

Discrete Structure Assignment 3

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Question 1

(a) Pigeon : Students

Pigeonhole : Scores 0 to 100

$$k = 2.$$

$$\left\lceil \frac{n}{101} \right\rceil = 2$$

$$n = 102.$$

(b) Pigeon : Students.

Pigeonhole : {A, B, C, D, F}.

$$\left\lceil \frac{n}{5} \right\rceil = 6.$$

$$n = 5 \times (6 - 1) + 1$$

$$n = 26$$

Question 2.

$$(a) P(A) = \frac{70}{70+30} = 0.7$$

$$(b) P(B) = \frac{30}{70+30} = 0.3$$

$$(c) P(W|A) = 0.2$$

$$(d) P(W|A) = \frac{P(A \cap W)}{P(A)}$$

$$P(A \cap W) = P(W|A) P(A)$$

$$= 0.2 (0.7)$$

$$P(A \cap W) = 0.14$$

$$(e) P(W|B) = \frac{P(B \cap W)}{P(B)}$$

$$P(B \cap W) = P(W|B) P(B)$$

$$= 0.4 (0.3)$$

$$= 0.12$$

$$(f) P(W) = P(W|A) P(A) + P(W|B) P(B)$$

$$= 0.2 (0.7) + 0.4 (0.3)$$

$$P(W) = 0.26$$

$$(g) P(A|W) = \frac{P(W|A) P(A)}{P(W)}$$

$$= \frac{0.2 (0.7)}{0.26}$$

$$P(A|W) = 0.5386$$

Question 3.

(a) Vertices : points in a graph.

(b) Edges : connecting line between vertices.

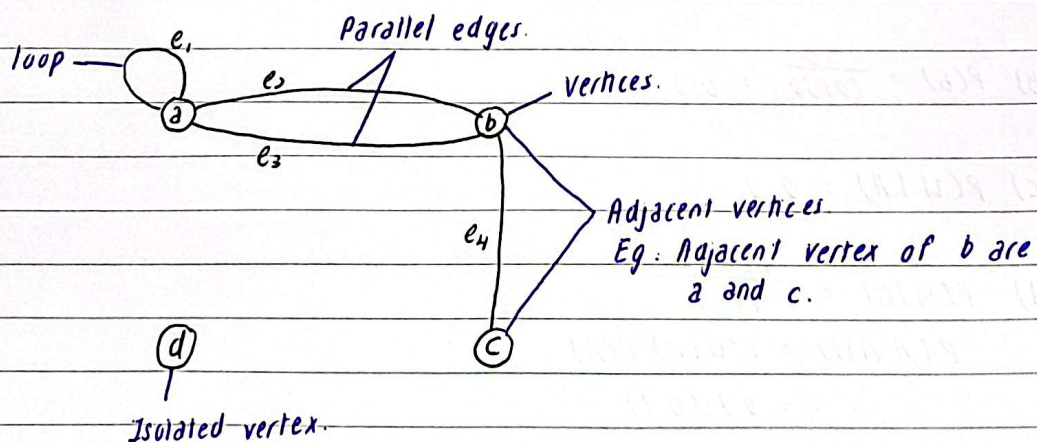
(c) Adjacent vertices : vertices that are connected to the same edge.

(d) Incident edges : edges that are incident to a vertex.

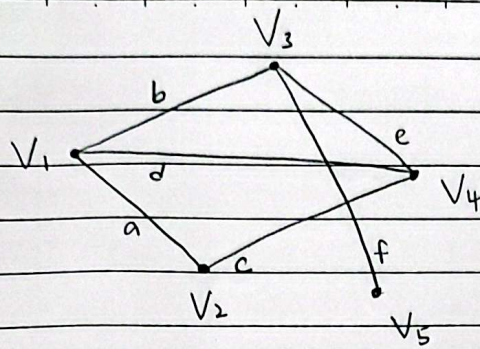
(e) Isolated vertex : vertex that has no connection at all

(f) Loop : edges that go out and end at the same vertex.

(g) Parallel edges : two or more edges that are connected to the same two vertices.



4.



$$d(V_1) = 3$$

$$d(V_2) = 2$$

$$d(V_3) = 3$$

$$d(V_4) = 3$$

$$d(V_5) = 1$$

5. i) incident matrix

| | a | b | c | d | e | f | g | h | i | k |
|-------|---|---|---|---|---|---|---|---|---|---|
| V_1 | 1 | 2 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| V_2 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| V_3 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |
| V_4 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 0 |
| V_5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 |
| V_6 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 1 |

ii) adjacent matrix

| | V_1 | V_2 | V_3 | V_4 | V_5 | V_6 |
|-------|-------|-------|-------|-------|-------|-------|
| V_1 | 1 | 0 | 2 | 1 | 0 | 0 |
| V_2 | 0 | 0 | 0 | 1 | 0 | 0 |
| V_3 | 2 | 0 | 0 | 1 | 1 | 1 |
| V_4 | 1 | 1 | 1 | 0 | 0 | 1 |
| V_5 | 0 | 0 | 1 | 0 | 0 | 1 |
| V_6 | 0 | 0 | 1 | 1 | 1 | 0 |

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Question 6.

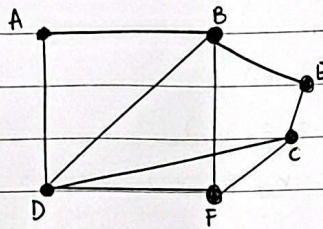
Both graphs have 6 vertices and 9 edges.

Both graphs have : 2 vertices = 4 degrees.

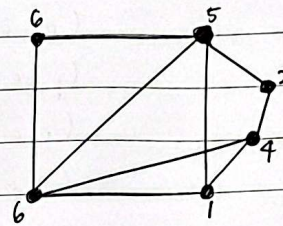
2 vertices = 3 degrees.

2 vertices = 2 degrees.

Define : $f(A)=6$, $f(B)=5$, $f(C)=4$, $f(D)=3$, $f(E)=2$, $f(F)=1$



Graph Y



Graph Z

A_1 rows and column are labeled : A, B, C, D, E, F

A_2 rows and column are labeled : 6, 5, 4, 3, 2, 1

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| A | 0 | 1 | 0 | 1 | 0 | 0 |
| B | 1 | 0 | 0 | 1 | 1 | 1 |
| C | 0 | 0 | 0 | 1 | 1 | 1 |
| D | 1 | 1 | 1 | 0 | 0 | 1 |
| E | 0 | 1 | 1 | 0 | 0 | 0 |
| F | 0 | 1 | 1 | 1 | 0 | 0 |

| | 6 | 5 | 4 | 3 | 2 | 1 |
|---|---|---|---|---|---|---|
| 6 | 0 | 1 | 0 | 1 | 0 | 0 |
| 5 | 1 | 0 | 0 | 1 | 1 | 1 |
| 4 | 0 | 0 | 0 | 1 | 1 | 1 |
| 3 | 1 | 1 | 1 | 0 | 0 | 1 |
| 2 | 0 | 1 | 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 | 1 | 0 | 0 |

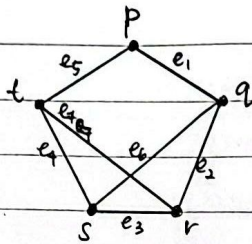
\therefore Since A_1 and A_2 are the same,

thus Graph Y and Z are isomorphic.

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Question 7

Graph :

i) Path from p to t : (p, e_5, t) $(p, e_1, q, e_6, s, e_4, t)$ $(p, e_1, q, e_2, r, e_7, t)$ $(p, e_1, q, e_2, r, e_3, s, e_4, t)$ $(p, e_1, q, e_6, s, e_3, r, e_7, t)$ ii) Trail from p to t : (p, e_5, t) $(p, e_1, q, e_6, s, e_4, t)$ $(p, e_1, q, e_2, r, e_7, t)$ $(p, e_1, q, e_2, r, e_3, s, e_4, t)$ $(p, e_1, q, e_6, s, e_3, r, e_7, t)$ $(p, e_5, t, e_4, s, e_3, r, e_7, t)$ $(p, e_5, t, e_7, r, e_3, s, e_4, t)$ $(p, e_5, t, e_4, s, e_6, q, e_2, r, e_7, t)$ $(p, e_5, t, e_7, r, e_2, q, e_6, s, e_4, t)$ iii) Shortest path : (p, e_5, t) longest path : $(p, e_1, q, e_2, r, e_3, s, e_4, t)$ and $(p, e_1, q, e_6, s, e_3, r, e_7, t)$ iv) Shortest trail : (p, e_5, t) longest trail : $(p, e_5, t, e_4, s, e_6, q, e_2, r, e_7, t)$ and $(p, e_5, t, e_7, r, e_2, q, e_6, s, e_4, t)$