVO, it already adapted the true parameters for optimization. So it has the highest ability to identify potential short selling opportunities than any of the other EFs. The actual EF is a hybrid of estimated EF and true EF. It consists of the true portfolio mean and variance points corresponding to portfolios on the estimated frontier. So it make sense that its ability to identify short selling opportunities is between estimated EF and true EF.

Reference:

"Companies Ranked by Market Cap." *CompaniesMarketCap.com - Companies Ranked by Market Capitalization*, https://companiesmarketcap.com/.

"Yahoo Finance - Stock Market Live, Quotes, Business & Finance News." *Yahoo! Finance*, Yahoo!, https://finance.yahoo.com/.

"10 Year Treasury Rate - 54 Year Historical Chart." *MacroTrends*, https://www.macrotrends.net/2016/10-year-treasury-bond-rate-yield-chart.