Programme	SECPH - Bachelor of Computer Science (Data Engineering) with Honours
Semester	2023/24-1
Section	Section 02
Course Name	SECI Discrete Structure
Lecturer's Name	Dr. Noorfa Haszlinna Binti Mustaffa
Assignment Topic	**Assignment 2**
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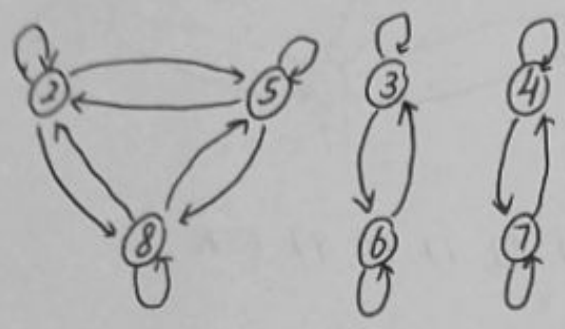
Question 1

1. $A = \{1, 3, 4, 5, 6, 7, 8\}$

$R = \{(2,5), (5,2), (2,8), (8,2), (3,6), (6,3), (4,7), (7,4), (5,8), (8,5), (2,2), (3,3), (4,4), (5,5), (6,6), (7,7), (8,8)\}$.

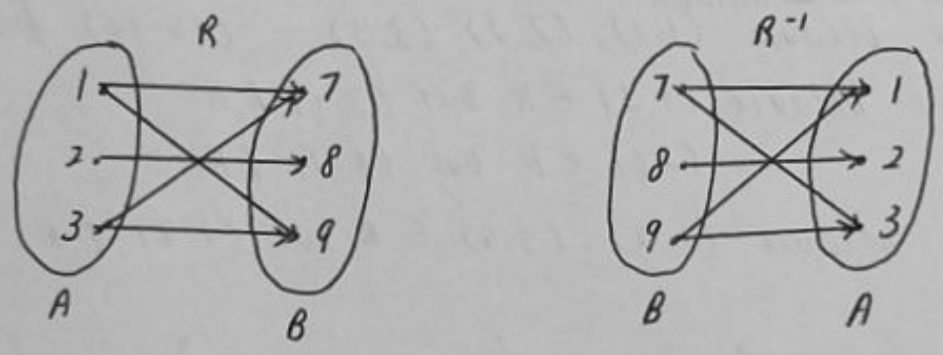


2. $A = \{1, 2, 3\}$, ~~B~~ $B = \{9, 8, 7\}$

(a) $R = \{(1,9), (1,7), (2,8), (3,9), (3,7)\}$

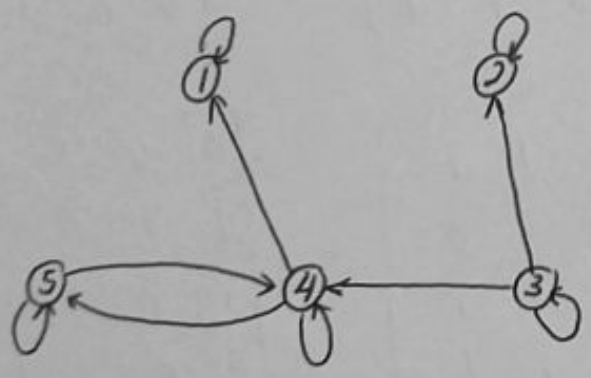
$R^{-1} = \{(9,1), (7,1), (8,2), (9,3), (7,3)\}$.

(b)



(c) R^{-1} is a relation from B to A such that $bRa \leftrightarrow b+a$ is an even number and $\neq$ for all $(b, a) \in B \times A$

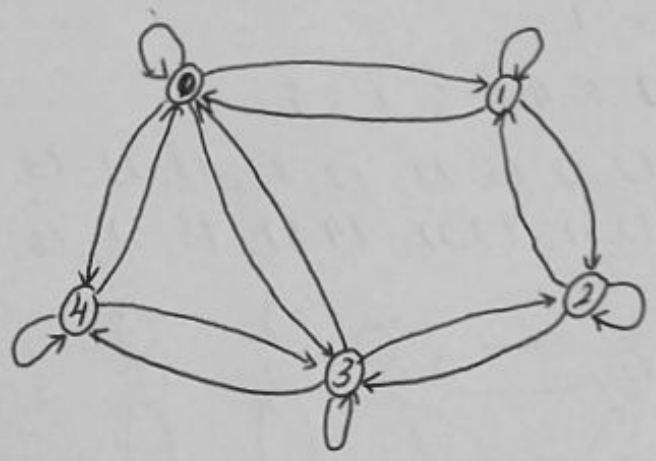
3.



	1	2	3	4	5
In-degree	2	2	1	3	2
Out-degree	1	1	3	3	2

4.

	0	1	2	3	4
0	1	1	0	1	1
1	1	1	1	0	0
2	0	1	1	1	0
3	1	0	1	1	1
4	1	0	0	1	1



Reflexive because $(0,0), (1,1), (2,2), (3,3), (4,4) \in R$

Symmetric ~~because~~ because $M_R = M_R^T$

Not transitive because $(0,1), (1,2) \in R$ but $(0,2) \notin R$

5. $R = \{(1,3), (2,6), (3,9), (4,12)\}$.

(a) Not reflexive, since it is irreflexive because $(1,1), (2,2), (3,3) \dots (14,14) \notin R$

(b) Not symmetric because $(1,3) \in R$ but $(3,1) \notin R$
$(2,6) \in R$ but $(6,2) \notin R$

(c) Not transitive because $(1,3), (3,9) \in R$ but $(1,9) \notin R$

(b) $RS = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

(b) SR $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} \otimes \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

Q2. Function.

7. A relation is a subset of a Cartesian product $X \times Y$ where it does not have any restrictions or properties.

A function is a relation from X to Y with having the properties;

- The domain of f (function) is X
- And if $(x,y), (x,y') \in f$, then $y = y'$.
- Function can be many to one and one-to-one only.

8. (i) $R = \{(2,3), (3,4), (4,5), (5,2)\}$

R is a function.

Domain $= \{2,3,4,5\} = A$

(ii) $R = \{(2,4), (3,4), (5,4), (4,4)\}$

R is a function.

Domain $= \{2,3,4,5\} = A$

(iii) $R = \{(2,3), (2,4), (5,4)\}$

R is not a function

Domain $= \{2,5\} \neq A$

$(2,3)$ and $(2,4) \in R$ but $3 \neq 4$

(iv) $R = \{(2,3), (3,5), (4,5)\}$

R is not a function

Domain $= \{2,3,4\} \neq A$

(v) $R = \{(2,2), (2,3), (4,4), (4,5)\}$

R is not a function

Domain $= \{2,4\} \neq A$

$(2,2)$ and $(2,3) \in R$ but $2 \neq 3$

$(4,4)$ and $(4,5) \in R$ but $4 \neq 5$

9. $R = \{(x,y) \mid y = x+5,\ x \text{ is } Z^+ \text{ less than } 6\}$

$R = \{(1,6), (2,7), (3,8), (4,9), (5,10)\}$

Domain of $R = \{1,2,3,4,5\}$

Range of $R = \{6,7,8,9,10\}$

10. (v) $f = R \to R,\ f(x) = 1-2x$

$f(x_1) = f(x_2)$

$1-2x_1 = 1-2x_2 \quad (-1)$

$-2x_1 = -2x_2 \quad (\div(-2))$

$x_1 = x_2$

f is one-to-one function

$y = 1-2x$

$\frac{1-y}{2} = x$

$f\left(\frac{1-y}{2}\right) = 1-2\left(\frac{1-y}{2}\right)$

$= 1-1+y$

$= y$

~~f is onto~~ $f(x) = y$, hence f is onto

∴ since f is one-to-one and onto, hence f is ~~bijection~~ bijective.

(vi) $f = R \to R,\ f(x) = 5x^2 - 1$

$f(x_1) = f(x_2)$

$5x_1^2 - 1 = 5x_2^2 - 1 \quad (+1)$

$5x_1^2 = 5x_2^2 \quad (\div 5)$

$x_1^2 = x_2^2 \quad (\sqrt{x^2})$

$\pm x_1 = \pm x_2$

since $x_1 \neq x_2$, f is not one-to-one function.

$y = 5x^2 - 1$

$x = \sqrt{\frac{y+1}{5}}$

$f\left(\sqrt{\frac{y+1}{5}}\right) = 5\left(\sqrt{\frac{y+1}{5}}\right)^2 - 1$

$= (y+1) - 1$

$= y$

$f(x) = y$ hence f is onto

∴ since f is not one-to-one, thus f is not bijective.

(vii) $f = R \to R,\ f(x) = x^4$

$f(x_1) = f(x_2)$

$x_1^4 = x_2^4 \quad (\sqrt[4]{x^4})$

$\pm x_1 = \pm x_2$

since $x_1 \neq x_2$, f is not one-to-one function

$y = x^4$

$x = \sqrt[4]{y}$

$f(\sqrt[4]{y}) = (\sqrt[4]{y})^4$

$= y$

$f(x) = y$, hence f is onto.

∴ since f is not one-to-one, f is not bijective.

10. (viii) $f = R \to R$, $f(x) = \left(\frac{x-2}{x-3}\right)$

$$f(x_1) = f(x_2)$$

$$\frac{x_1-2}{x_1-3} = \frac{x_2-2}{x_2-3}$$

$$(x_1-2)(x_2-3) = (x_2-2)(x_1-3)$$

$$x_1x_2 - 3x_1 - 2x_2 + 6 = x_2x_1 - 3x_2 - 2x_1 + 6 \quad \text{(All left to right)}$$

$$-x_1 + x_2 = 0 \quad (-x_1 \text{ to right})$$

$$x_2 = x_1$$

f is one-to-one function.

If $y = \frac{x-2}{x-3}$

$$y(x-3) = x-2$$
$$yx - 3y + 2 = x$$
$$2 - 3y = x - yx$$
$$2 - 3y = x(1-y)$$
$$\frac{2-3y}{1-y} = x$$

$$f\left(\frac{2-3y}{1-y}\right) = \frac{\left(\frac{2-3y}{1-y}\right)-2}{\left(\frac{2-3y}{1-y}\right)-3}$$

$$= \frac{\left(\frac{2-3y-2+2y}{1-y}\right)}{\left(\frac{2-3y-3+3y}{1-y}\right)}$$

$$= \frac{-y}{-1}$$

$$= y$$

$f(x) = y$, hence f is onto.

∴ f is bijective function. ✱

11. find $f(g(x))$ and the value if $x = \{0,1,2,3\}$.

(ix) $f(x) = 3x-1$, $g(x) = x^2-1$

$$f(g(x)) = 3(x^2-1) - 1$$
$$= 3x^2 - 3 - 1$$
$$\therefore f(g(x)) = 3x^2 - 4$$

$f(g(0)) = 3(0)^2 - 4 = -4$

$f(g(1)) = 3(1)^2 - 4 = -1$

$f(g(2)) = 3(2)^2 - 4 = 8$

$f(g(3)) = 3(3)^2 - 4 = 23$

$\therefore f \circ g = \{(0,-4), (1,-1), (2,8), (3,23)\}$ ✱

(x) $f(x) = x^2$; $g(x) = 5x-6$

$$f(g(x)) = (5x-6)^2$$
$$f(g(x)) = 25x^2 - 30x - 30x + 36$$
$$\therefore f(g(x)) = 25x^2 - 60x + 36$$

$f(g(0)) = 25(0)^2 - 60(0) + 36 = 36$

$f(g(1)) = 25(1)^2 - 60(1) + 36 = 1$

$f(g(2)) = 25(2)^2 - 60(2) + 36 = 16$

$f(g(3)) = 25(3)^2 - 60(3) + 36 = 81$

$\therefore f \circ g = \{(0,36), (1,1), (2,16), (3,81)\}$ ✱

11. (xi) $f(x) = x - 1$; $g(x) = x^3 + 1$

$$f(g(x)) = (g(x)) - 1$$
$$= (x^3 + 1) - 1$$
$$\therefore f(g(x)) = x^3$$

$f(g(0)) = (0)^3 = 0$

$f(g(1)) = (1)^3 = 1$

$f(g(2)) = (2)^3 = 8$

$f(g(3)) = (3)^3 = 27$

$\therefore f \circ g = \{(0,0), (1,1), (2,8), (3,27)\}$ ∆

Q3

12. xii) $a_n = 6a_{n-1} - 9a_{n-2}$, $a_0 = 1$, $a_1 = 6$

$a_2 = 6(a_1) - 9(a_0) = 27$

$a_3 = 6(a_2) - 9(a_1) = 108$

$a_4 = 6(a_3) - 9(a_2) = 405$

1, 6, 27, 108, 405, ...

xiii) $a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$, $a_0 = 2$, $a_1 = 5$, $a_2 = 15$

$a_3 = 6(a_2) - 11(a_1) + 6(a_0) = 47$

$a_4 = 6(a_3) - 11(a_2) + 6(a_1) = 147$

$a_5 = 6(a_4) - 11(a_3) + 6(a_2) = 455$

2, 5, 15, 47, 147, 455, ...

xiv) $a_n = -3a_{n-1} - 3a_{n-2} + a_{n-3}$, $a_0 = 1$, $a_1 = -2$, $a_2 = -1$

$a_3 = -3(a_2) - 3(a_1) + a_0 = 10$

$a_4 = -3(a_3) - 3(a_2) + a_1 = -29$

$a_5 = -3(a_4) - 3(a_3) + a_2 = 56$

1, -2, -1, 10, -29, 56, ...

13. $a_{n+1} = 5a_n - 3$, $a_1 = k$

i) $a_2 = 5a_1 - 3$
$= 5k - 3$

$a_3 = 5a_2 - 3$
$= 5(5k-3) - 3$
$= 25k - 18$

$a_4 = 5a_3 - 3$
$= 5(25k - 18) - 3$
$= 125k - 93$

ii) $a_4 = 125k - 93 = 7$

$125k = 100$

$k = 0.8$