Mini course

Bayesian Methods in Neuroscience

Free Energy Principle and Active inference

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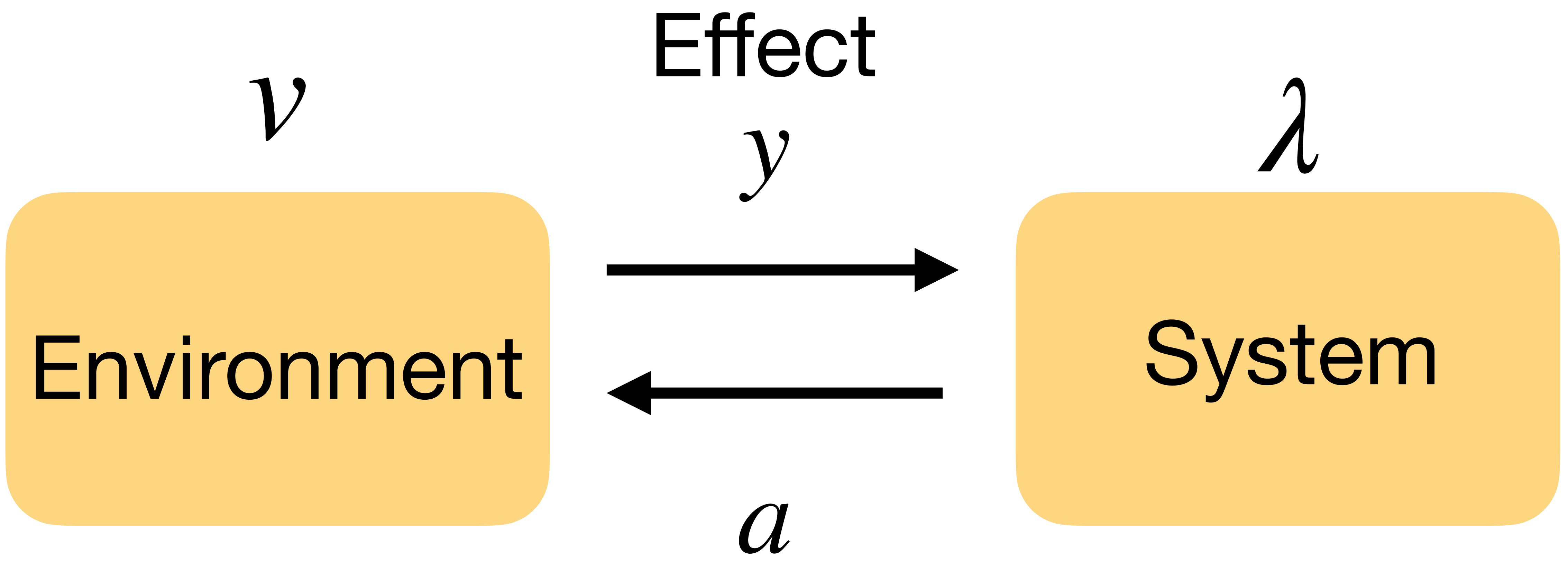
Free energy principle

Free energy



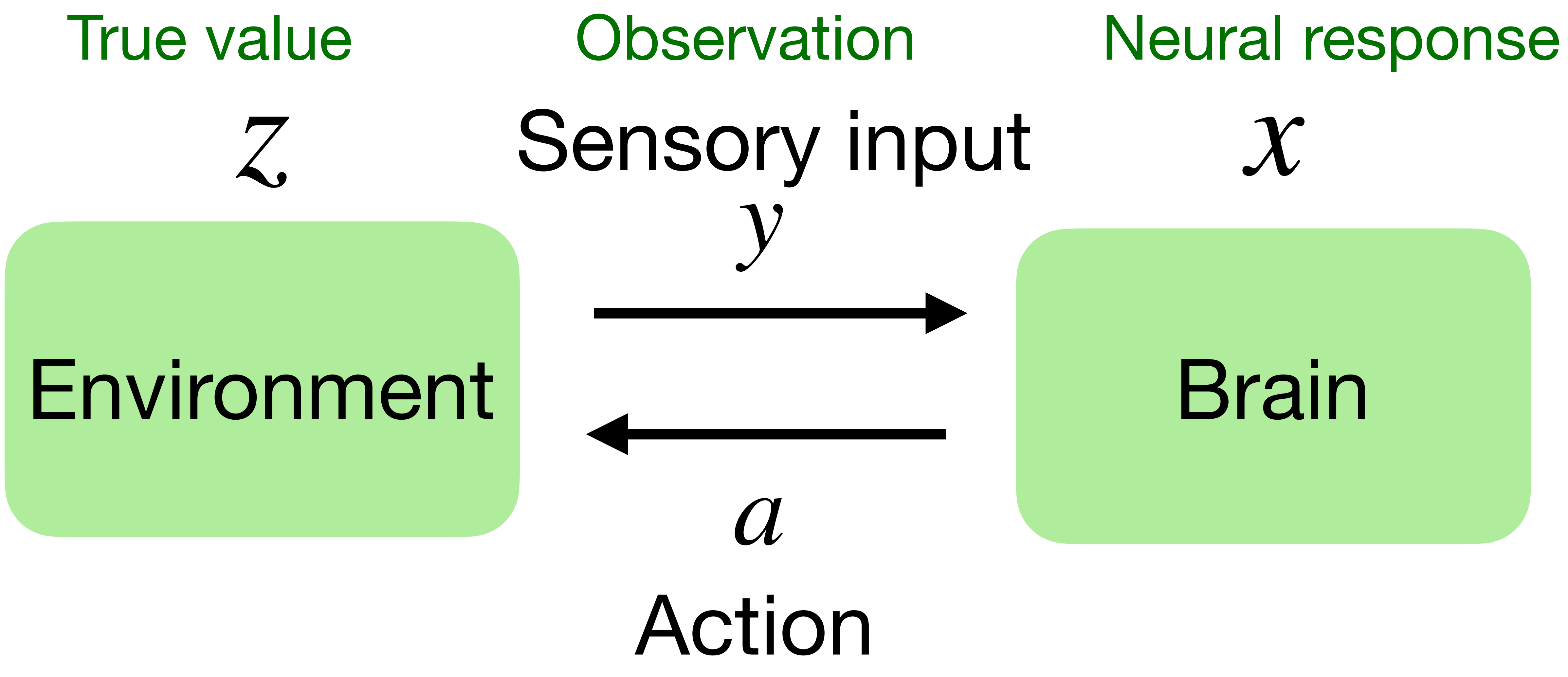
based on (Friston, et al., 2006)

Free energy:

$$F = -\int q(v) \ln \frac{p(y, v)}{q(v)} dv$$

$$D_{KL}(q(v) \parallel p(y, v))$$

Change notation



Free energy:

$$F = -\int q(z) \ln \frac{p(y, z)}{q(z)} dz$$

$$D_{KL}(q(z) \parallel p(y, z))$$

Free energy principle

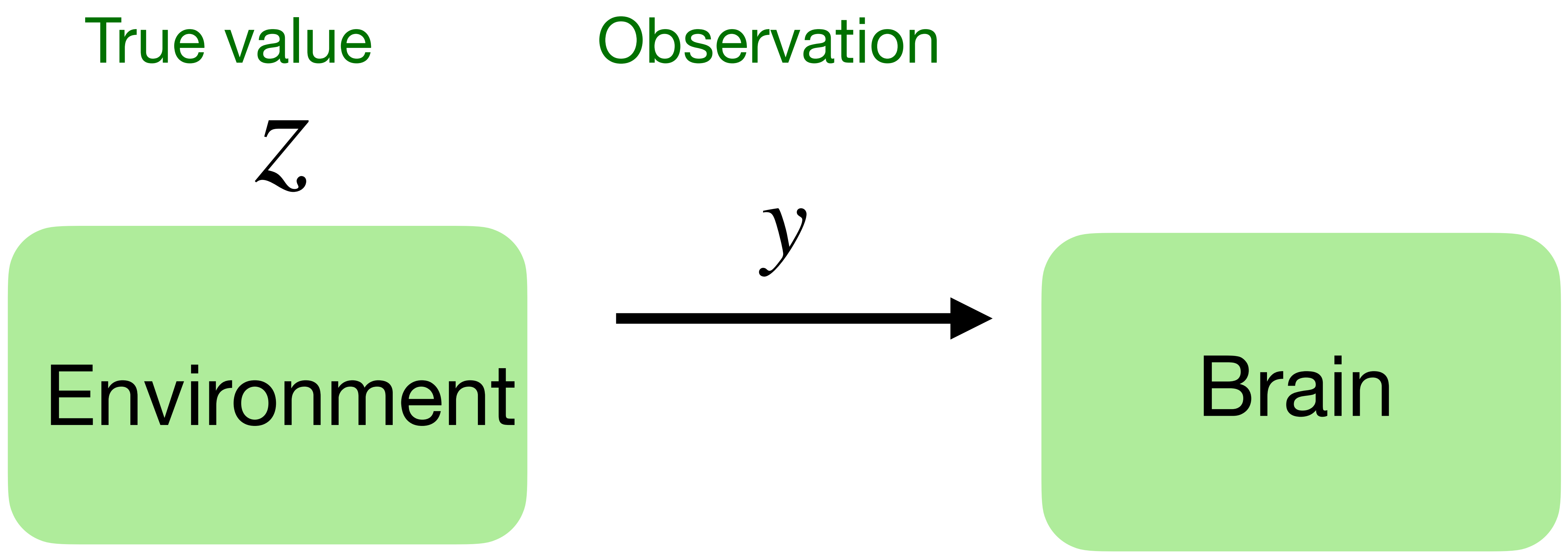
Perception

$$F = D_{KL}(q(z) \parallel p(y, z))$$

$q(z)$ True distribution of environmental z

$$p(y, z) = p(y \mid z)p(z)$$

Generative density, generate samples y based on environment z



Goal: find $p(y, z)$ as close as possible to $q(z)$

$\longrightarrow$ **Minimize** $D_{KL}(q(z) \parallel p(y, z))$

Free energy principle

Reform

$$F = -\int q(z) \ln \frac{p(y,z)}{q(z)} dz \longleftarrow D_{KL}(q(z) \parallel p(y,z))$$

$$= -\int q(z) \ln p(y,z) dz + \int q(z) \ln q(z) dz$$

Entropy of $p(y,z)$ under $q(z)$ Negative entropy of $q(z)$ under $q(z)$

⬇

Energy in the brain perception − Energy in the environment

⬇

Free energy

Minimize $D_{KL}(q(z) \parallel p(y,z))$ ➡ Minimize Free energy

Free energy principle

Optimize perception

$p(y, z) = p(z \mid y)p(y)$

$$F = -\int q(z)\ln\frac{p(y, z)}{q(z)}dz$$

$$= -\int q(z)\ln\frac{p(z \mid y)p(y)}{q(z)}dz$$

$$= -\int q(z)\ln p(y)dz - \int q(z)\ln\frac{p(z \mid y)}{q(z)}dz$$

$$= -E[\ln p(y)]_q + D_{KL}(q(z) \parallel p(z \mid y))$$

known

Always non-negative

$$F \geq -E[\ln(p(y))]_q$$

Minimize F —> Minimize $D_{KL}(q(z) \parallel p(z \mid y))$

—>$q(z) \approx p(z \mid y)$

Finding expectation of $q(z)$

to approximate posterior $p(z \mid y)$

Recall: Variational Bayes

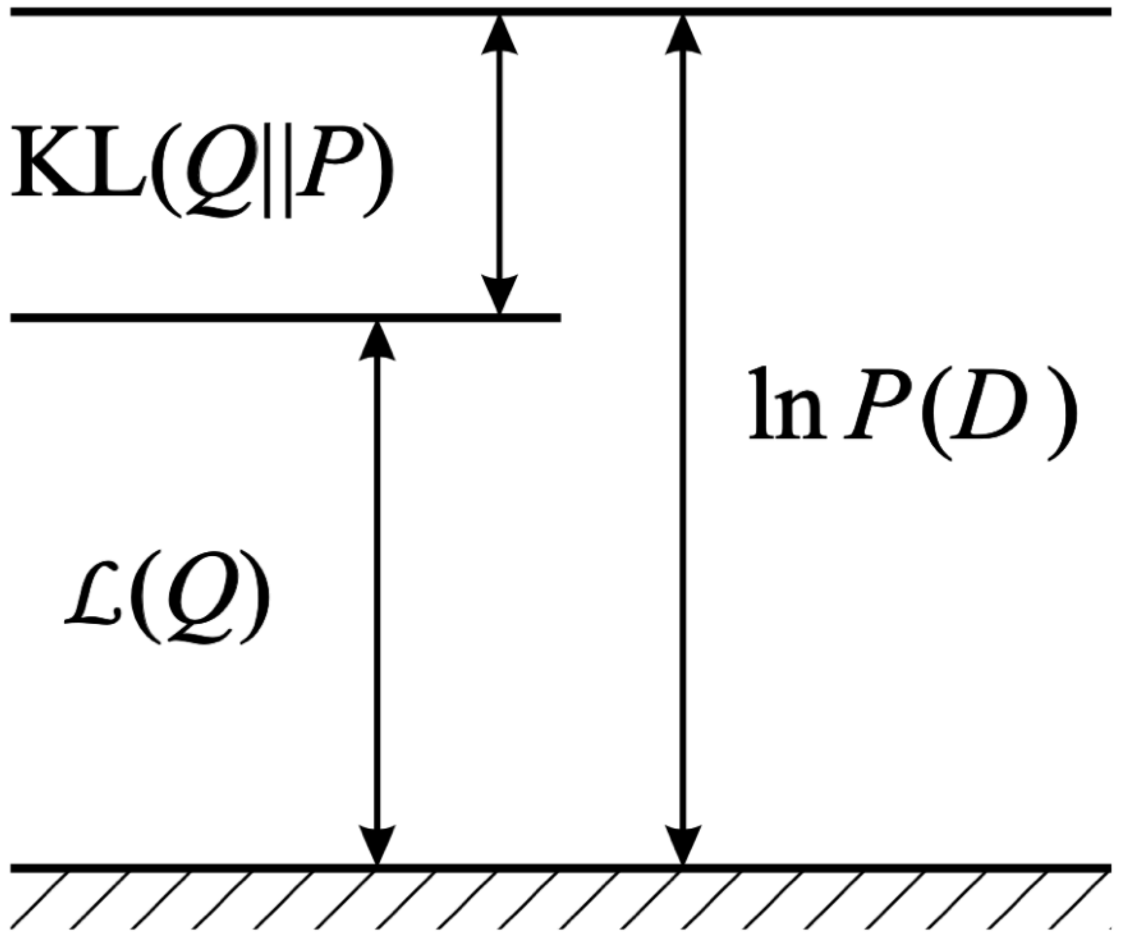
Lower bound

Data: D, Model parameters: θ

$$
\begin{aligned}
\ln P(D) &= \ln \int P(D,\theta) d\theta \\
&= \ln \int Q(\theta) \frac{P(D,\theta)}{Q(\theta)} d\theta \\
&= \int Q(\theta) \ln \frac{P(D,\theta)}{Q(\theta)} d\theta - \int Q(\theta) \ln \frac{P(\theta \mid D)}{Q(\theta)} d\theta \\
&\geq \int Q(\theta) \ln \frac{P(D,\theta)}{Q(\theta)} d\theta \\
&= L(Q)
\end{aligned}
$$

Always non-negative

Lower bound



Maximize Lower bound

$$\ln P(D) \approx L(Q)$$

Recall: Variational Bayes

Factorized approximation

$$Q(\boldsymbol{\theta}) = \prod_i Q_i(\theta_i).$$

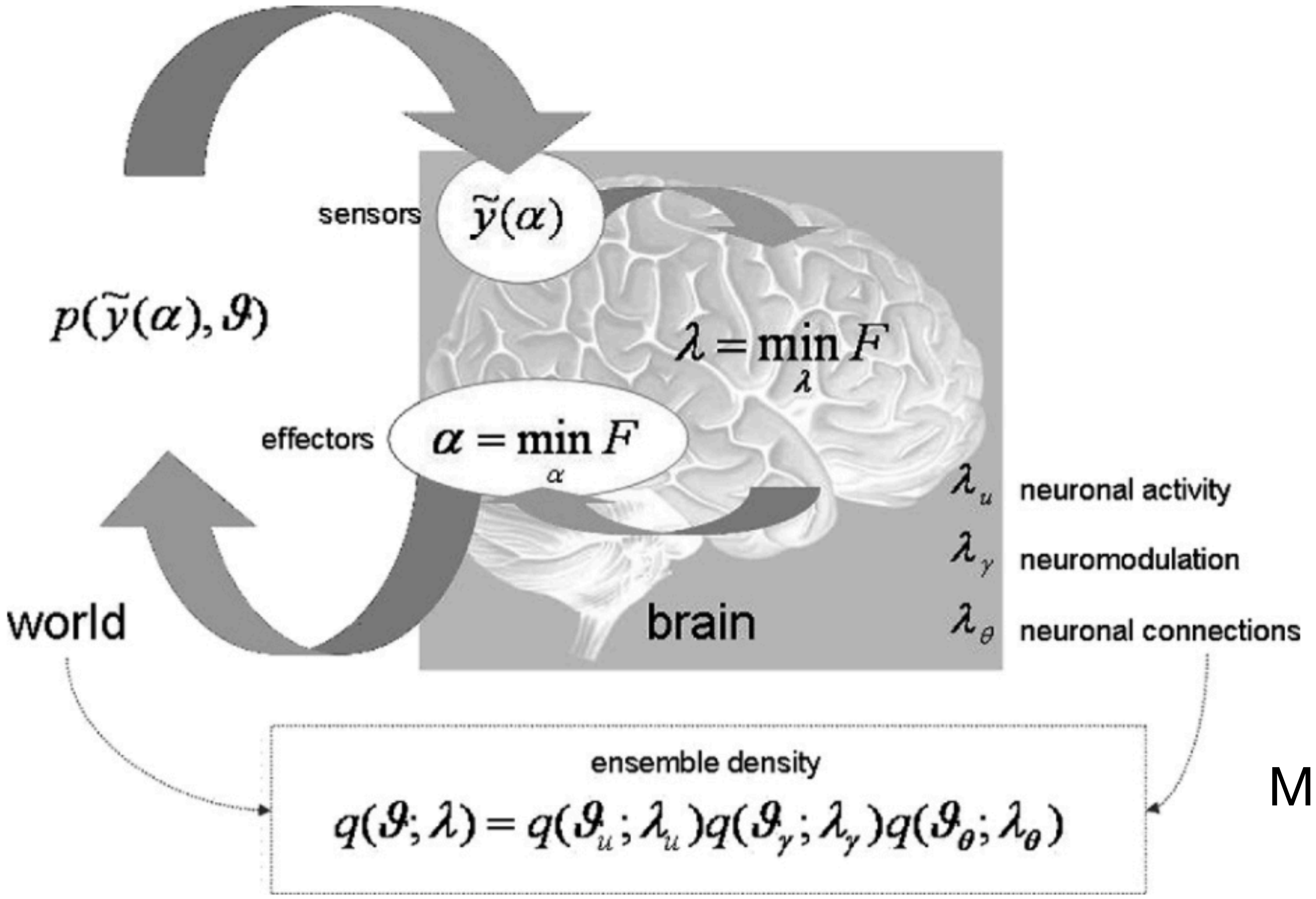
using simpler distribution $Q_i(\theta_i)$

Solution for each θ_i by maximizing $Q(\theta)$:

$$Q_i(\theta_i) = \frac{\exp \left\langle \ln P(D, \boldsymbol{\theta}) \right\rangle_{k \neq i}}{\int \exp \left\langle \ln P(D, \boldsymbol{\theta}) \right\rangle_{k \neq j} d\theta_j}$$

Free energy principle

Factorized approximation



(Friston, et al., 2006)

v

Environment $\xrightarrow{y}$ System

$$q(\vartheta_i) \propto \exp(I(\vartheta_i))$$

$$I(\vartheta_i) = \langle \ln p(\tilde{y}, \vartheta) \rangle_{q_{\setminus i}}$$

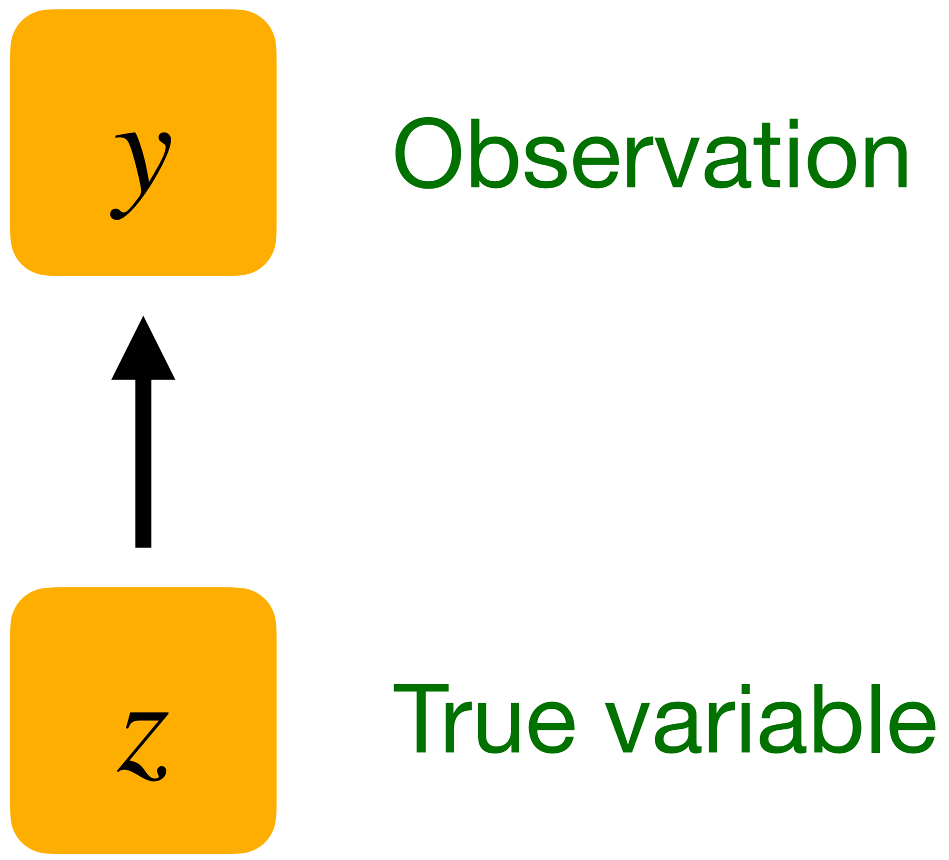
Variational energy

Minimize F: $q(z) \approx p(z \mid y)$

Maximize $q(v_i)$: Maximize Variational energy $I(v_i)$

Free energy principle

Recall Bayesian model



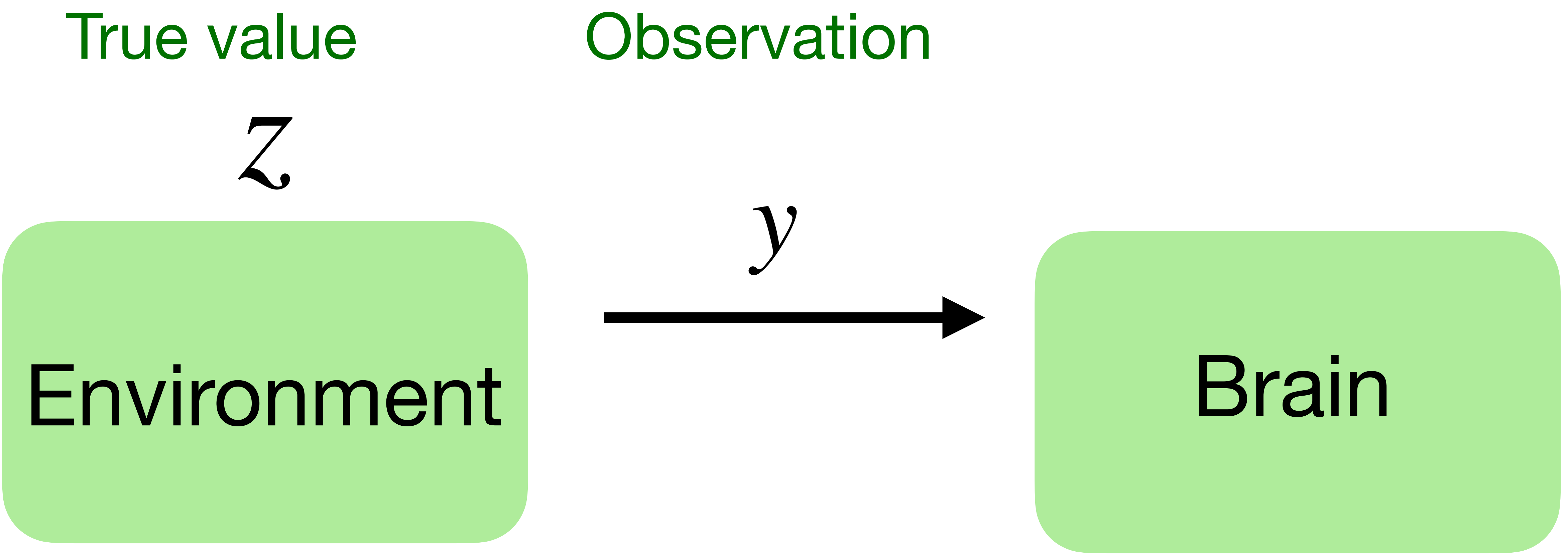
Goal: estimate z

$p(z)$ Prior

$p(y \mid z)$ Likelihood

$p(z \mid y)$ Posterior

Solution: Maximize posterior $p(z \mid y)$



Minimize F —> Maximize variational energy I_i

$q(z_i) \propto exp(I_i)$ **approximation to posterior**

(Variational) Free energy principle is

inherently Variational Bayesian

Examples of free energy principle in brain

Free energy principle in fMRI study

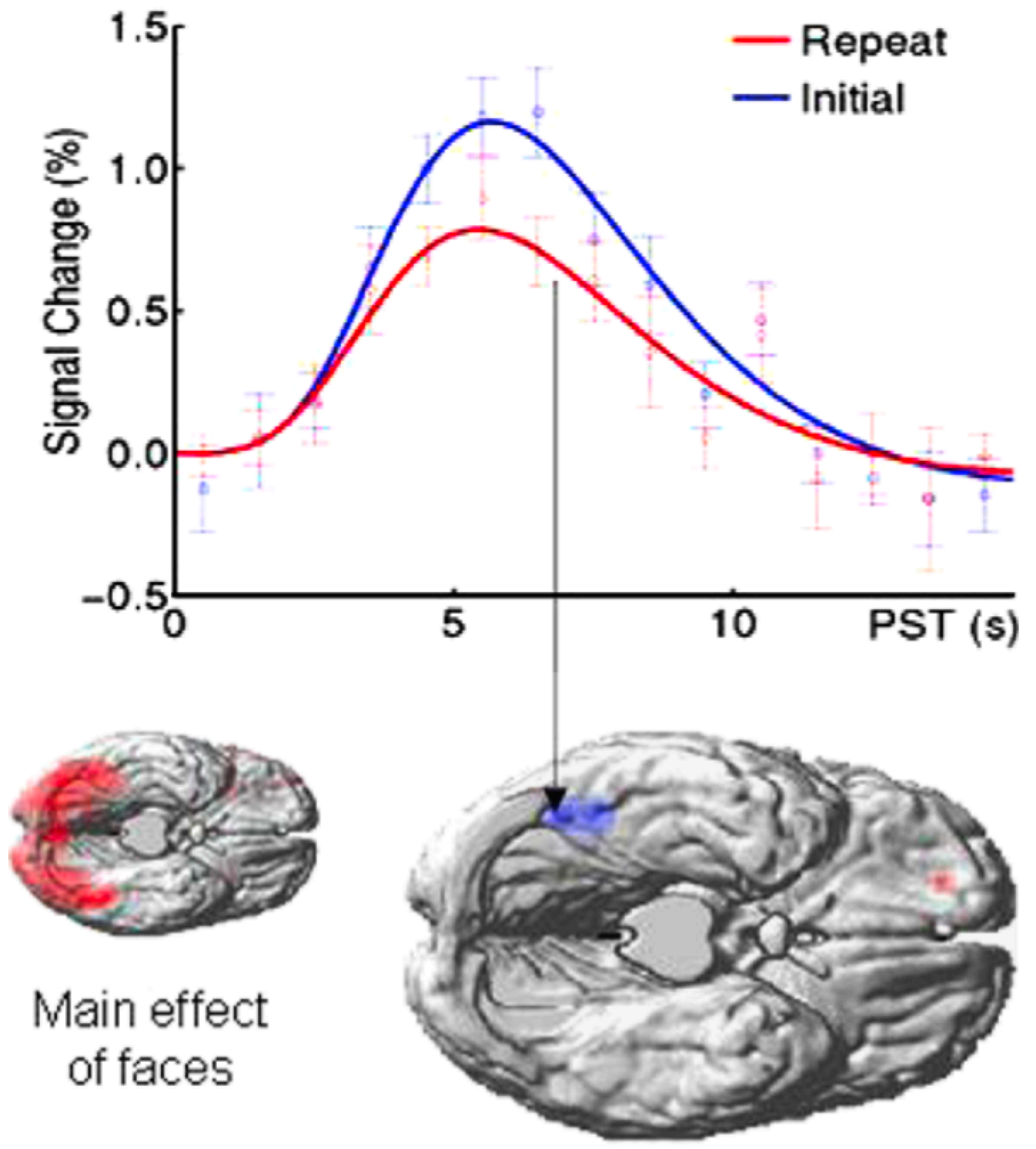
Suppression free energy in brain

Minimize Free energy:

$$F = -\int q(z) \ln \frac{p(y,z)}{q(v)} dz$$

$$D_{KL}(q(z) \parallel p(y,z))$$

Surprise, prediction error, …



Suppression of inferotemporal responses to repeated faces

(Henson, et al., 2006)

Decrease in Bold signals when repeating stimuli

Free energy principle In vitro

Validation in vitro neural networks

nature communications

Article

https://doi.org/10.1038/s41467-023-40141-z

Experimental validation of the free-energy principle with in vitro neural networks

Received: 12 October 2022

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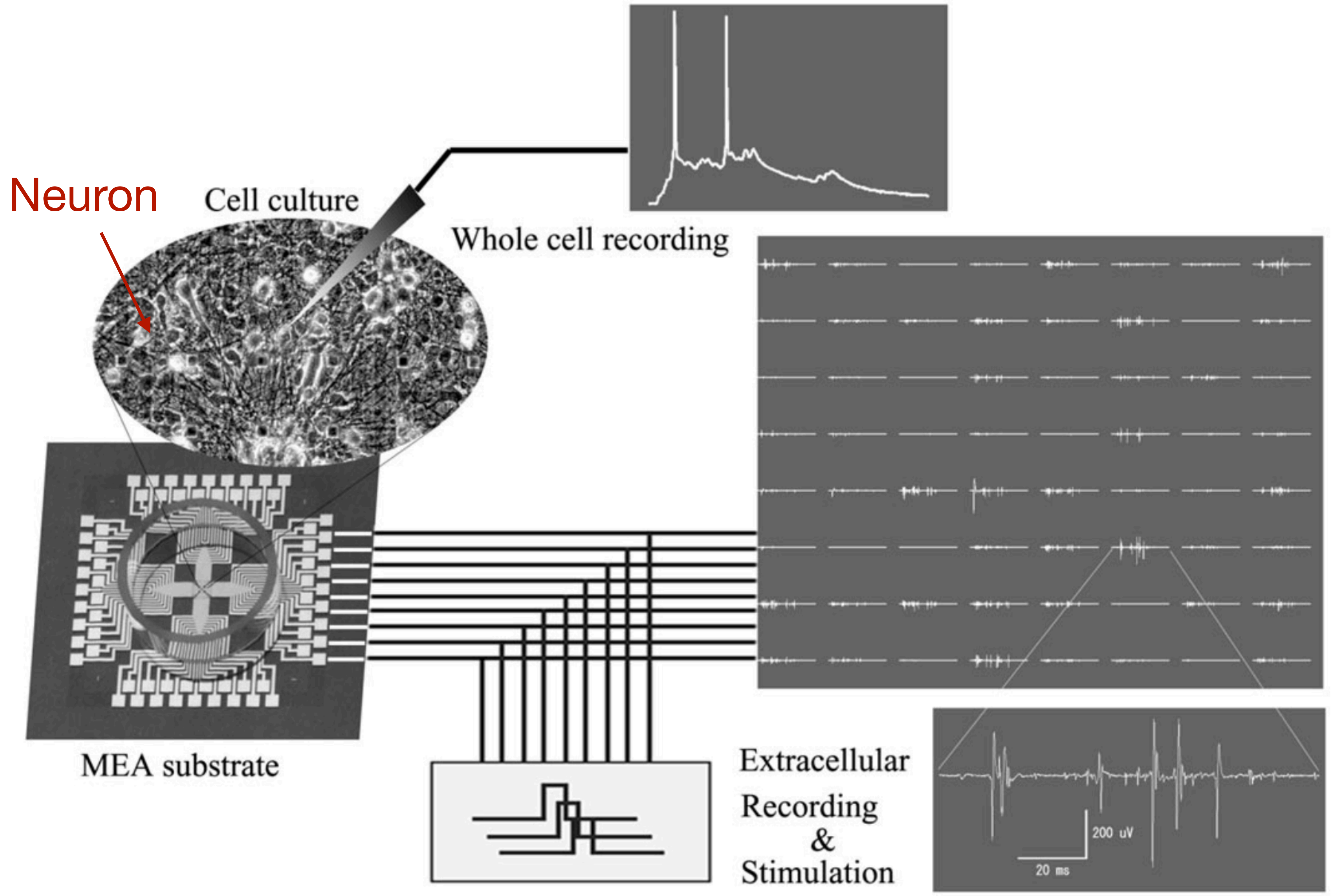
Takuya Isomura [1], Kiyoshi Kotani[2], Yasuhiko Jimbo[3] & Karl J. Friston [4,5]

(Isomura, et al., 2023)

Free energy principle In vitro

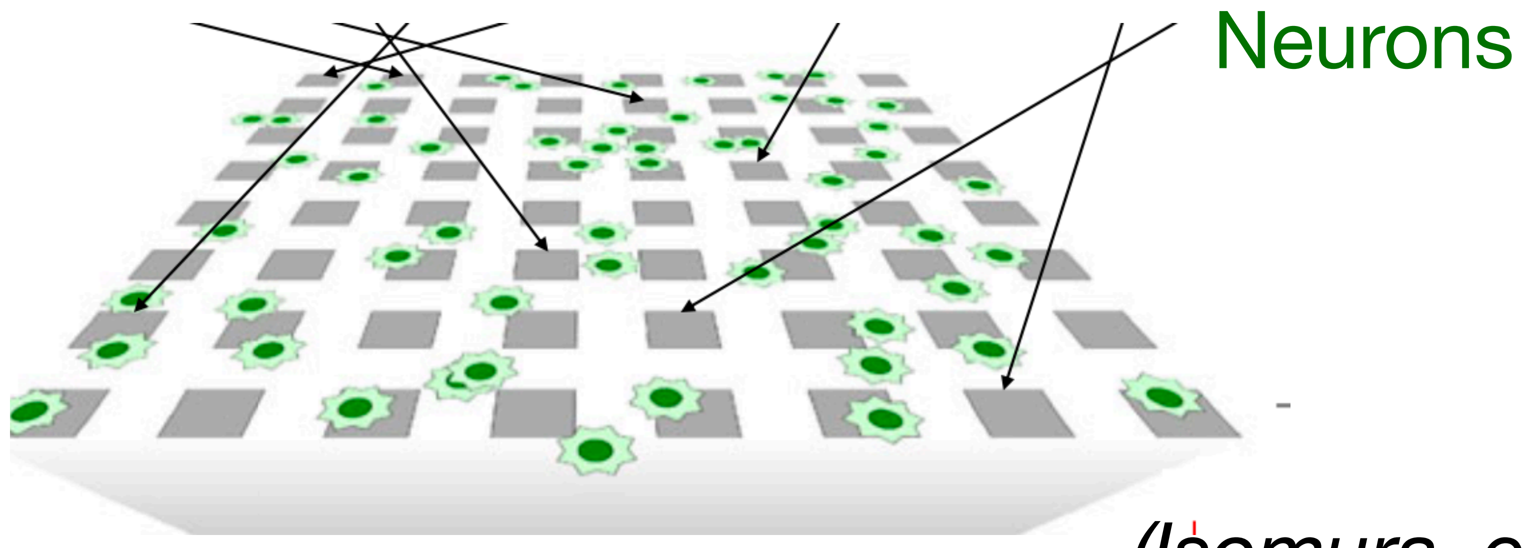
In vitro neuron culture with electric stimulation

MEA: microelectrode arrays



(Jimbo, et al., 2003)

- 8*8 MEA *(Isomura, et al., 2015)*
- random select 32 electrodes for simulation

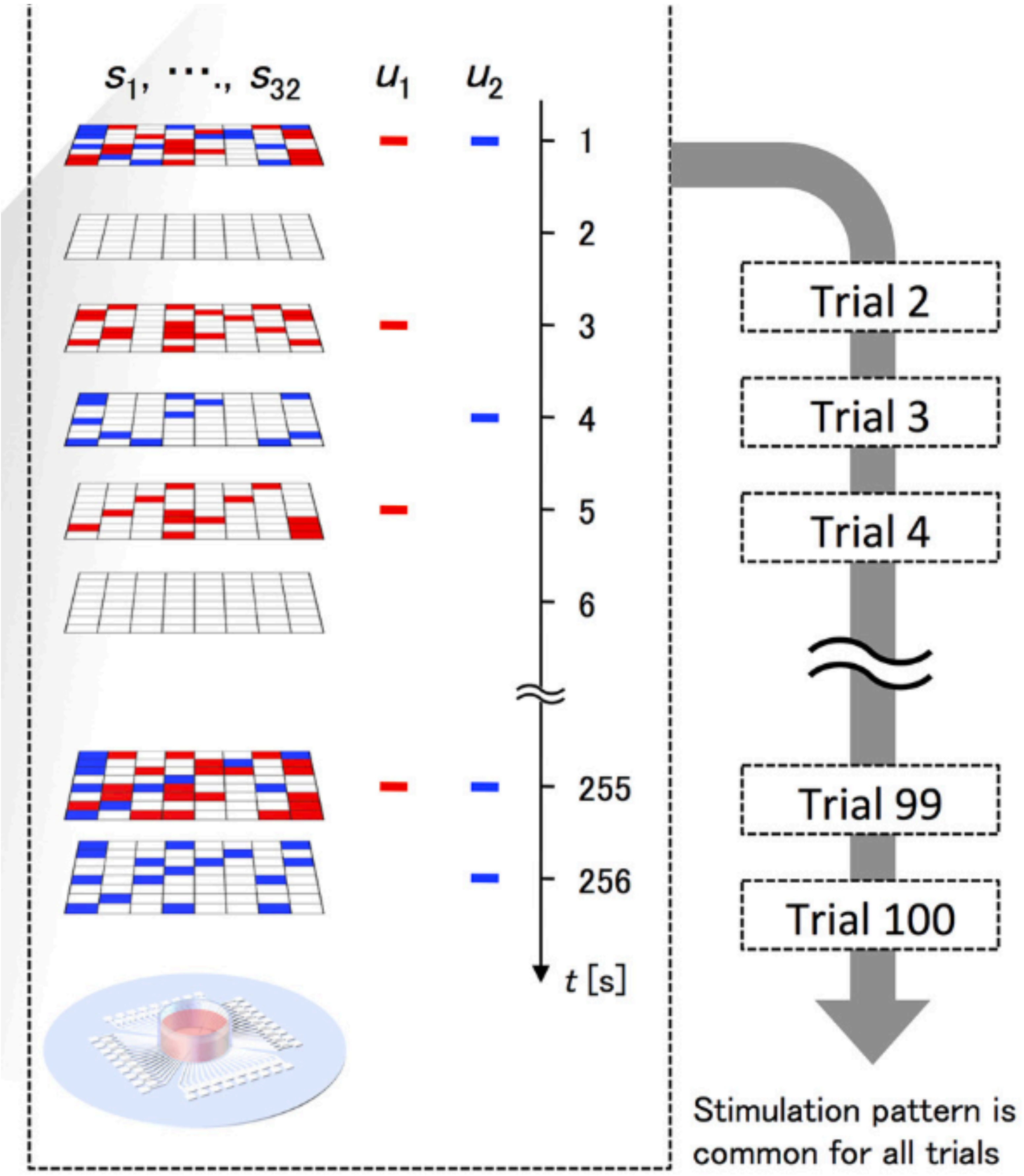


(Isomura, et al., 2023)

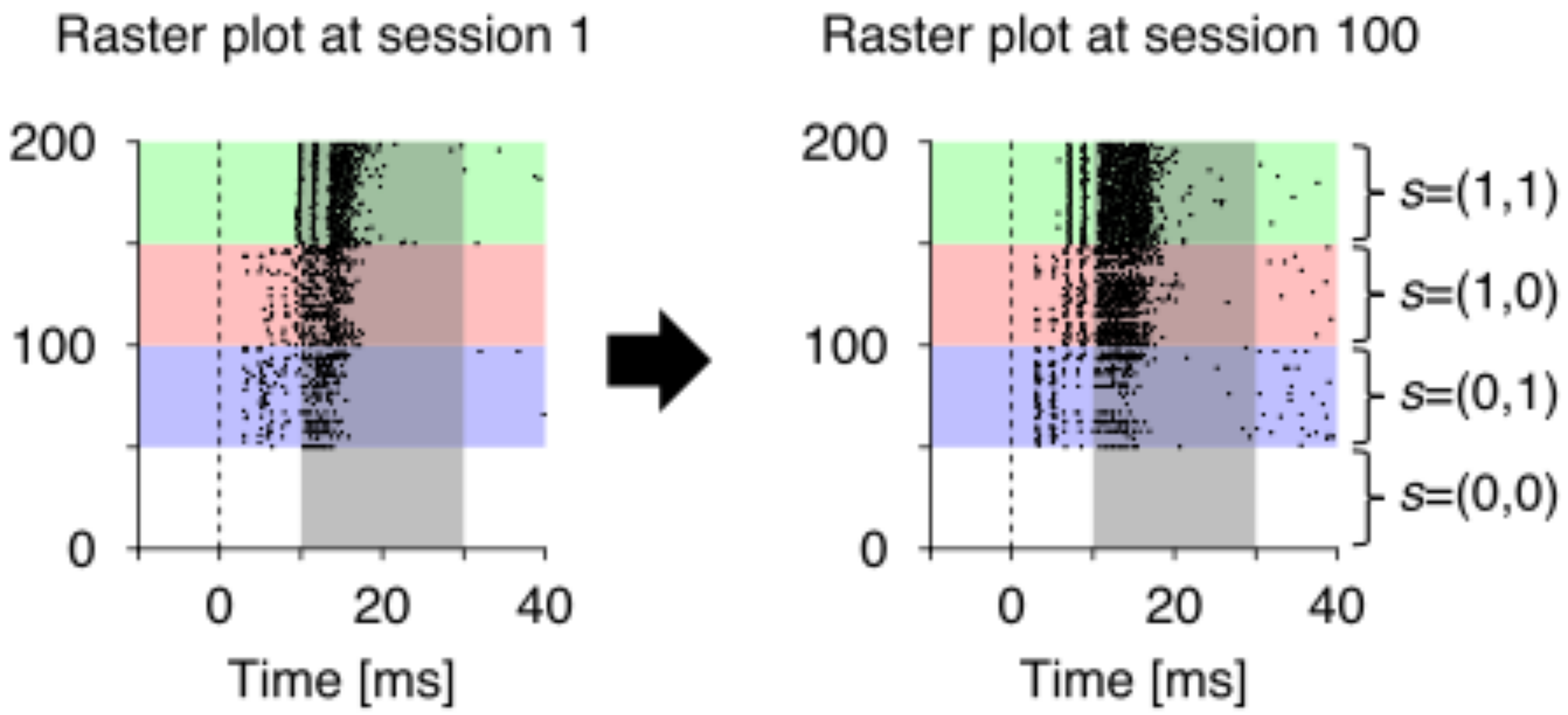
Free energy principle in vitro

Two source stimulation with uncertainty

Two stimuli sources 75%,25%



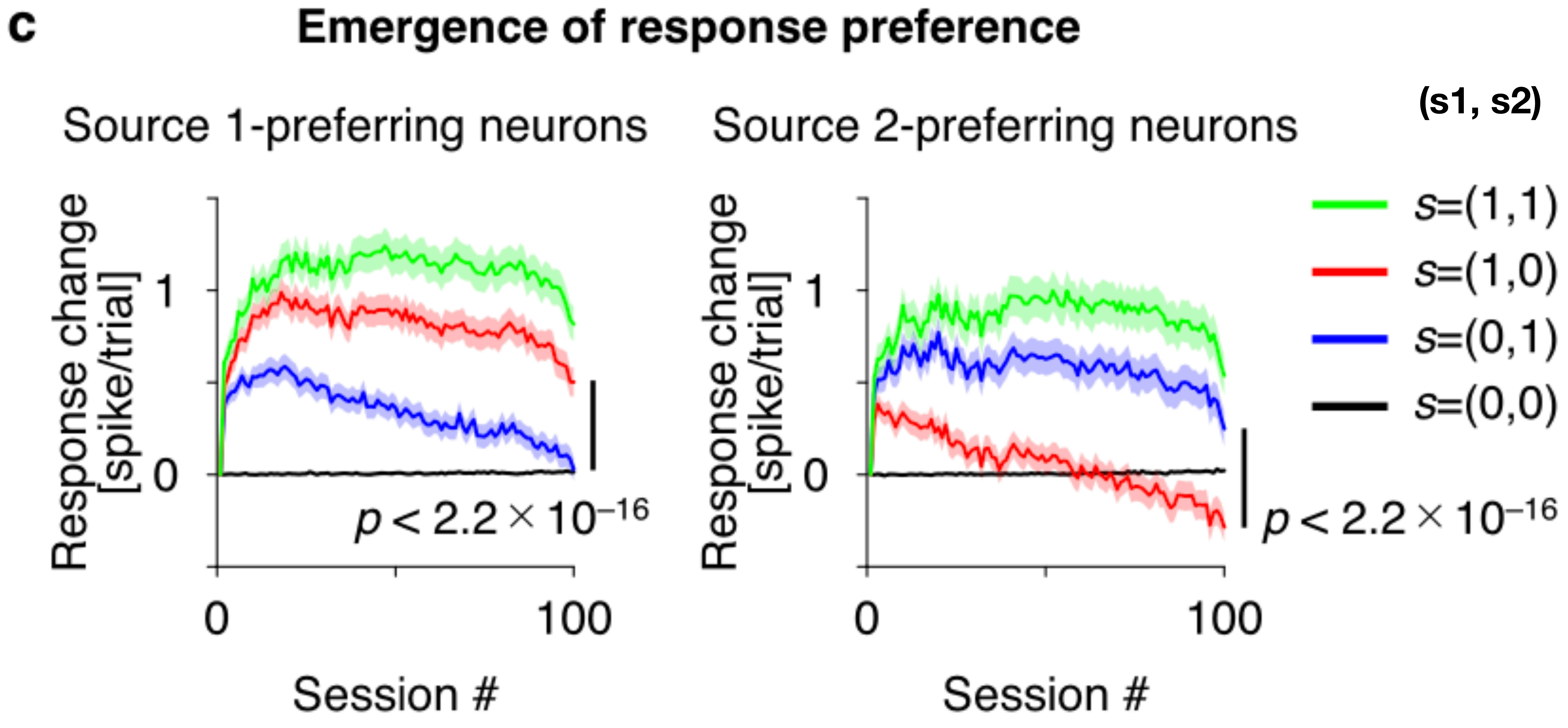
Spike recordings



- two stimuli sources 75%,25%
- culture for 10 days
- neurons self-origanize to a neural network

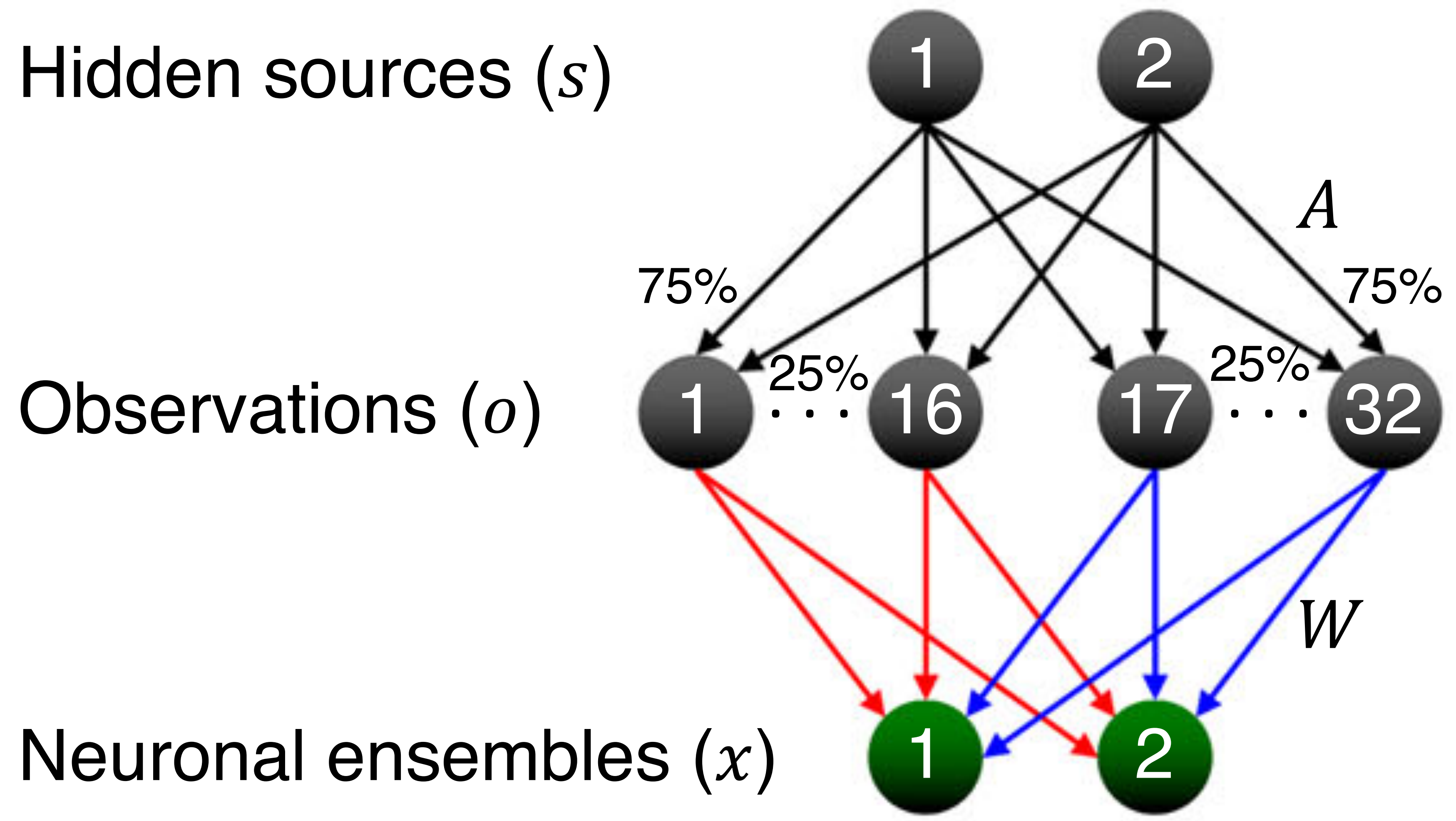
Free energy principle in vitro

Neurons has response preference



Free energy principle in vitro

Estimate synaptic weights from neural recording



$0 \leq x_1, x_2 \leq 1$

Neuronal response:

$$\dot{x}_t \propto -\mathrm{sig}^{-1}(x_t) + W o_t + h$$

$$x_{t+1} = x_t - \dot{x}_t$$

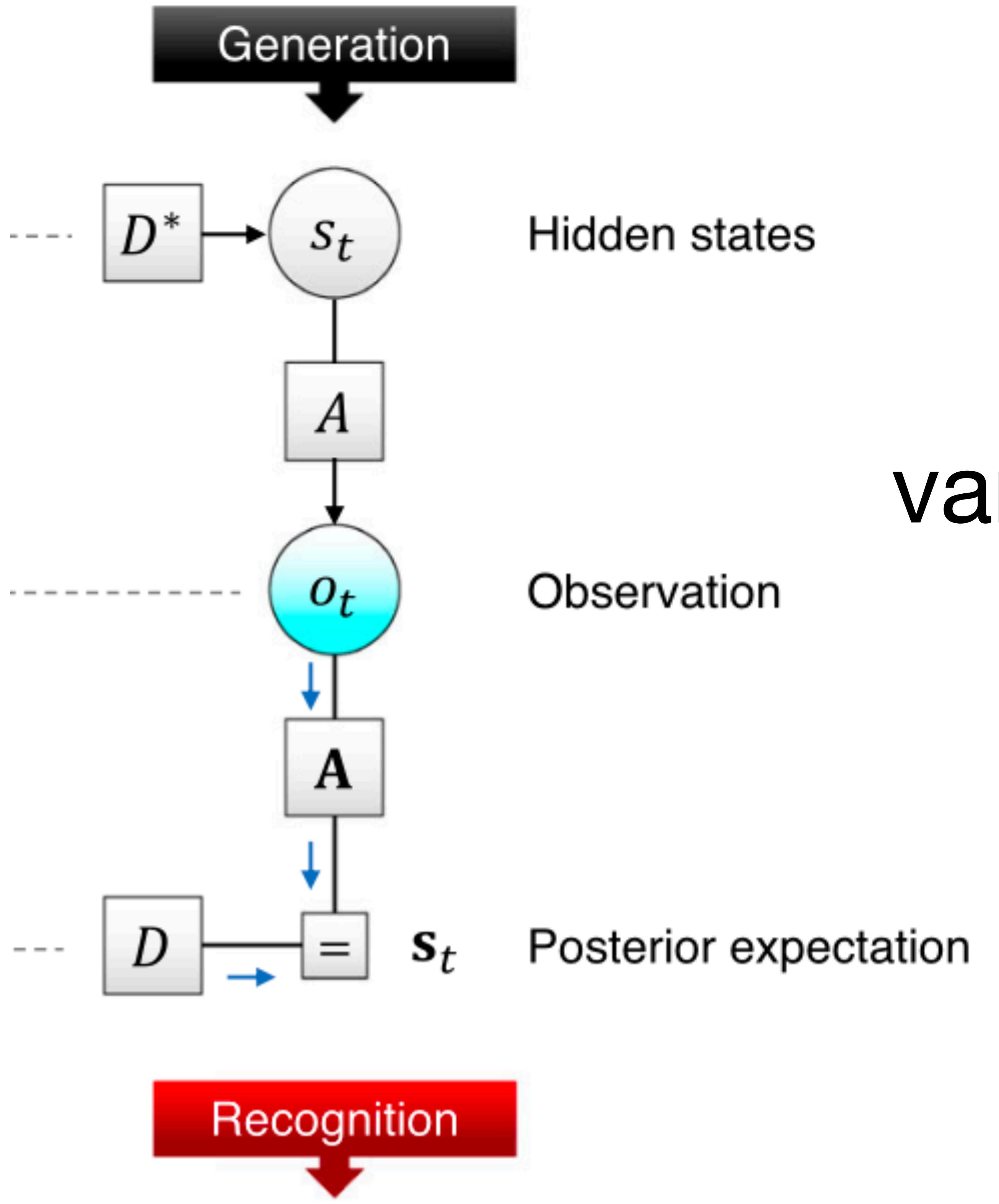
Revere generative model:

$$\begin{cases} W_1 = \mathrm{sig}^{-1}\left(\left\langle x_t o_t^{\mathrm{T}} \right\rangle \oslash \left\langle x_t \vec{1}^{\mathrm{T}} \right\rangle\right) \\ W_0 = \mathrm{sig}^{-1}\left(\left\langle \overline{x_t} o_t^{\mathrm{T}} \right\rangle \oslash \left\langle \overline{x_t} \vec{1}^{\mathrm{T}} \right\rangle\right) \end{cases}$$

Free energy principle in vitro

Neural network based on free energy principle

Solution by variational Bayesian:



Generative model:

$$P(o_{1:t}, s_{1:t}, A) = P(A) \prod_{\tau=1}^{t} P(s_\tau) P(o_\tau | s_\tau, A)$$

variational Bayesian:

$$Q(s_{1:t}, A) = Q(A) \prod_{\tau=1}^{t} Q(s_\tau)$$

free energy:

$$F = \sum_{\tau=1}^{t} \mathbf{s}_\tau \cdot (\ln \mathbf{s}_\tau - \ln \mathbf{A} \cdot o_\tau - \ln D) + \mathcal{O}(\ln t)$$

solution by gradient descent:

$$\dot{x}_t \propto -\mathrm{sig}^{-1}(x_t) + W o_t + h$$

Free energy principle in vitro

Comparison

Experimental estimated synaptic weight

Neural network weight based on free energy principle

Neural activity

$\dot{x} \propto -f(x) + Wo + h$

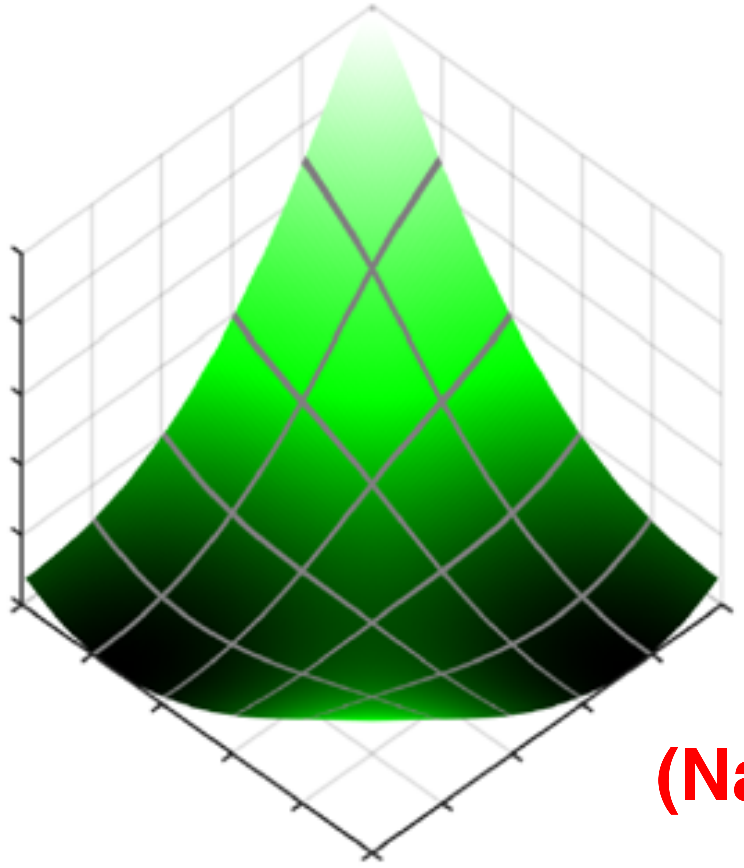
Integral with respect to x

Derivative with respect to W

Synaptic plasticity

$\dot{W} \propto pre \times post - homeostatic$

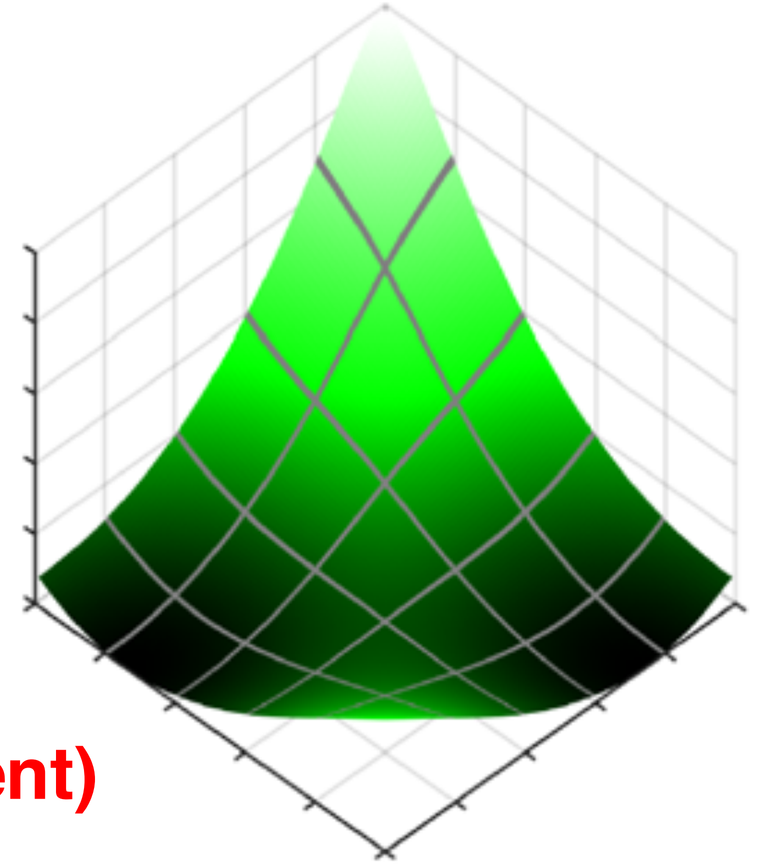
Neural network cost function *L*



Mathematically equivalent

(Naturally equivalent)

Variational free energy *F*



Inference

$\mathbf{s}_t = \sigma(\ln \mathbf{A} \cdot o_t + \ln D)$

Minimisation with respect to $\mathbf{s}_t$

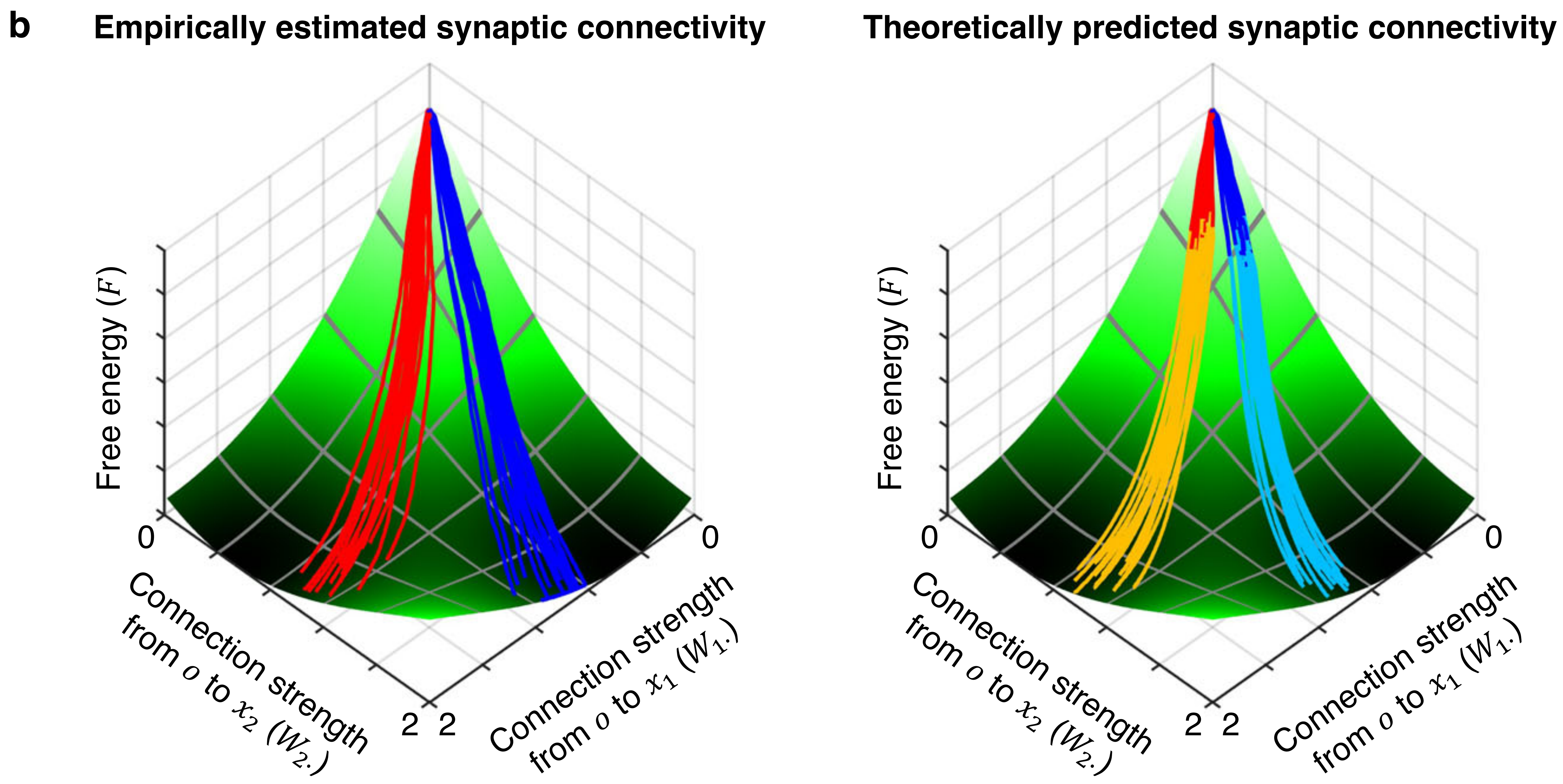
Minimisation with respect to $\mathbf{a}$

Learning

$\mathbf{a} = a + \sum_{\tau=1}^{t} o_\tau \otimes \mathbf{s}_\tau$

Free energy principle in vitro

Results

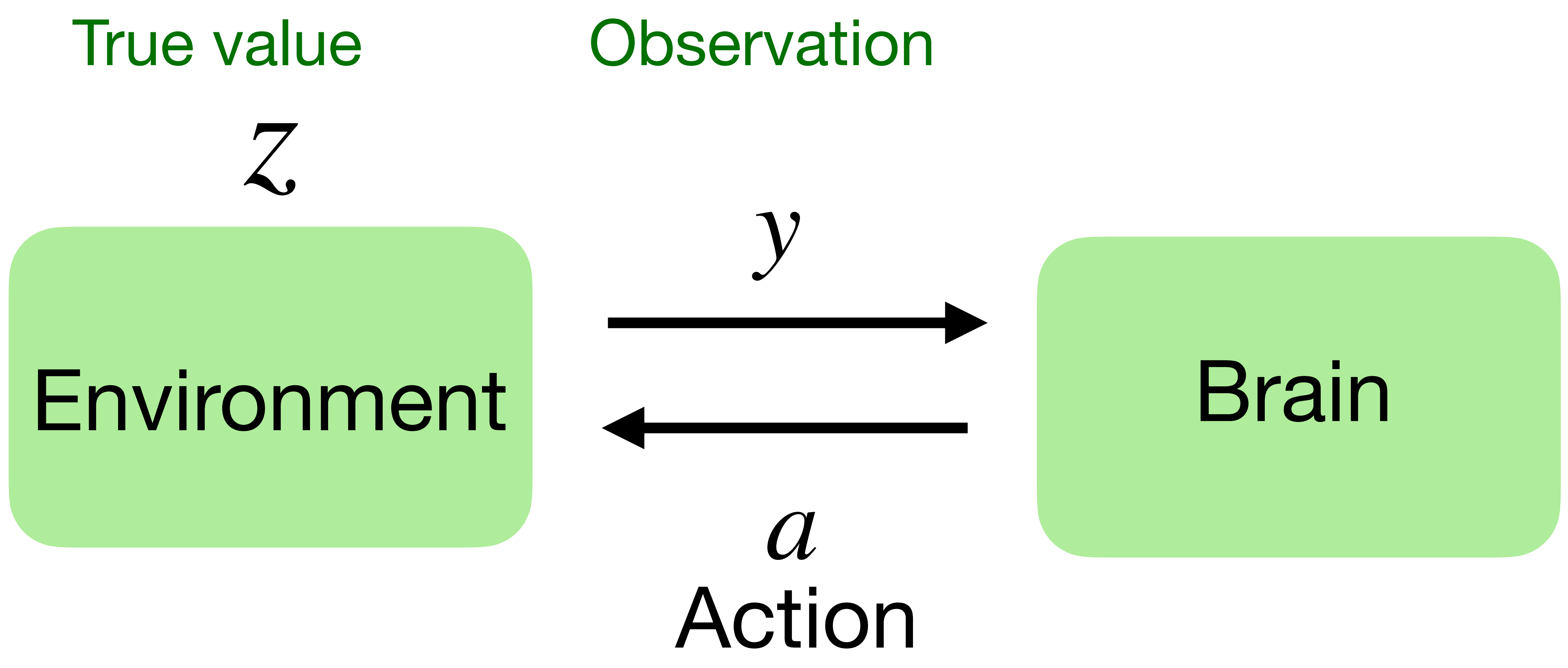


(Isomura, et al., 2023)

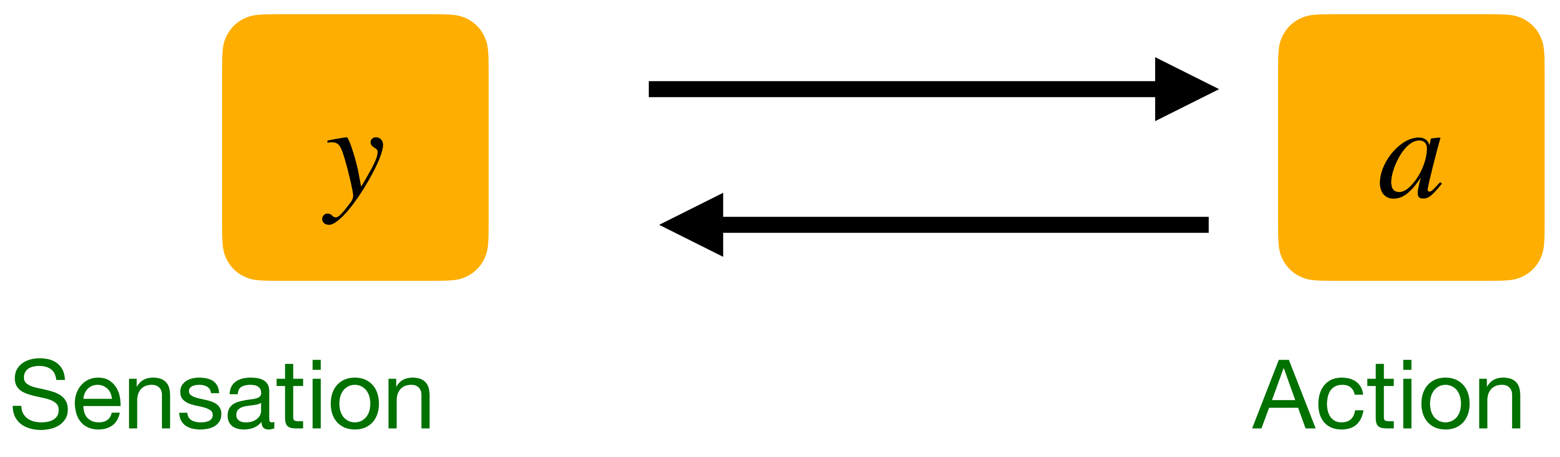
Free energy principle

Optimize action

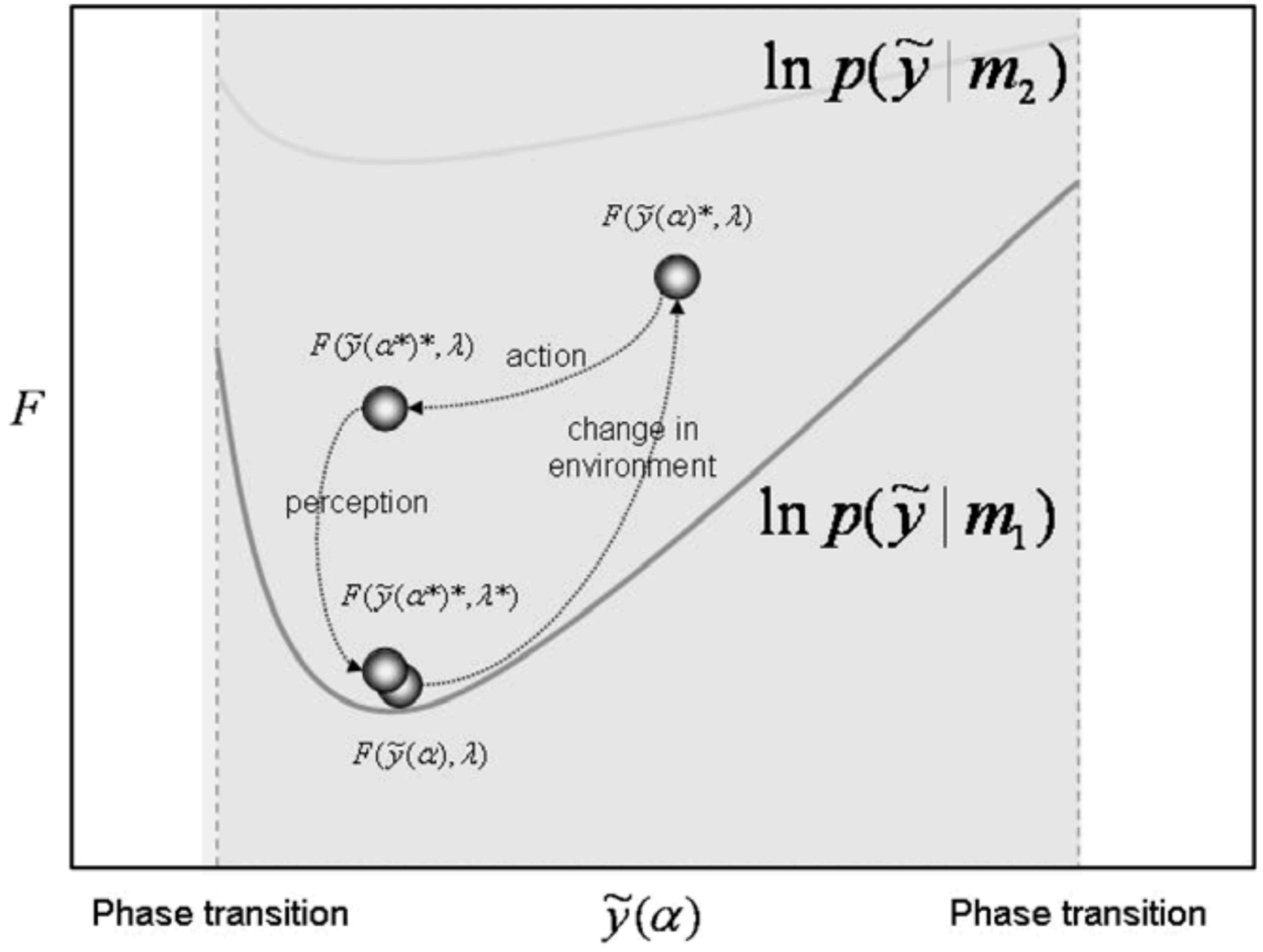
$$F = -E[\ln p(y(a)]_q + D_{KL}(q(z) \parallel p(z \mid y))$$



Free energy principle involves action.



Active inference



(Friston, et al., 2006)

Learning by gradient descent:

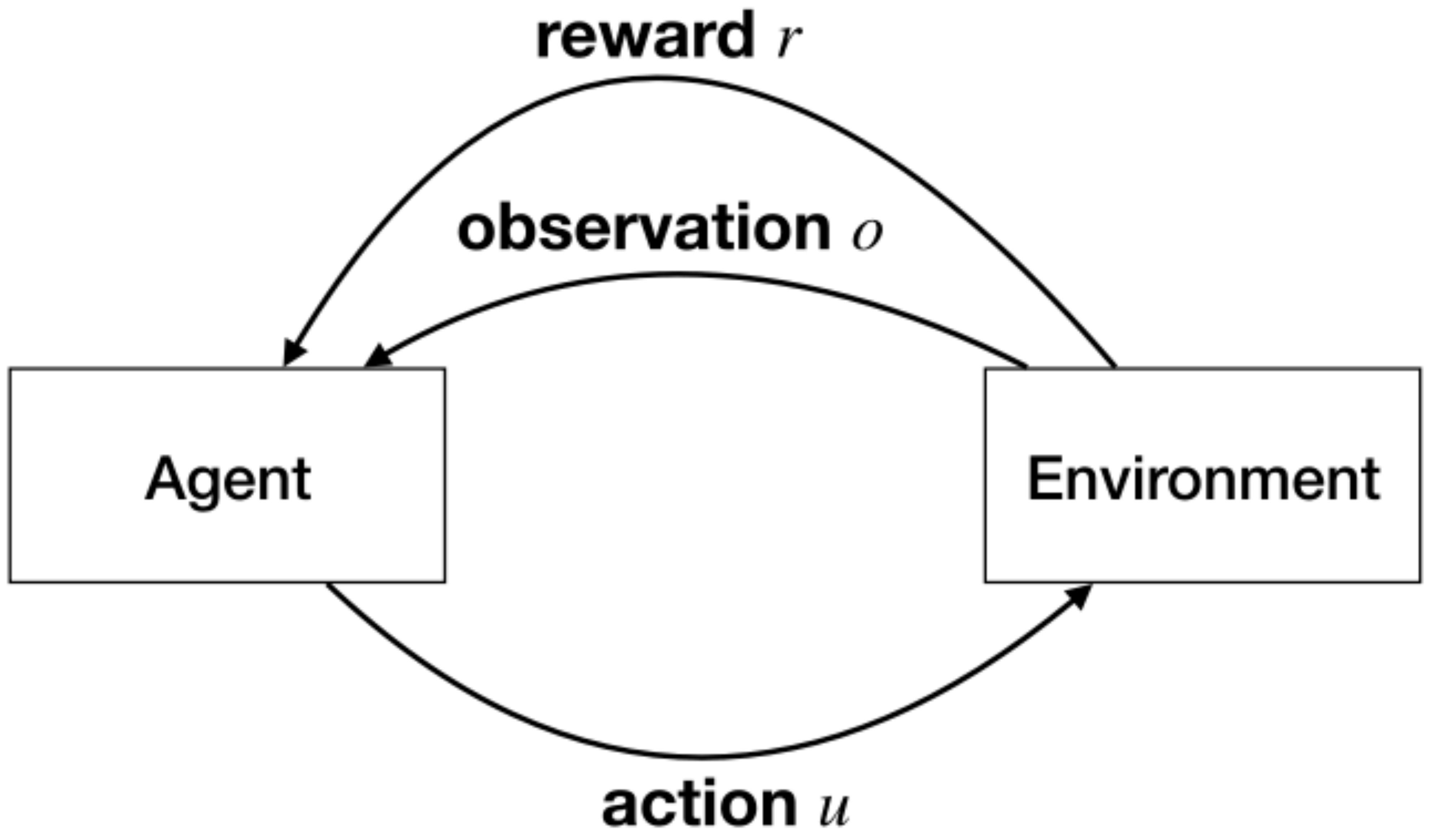
$$\dot{\mu}_\theta = \kappa \partial I(\vartheta_\theta) / \partial \vartheta_\theta$$

$$I(\vartheta_\theta) = \langle \ln p(\tilde{y}, \vartheta) \rangle_{q_u q_\gamma}$$

Active inference

Recall reinforcement learning

Reinforcement learning

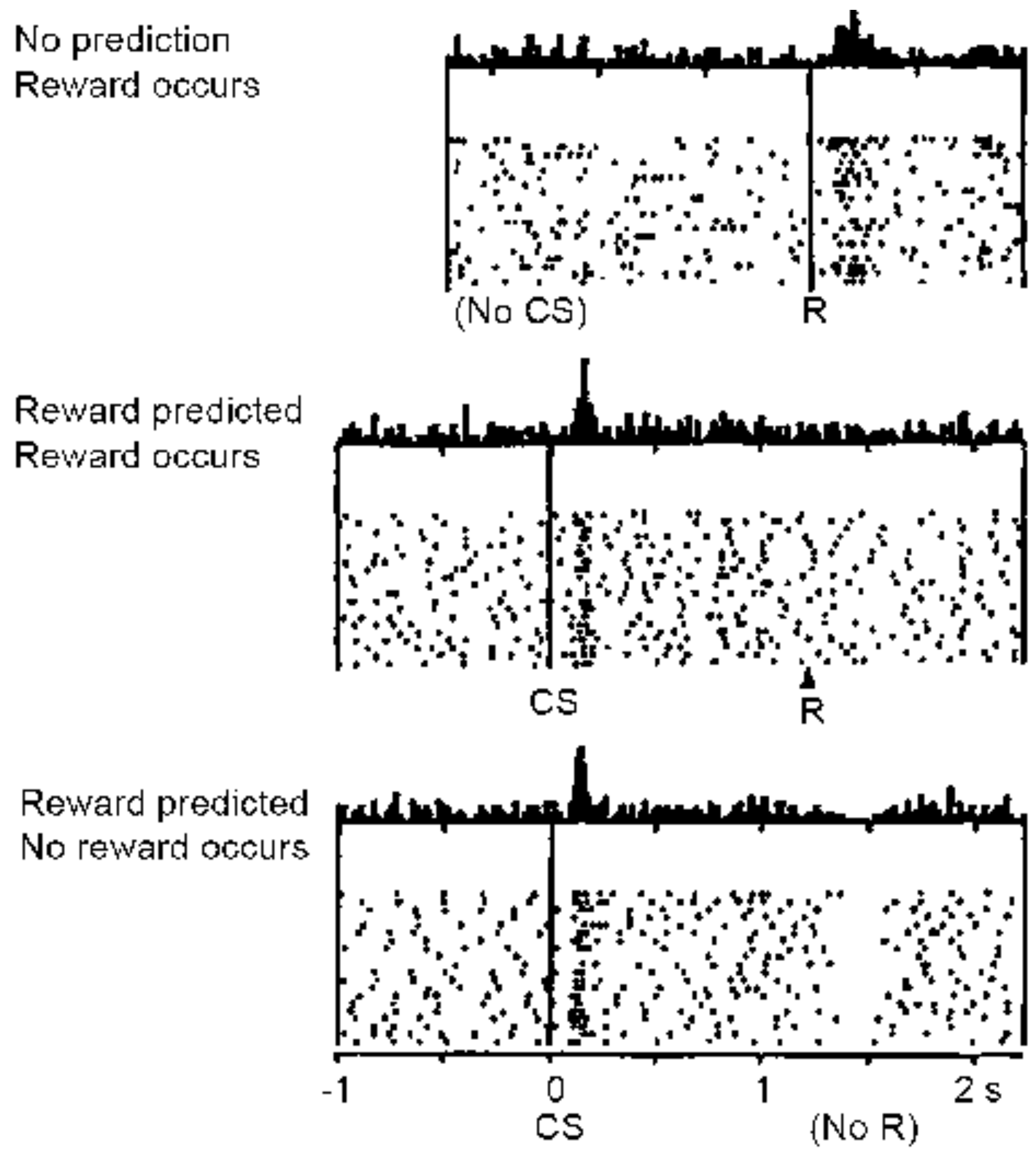


$$u_t^* = \arg\max_{u_t} V(s_t, u_t) = \pi(s_t)$$

$$V(s_t, u_t) = \sum_{s_{t+1}} (r_{t+1} + \max_{u_{t+1}} V(s_{t+1}, u_{t+1})) P(s_{t+1} | s_t, u_t)$$

Prediction of reward

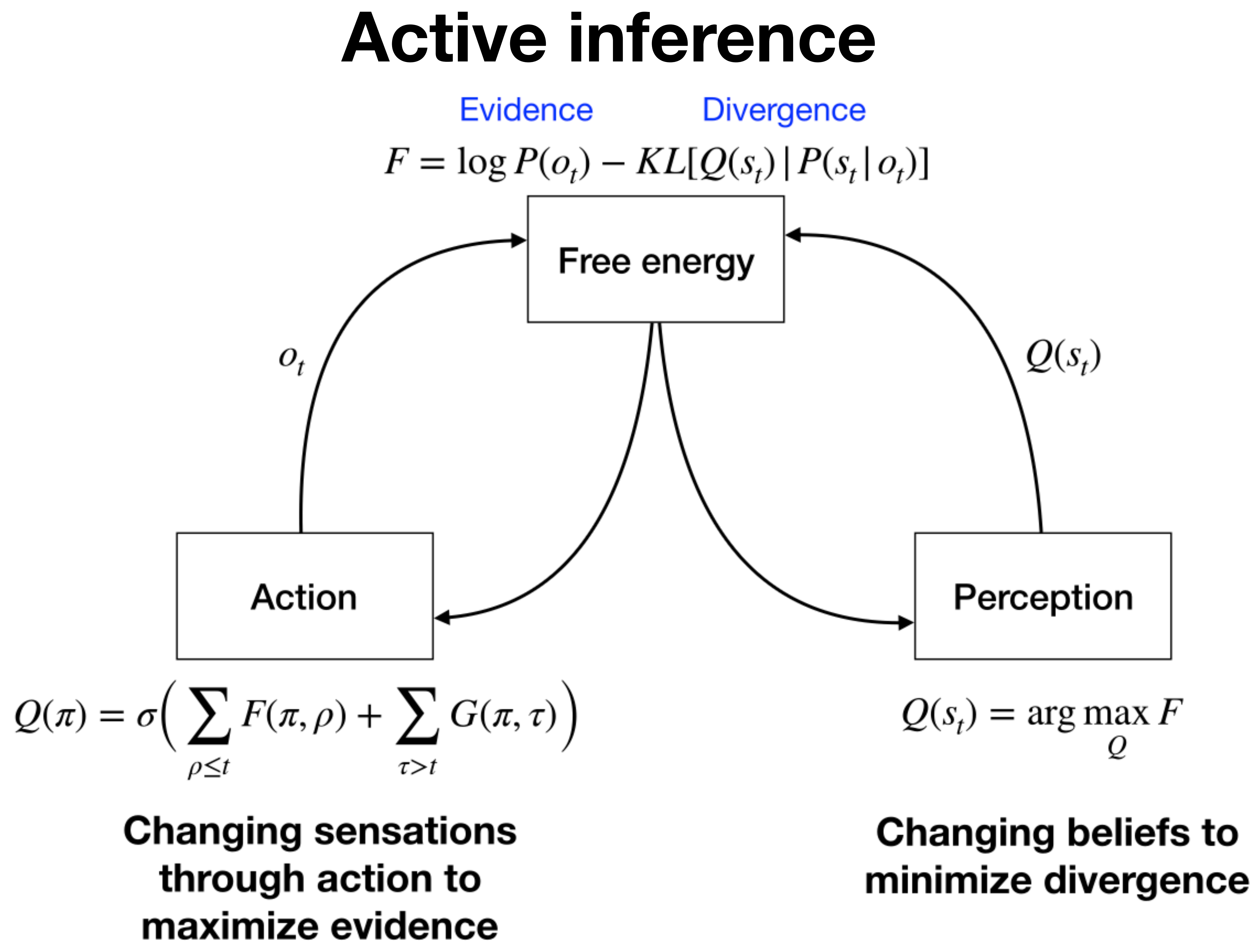
Dopamine signal: prediction error



(Schultz, et al., 1997)

Active inference

Compare with Active inference



Need to re-interpret Dopamine signal:

precision of prediction error on

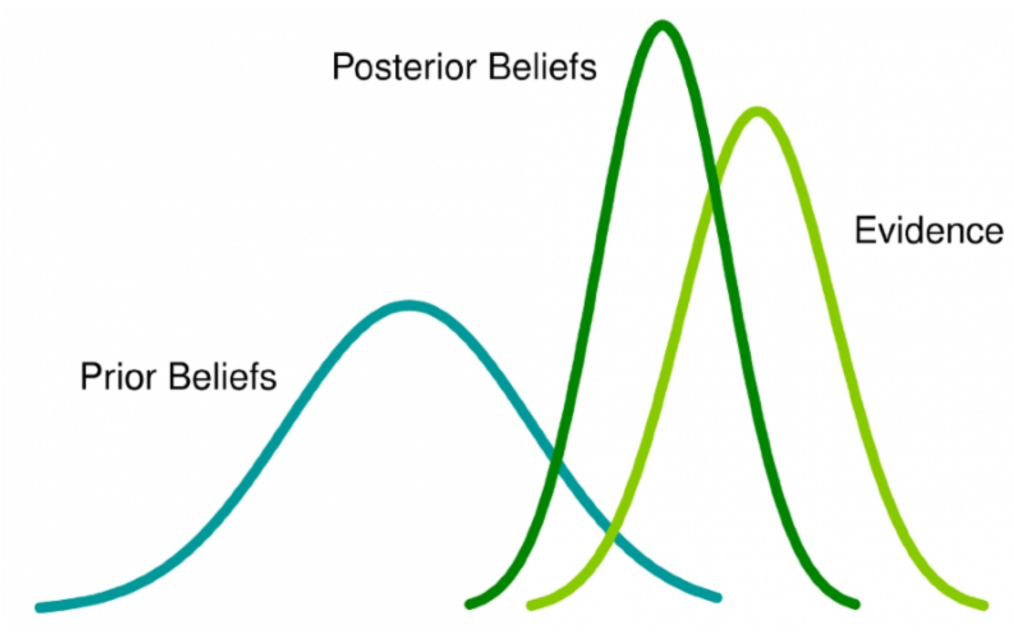
sensory state

(Friston, 2008)

Summary

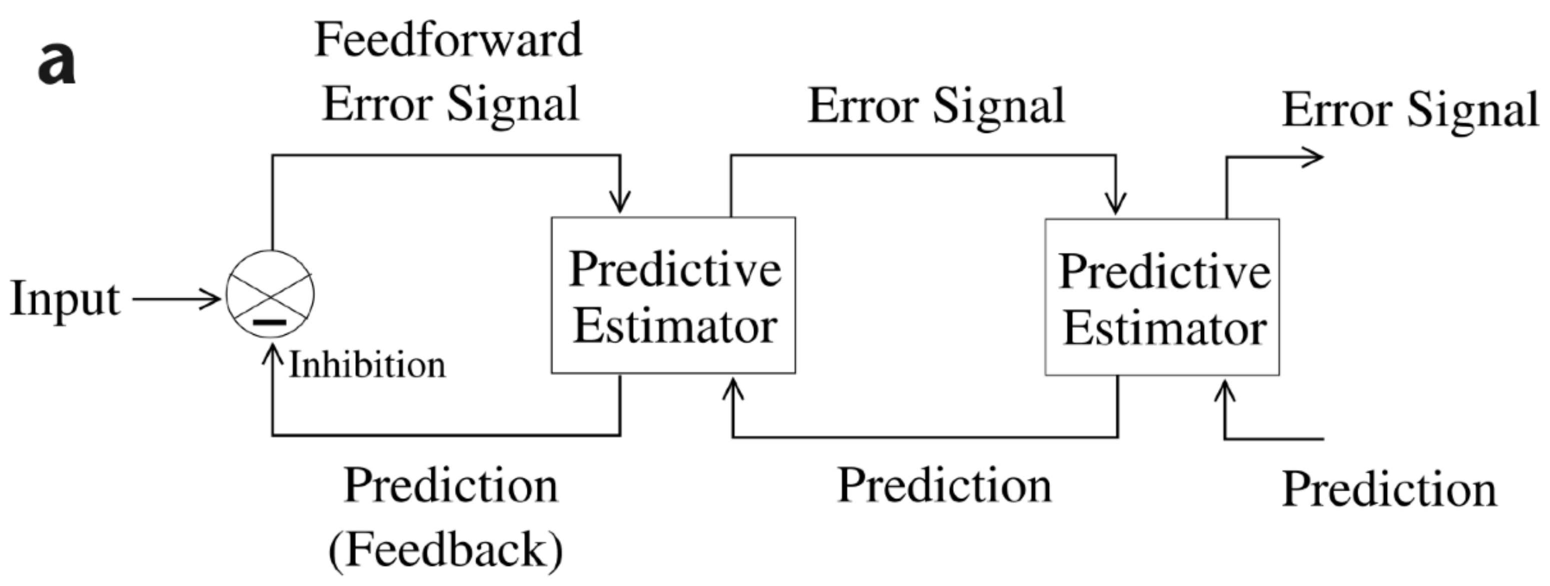
All Bayesian

Bayesian



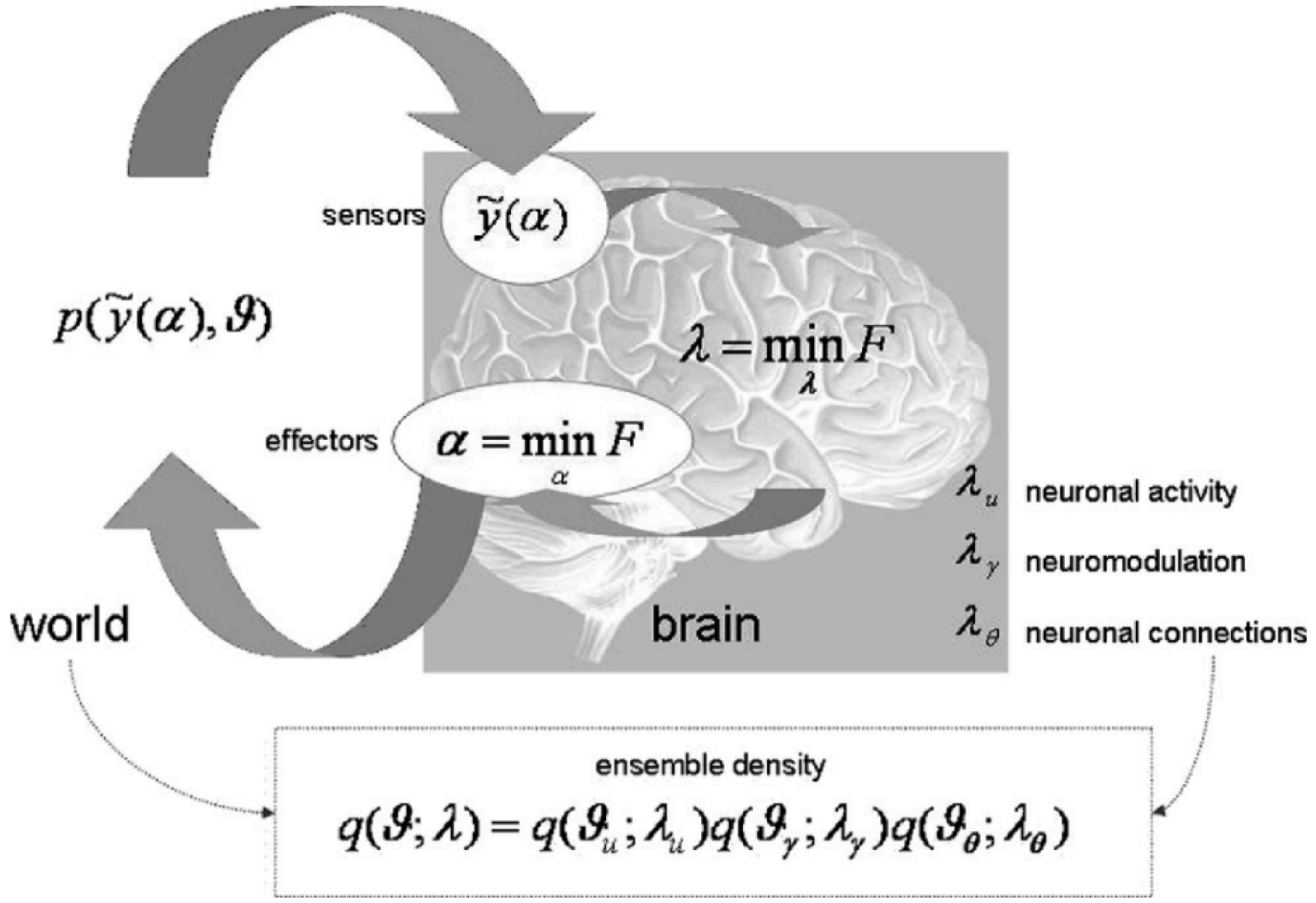
- Probability distribution

Predictive coding



- Update based on prediction error signals

Free energy principle

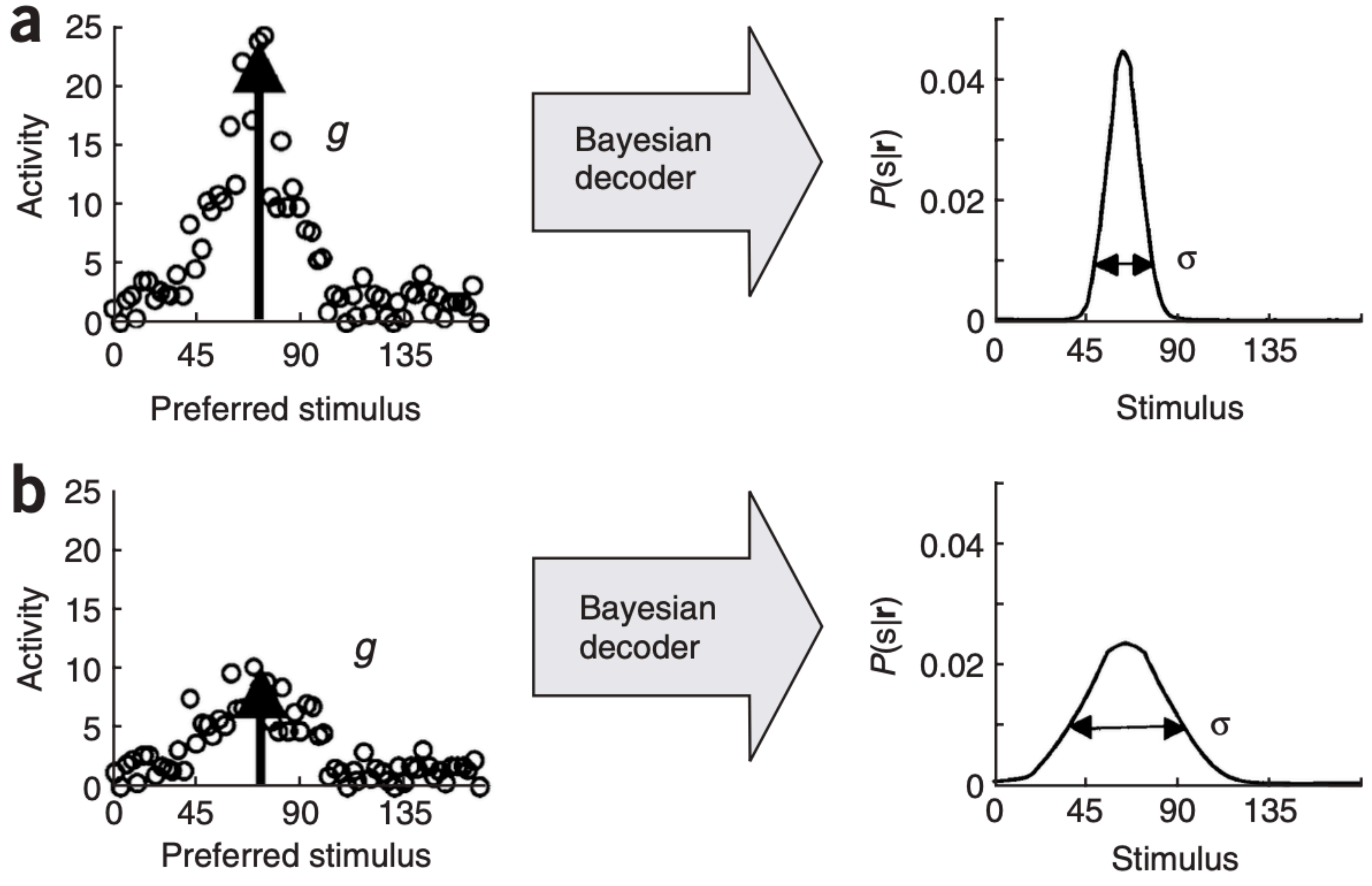


- Variational Bayesian

Summary

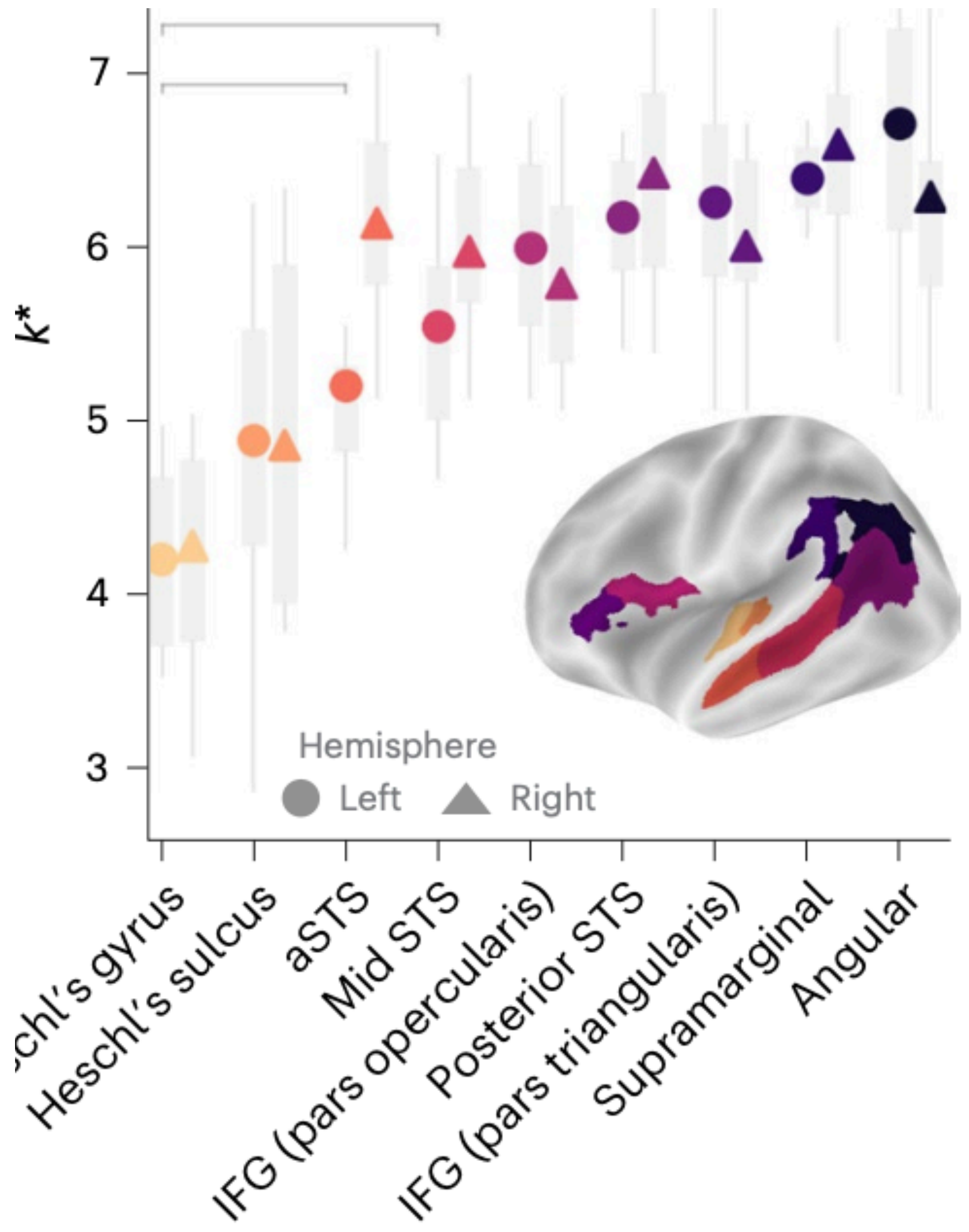
Validation in brain

Bayesian



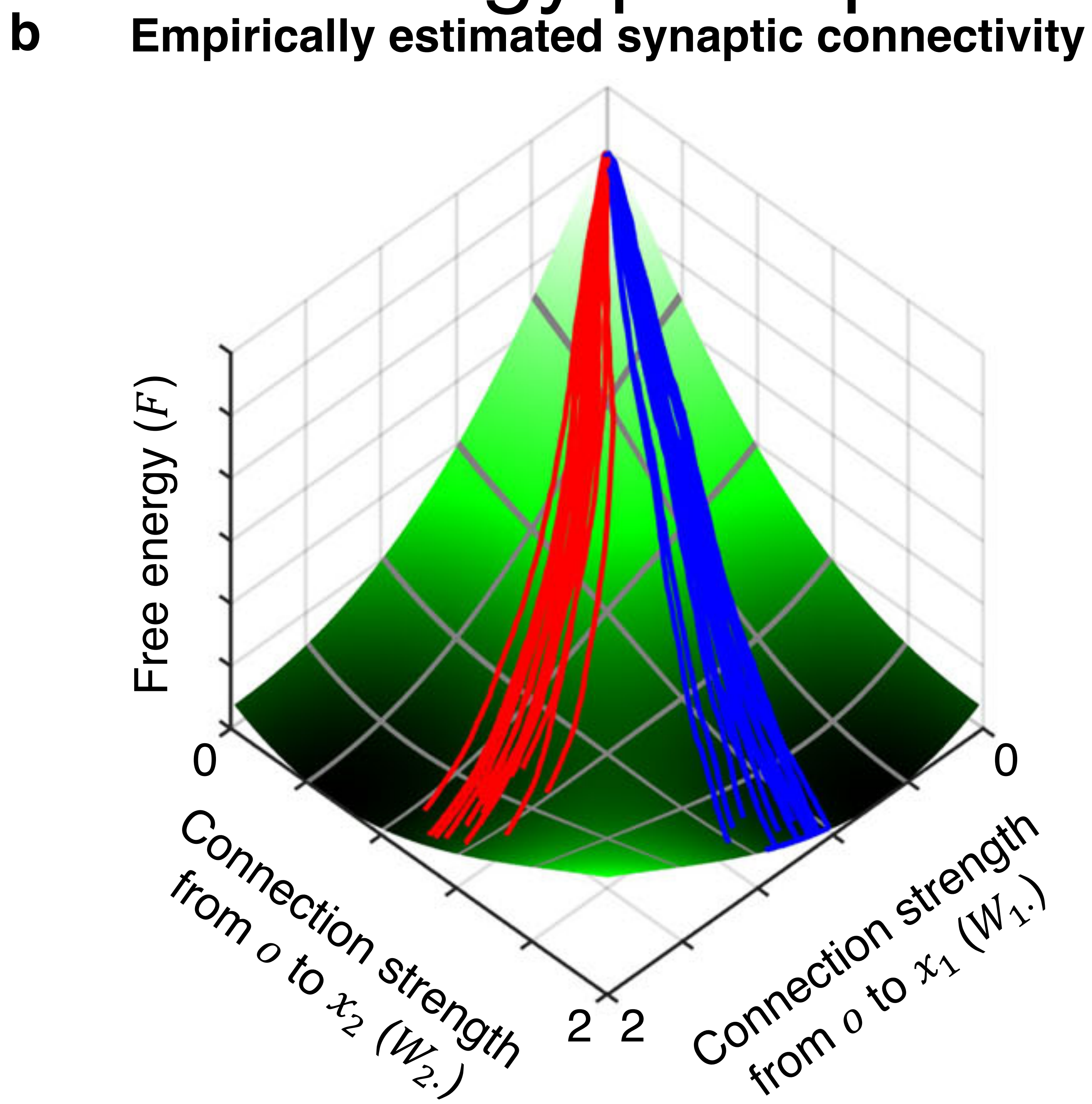
- Turning curve based likelihood
- MAP to predict

Predictive coding



- Forecast depth by comparing a GPT-2 model

Free energy principle



- Estimate synaptic weight

Thank you!