Example: MVO model

- Consider 3 securities with expected return given in Table 7.1 and with covariance given in Table 7.2

Table 7.1 Expected security returns

expected return	security 1 (i=1)	security 2 (i=2)	security 3 (i=3)
μ_i	9.73%	6.57%	5.37%

Table 7.2 Covariance of returns

covariance σ_{ij}	$i = 1$	$i = 2$	$i = 3$
$i = 1$	0.02553	0.00327	0.00019
$i = 2$		0.013400	-0.00027
$i = 3$			0.00125

Example: MVO model

- If the goal return of portfolio is 5.5%, and short selling is allowed, then the corresponding MVO is

$$\min\ (0.02553)x_1^2 + (0.01340)x_2^2 + (0.00125)x_3^2$$
$$+ 2(0.00327)x_1x_2 + 2(0.00019)x_1x_3 + 2(-0.00027)x_2x_3$$
$$\text{s.t. } (0.0972)x_1 + (0.0657)x_2 + (0.0537)x_3 \geq 0.055$$
$$x_1 + x_2 + x_3 = 1$$

- Solving the model give the optimal portfolio

$$x_1 = 0.0240,\ x_2 = 0.0928,\ x_1 = 0.8832$$

with risk (variance) $\sigma_p^2 = 0.033069$.

Example: MVO model

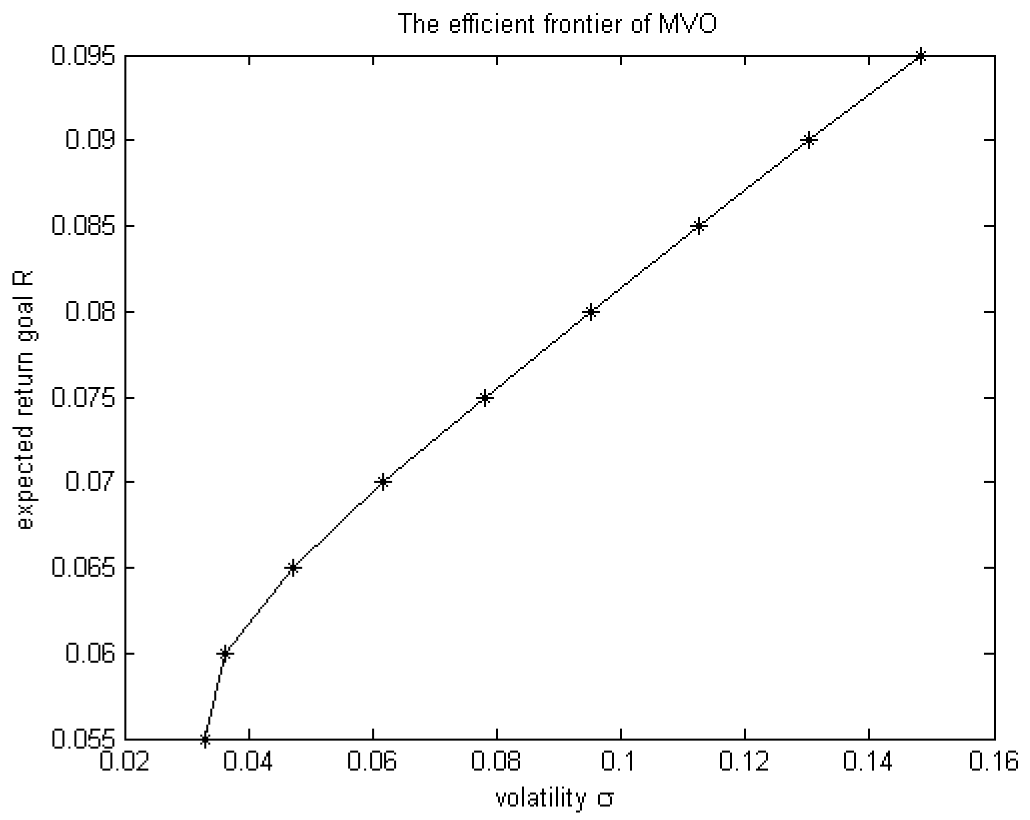
- Table 7.3 lists the optimal portfolio under different R:

Table 7.3 Optimal Portfolio for different R

return goal R	x_1	x_2	x_3	σ_P
5.50%	0.02398119	0.09280359	0.88321522	0.03291684
6.00%	0.11419877	0.11007779	0.77572344	0.03629413
6.50%	0.22313676	0.13093644	0.64592680	0.04714421
7.00%	0.33207475	0.15179509	0.51613017	0.06172131
7.50%	0.44101273	0.17265374	0.38633353	0.07796214
8.00%	0.54995072	0.19351238	0.25653690	0.09501737
8.50%	0.65888871	0.21437103	0.12674026	0.11251728
9.00%	0.76782669	0.23522968	-0.00305637	0.13028280
9.50%	0.87676468	0.25608833	-0.13285301	0.14821845

Example: MVO model

- Figure 7.1 shows the efficient frontier of MVO:



Solving QP using MATLAB

- The statement

  [*x*, *fval*] = quadprog(Q, f, A, b, Aeq, beq, lb, ub)

  returns optimal solution *x* and optimal objective value *fval* for quadratic program.
- Consider above MVO example with

$$f=\begin{bmatrix}0\\0\\0\end{bmatrix}, Q=\begin{bmatrix}0.02553 & 0.00327 & 0.00019\\ 0.00327 & 0.01340 & -0.00027\\ 0.00019 & -0.00027 & 0.00125\end{bmatrix},$$

$$A=\begin{bmatrix}-0.0972 & -0.0657 & -0.0537\end{bmatrix}, b=\begin{bmatrix}-0.055\end{bmatrix},$$

$$Aeq=\begin{bmatrix}1 & 1 & 1\end{bmatrix}, beq=\begin{bmatrix}1\end{bmatrix}$$

Solving QP using MATLAB

- Solving the problem by following Matlab code:

```
>> Q=[0.02553, 0.00327,  0.00019;
        0.00327, 0.013400, -0.00027;
        0.00019, -0.00027, 0.00125];
>> f=[0,0,0];
>> A=[-0.0972,-0.0657,-0.0537]; b=[-0.055];
>> Aeq=[1,1,1]; beq=[1];
>> [x, fval] = quadprog(Q,f, A, b, Aeq, beq, [ ], [ ])
   x =
     0.0240
     0.0928
     0.8832
   fval=
     5.4176e-004
```

Solving QP using MATLAB

- Generating Table 7.3 and efficient frontier Figure 7.1 by following code:

```
 n=3;
 mu=[9.73 6.57 5.37]/100; % expected returns of assets
 Q=[.02553  .00327  .00019;
      .00327  .01340 -.00027;
      .00019 -.00027  .00125]; % covariance matrix
 goal_R=[5.5:.5:9.5]/100; % expected return goals range from 5.5% to 9.5%
 c=zeros(n,1);
 for a=1:length(goal_R)
     A=-mu;  b=-goal_R(a);  Aeq=[ones(1,n)];  beq=[1];
    [x(a,:), fval(a,1)] = quadprog(Q, c, A,b, Aeq,beq, [],[]);
    std_devi(a,1)=(x(a,:)*Q*x(a,:)')^.5; % standard deviation = (x'*Q*x)^.5
end
plot(std_devi, goal_R, '-k*')
xlabel('volatility \sigma')
ylabel('expected return goal R')
title('The efficient frontier of MVO')
```