Introduction to Machine Learning
Lab 7: Gaussian Mixture Model for Point Cloud Alignment

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1 Motivation

- Implement the EM algorithm for GMM and get some feelings about the pipeline.
- Based on the EM algorithm, implement a non-trivial application of GMM — affine registration of point clouds.
- Get to know that GMM is not limited to classic clustering problems.

2 Tasks

Please read Lecture 9 carefully before doing this lab work.

- Given a source point cloud $\boldsymbol{Y} \in \mathbb{R}^{M\times D}$, we would like to align/match it to a target point cloud $\boldsymbol{X} \in \mathbb{R}^{N\times D}$ via an affine transformation (projection + translation): for each row of $\boldsymbol{Y}$, i.e., $\boldsymbol{y} \in \mathbb{R}^{D}$,

$$\boldsymbol{x} = \boldsymbol{A}\boldsymbol{y} + \boldsymbol{t} + \boldsymbol{\epsilon}, \quad \boldsymbol{\epsilon} \sim \mathcal{N}(0, \sigma^2 \boldsymbol{I}_D) \tag{1}$$

  where $\boldsymbol{A} \in \mathbb{R}^{D\times D}$ is the projection matrix and $\boldsymbol{t} \in \mathbb{R}^{D}$ is the translation vector, σ^2 is the variance of noise.

- Suppose that $\boldsymbol{X}$ is the observed data of a GMM model with M Gaussian components:

$$\mathcal{N}(\boldsymbol{A}\boldsymbol{y}_m + \boldsymbol{t}, \sigma^2 \boldsymbol{I}_D), \quad \forall \boldsymbol{y}_m \in \boldsymbol{Y} \tag{2}$$

  and the components are with the same weight, *i.e.*, $w_m = \frac{1}{M}$ for $m = 1, ..., M$.

- **Task: Align $\boldsymbol{Y}$ to $\boldsymbol{X}$ via an EM algorithm, 1) learn the model parameters $\{\boldsymbol{A}, \boldsymbol{t}, \sigma^2\}$ and 2) estimate the correspondence between $\boldsymbol{Y}$ and $\boldsymbol{X}$.**
  1. Design the E-step to estimate the correspondence between $\boldsymbol{Y}$ and $\boldsymbol{X}$
  2. Design the M-step to learn the model parameters
  3. Design the EM algorithm, including the two modules above and an initialization strategy, to align two point clouds.