Introduction to Machine Learning

Lecture 13 Classification - Information Theory, Decision Tree, and Practical Challenges

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Outline

Review

- Support-vector machine (SVM)
- Kernelized SVM

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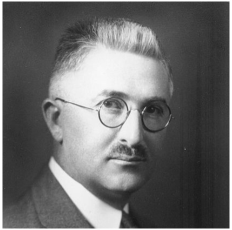
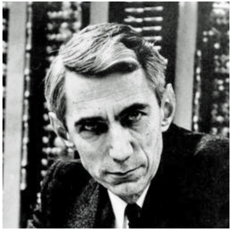
- Kernelized SVM

Today

- Information theory in Statistical ML
- Decision tree model and its learning

Information Theory: Origin





Ralph Hartley and Harry Nyquist 1920's   Claude Shannon 1940's

Claude E. Shannon, "A Mathematical Theory of Communication" Bell System Technical Journal in July and October 1948.

The Most Important Concept in Information Theory: Entropy

Motivation:

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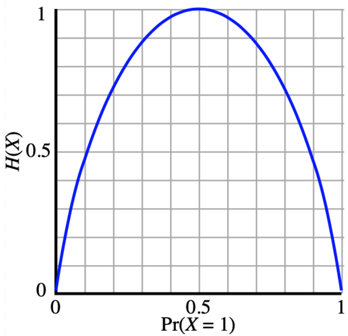

Extension: Joint Entropy

- The entropy of paired symbols (random variables):

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- We more care about the case that X and Y are dependent, and how much can we know X given Y?

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- Therefore, in general we have

$$H(X|Y) = H(X,Y) - H(Y) \tag{6}$$

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$$\begin{aligned} I(X;Y) &= H(X) - H(X|Y) \\ I(X;Y) &= I(Y;X) = H(X) + H(Y) - H(X,Y) \end{aligned} \tag{8}$$

Extension: Information Gain / Relative Entropy / KL-Divergence

- A way to compare two r.v.'s distributions
- Given a true distribution $p(X)$ and its estimation $q(X)$:

$$\mathrm{KL}(p_X \| q_X) = \mathbb{E}_{x \sim p_X}[\log \frac{p(x)}{q(x)}] = \sum_{x \in \mathcal{X}} p(x) \log \frac{p(x)}{q(x)}$$

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- In other words, maximize mutual information = maximize the KL-divergence between $p(X,Y)$ and $p(X)p(Y)$ = ensure that X and Y are heavily dependent.

Is KL-Divergence Always Nonnegative?

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Revisit Some Methods/Models through Information Theory

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- The decision criterion used in Decision Tree Models

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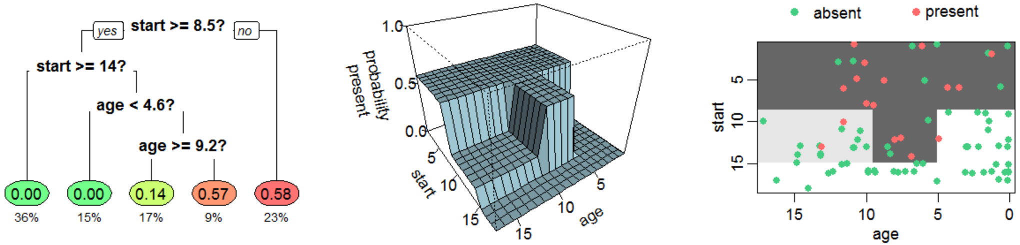

Decision Tree Learning

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Principle:

- Tree-generation algorithms: Generate a tree by the splitting driven by information gain maximization

Decision Tree Learning

- Given classified training data, $\mathcal{S} = \cup_{i=1}^{C} \mathcal{S}_i$ and $\boldsymbol{x}_n = [x_{1,n}, \ldots, x_{D,n}] \in \mathcal{S}_i$ is the n-th D-dimensional sample of the i-th class.

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- The above process will continue until all children nodes have consistent data or until the information gain is 0.

Recall The Example

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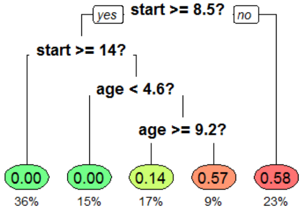

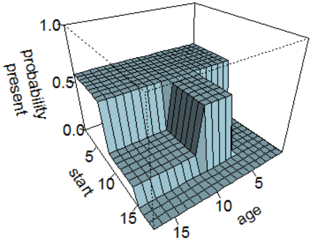

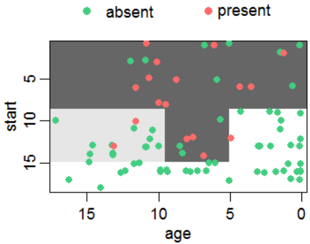

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Refer to https://en.wikipedia.org/wiki/Decision_tree_learning

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- But can be suppressed by ensemble modeling (Next lecture)

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Fairness, Interpretability, Privacy, ...

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- If a generative model is applicable, it may work better.

Ensemble Model: Bagging (or Called Bootstrap Aggregating)

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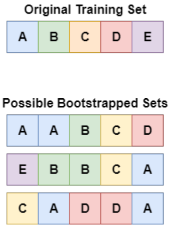
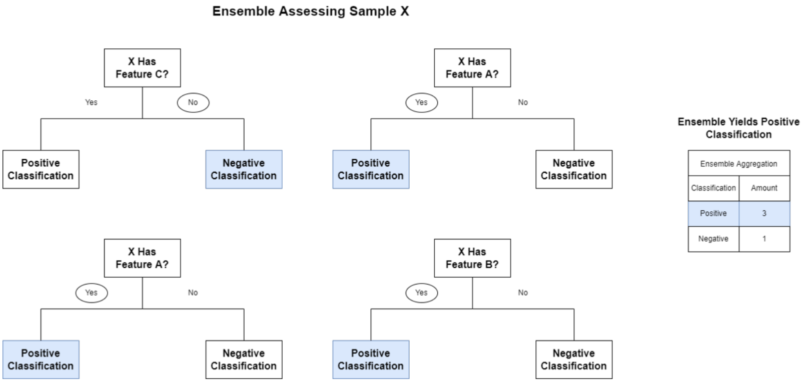
Principle:

- Applying bootstrap to generate bootstrapped datasets.



Ensemble Model: Bagging (or Called Bootstrap Aggregating)

- Training several models based on the bootstrapped datasets.
- In the testing phase, output the voting results of the models.



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- Training each new model to emphasize the samples that previous models mis-classified.

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- Training each new model to emphasize the samples that previous models mis-classified.

$$F_T(x) = \sum_{t=1}^{T} f_t(x) \tag{15}$$

- f_t is a weak learner/classifier created at step t.
- F_T is the ensemble model obtained after T steps.

Ensemble Model: Boosting (e.g., AdaBoost)

At each step t,

- **Training:** A model (hypothesis h) is selected and assigned a coefficient α_t, and we learn it via minimizing the total training error

$$h^* = \arg\min_h \sum_{(y,x)\in\mathcal{D}} \mathrm{loss}(y, F_{t-1}(x) + \alpha_t h(x))$$

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Note: we need to ensure each $\epsilon_t < 0.5$ (weak, but cannot weaker anymore.)

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- Boosting often shows better performance on bagging.
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- If each model is a decision tree:
  - Bagging: Random forest
  - Boosting: Adaptive boosting (AdaBoost)

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What we missed in this course (but you will learn it in the future)

- Neural networks, deep generative modeling, semi-supervised learning, contrastive learning, meta-learning, few/zero-shot learning, self-supervised learning, transfer learning, domain adaptation, Bayesian inference, Bayesian optimization, probabilistic graphical model, causal inference, metric learning, large-scale pretraining, distributed learning, federated learning, lifelong learning, reinforcement learning, time series, PAC learning theory, stochastic control theory, ...

In Summary

- Basic concepts of information theory: entropy, mutual information, etc.
- Revisit some loss functions and machine learning models from the viewpoint of information theory
- Decision tree model and its learning
- More practical problems and potential solutions.

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Next...

- Review and happy ending of this course:)
- Review all homework