Introduction to Machine Learning Lab 3: Regularized Linear Regression

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1 Motivation

- Implement the commonly-used regularization methods for learning regression.
- Implement commonly-used learning algorithm, such as soft-thresholding for lasso, and iteratively reweighted least squares (IRLS) for robust linear regression.
- Quantitatively analyze the performance of different methods on model estimation and data prediction.

2 Tasks

Please read Lecture 4 carefully before doing this lab work.

1. Given the problem

$$\min_{\boldsymbol{w}} \|\boldsymbol{y} - \boldsymbol{X}\boldsymbol{w}\|_2^2 + \gamma \|\boldsymbol{w}\|_2^2, \tag{1}$$

implement the function "training" to achieve its closed-form solution.

- 2. Given the problem in (1), implement the function "training_sgd" to learn the model via stochastic gradient descent.
- 3. Implement the iterative soft-thresholding algorithm to solve the lasso problem

$$\min_{\boldsymbol{w}} \|\boldsymbol{y} - \boldsymbol{X}\boldsymbol{w}\|_{2}^{2} + \gamma \|\boldsymbol{w}\|_{1}. \tag{2}$$

4. Implement an algorithm to solve the linear regression with elastic net regularization

$$\min_{\boldsymbol{w}} \|\boldsymbol{y} - \boldsymbol{X}\boldsymbol{w}\|_{2}^{2} + \gamma_{1} \|\boldsymbol{w}\|_{1} + \gamma_{2} \|\boldsymbol{w}\|_{2}^{2},$$
 (3)

(Hint: Reformulate it to Lasso.)

5. Implement the iteratively reweighted least squares (IRLS) to solve robust linear regression:

$$\min_{\boldsymbol{w}} \|\boldsymbol{y} - \boldsymbol{X}\boldsymbol{w}\|_1 \tag{4}$$