OIST Mini course

Bayesian Methods in Neuroscience

Bayesian Inference and Prediction Coding

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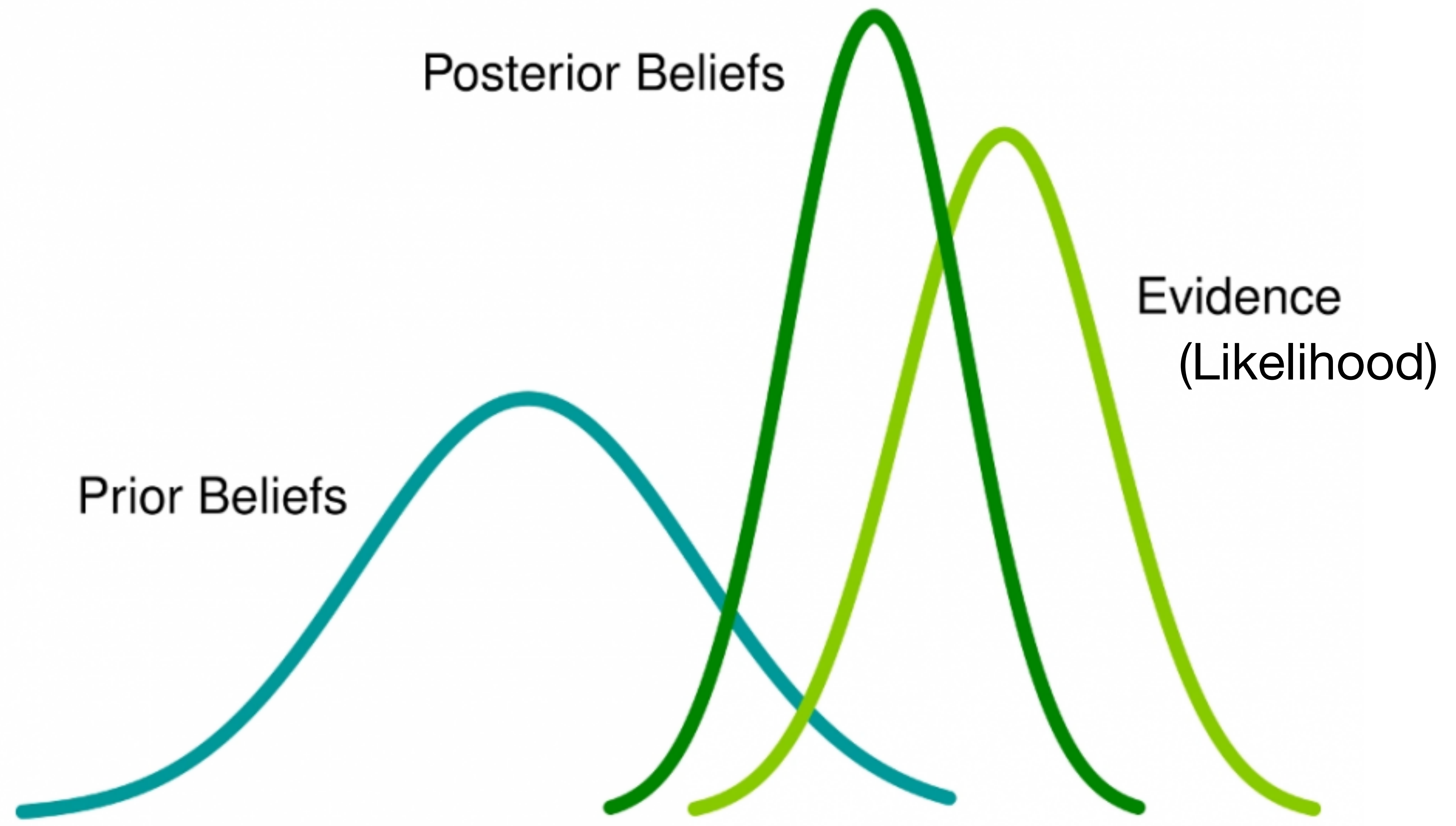
Recap: Bayesian Theory

Bayesian Inference



$$\underbrace{\boxed{P(\text{Hypothesis}|\text{Data})}}_{\text{Posterior}} = \frac{\underbrace{\boxed{P(\text{Hypothesis})}}_{\text{Prior}} \times \underbrace{\boxed{P(\text{Data}|\text{Hypothesis})}}_{\text{Likelihood}}}{\underbrace{\boxed{P(\text{Data})}}_{\text{Marginal}}}$$

$$P(\text{Hypothesis}|\text{Data}) \propto P(\text{Hypothesis}) \times P(\text{Data}|\text{Hypothesis})$$

Recap: Bayesian method in classification

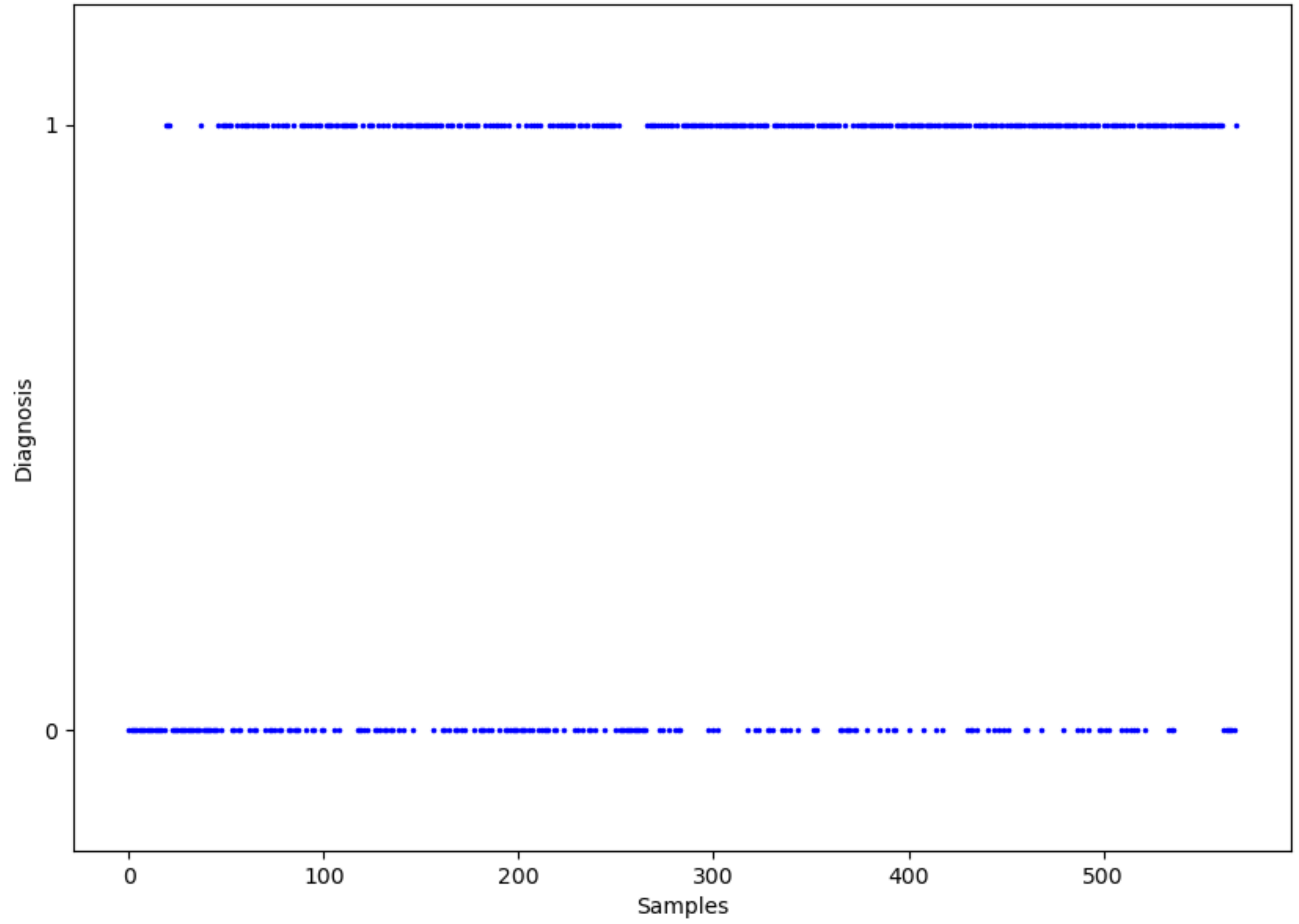
Breast cancer wisconsin (diagnostic) dataset

Features

	mean radius	mean texture	mean perimeter	mean area	mean smoothness	mean compactness	mean concavity	mean concave points	mean symmetry	mean fractal dimension	...
0	17.99	10.38	122.80	1001.0	0.11840	0.27760	0.30010	0.14710	0.2419	...	
1	20.57	17.77	132.90	1326.0	0.08474	0.07864	0.08690	0.07017	0.1812	...	
2	19.69	21.25	130.00	1203.0	0.10960	0.15990	0.19740	0.12790	0.2069	...	
3	11.42	20.38	77.58	386.1	0.14250	0.28390	0.24140	0.10520	0.2597	...	
4	20.29	14.34	135.10	1297.0	0.10030	0.13280	0.19800	0.10430	0.1809	...	
...	...	...	...	...	...	...	...	...	...	...	
564	21.56	22.39	142.00	1479.0	0.11100	0.11590	0.24390	0.13890	0.1726	...	
565	20.13	28.25	131.20	1261.0	0.09780	0.10340	0.14400	0.09791	0.1752	...	
566	16.60	28.08	108.30	858.1	0.08455	0.10230	0.09251	0.05302	0.1590	...	
567	20.60	29.33	140.10	1265.0	0.11780	0.27700	0.35140	0.15200	0.2397	...	
568	7.76	24.54	47.92	181.0	0.05263	0.04362	0.00000	0.00000	0.1587	...	

Number of Instances: 569
Number of Attributes: 30

Diagnosis results



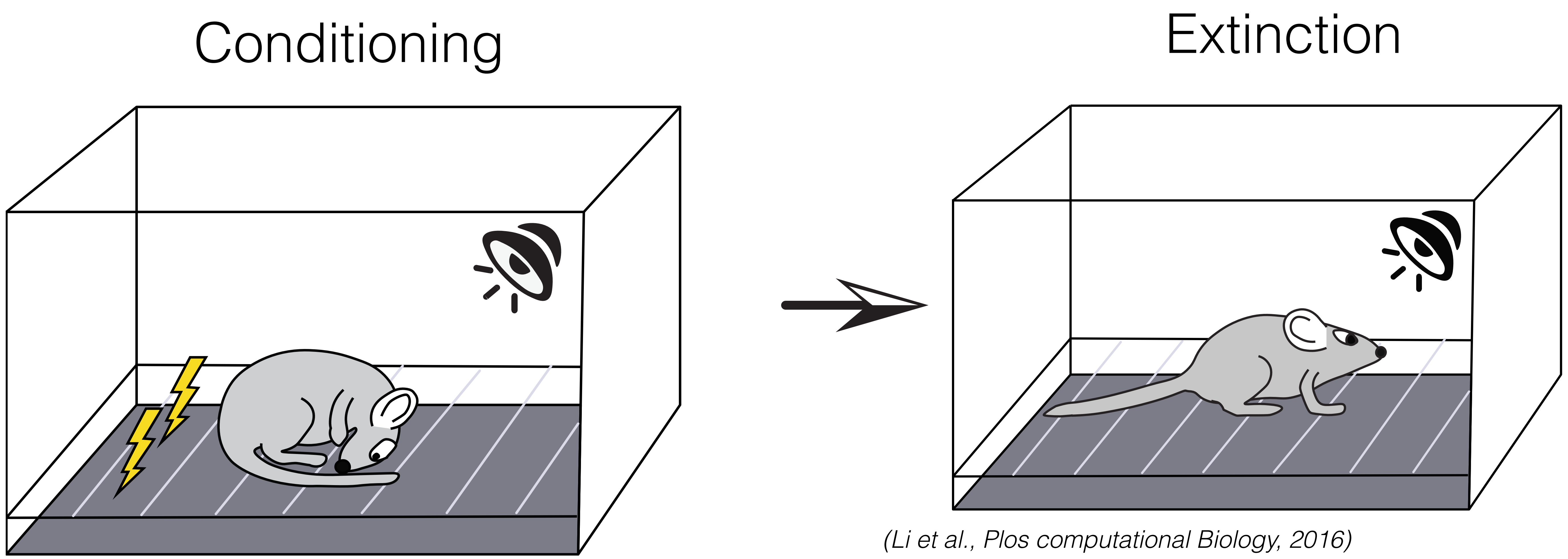
Goal: $P(y|X)$

Likelihood: $P(y|X,w)$

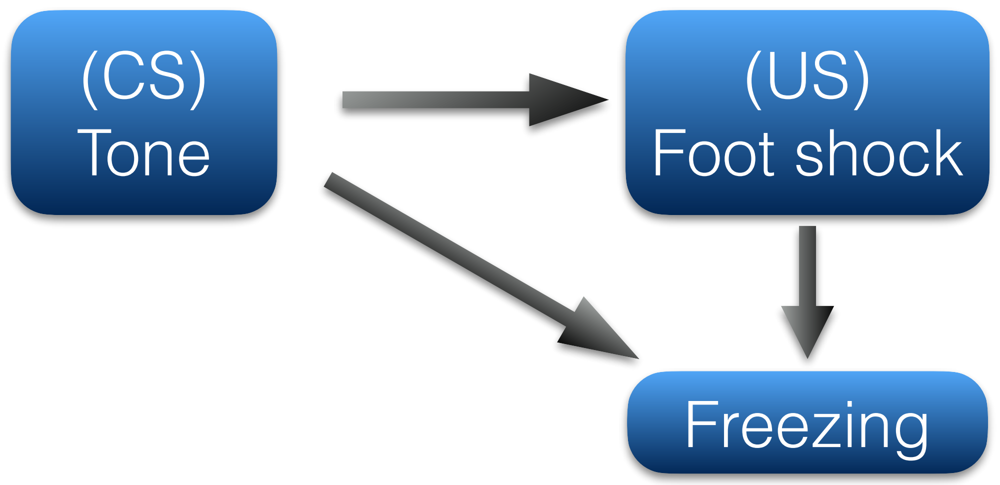
Posterior: $P(w|y,X) \propto P(w)P(y|X,w)$

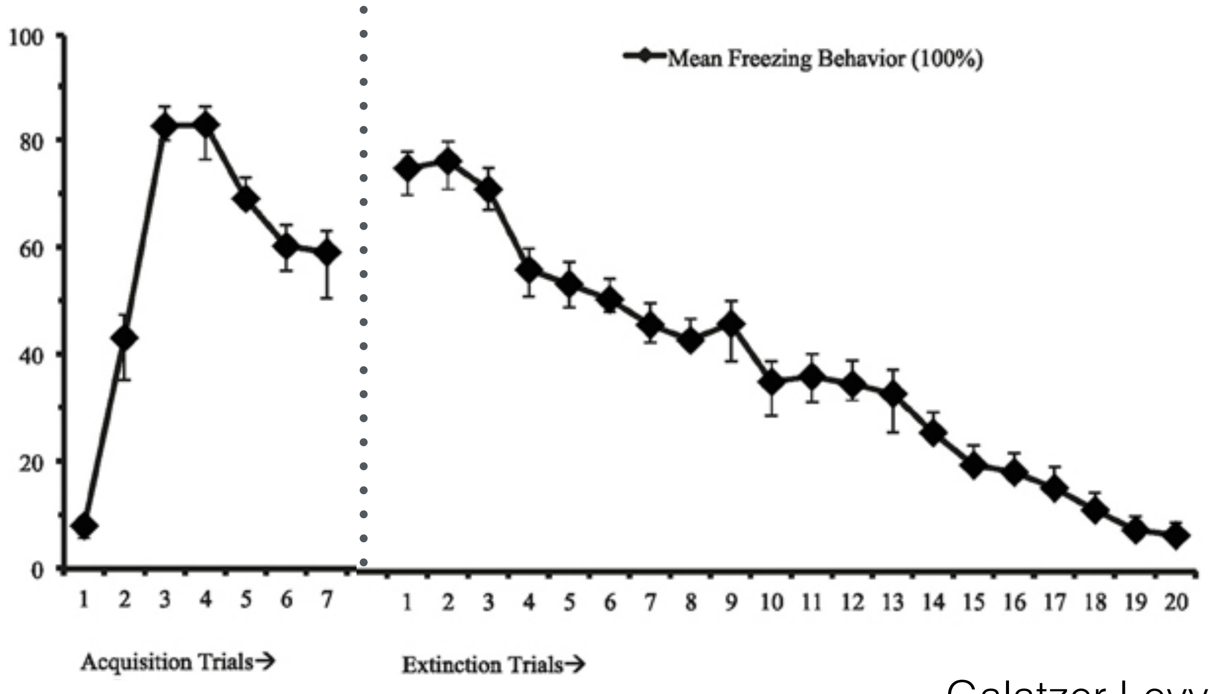
Recap: Bayesian model on sequential behavior

Fear conditioning



(Li et al., Plos computational Biology, 2016)





Galatzer-Levy et al.,
Front. Behav. Neurosci., 2013

Goal: $P(US_{t+1} \mid US_t)$

likelihood:

$$P(US_{t+1} \mid U_t, w)$$

posterior:

$$P(w \mid US) \propto P(US_{t+1} \mid U_t, w)P(w)$$

I. Applications of Bayesian Inference in interpreting Brain function

Example 1: Bayesian model in the fMRI data

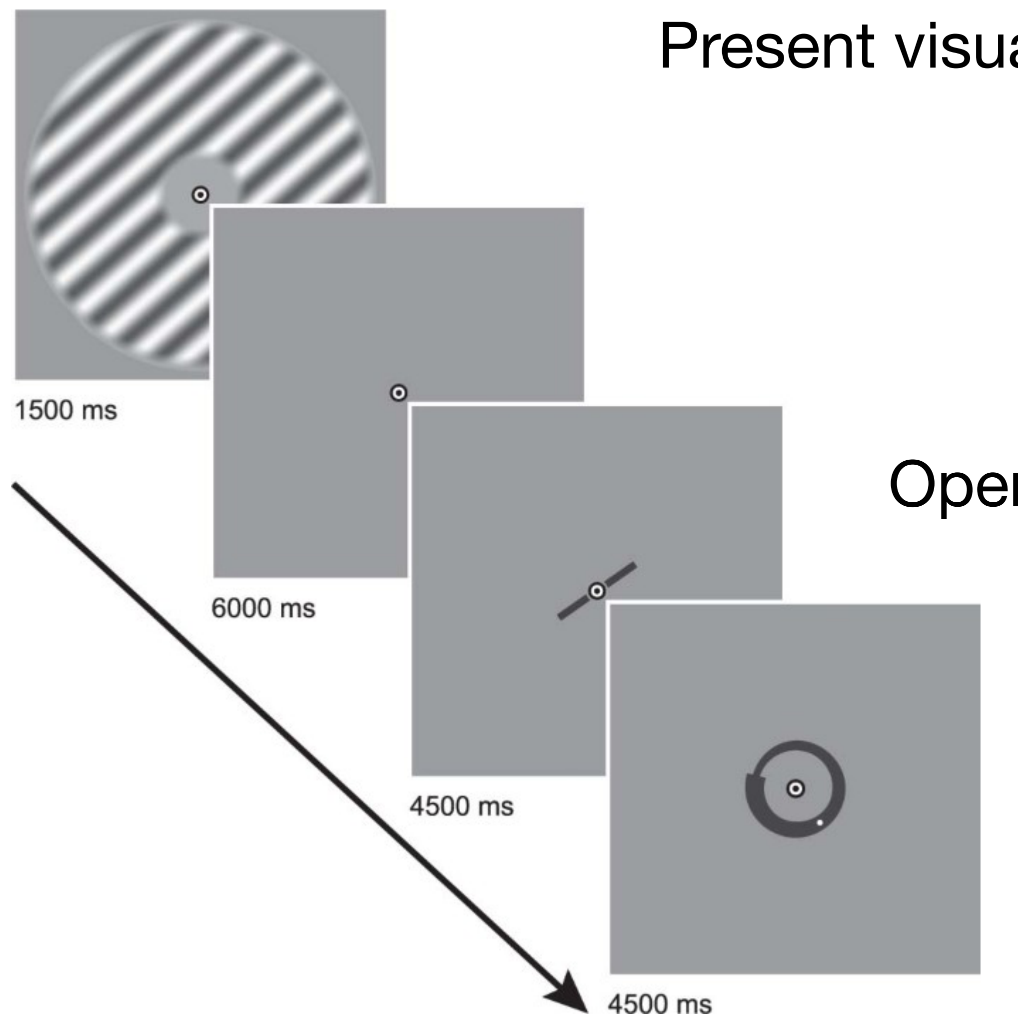
Bayesian model in the fMRI data

Vision-motion task

ARTICLES
https://doi.org/10.1038/s41562-021-01247-w
nature human behaviour
Check for updates

Subjective confidence reflects representation of Bayesian probability in cortex

Laura S. Geurts[1], James R. H. Cooke[1], Ruben S. van Bergen[1,2] and Janneke F. M. Jehee[1] ✉



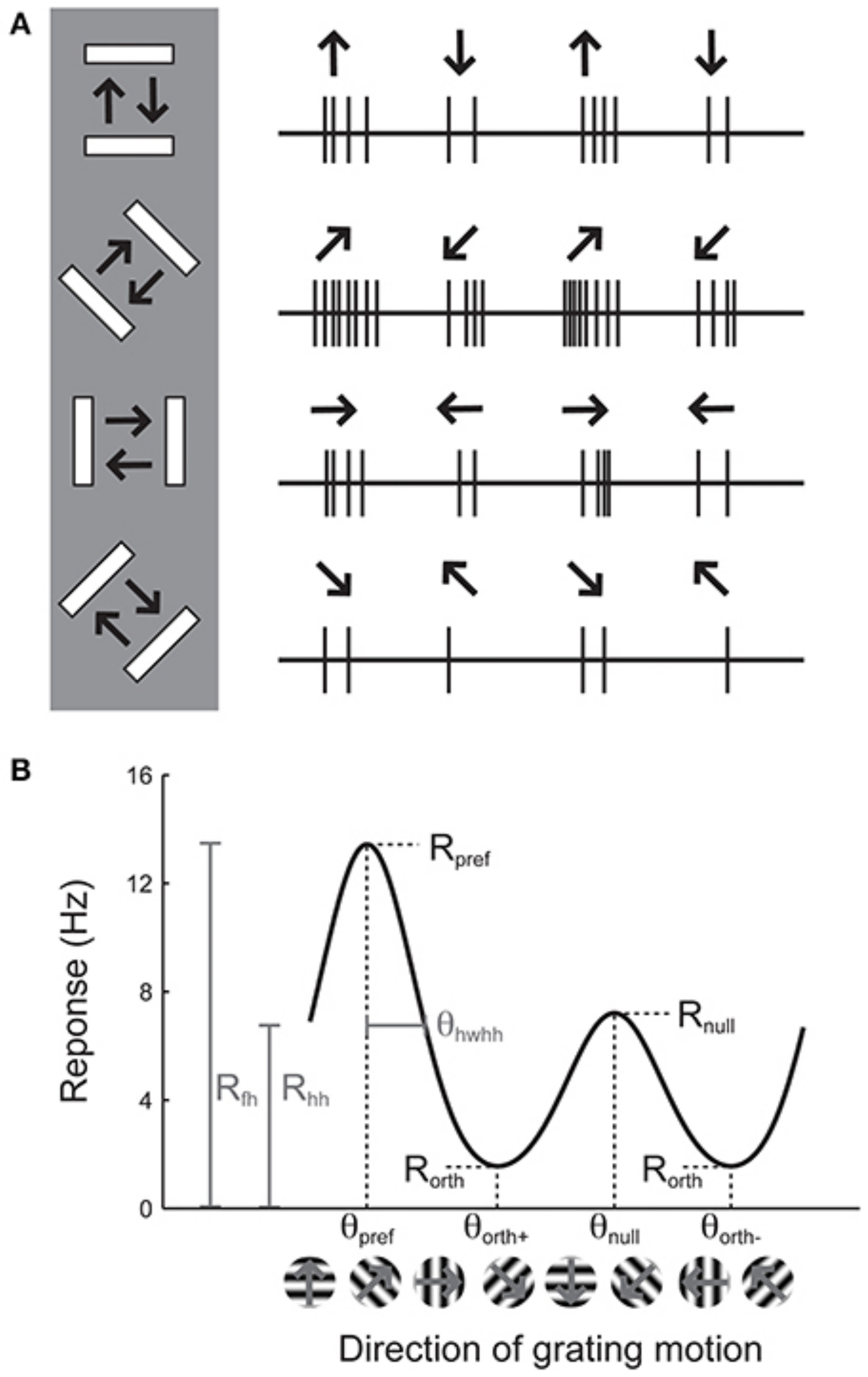
Present visual orientation stimulus

Operate a bar to recreate the orientation

report confidence of the recreation

Bayesian model in the fMRI data

Bayesian model measure orientation stimulus



Tuning curve f(s)

(*Mazurek, et al., Front. Neural Circuits, 2014*)

Goal: estimate the orientation stimulus

stimulus (s) —>fMRI Bold signal (b)

Likelihood: $$b_i = \sum_{k}^{K} W_{ik}\left(f_k(s) + \eta_k\right) + \nu_i$$

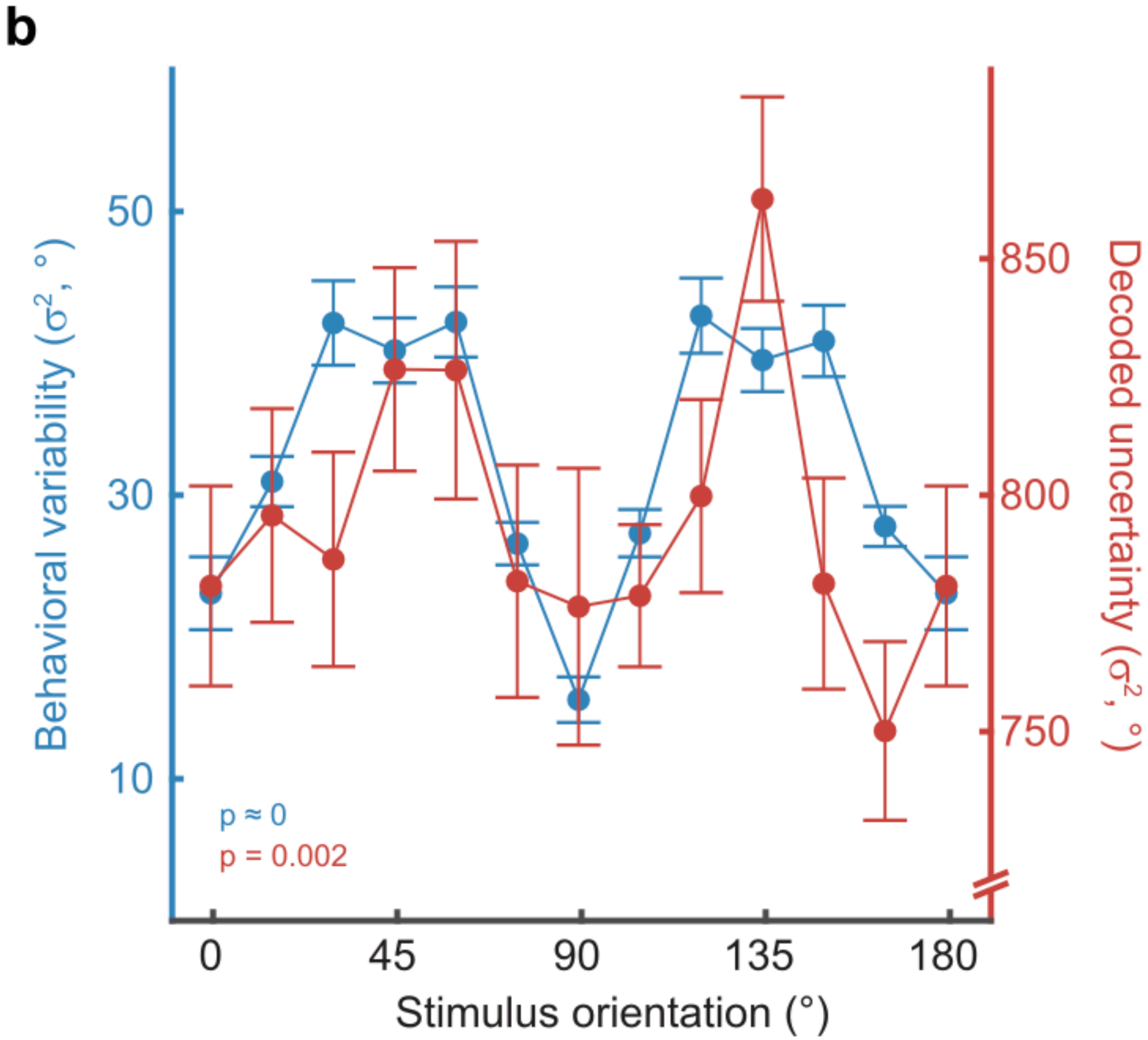
Posterior: $$p\left(s \middle| \mathbf{b};\hat{\boldsymbol{\theta}}\right) = \frac{p\left(\mathbf{b} \middle| s;\hat{\boldsymbol{\theta}}\right) p(s)}{\int p\left(\mathbf{b} \middle| s;\hat{\boldsymbol{\theta}}\right) p(s)\, ds}$$

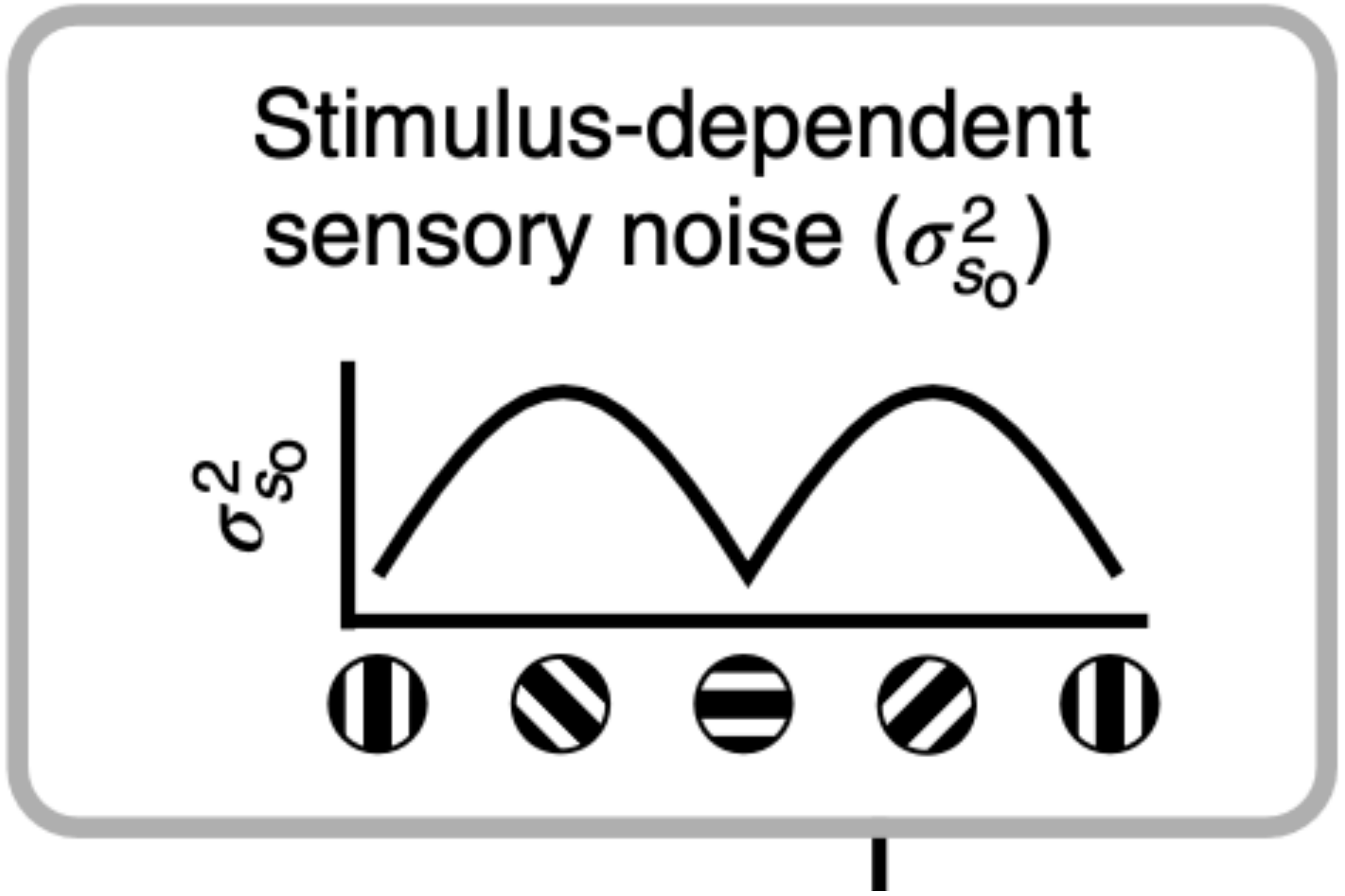
Decode Uncertainty: variance of the posterior

Bayesian model in the fMRI data

Decoded uncertainty

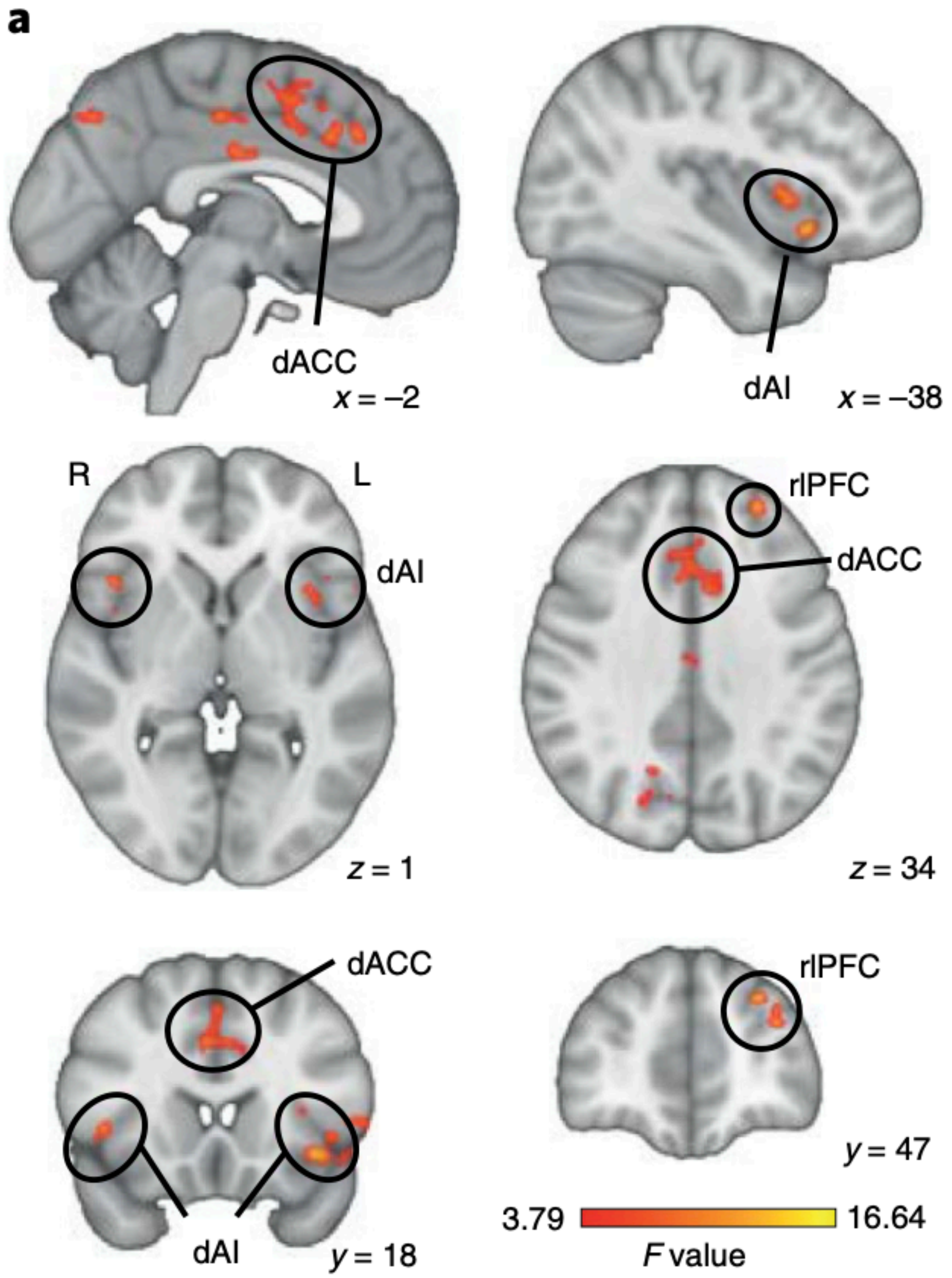
Decoded Uncertainty v.s. reported confidence



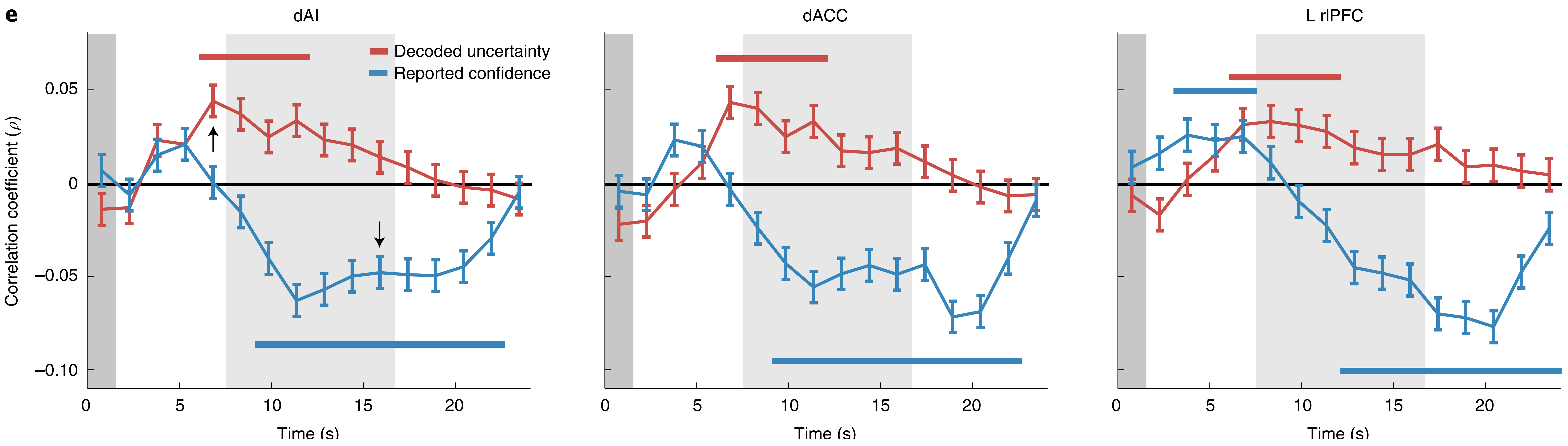


Bayesian model in the fMRI data

Decode uncertainty from fMRI

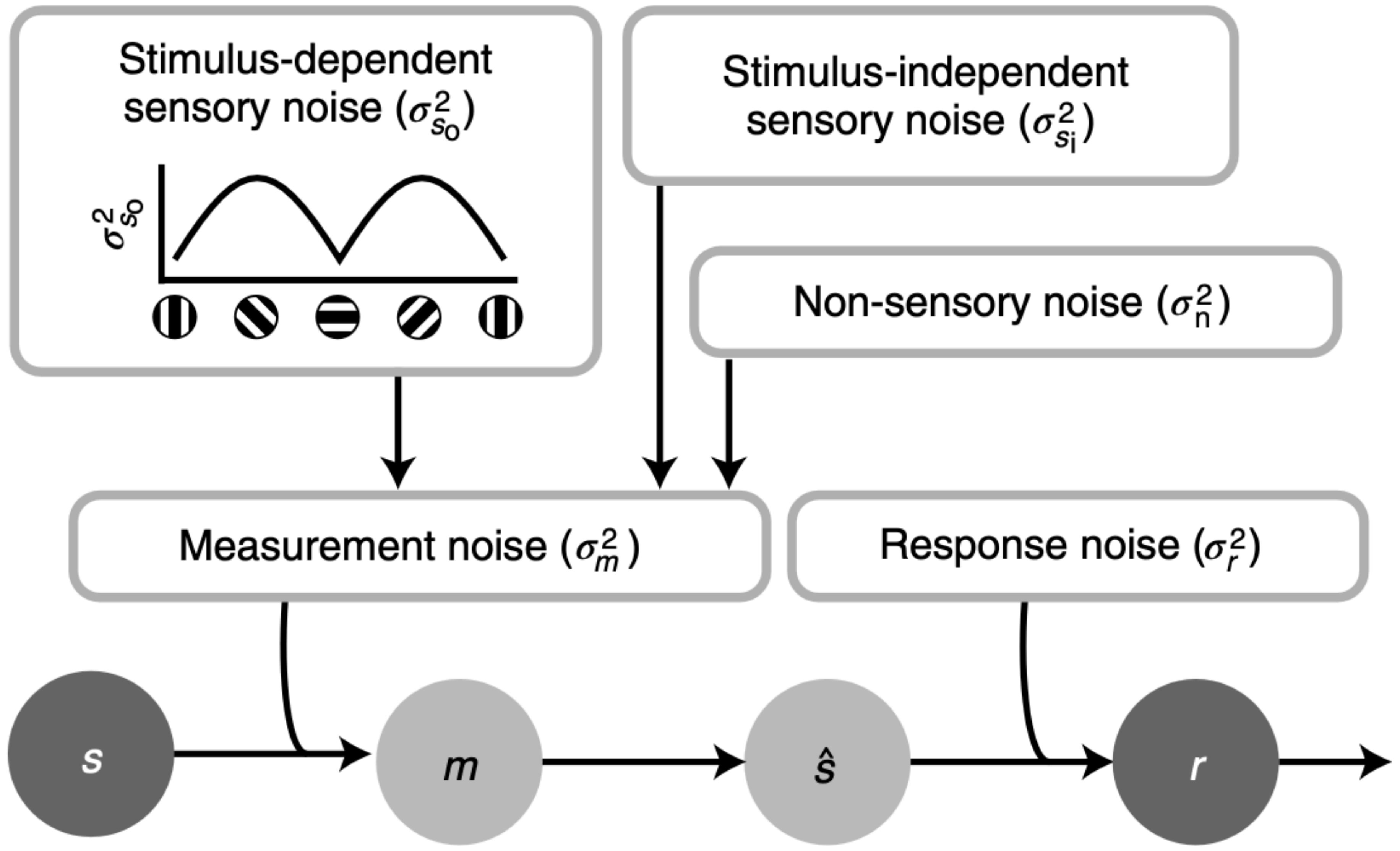


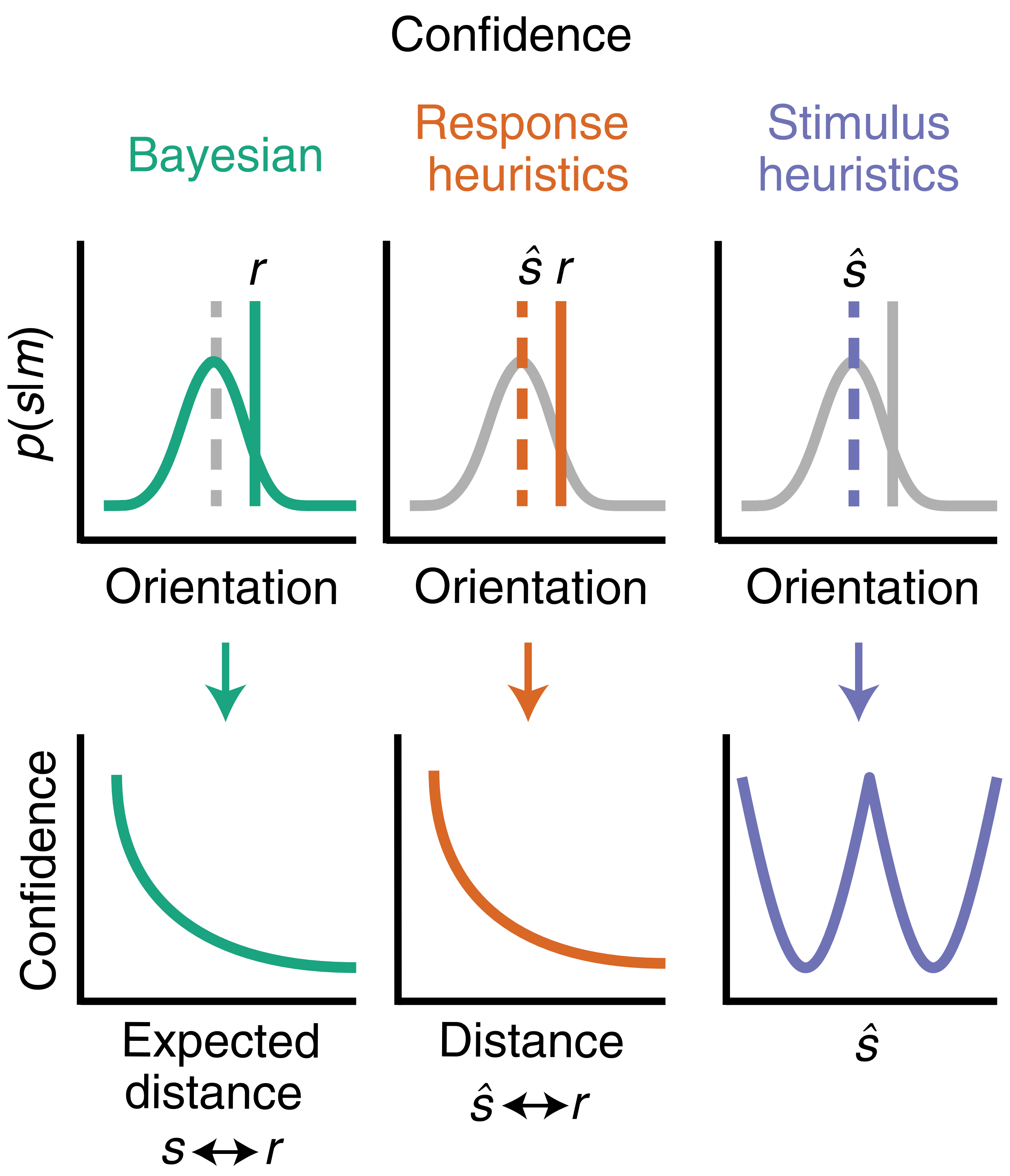
Decoded Uncertainty v.s. reported confidence



Bayesian model in the fMRI data

Three possible models

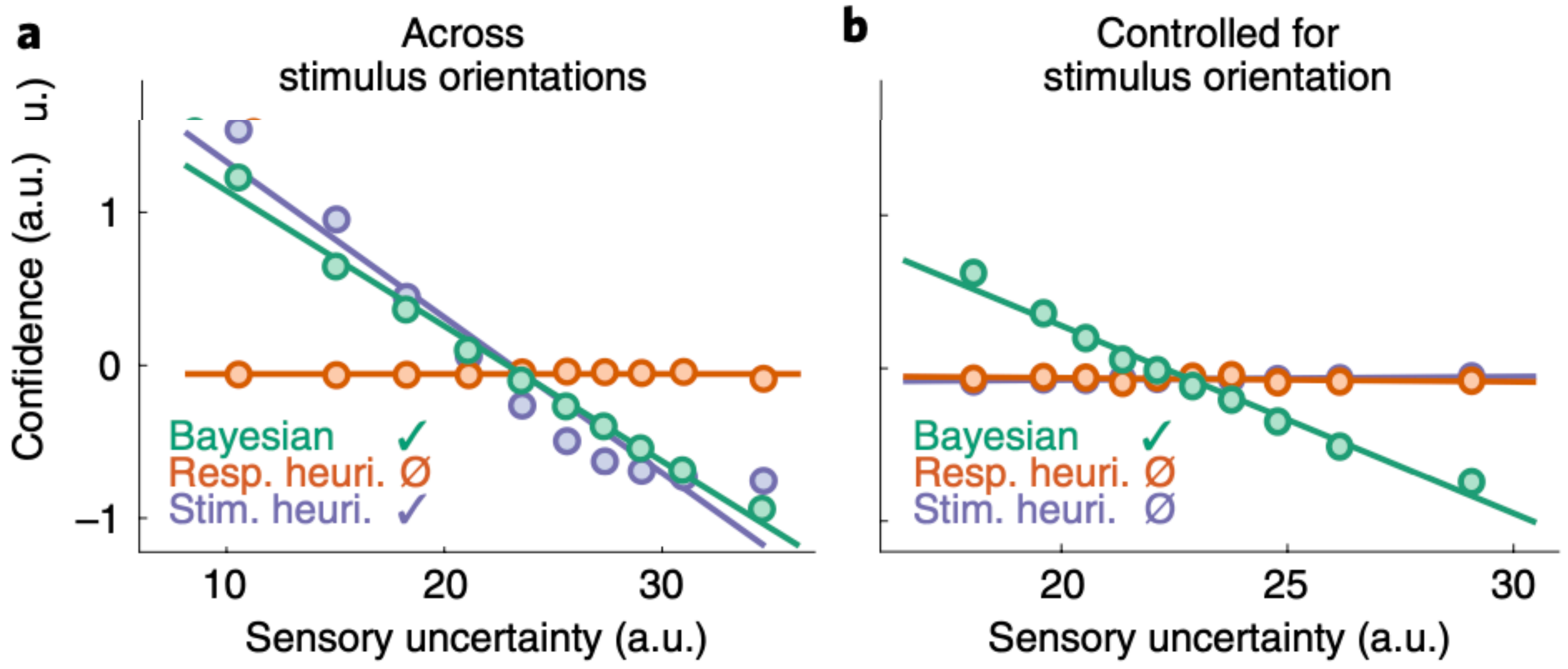




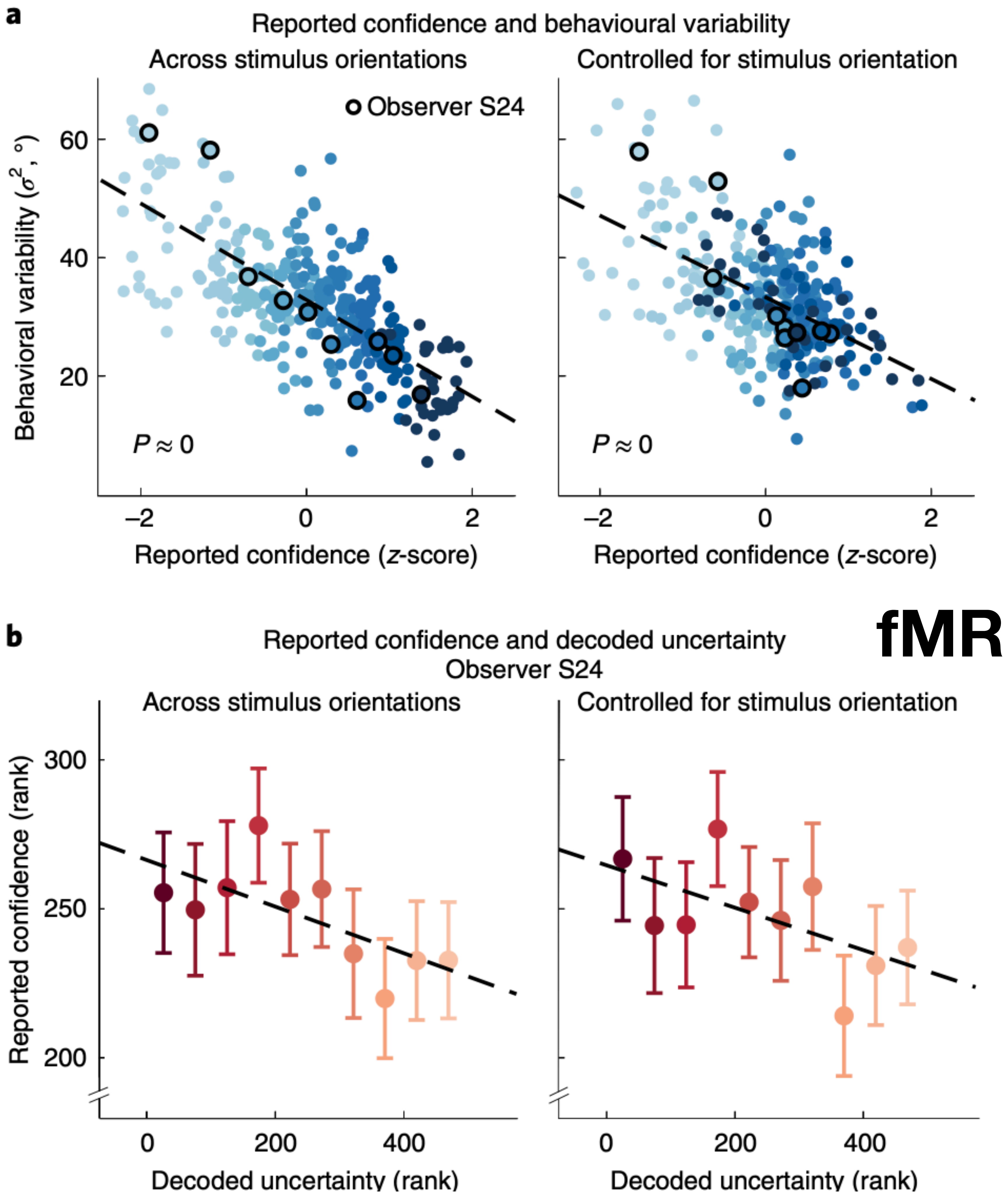
Bayesian model in the fMRI data

Compare models with analysis based on fMRI data

Simulation



Self report



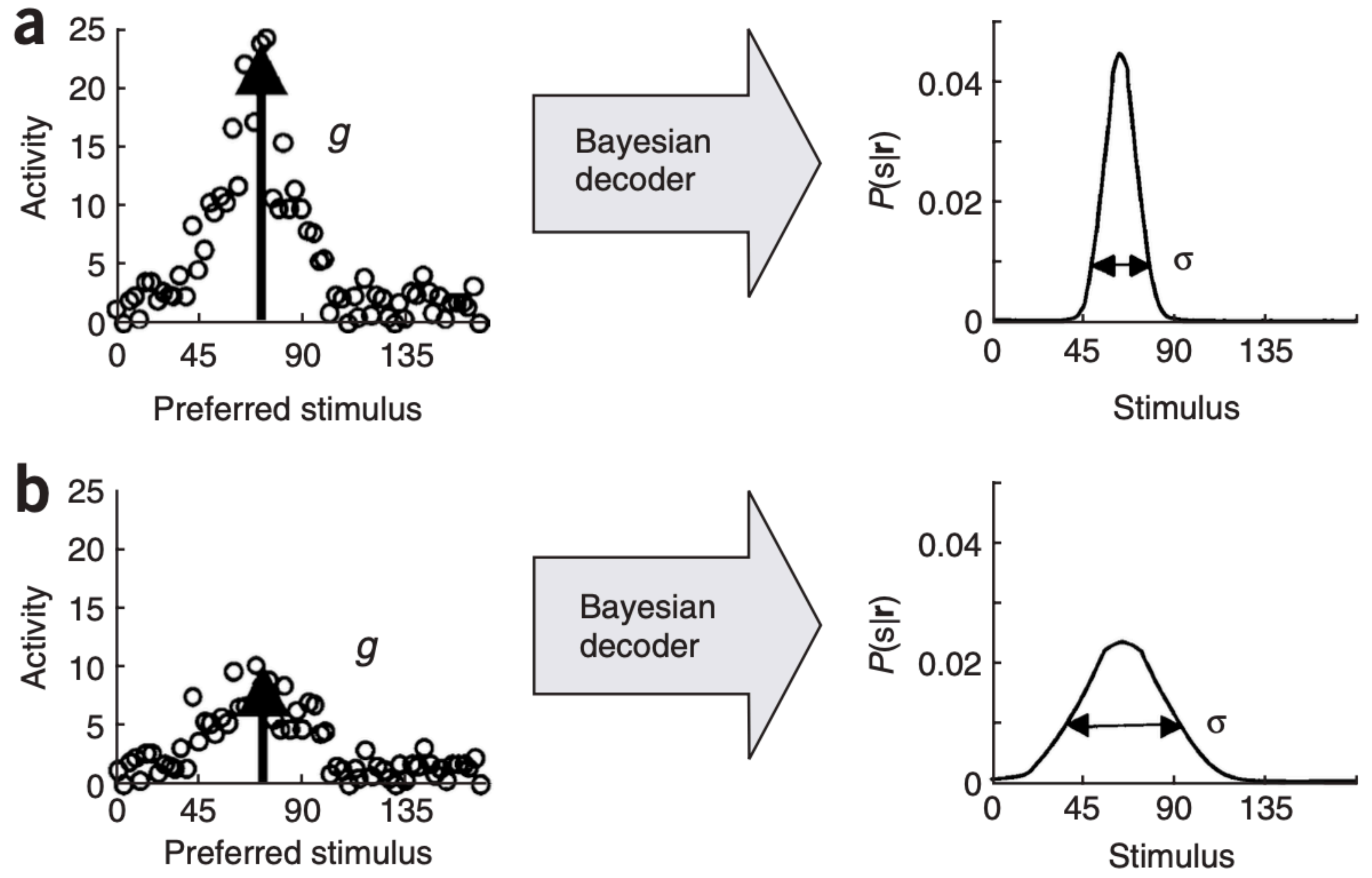
fMRI data

Bayesian observer model won

Example 2: Bayesian Inference in population neural activities

Bayesian Population coding

Tuning curve f(r|s)



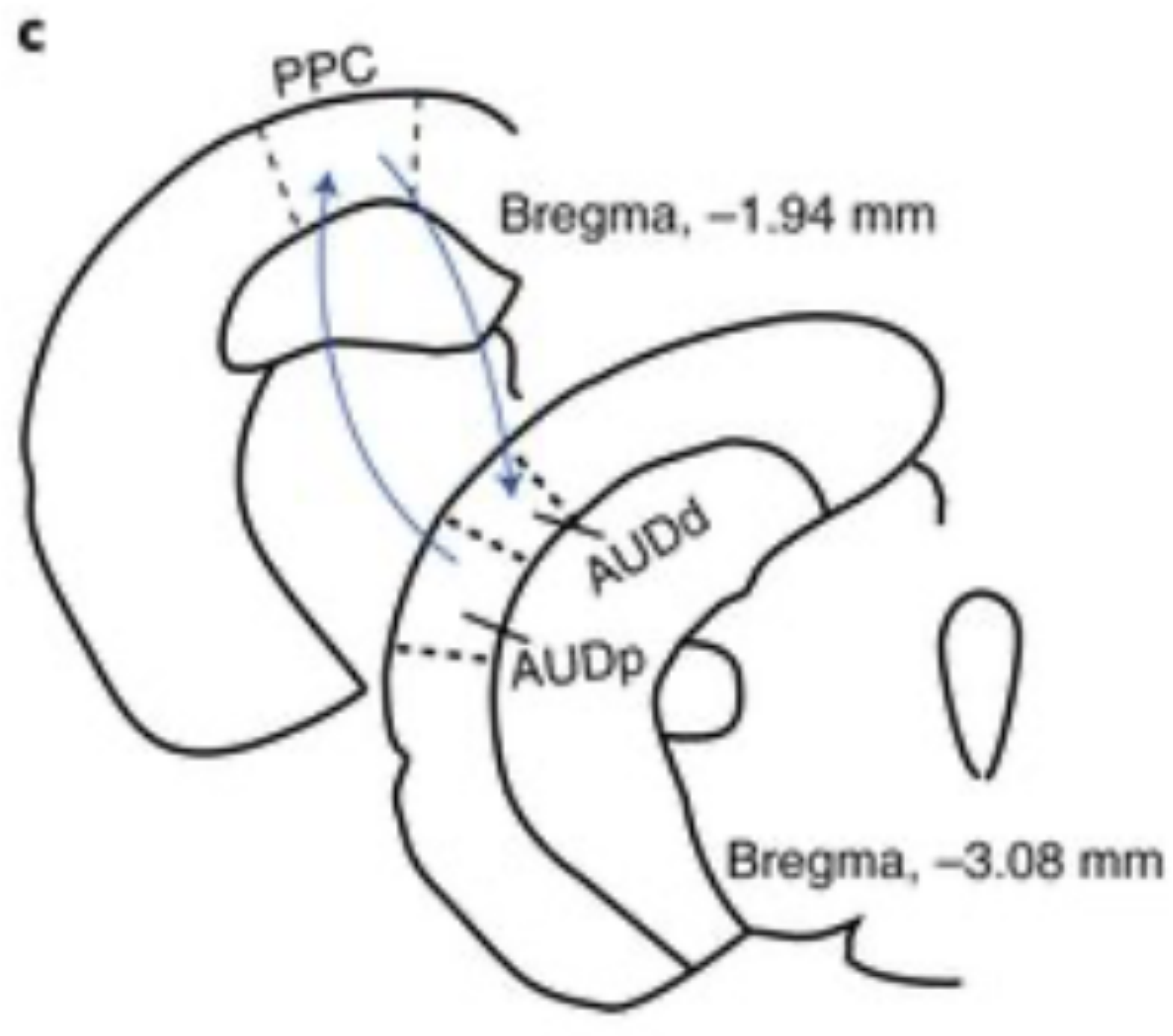
(Ma, et al, Nature neuroscience, 2006)

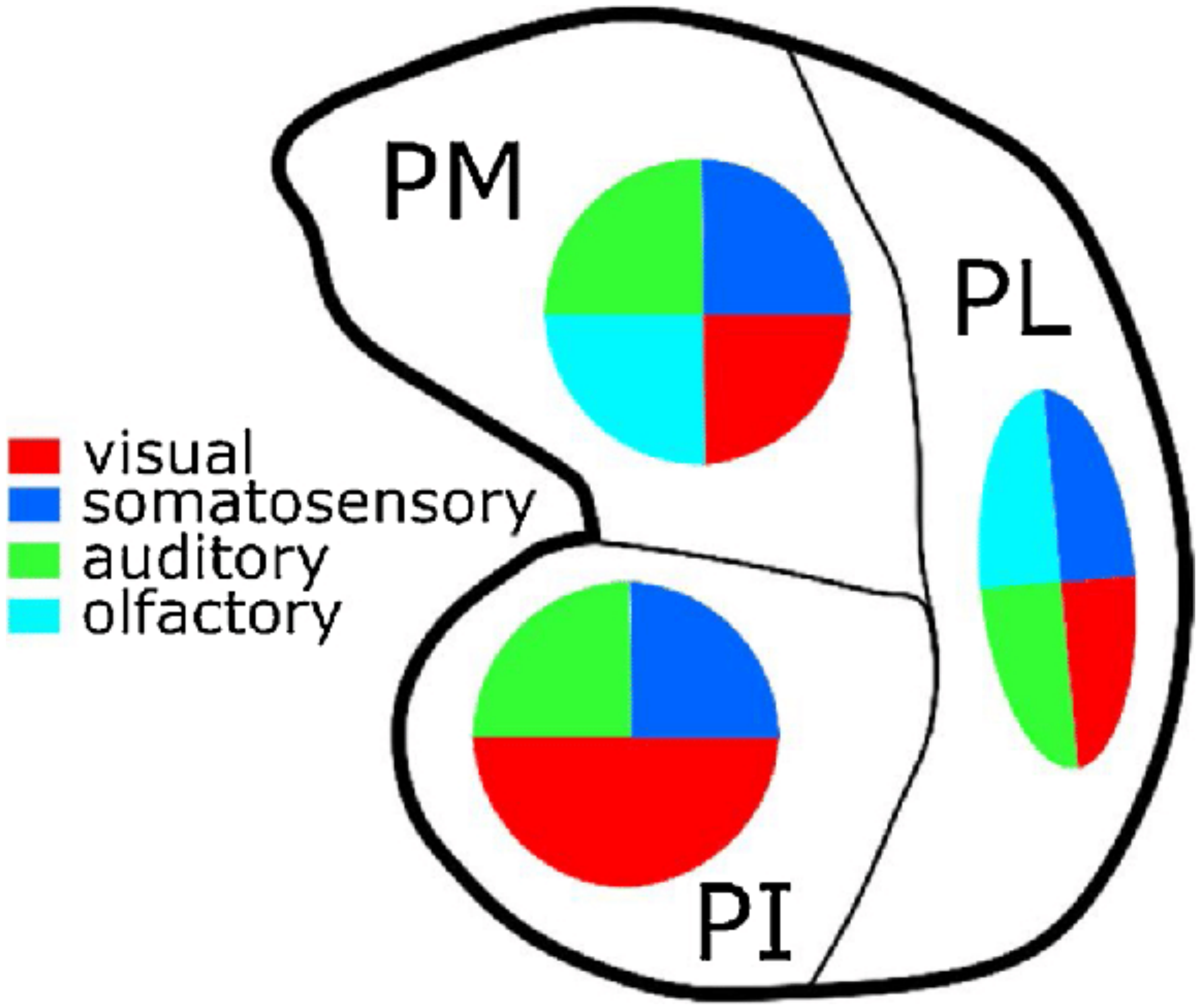
Poisson distribution

Likelihood: $p(r \mid s) = \prod_i \frac{e^{-f_i(s)} f_i(s)^{r_i}}{r_i!}$

Posterior: $p(s|\mathbf{r}) \propto \prod_i \frac{e^{-f_i(s)} f_i(s)^{r_i}}{r_i!} p(s),$

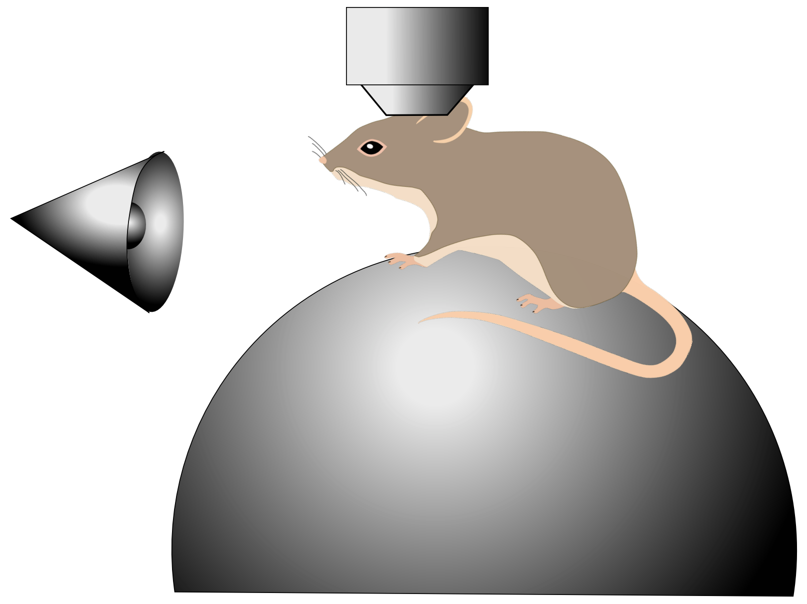
Posterior Parietal Cortex (PPC) and medial pulvinar nucleus (PM)

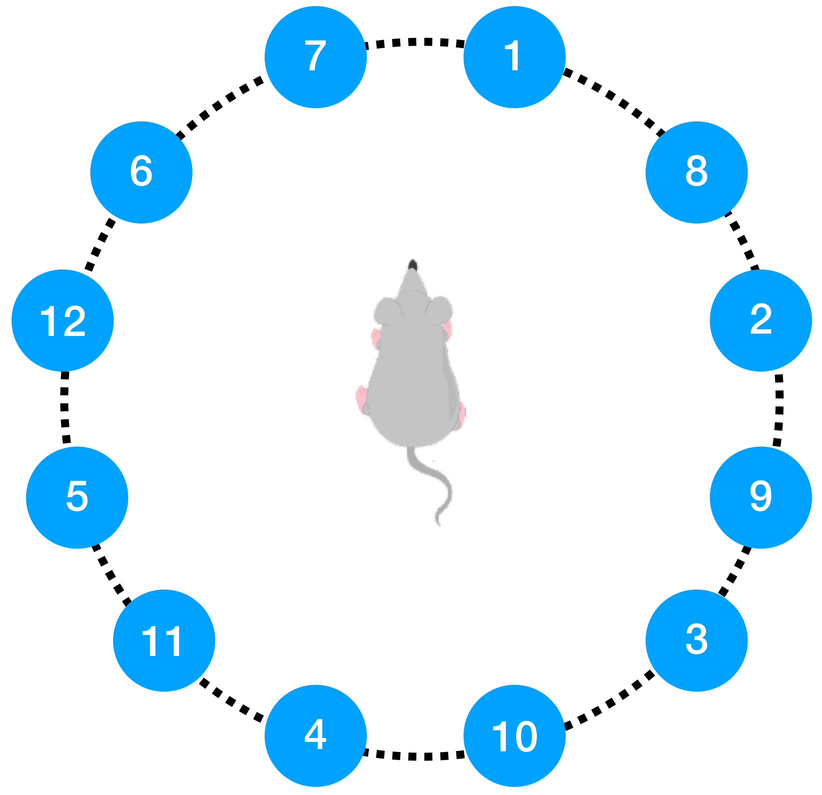


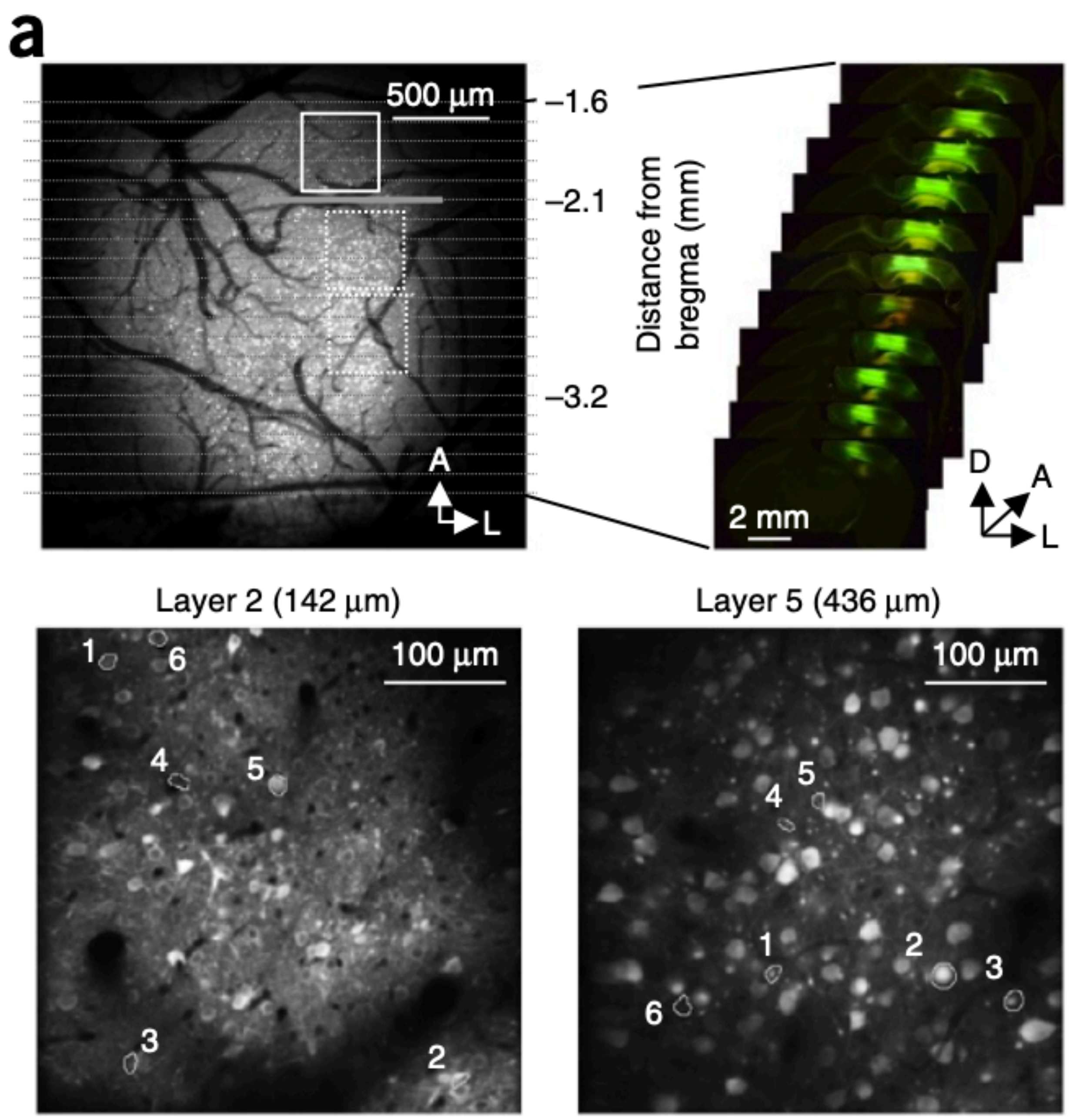


PPC-to-auditory cortex projections are necessary for categorical decision-making on new sensory stimuli

Bayesian inference in PPC







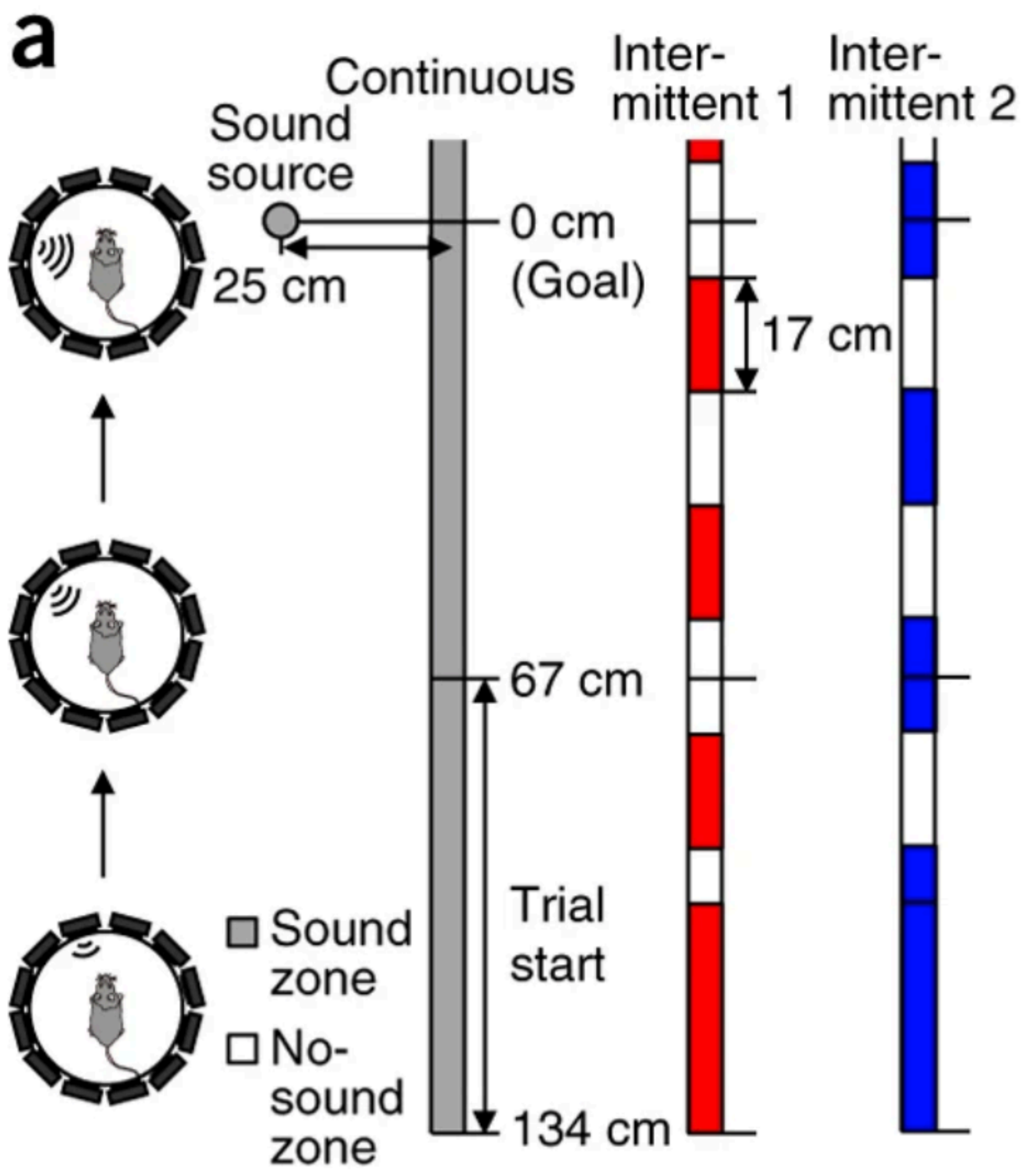
(Funamizu, et al, Nature neuroscience, 2016)

ARTICLES

nature neuroscience

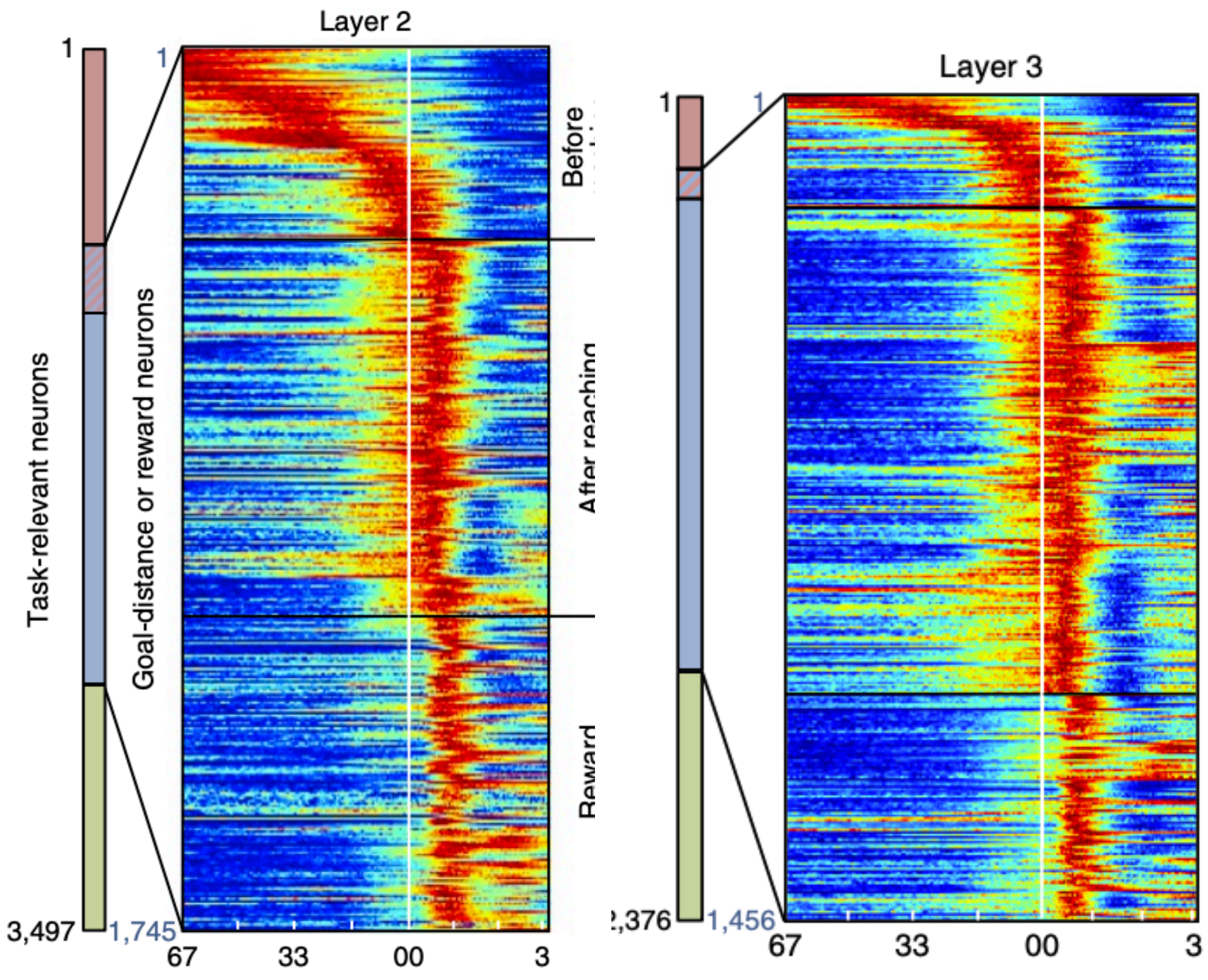
Neural substrate of dynamic Bayesian inference in the cerebral cortex

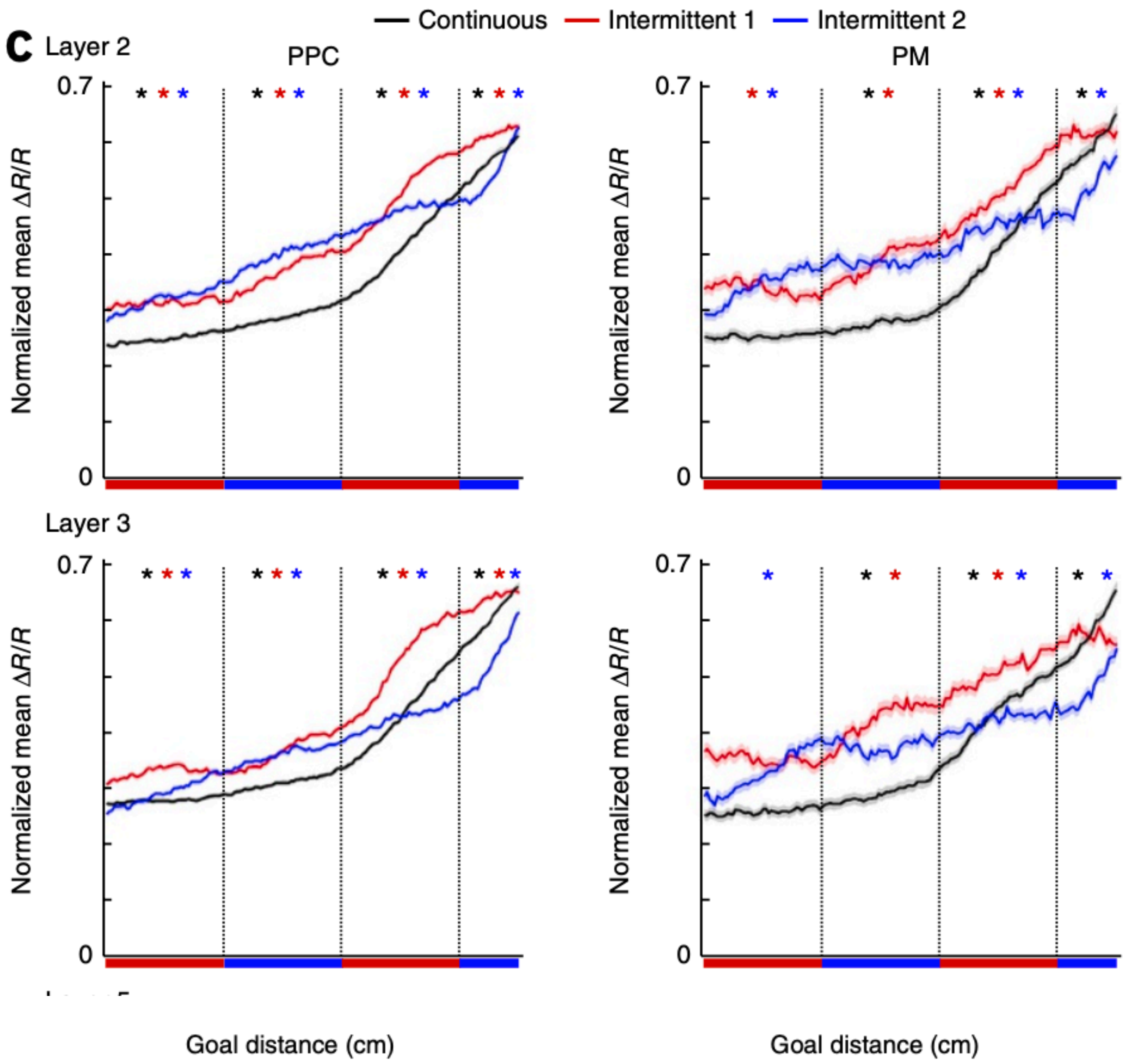
Akihiro Funamizu[1,2], Bernd Kuhn[2] & Kenji Doya[1]



Bayesian inference in PPC and PM

Distance - activition

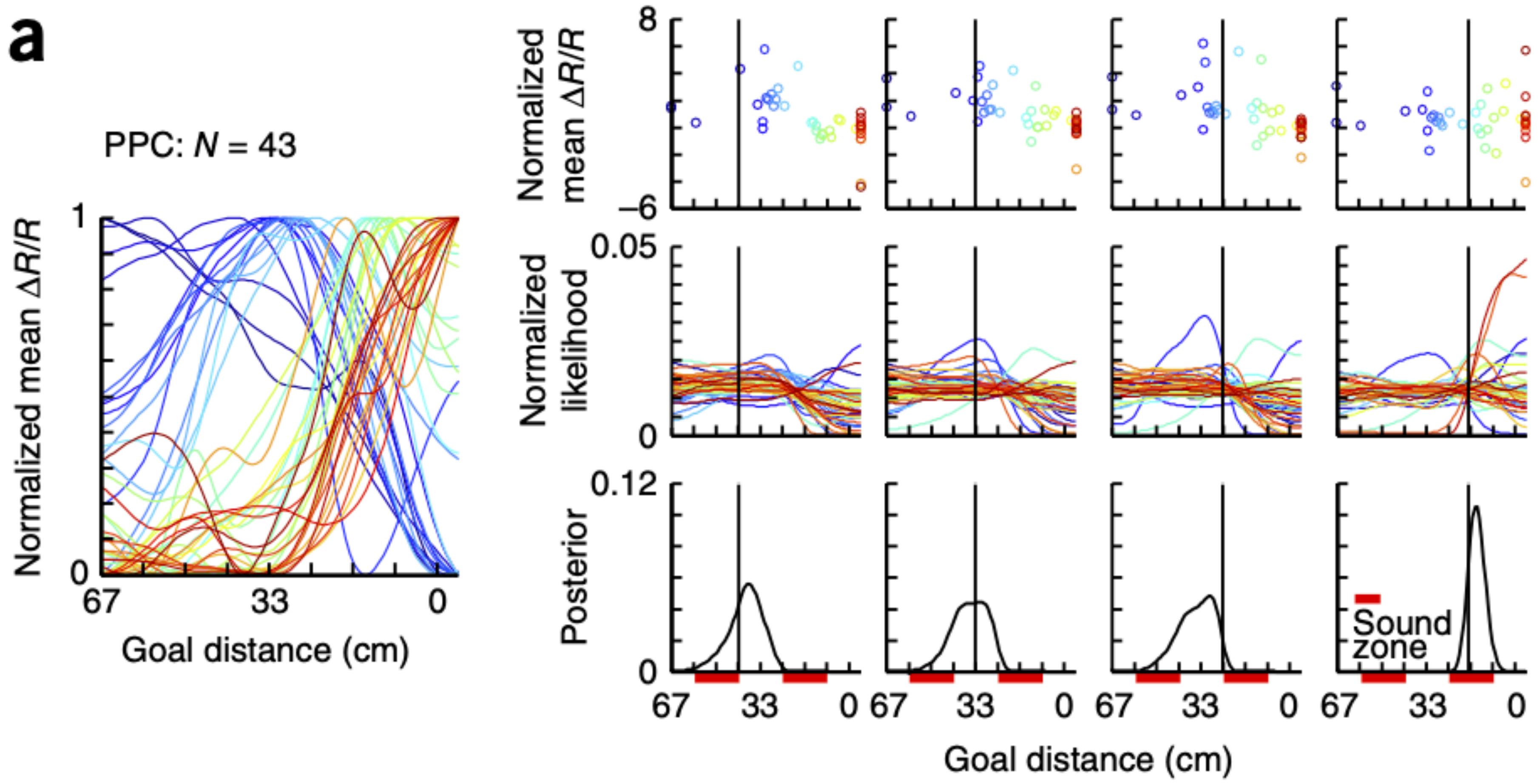


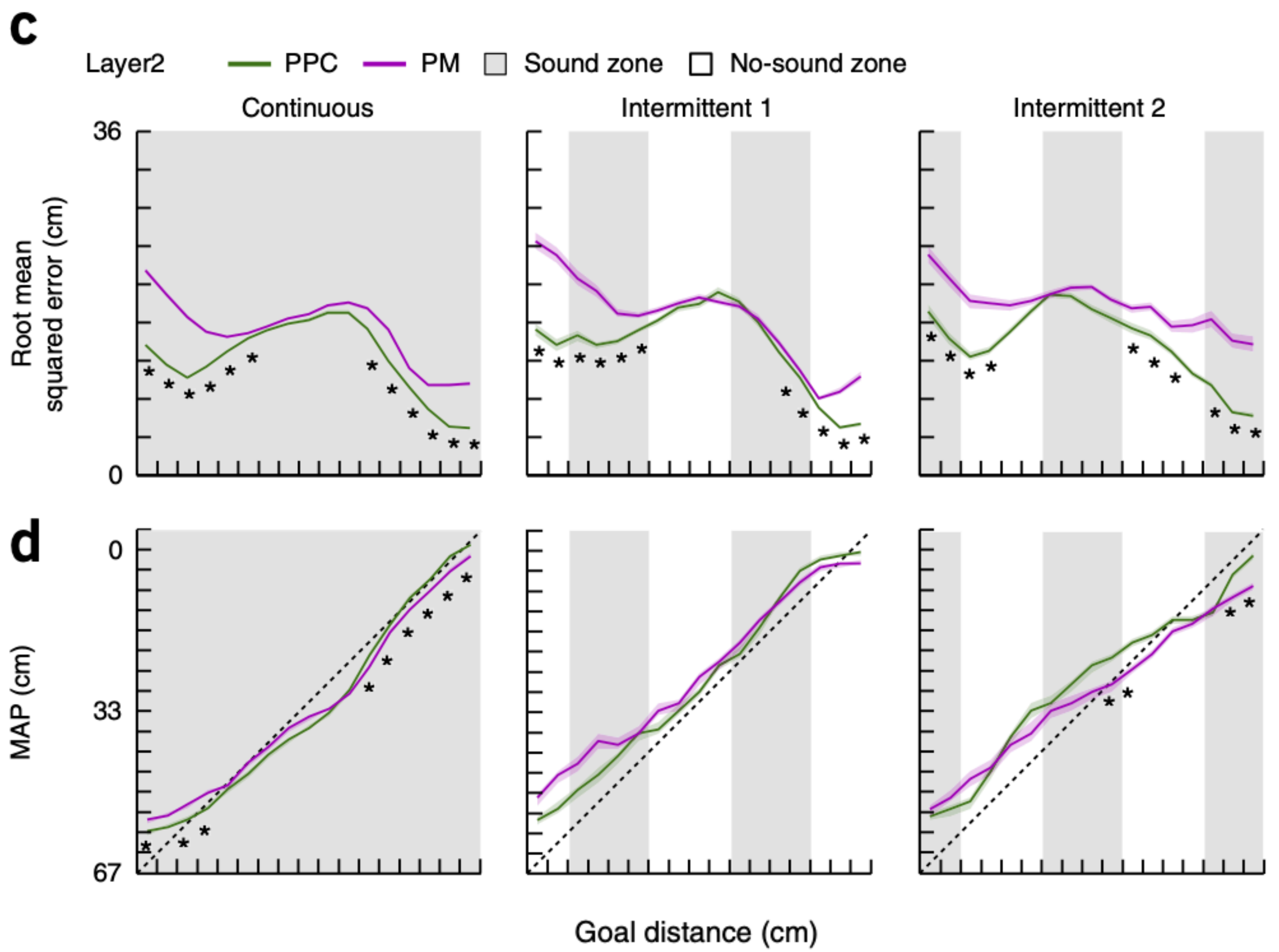


(Funamizu, et al, Nature neuroscience, 2016)

Bayesian inference in PPC and PM

Tuning curves





Likelihood: $P(n_{i,t} \mid x) = \frac{1}{n_{i,t}\sigma_{i,x}\sqrt{2\pi}} \exp\left[-\frac{\{\ln(n_{i,t}) - \mu_{i,x}\}^2}{2\sigma_{i,x}^2}\right]$

Posterior: $P(x \mid n_{all,t}) = \frac{P(n_{all,t} \mid x)P(x)}{\sum_x P(n_{all,t} \mid x)P(x)} \propto P(n_{all,t} \mid x)P(x)$

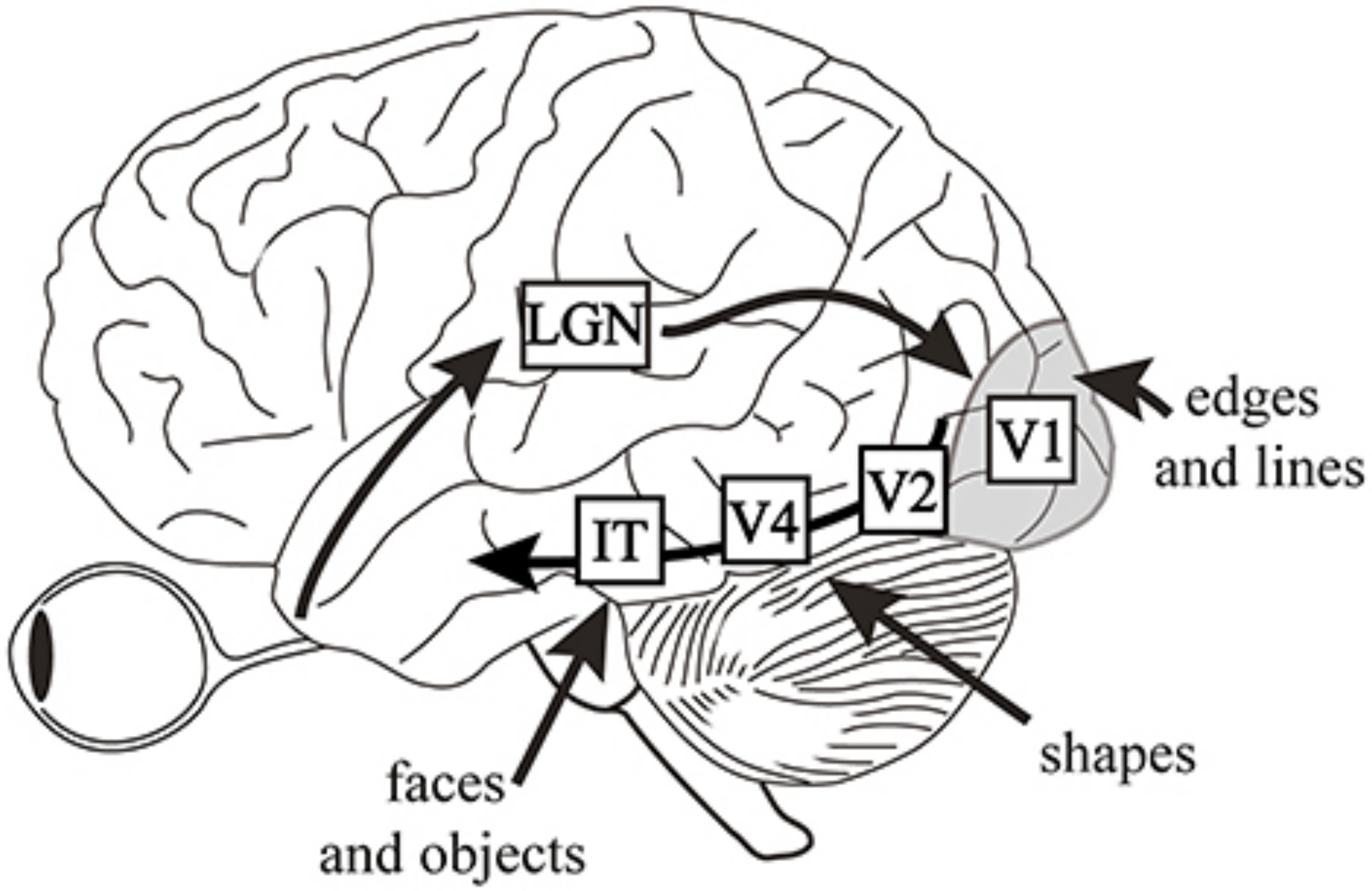
(Funamizu, et al, Nature neuroscience, 2016)

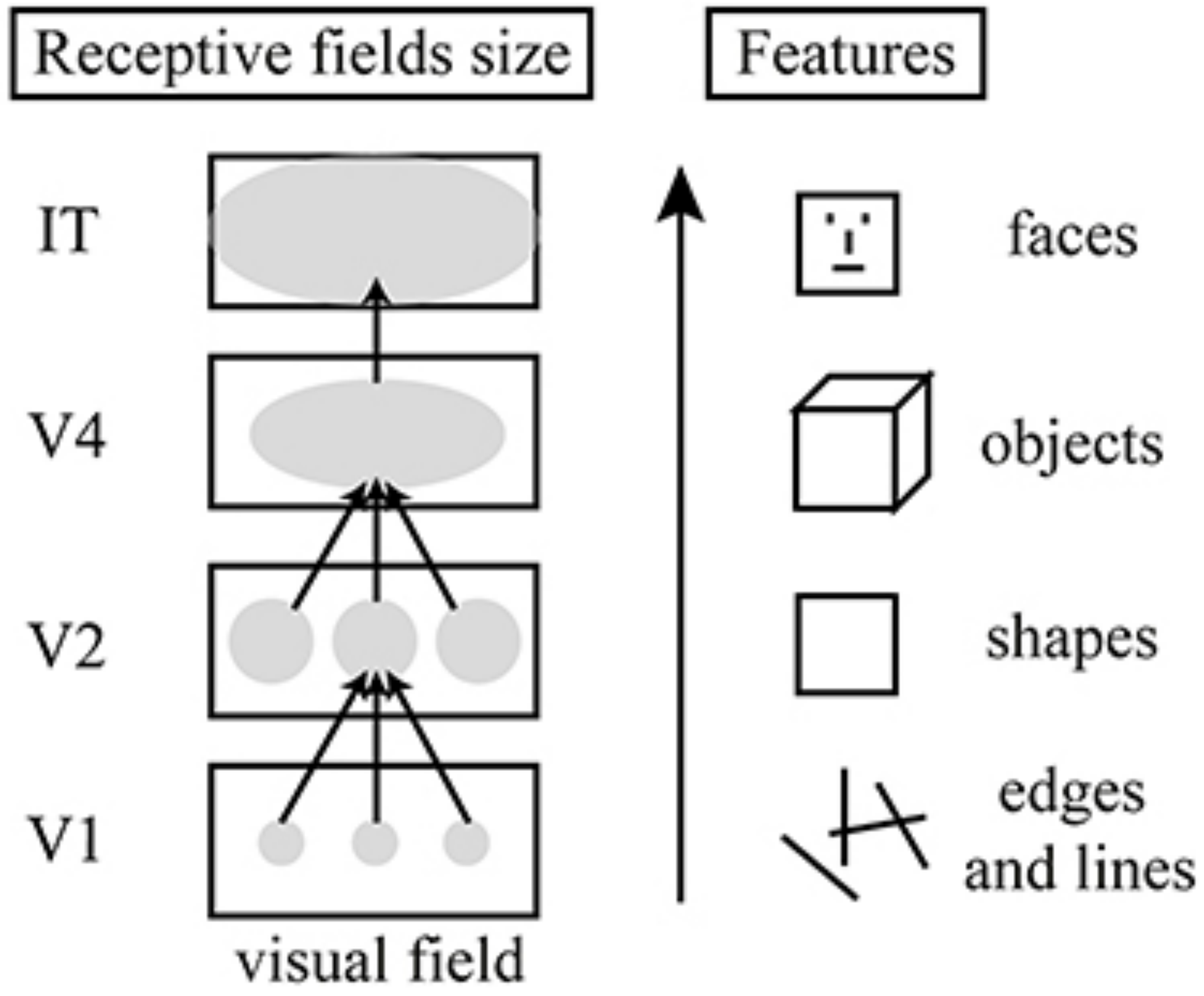
II. Predictive coding

Predictive coding

(Rao et al, Nature Neuroscience, 1999)

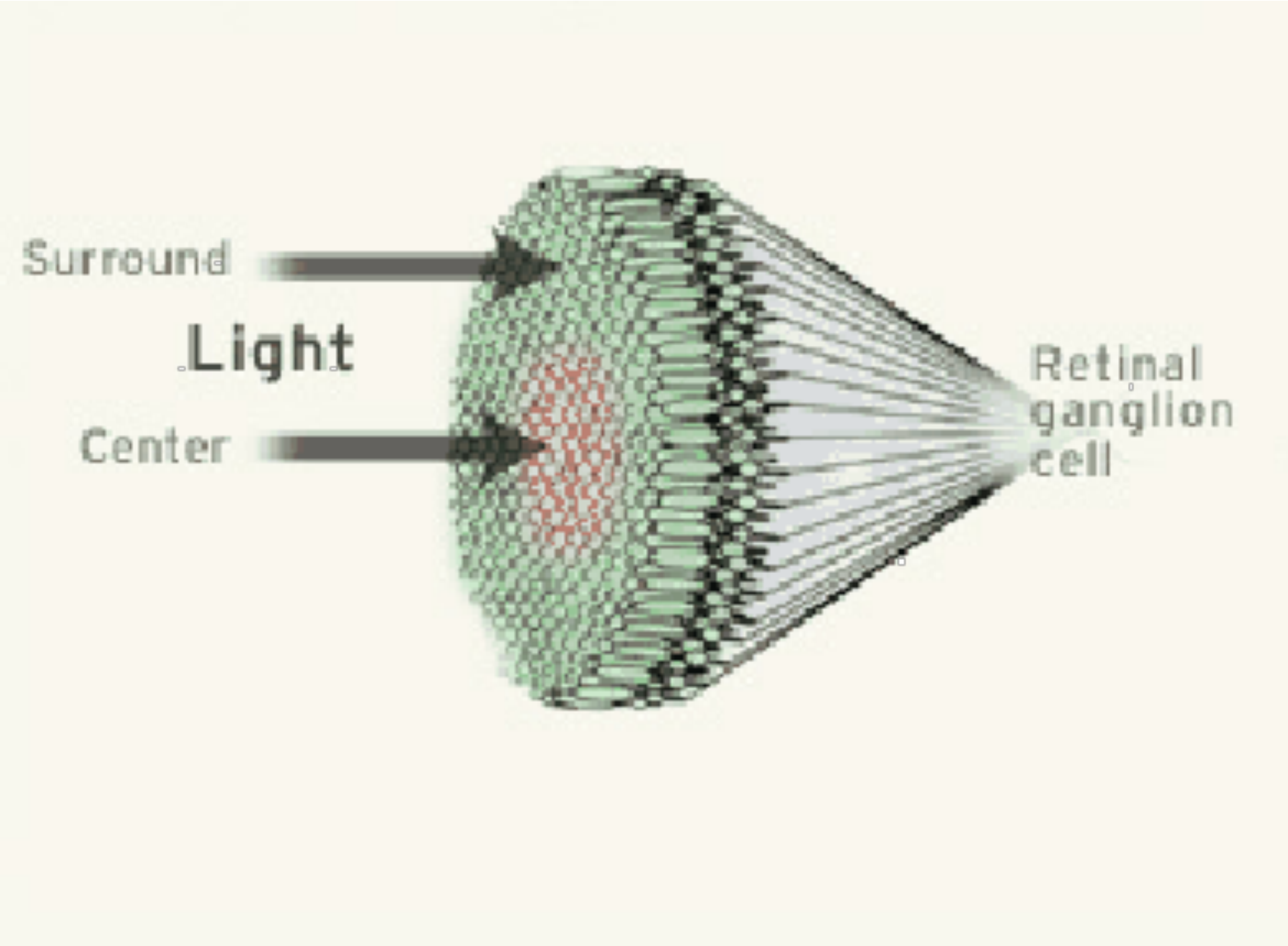
- A theory in visual cortex



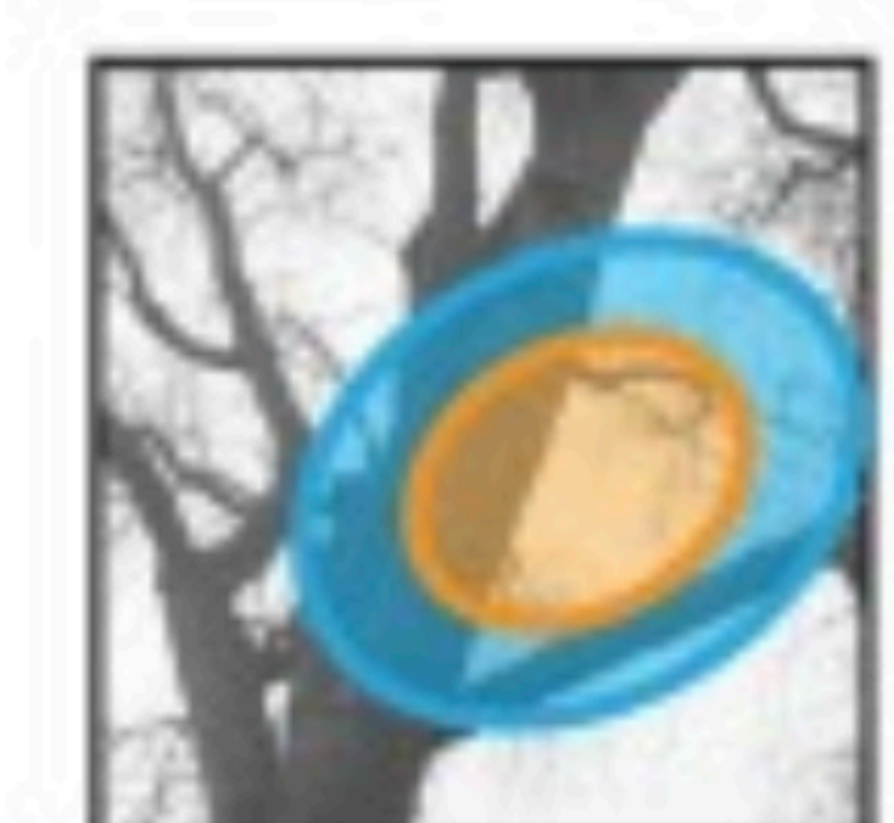


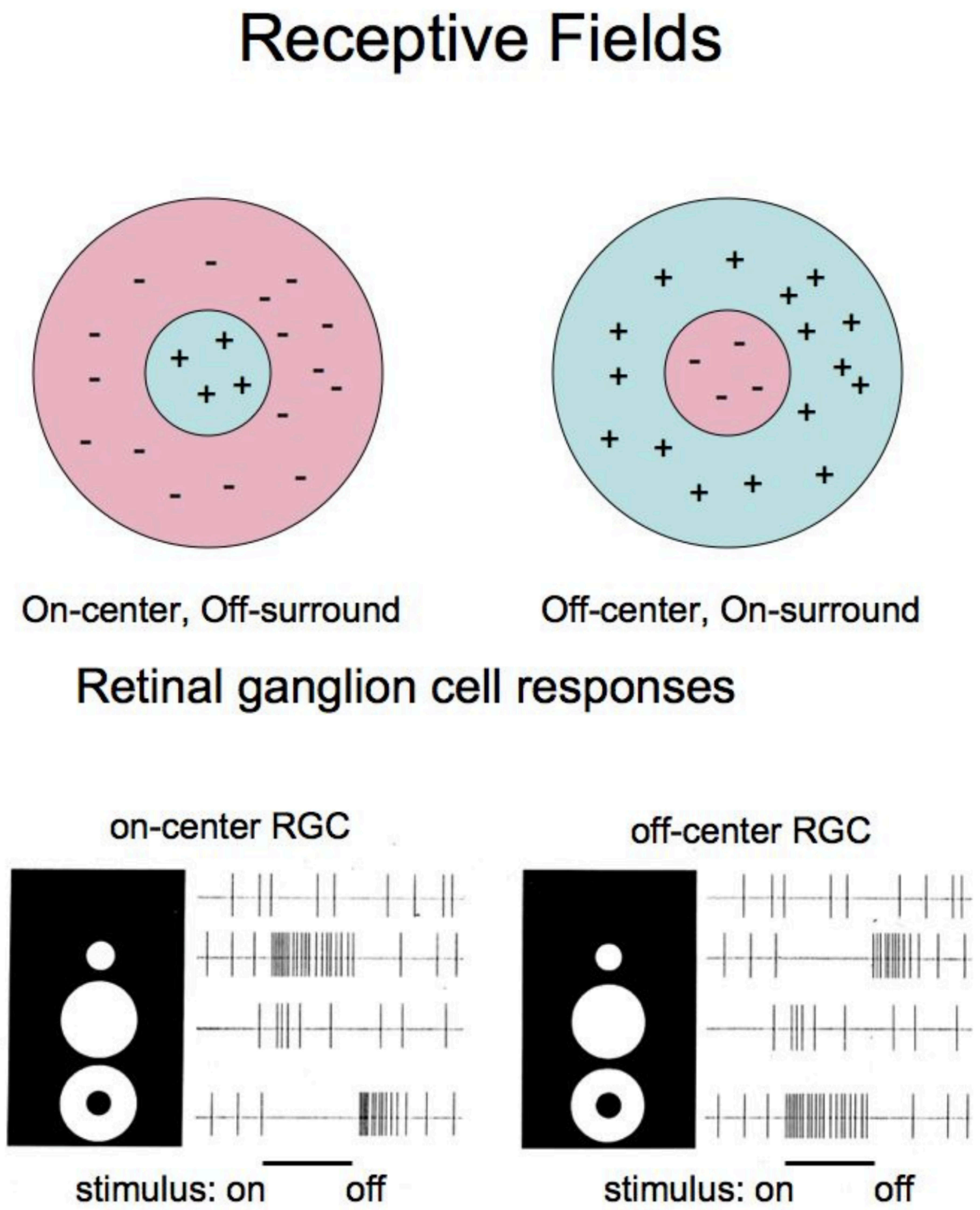
(Herzog, Front. Comput. Neurosci.,2014)

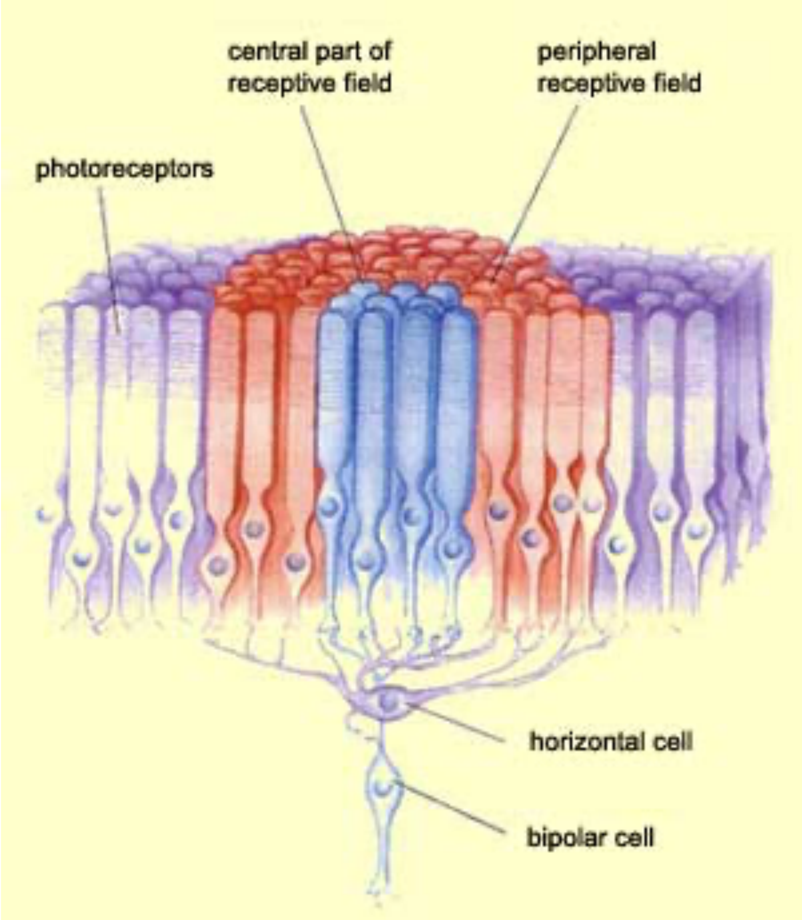
Receptive Field

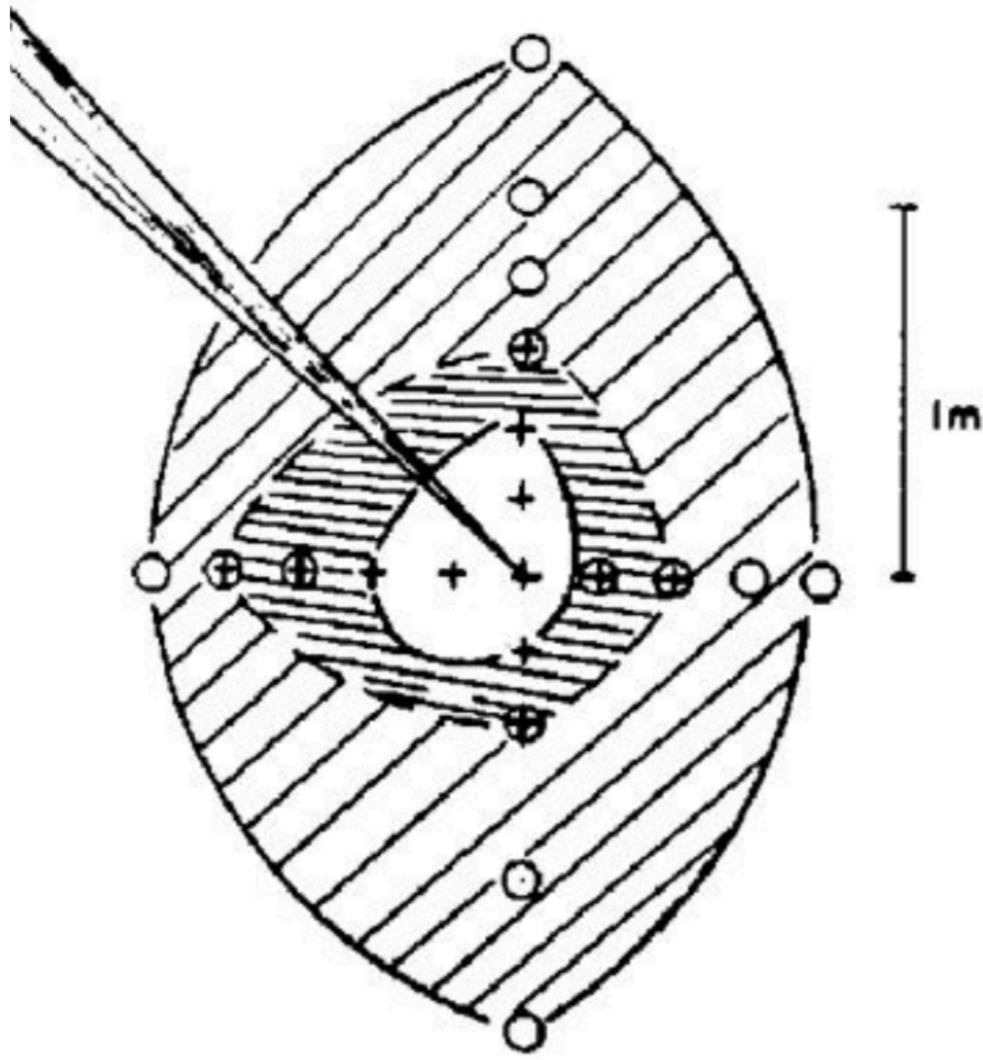


(LaValle, et al., 2020)





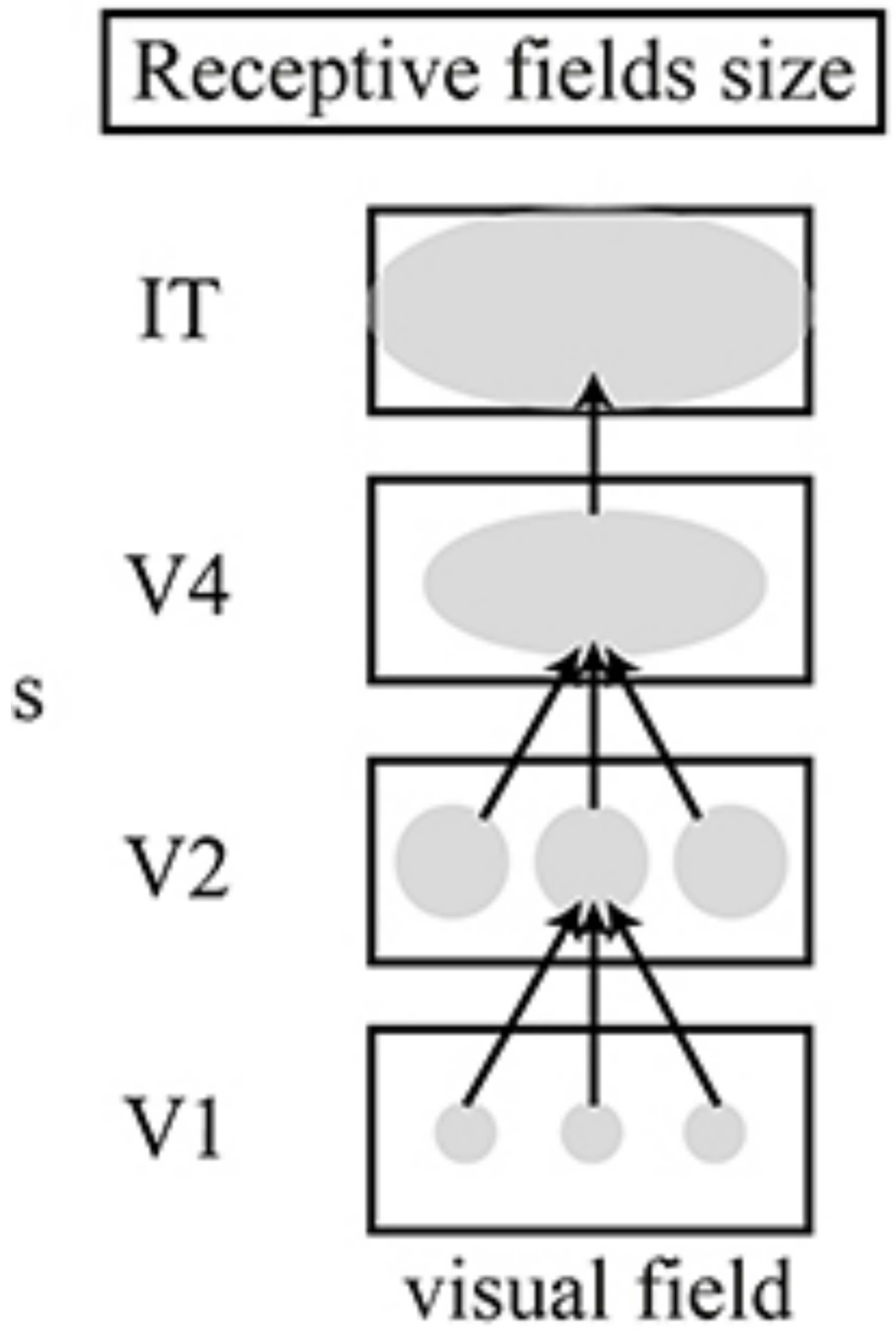




(Heeger, 2016)

Predictive coding

Generative model



Higher

Lower

From brain higher area to lower area

$\boxed{r^h}$ → $\boxed{r}$ → $\boxed{\text{I}}$

neural response from higher area (r^h)

↓

neural response (r) $r = f(U^h r^h) + n^{td}$

↓

Image (I)

$$\mathbf{I} = f(\mathbf{Ur}) + \mathbf{n}$$

$$f(\mathbf{Ur}) = f\left(\sum_{j=1}^{k} \mathbf{U}_j r_j\right)$$

Predictive coding

Prediction Error

Goal: to minimize reconstruction error:

$$\mathbf{I} = f(\mathrm{U}\mathbf{r}) + \mathbf{n}$$

$$E = (I - f(Ur))^T(I - f(Ur))$$

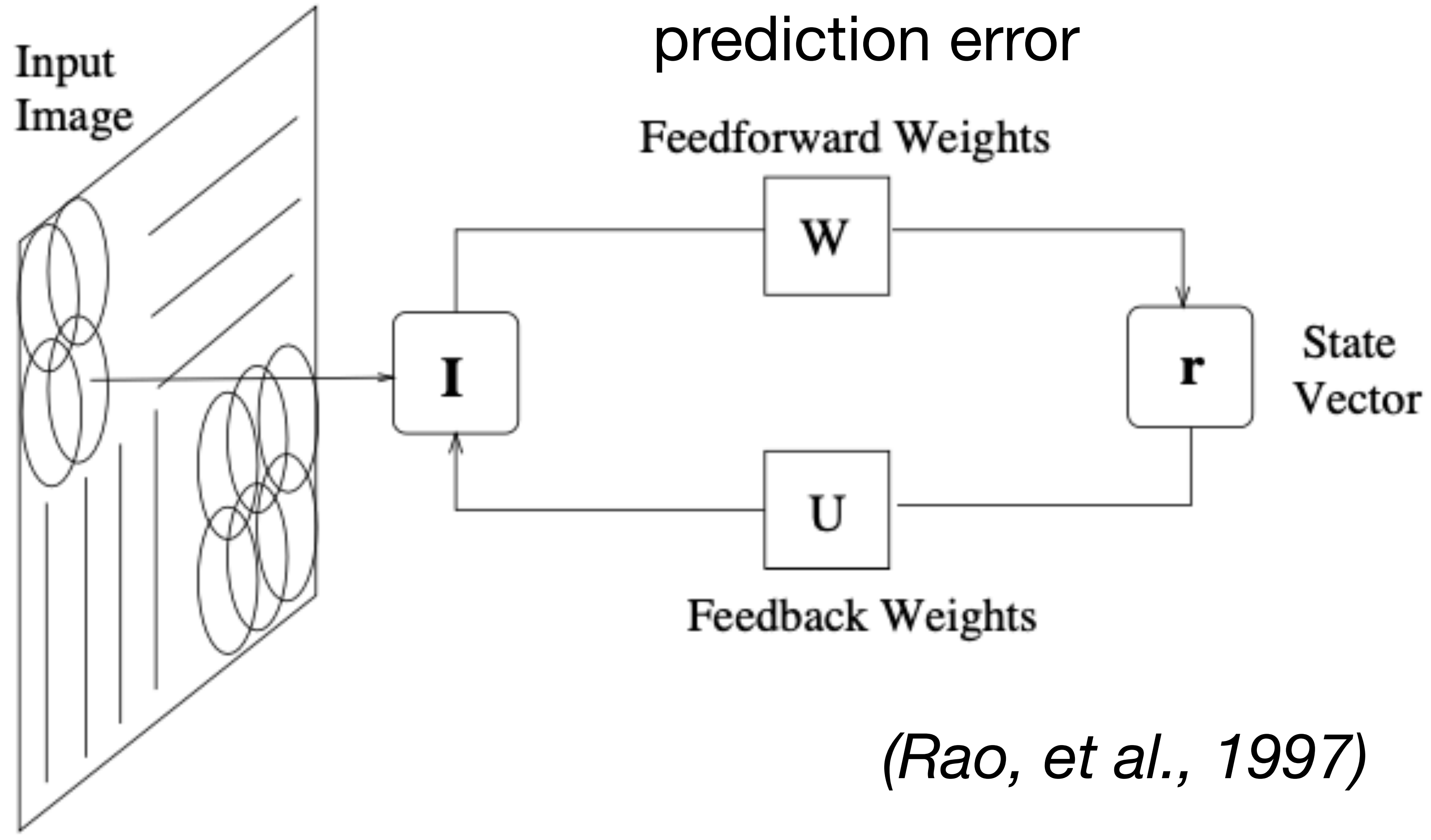
$$\frac{\partial E}{\partial U} = (I - f(Ur))r^T$$

$$\frac{\partial E}{\partial r} = U^T\boxed{(I - f(Ur))}$$

Prediction error

$$U^{t+1} = U^t + k_1(I - f(Ur))r^T$$

$$r^{t+1} = r^t + k_2U^T(I - f(Ur))$$

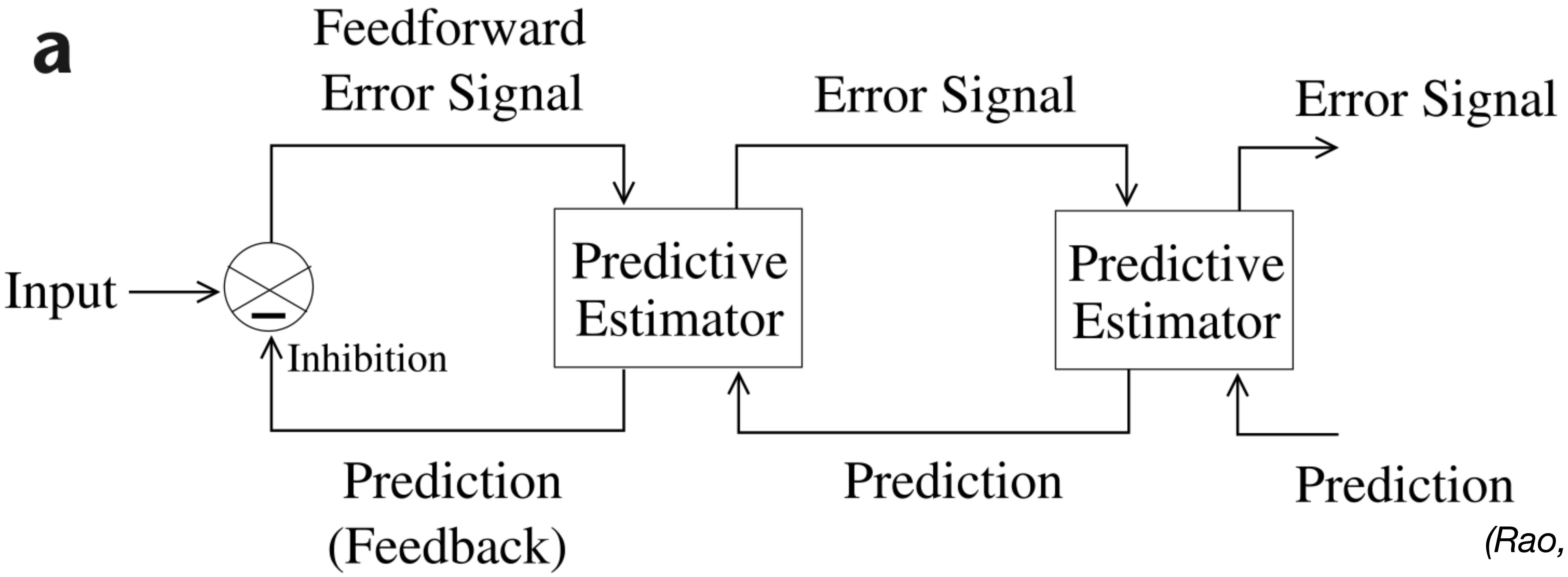


(Rao, et al., 1997)

Predictive coding

Hierarchical structure

Level 1 Level 2 Level 3



(Rao, et al., 1999)

$$E_1 = \frac{1}{\sigma^2}(\mathbf{I} - f(\mathbf{U}\mathbf{r}))^{\mathrm{T}}(\mathbf{I} - f(\mathbf{U}\mathbf{r})) + \frac{1}{\sigma_{td}^2}(\mathbf{r} - \mathbf{r}^{td})^{\mathrm{T}}(\mathbf{r} - \mathbf{r}^{td})$$

-> —log(likelihood)

$$E = E_1 - \log P(r) - \log P(U)$$

—> similar to MAP

Predictive coding v.s. Bayesian model

Minimizing E is equivalent to MAP

$$E_1 = \frac{1}{\sigma^2}(\mathbf{I} - f(\mathrm{U}\mathbf{r}))^{\mathrm{T}}(\mathbf{I} - f(\mathrm{U}\mathbf{r})) + \frac{1}{\sigma_{td}^2}(\mathbf{r} - \mathbf{r}^{td})^{\mathrm{T}}(\mathbf{r} - \mathbf{r}^{td})$$

-> —log(likelihood)

$$E = E_1 - \log P(r) - \log P(U)$$

—> -log(likelihood)-log (prior)

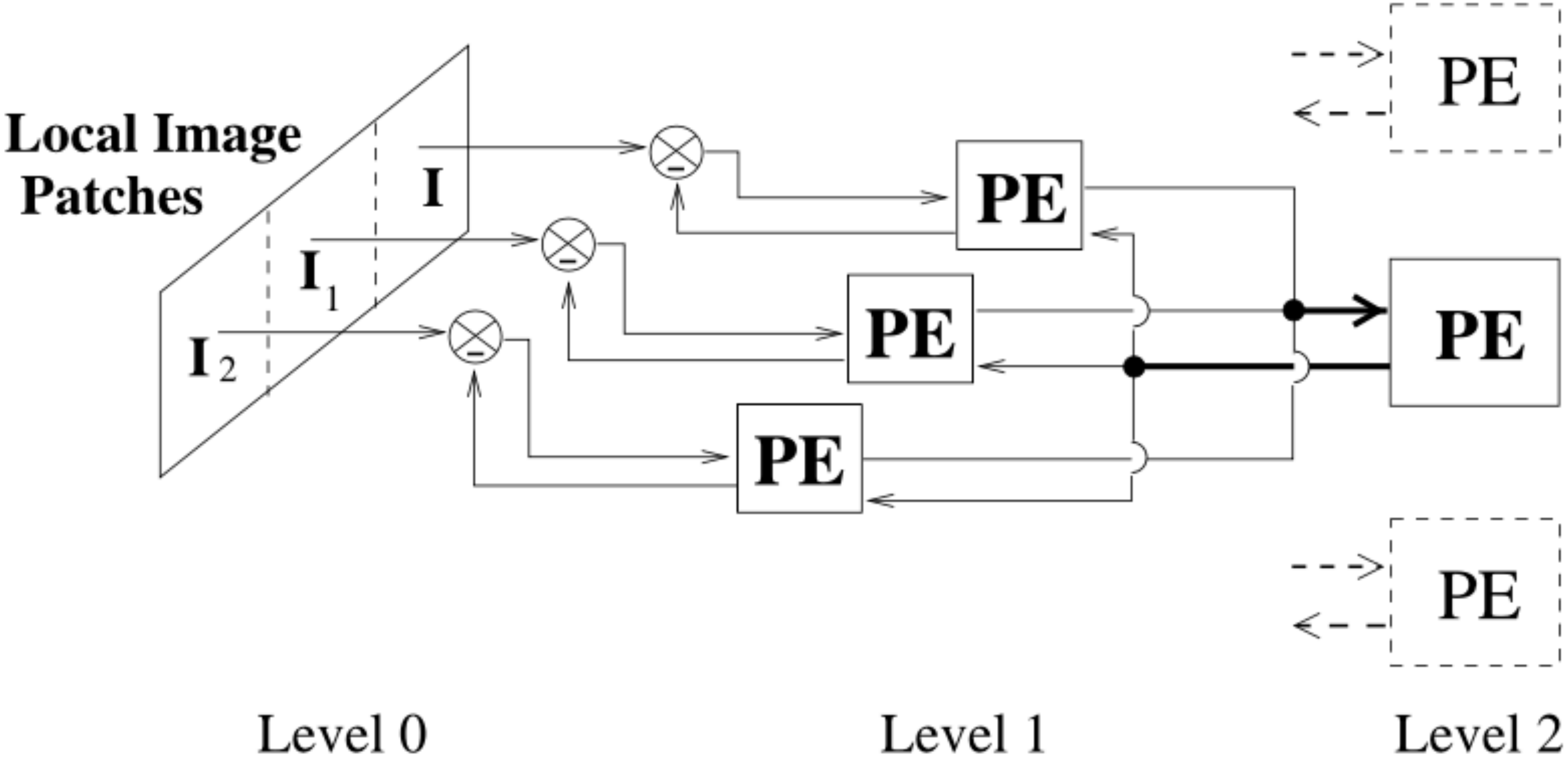
Minimize reconstruction error **—> maximize posterior (MAP)**

Posterior:

$$\begin{aligned}\log P(\mathbf{r} \mid \mathbf{I}) &= \log P(\mathbf{I} \mid \mathbf{r}) + \log P(\mathbf{r}) - \log P(\mathbf{I}) \\ &= -(\mathbf{I} - U\mathbf{r})^T \Sigma^{-1} (\mathbf{I} - U\mathbf{r}) - (\mathbf{r} - \bar{\mathbf{r}})^T M^{-1} (\mathbf{r} - \bar{\mathbf{r}}) + k \\ &= -E + k\end{aligned}$$

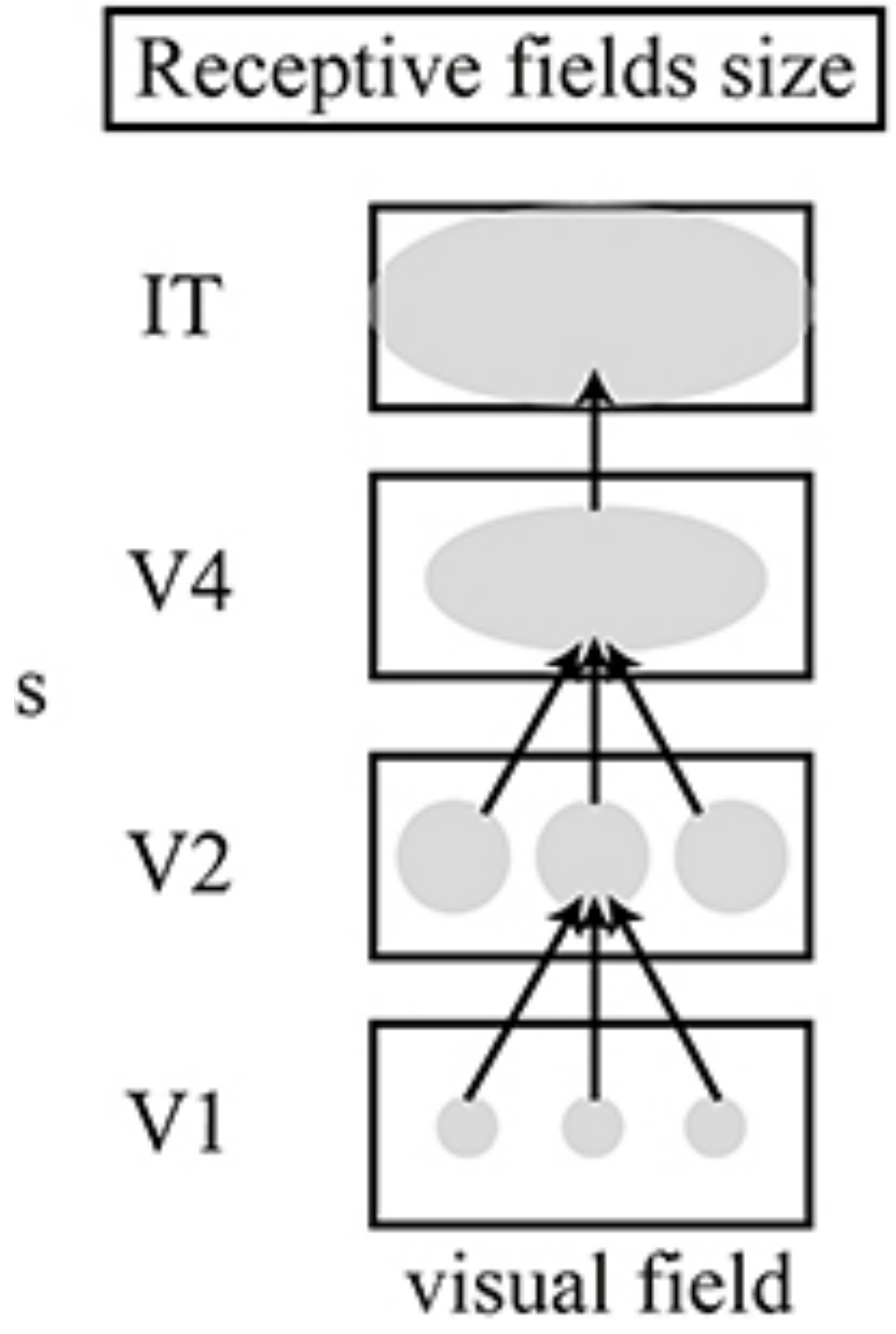
Example I: predictive coding in explaining receptive field (simulation)

Predictive coding neural network



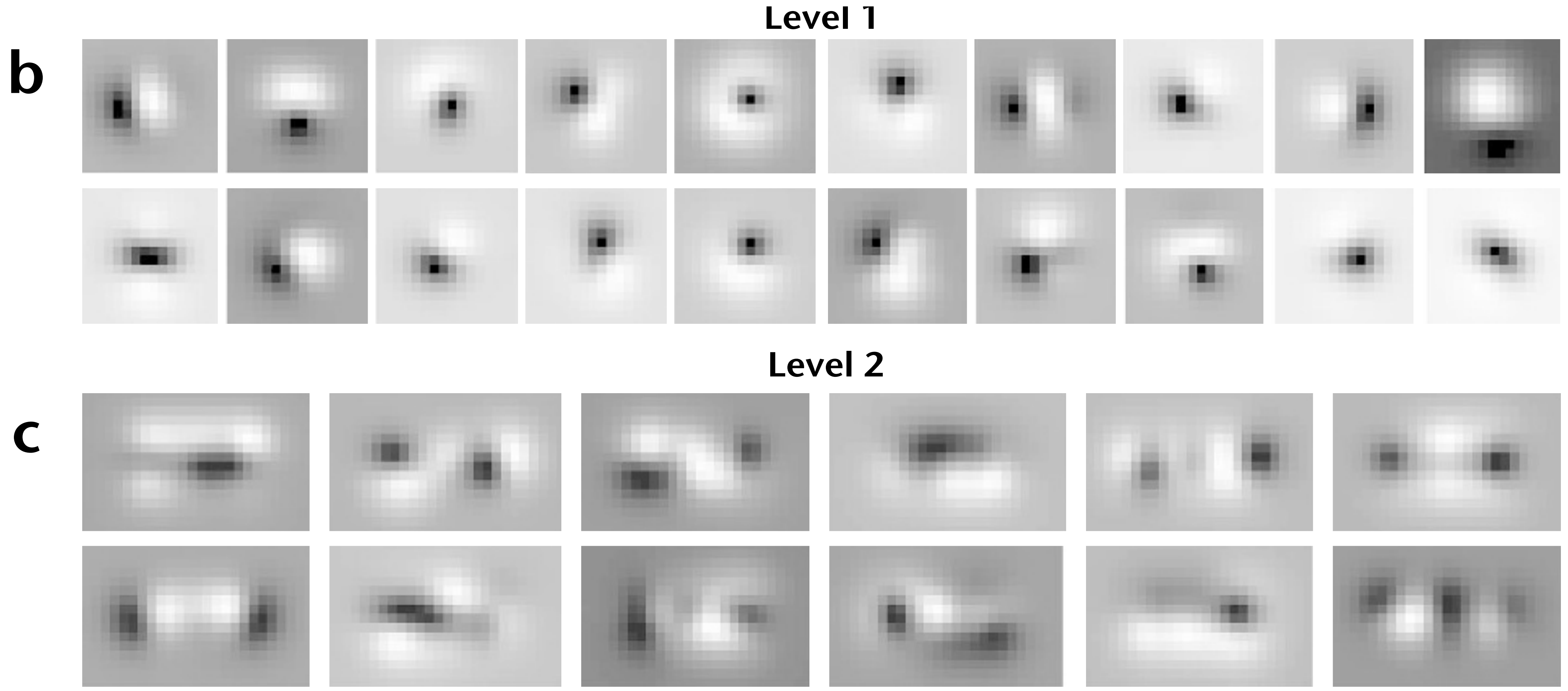
(Rao, et al., 1999)

Interpret Receptive field



Input

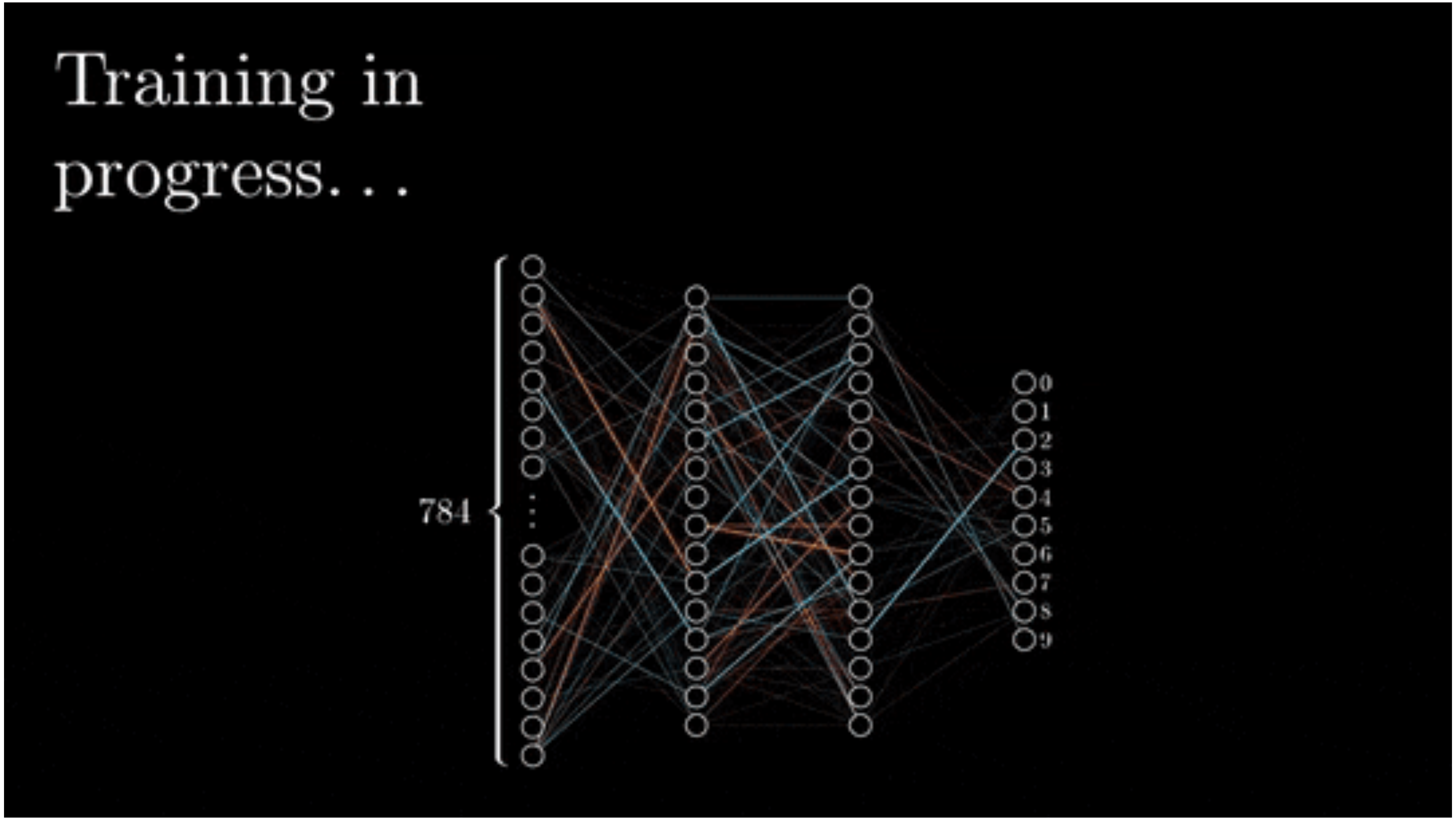
Synaptic weight after learning (Receptive field)



(Rao, et al., 1999)

Compare with Backpropagation

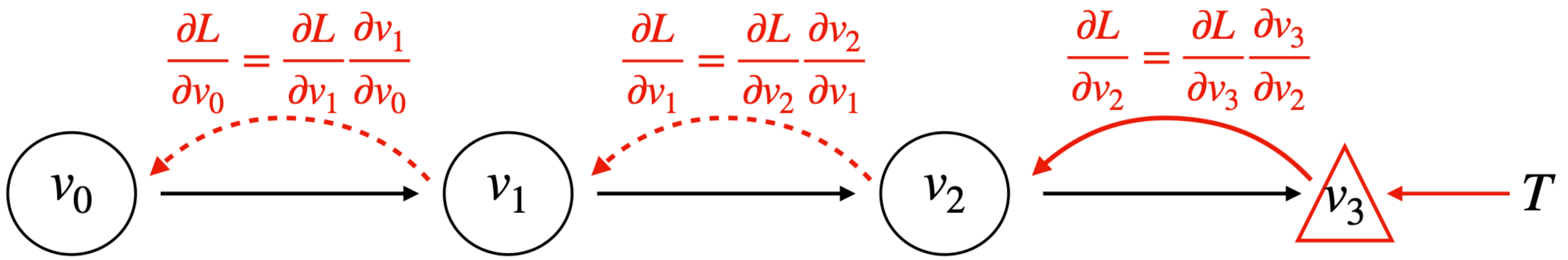
Backpropagation in training ANN



(AI mind, 2023)

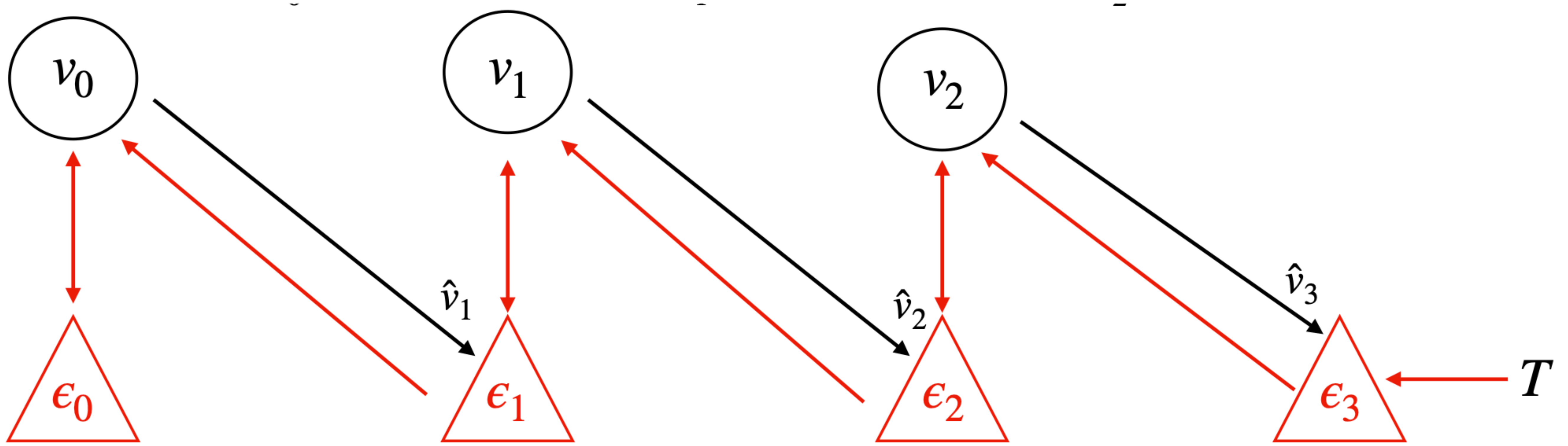
Predictive coding v.s. backpropagation

Backpropagation



Loss function from final output

Predictive coding



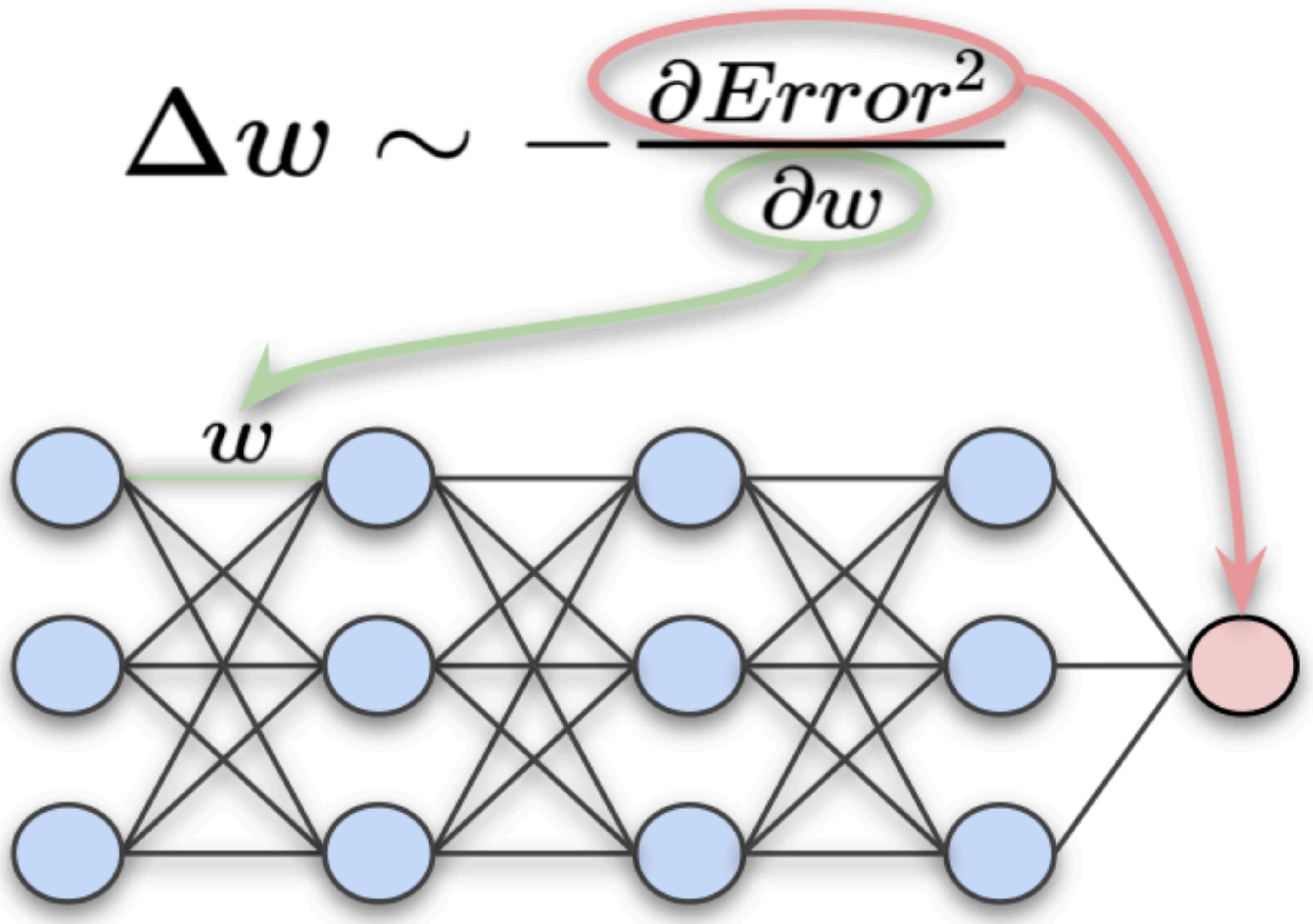
Loss function from each level/layer

(Millidge, et al., 2020)

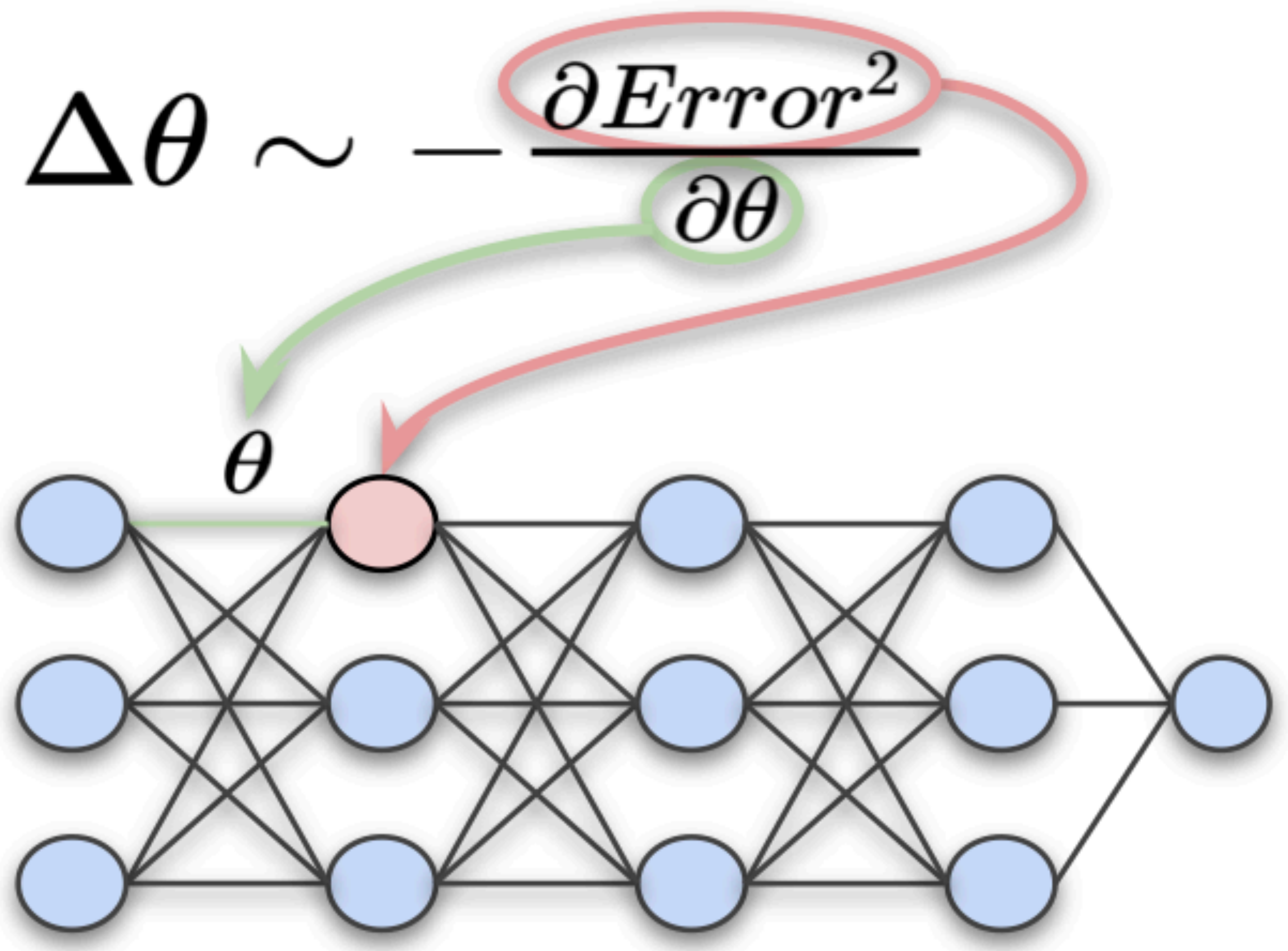
Predictive coding v.s. backpropagation

Alternative diagram

Backpropagation

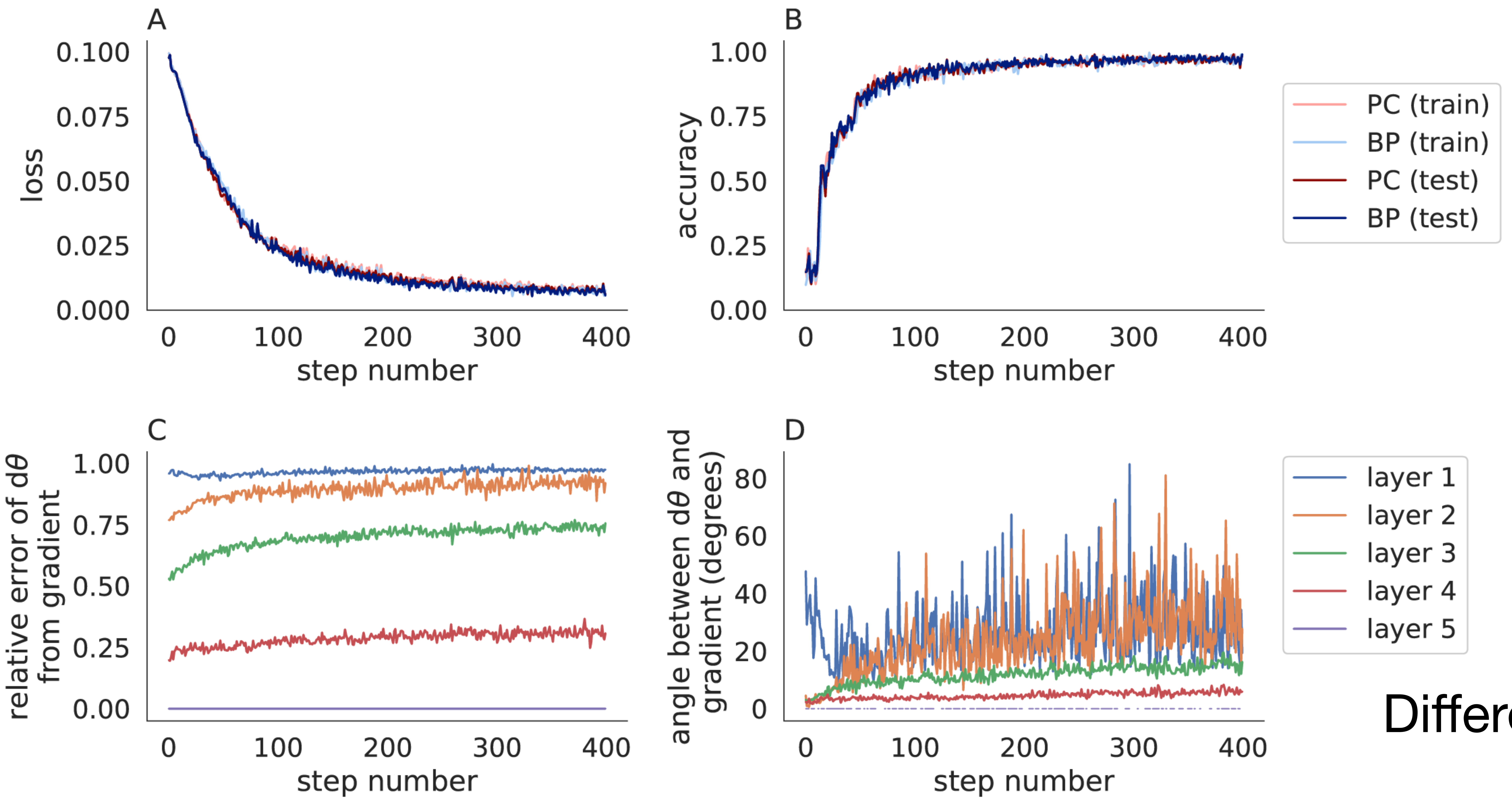


Predictive coding



(Millidge, et al., 2022)

Predictive coding v.s. backpropagation

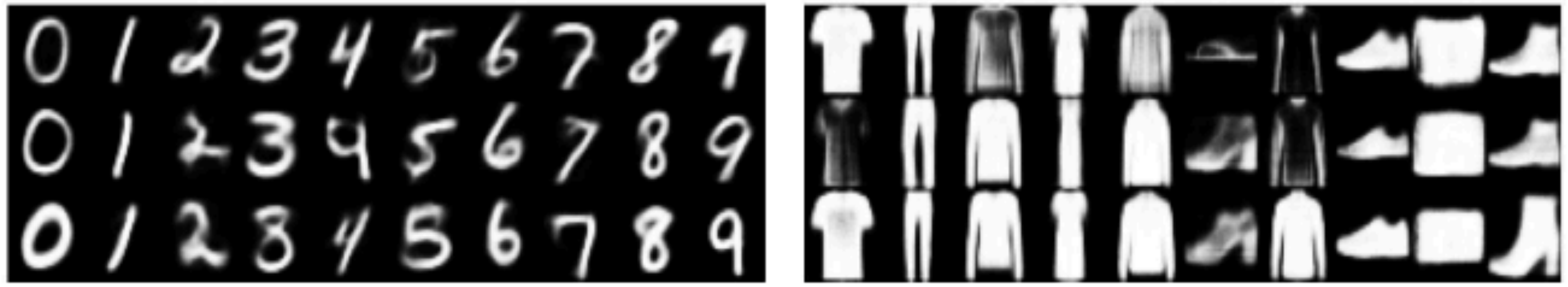


Similar accuracy

Different parameter update

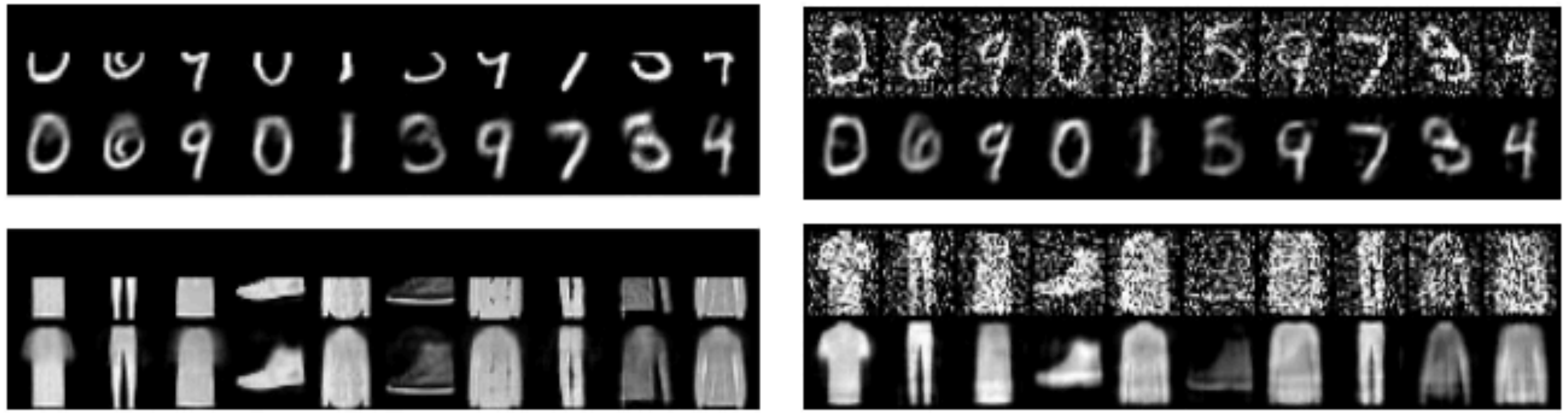
(Rosenbaum, Plos One, 2022)

Predictive coding works as good as DNN



(c) Generation

(Ororbia & Kifer, 2020)

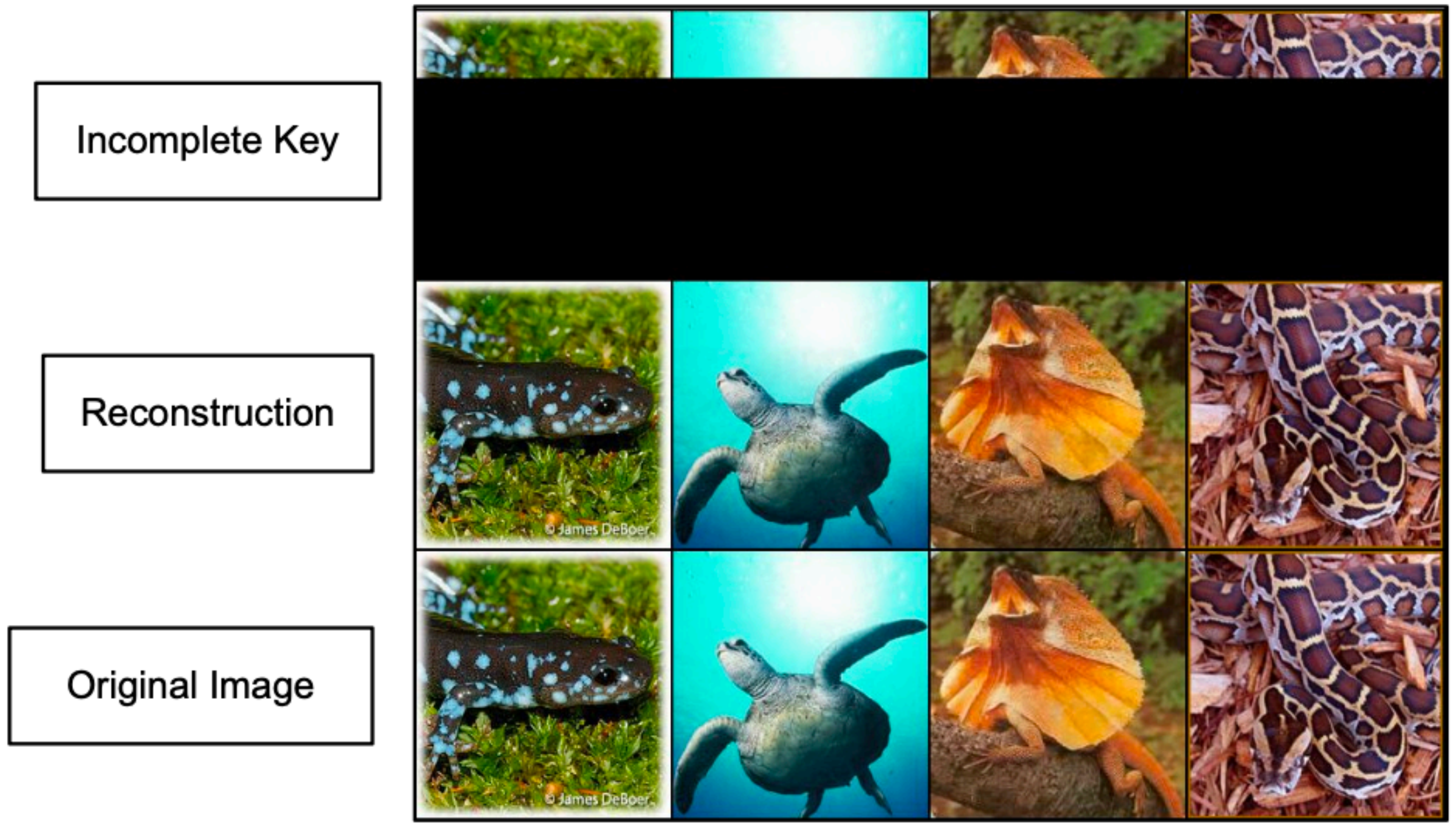


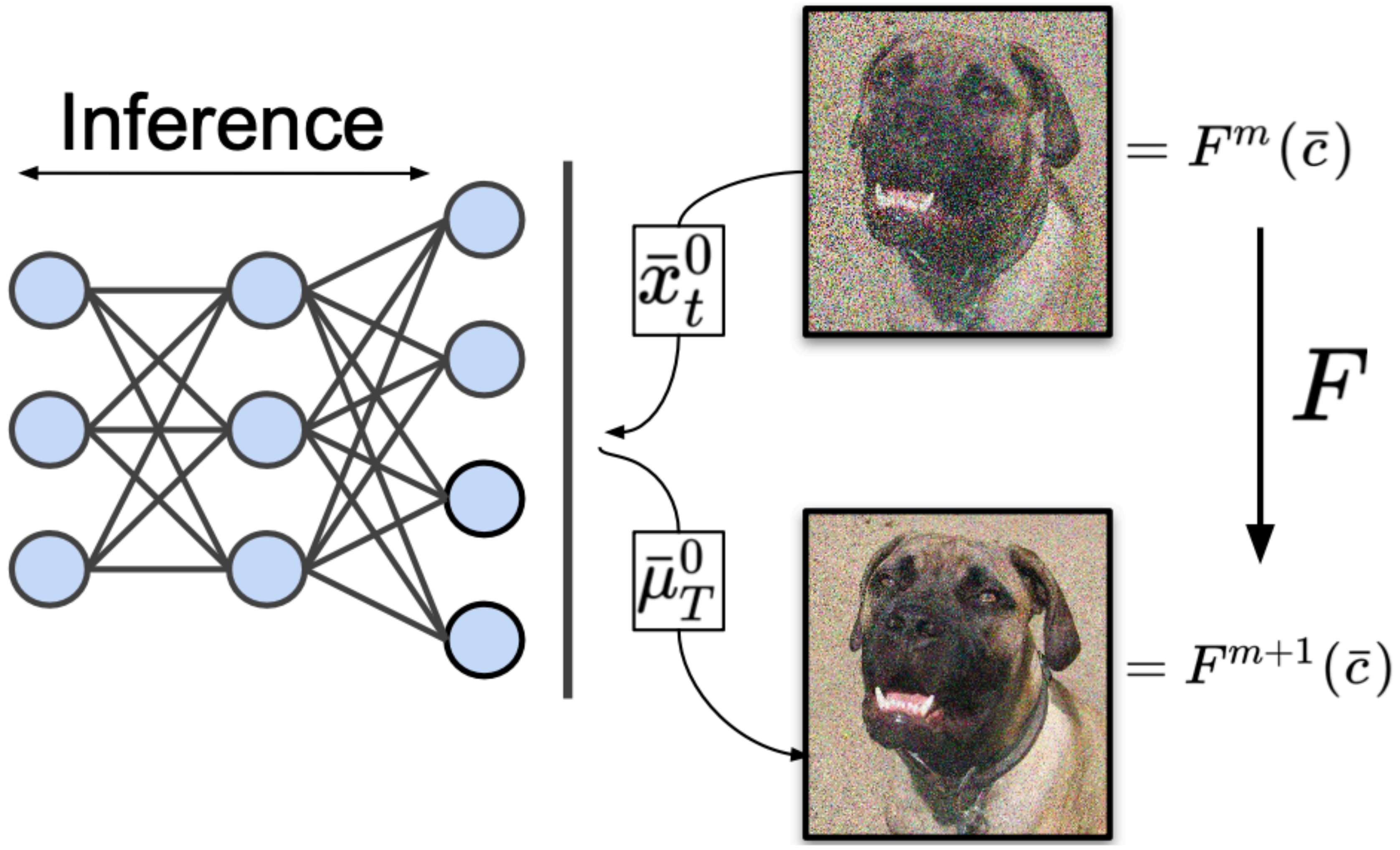
(d) Reconstruction and Denoising

(Salvatori et al., 2022)

Predictive coding outcome DNN with associative memory

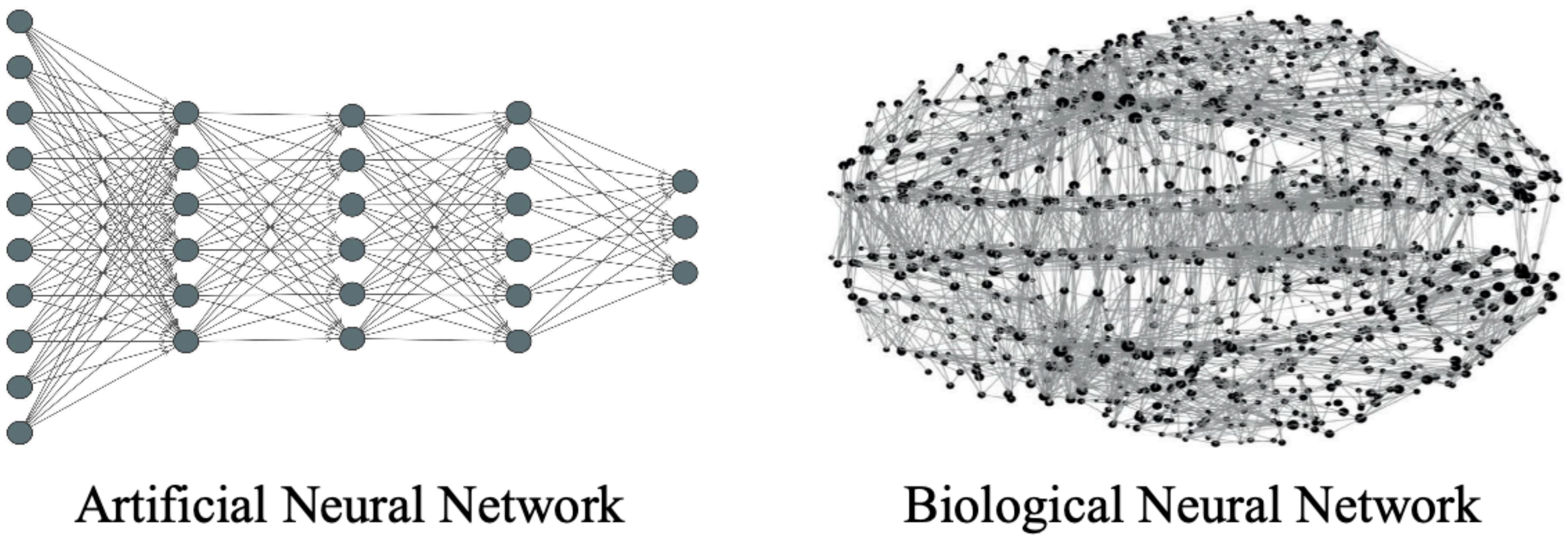
Corrupted Key





(Millidge, et al., 2022)

Predictive coding: Biological realistic learning algorithm



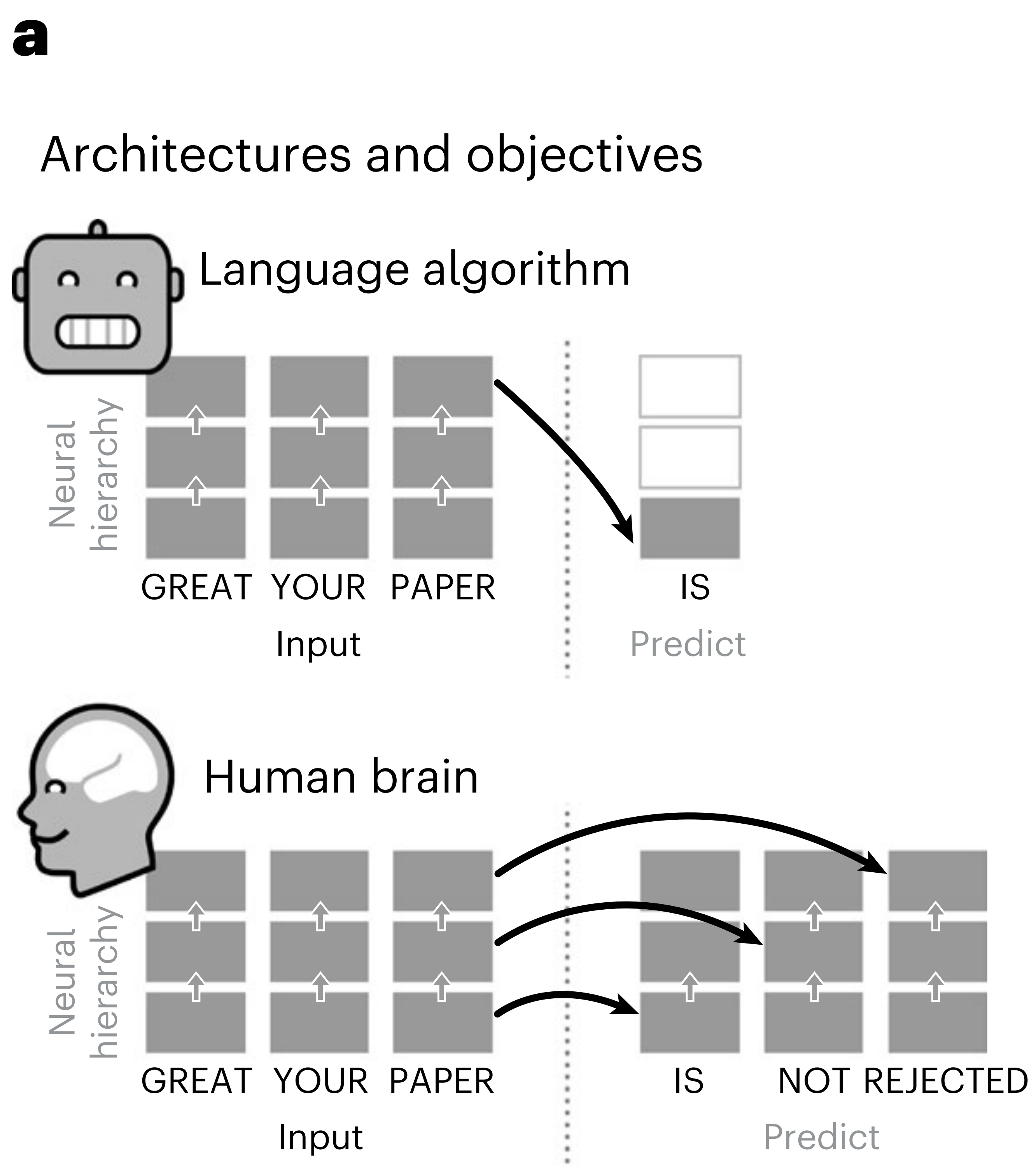
(Salvatori et al., 2022)

- Biological realistic
- Computational plausible

Example II: Predictive coding in fMRI

Predictive coding in fMRI

Train A Deep language model for long-range prediction



- Train a GPT-2 based deep language models with long range predications
  - predict word$_{t+8}$

fMRI data with 307 individuals listen to short stories


nature human behaviour

Article

https://doi.org/10.1038/s41562-022-01516-2

Evidence of a predictive coding hierarchy in the human brain listening to speech

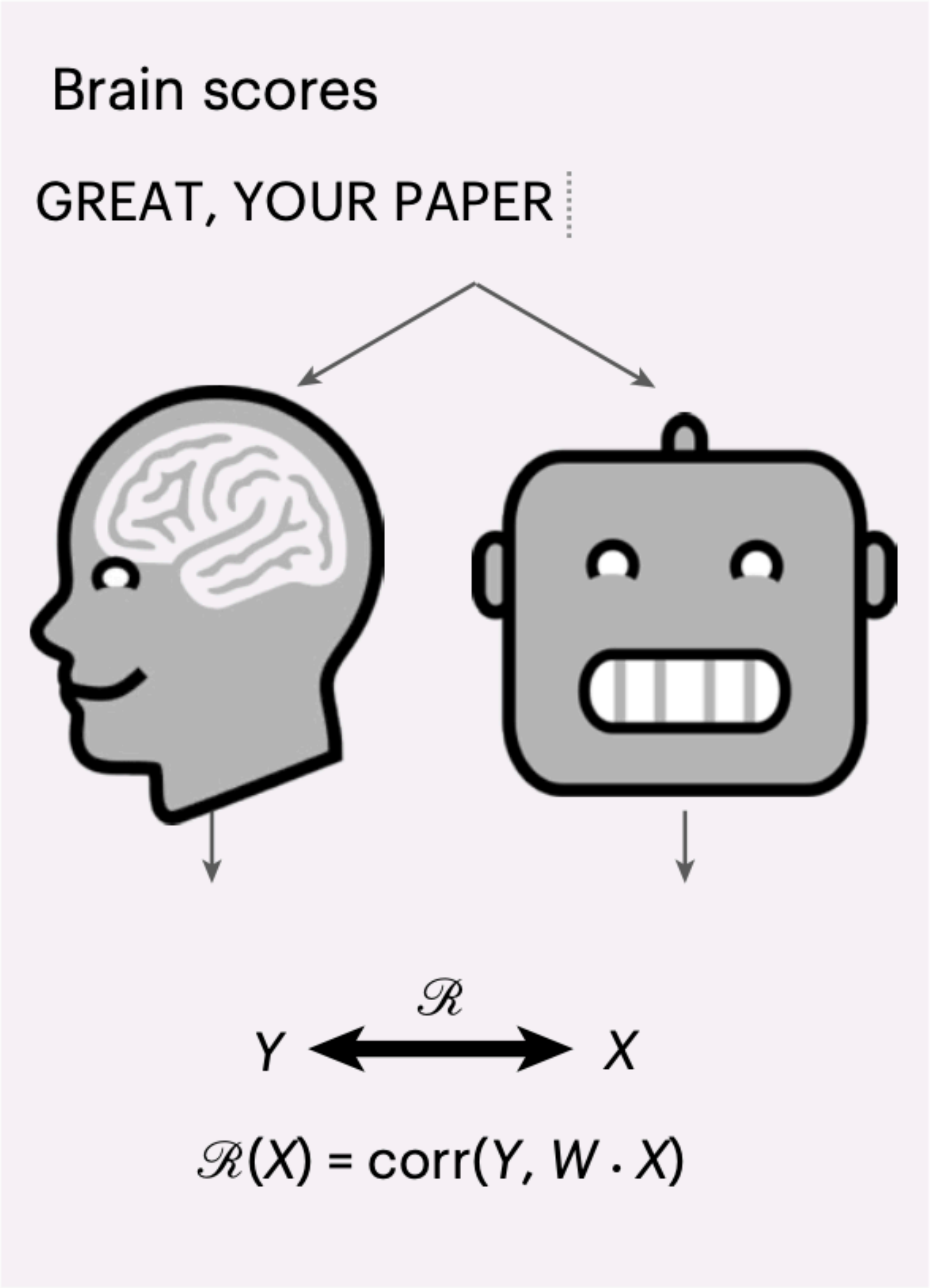
Received: 31 March 2022

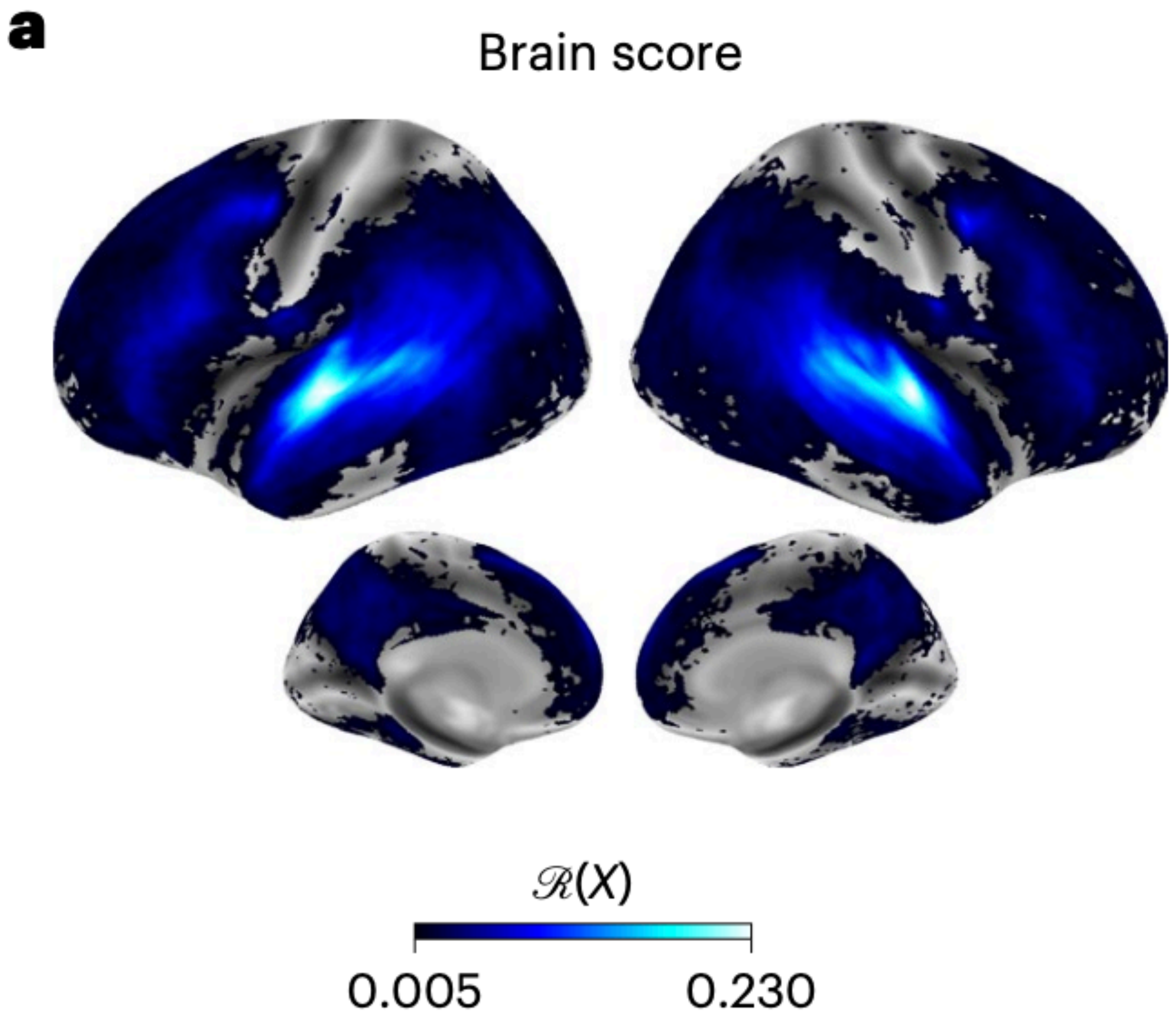
Charlotte Caucheteux[1,2], Alexandre Gramfort[1,2] & Jean-Rémi King[1,3]

(Caucheteux et al.,Nature Human Behaviour, 2022)

Predictive coding in fMRI

Brain score using regression from k-th layer of DNLP





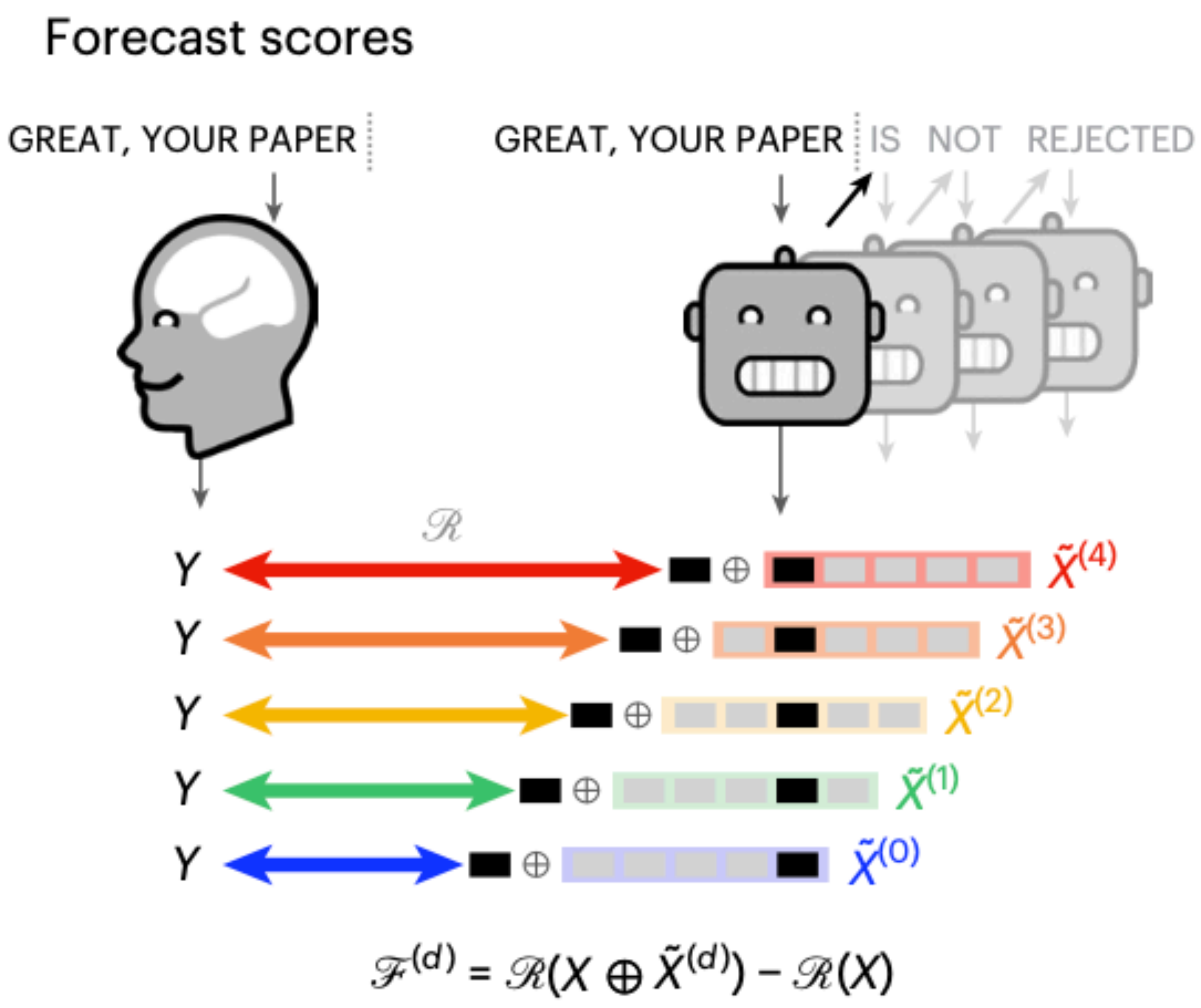
Use linear regression to fit fMRI voxel signals from the activation of GPT-2 model

Activation: output of k-layer of DNN model)

Predictive coding in fMRI

Find forecast distance

c



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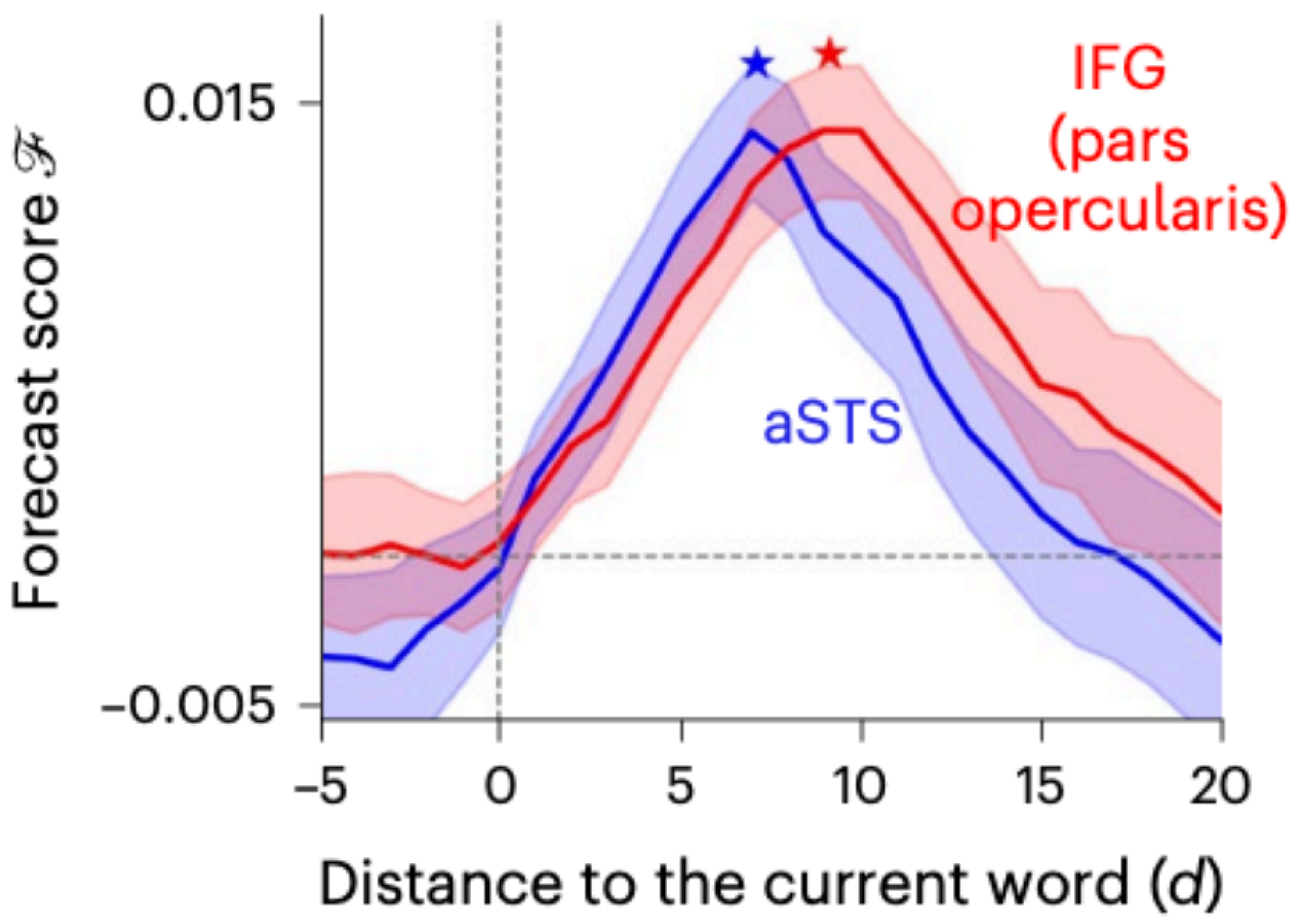

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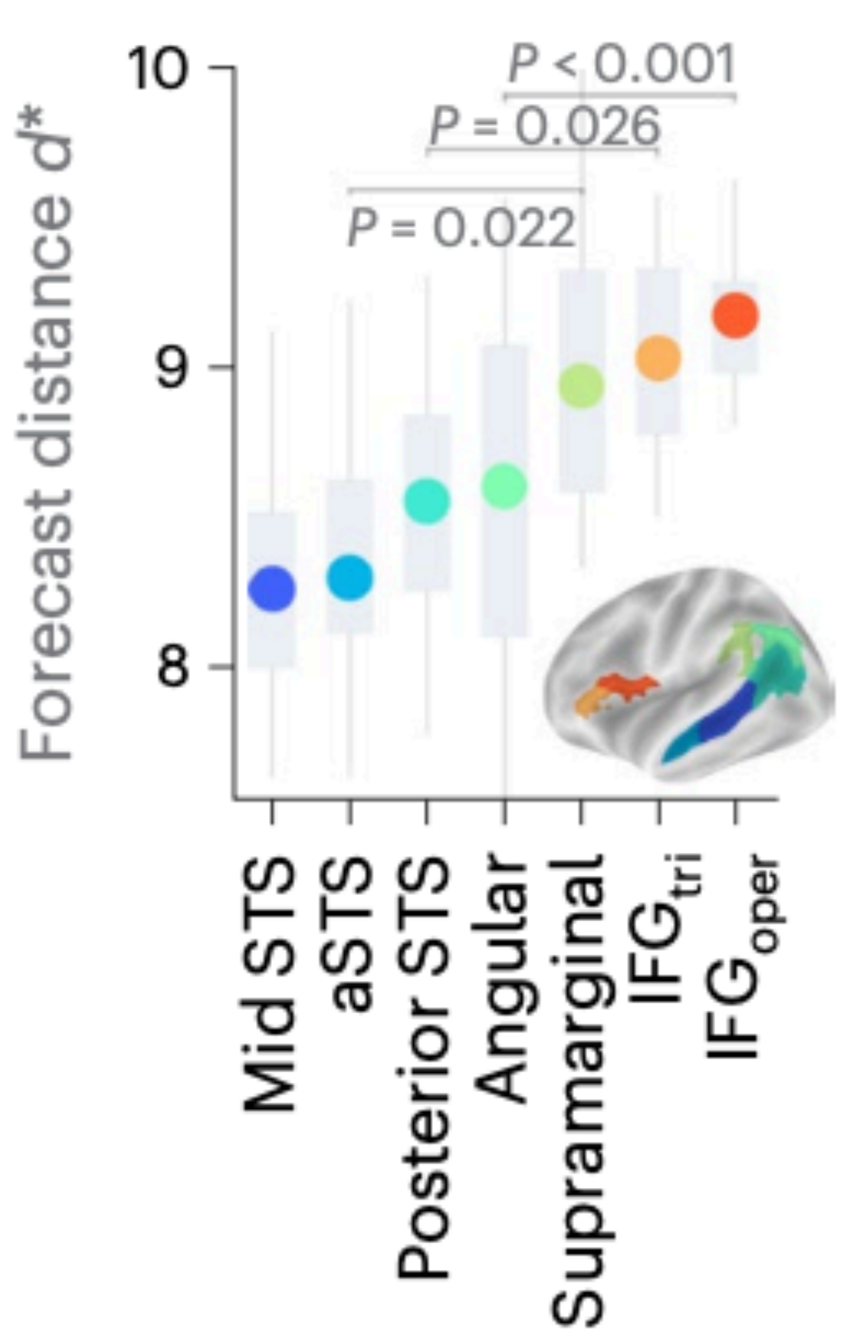
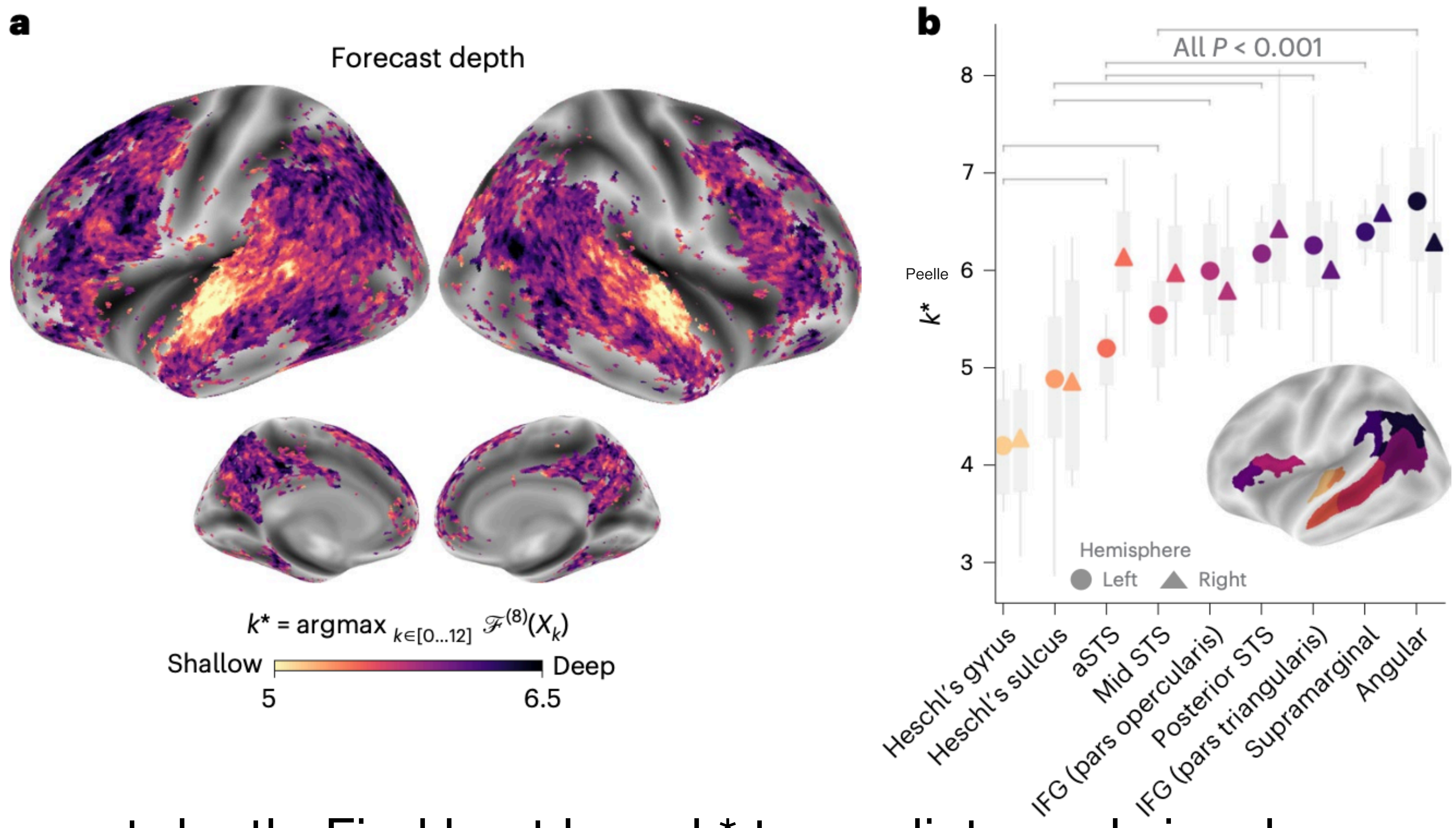
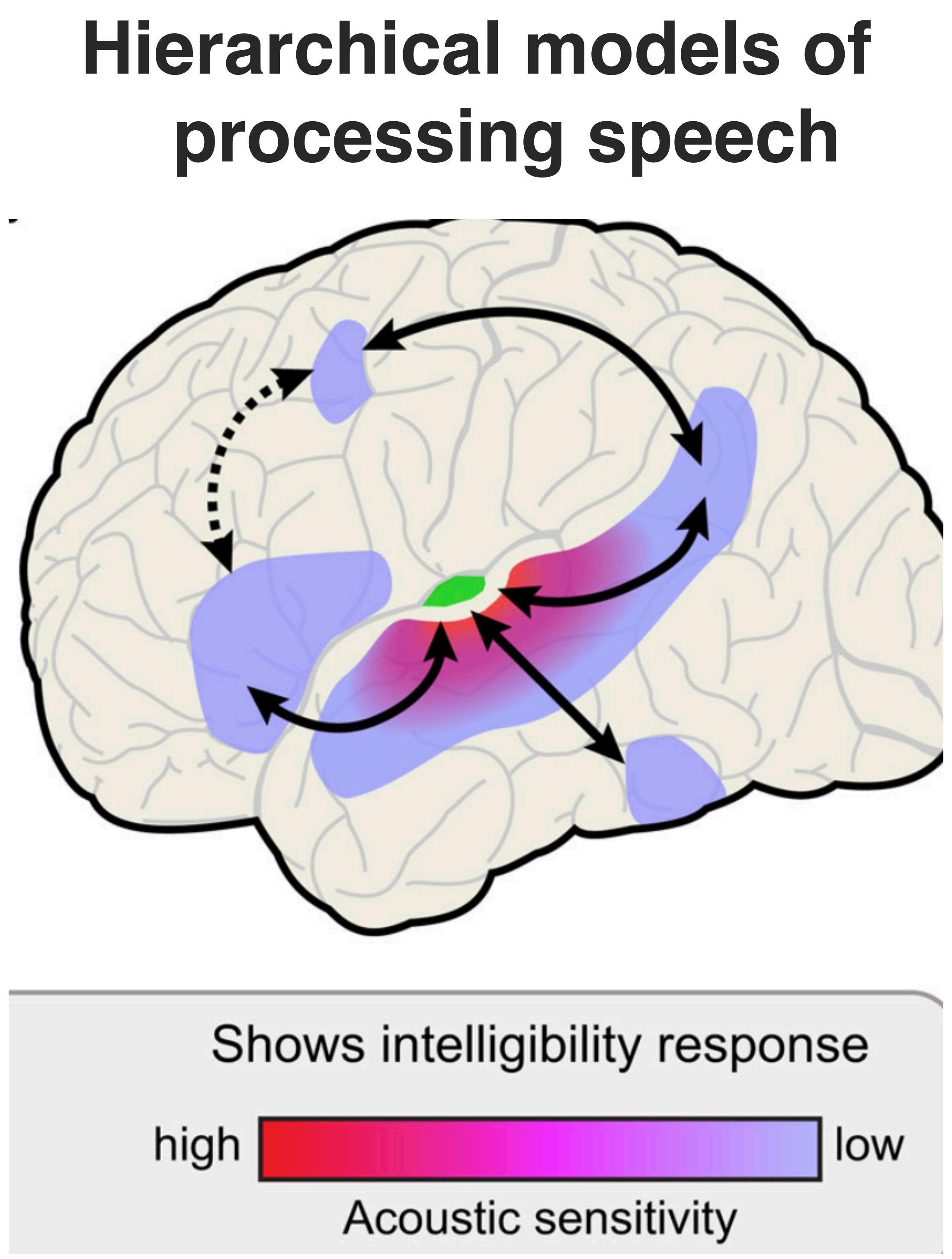


Predictive coding in fMRI

Forecast depth reflect hierarchy in brain





(Peele et al.,Frontier in Human Neuroscience, 2010)

Forecast depth: Find best layer k* to predict voxel signal

Summary

- **Bayesian inference in Population neurons**
  - Bayesian inference estimates orientation from fMRI data
  - Bayesian inference estimates distance from PPC (calcium imaging)
- **Predictive coding**
  - Predictive coding as a biological plausible learning algorithm for neural network
  - Predictive coding has evidence in brain (fMRI)

Thank you!