

Homework 4 Solutions

- 1a. $S = \frac{130\text{nm}}{32\text{nm}} = 4.0625$.
- Area scales by $1/S^2 = 0.06059$
- $A_{\text{new}} = 0.06059 \times A_{\text{old}} = 0.06059 \times 0.2 \mu\text{m}^2 = 0.01212 \mu\text{m}^2$.
- b. Capacitance scales by $1/S = 0.2462$.
- $C_{\text{new}} = 1/S \times C_{\text{old}} = 0.2462 \times 50 \text{ fF} = 12.31 \text{ fF}$.
- c. τ scales by $1/S = 0.2462$.
- $\tau_{\text{new}} = 1/S \times \tau_{\text{old}} = 0.2462 \times 20 \text{ ps} = 4.92 \text{ ps}$.
- d. R is constant in Denard scaling $\Rightarrow R_{\text{new}} = R_{\text{old}} = 0.1 \Omega$.

2a. $\text{Prob}(\text{all NAND2 are good}) = (1 - 0.000001)^{350,000} = 74.1\%$

$\text{Prob}(\text{all INV are good}) = (1 - 0.0000003)^{1,000,000} = 74.1\%$

$\text{Prob}(\text{all AOI22 are good}) = (1 - 0.0000045)^{200,000} = 40.7\%$

$\text{Prob}(\text{all NOR2 are good}) = (1 - 0.000001)^{200,000} = 13.5\%$

$\text{Yield} = \prod \text{Prob} = 3.02\%$

b. $\text{Prob}(\text{NAND2}) = (1 - 0.000001)^{150,000} = 86.1\%$

$\text{Prob}(\text{INV}) = (1 - 0.0000003)^{500,000} = 86.1\%$

$\text{Prob}(\text{AOI22}) = (1 - 0.0000045)^{100,000} = 63.8\%$

$\text{Prob}(\text{NOR2}) = (1 - 0.000001)^{100,000} = 36.8\%$

$\text{Yield} = 17.4\%$

c. $\text{Prob}(\text{NAND2}) = (1 - 0.000001)^{100,000} = 90.5\%$

$\text{Prob}(\text{INV}) = (1 - 0.0000003)^{333,333} = 90.5\%$

$\text{Prob}(\text{AOI22}) = (1 - 0.0000045)^{66,667} = 74.1\%$

$\text{Prob}(\text{NOR2}) = (1 - 0.000001)^{66,667} = 51.3\%$

$\text{Yield} = 31.1\%$

3a. Using Poisson's model:
 $Yield = e^{-AD} = e^{-2mm^2 \times 1/2mm^2} = e^{-1} = 36.8\%$

b. $Yield = e^{-1mm^2 \times 1/2mm^2} = e^{-1/2} = 60.65\%$

c. $Yield = e^{-0.5mm^2 \times 1/2mm^2} = e^{-1/4} = 77.88\%$

d. a. $Yield = e^{-2mm^2 \cdot 1/1mm^2} = e^{-2} = 13.53\%$

b. $Yield = e^{-1mm^2 \times 1/1mm^2} = e^{-1} = 36.8\%$

c. $Yield = e^{-0.5mm^2 \times 1/1mm^2} = e^{-0.5} = 60.65\%$

4. Inverter Variance = (standard deviation)² = (1ps)² = 1ps²

Expected delay of ring oscillator has a mean of $11 \times 10ps = 110ps$.

Ring Oscillator also has a variance of $11 \times 1ps^2 = 11ps^2$

\Rightarrow RO standard deviation = $\sqrt{11ps^2} = 3.317ps$

From table 7.9, Maximum of 100 ROs will have a mean of $110ps + (2.5 \times 3.22ps) = 118ps$.

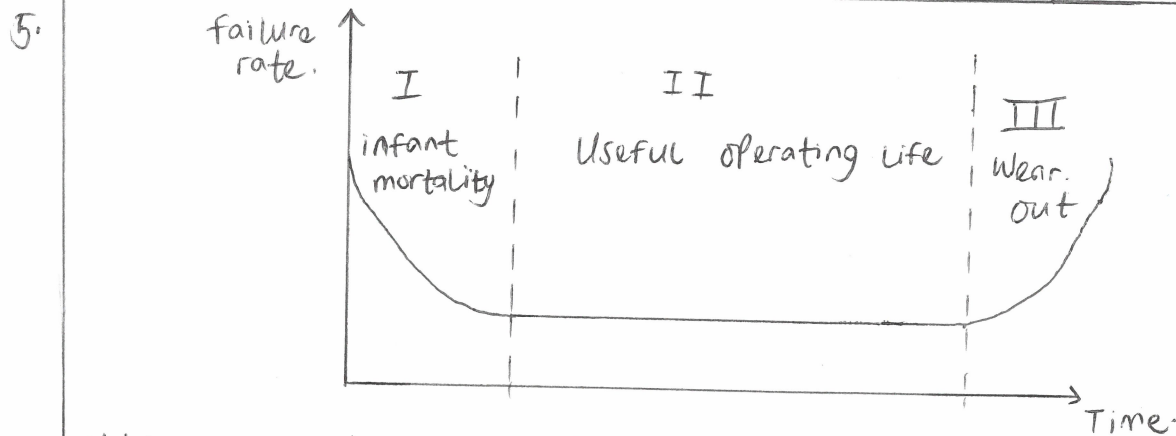
and standard deviation = $0.43 \times 3.22ps = 1.38ps$

a. Period = $2 \times 118ps = 236ps$ (Positive & negative edge).

frequency = $1/236ps = 4.24GHz$

b. 99.7% is $\mu + 3\sigma$ (mean + [3x standard deviation]) \Rightarrow Delay = $118 + (3 \times 1.38) = 122ps$.

Period = $2 \times 122 = 244ps \Rightarrow f = 1/244ps = 4.10GHz$



Chips are most likely to fail really early or have a long life. A good idea to weed out chips that will fail early is to stress chips during testing.