Introduction to Machine Learning
Lab 6: Nonlinear Dimensionality Reduction (LLE and Laplacian Eigenmaps)

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1 Motivation

- Implement two typical manifold learning methods.
- Try to get some feelings about the sensitivity of these two methods to the structure of their K-NN graphs.
- Try to overcome the numerical instability of LLE.

2 Tasks

Please read Lecture 7 and 8 carefully before doing this lab work.

1. Construct a K-NN graph of the data points.
2. Implement the Locally Linear Embedding (LLE) algorithm.

Step 1 Construct a K-NN graph, for each sample $\boldsymbol{x}_n \in \mathbb{R}^D$, find its K nearest neighbors $\boldsymbol{X}_n \in \mathbb{R}^{D\times K}$.

Step 2 Solve the locally linear self-representation problem: for $n = 1, ..., N$

$$\begin{aligned} &\min_{\boldsymbol{w}} \|\boldsymbol{x}_n - \boldsymbol{X}_n\boldsymbol{w}\|_2^2, \quad s.t. \sum_{k=1}^{K} w_k = 1 \\ \Rightarrow \quad &\boldsymbol{w}_n = \text{rescale}(\boldsymbol{C}^{-1}\mathbf{1}_K), \quad \boldsymbol{C} = (\boldsymbol{X}_n - \boldsymbol{x}_n\mathbf{1}_K^T)^T(\boldsymbol{X}_n - \boldsymbol{x}_n\mathbf{1}_K^T) \end{aligned} \tag{1}$$

Step 3 Construct the alignment matrix based on $\{\boldsymbol{w}_n\}_{n=1}^N$ and obtain the latent codes by eigenvalue decomposition.

3. Implement the Laplacian Eigenmaps. The Laplacian matrix can be derived based on the dense similarity matrix or the sparse similarity matrix modulated by the adjacency matrix of the K-NN graph.