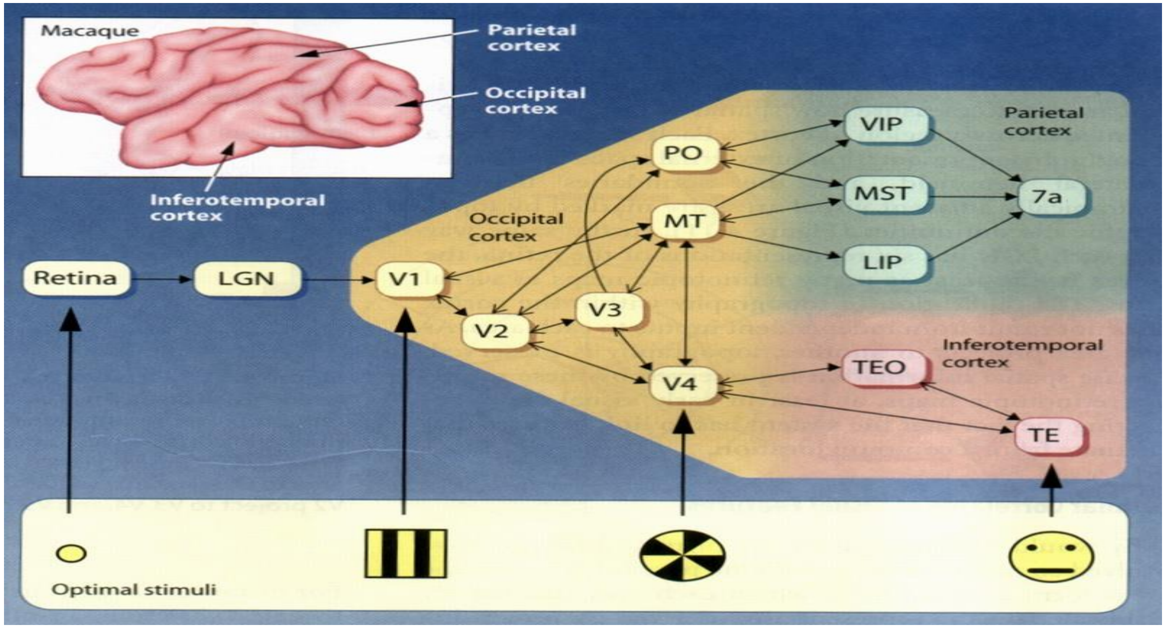
Neural Coding: Basis2

吴思
心理与认知科学学院
IDG/McGovern 脑科学研究所
北大-清华联合生命科学中心
北京大学

Efficient coding hypothesis

- Inspired by information theory, Barlow proposed that the sensory pathway is as a communication channel where neuronal spiking is an efficient code for representing sensory signals.
- The statistics of natural images shapes the coding scheme of sensory neurons.

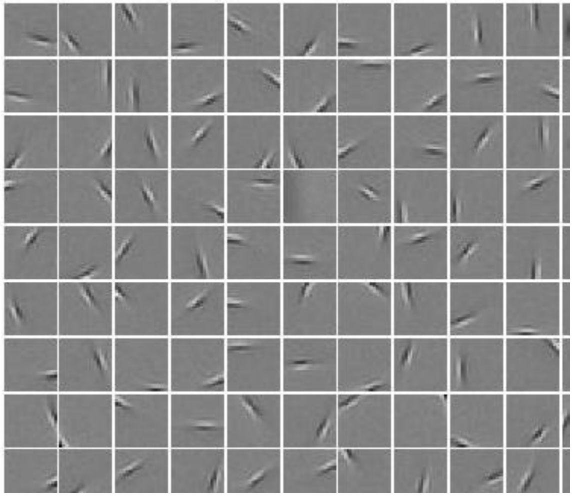


Spare coding hypothesis

A neural system encodes information using as less as possible number of neurons.

The cost function:
$E = \langle I(\mathbf{x}) - \mathbf{a} \cdot \Phi(\mathbf{x}) \rangle^2 + \lambda S(\mathbf{a})$
$=$ Reconstruction Error $+$ Spareness of neural activity

$I(\mathbf{x})$: the real image
$\Phi(\mathbf{x})$: the basis function (the neuronal receptive field)
$\mathbf{a}$: the neural activity
The sparseness constraint:
$S(a) = e^{-a^2}, \quad \ln(1 + a^2), \quad |a|, \quad \ldots$



The learned basis functions like Gabor filters

Iterative learning **a** and the basis function

Olshausen B.A., Field Nature, 381 (1996), pp. 607-609

A robust coding scheme

- A hypothesis: neural codes are constructed to be as robust as possible to noise.
- The cost function:

$$
\begin{aligned}
E &= \langle I(\mathbf{x}) - \mathbf{a} \cdot \Phi(\mathbf{x}) \rangle^2 + \lambda H(\mathbf{a}) \\
&= \text{Reconstruction Error} + \text{Representation variability}
\end{aligned}
$$

S. Li and S. Wu, Cognitive Neurodynamics 2007

On the choice of H(a)

- The total variability of neural responses given the set of natural images

$$H(\mathbf{a}) = \sum_l H(a_l|\mathbf{I})$$

Minimizing the reconstruction error + Restricting the total variability enforce the neural system to: 1) encode important features of external inputs; and 2) not encode unimportant components, i.e., robust to noise.

On the choice of H(a)

- The Sharon entropy

$$H(a_l|\mathbf{I}) = -\int p(a_l|\mathbf{I}) \ln p\,(a_l|\mathbf{I}) d a_l$$

- The quadratic Renyi’ s entropy

$$H(a_l|\mathbf{I}) = -\ln \int p(a_l|\mathbf{I})^2 d a_l$$

- The Parzon window approximation

$$p(a_l|\mathbf{I}) \approx \frac{1}{\sqrt{2\pi} dK} \sum_{k=1}^{K} \exp[-(a_l - a_l^k)^2/2d^2]$$

where $\{a_l^k\}$, for $k = 1, \ldots, K$, are sampling of a_l
when K natural images are presented

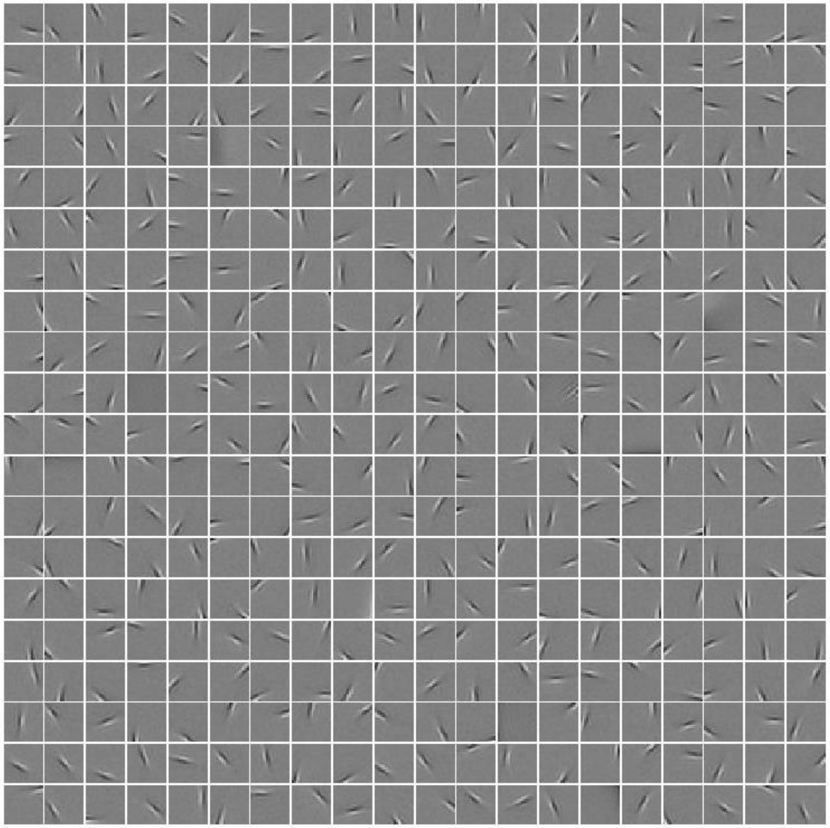
On the choice of H(a)

- The final form of H(a)

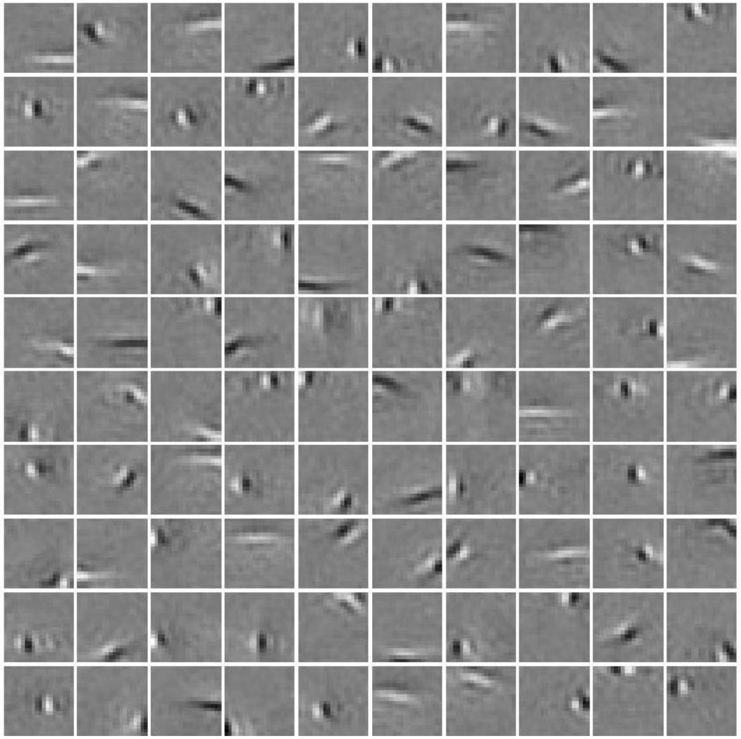
$$H(a_l|\mathbf{I}) \approx -\ln \frac{1}{\sqrt{2\pi} d K^2} \sum_{m=1}^{K} \sum_{k=1}^{K} \exp[-(a_l^m - a_l^k)^2 / 2d^2]$$

H(a) is now data-dependent; Sampling of **a** can be easily achieved in practice through batch learning.

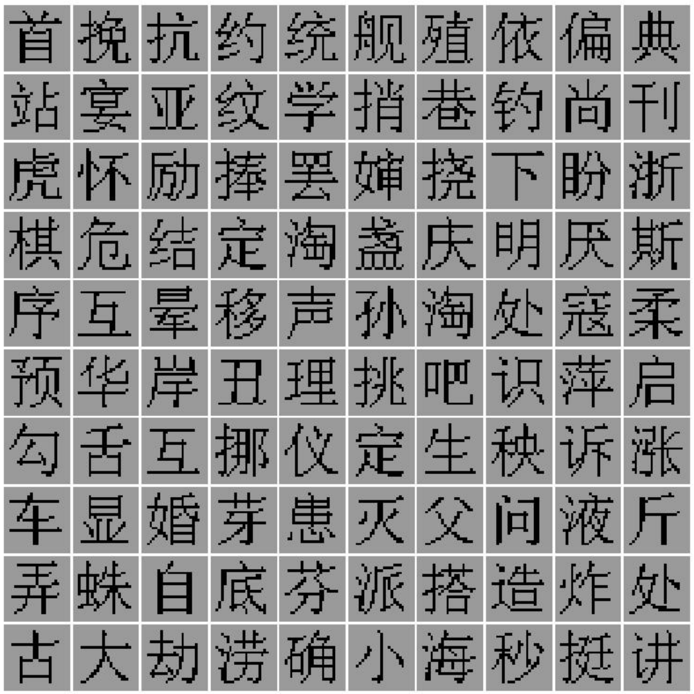
Optimal basis for natural scenes

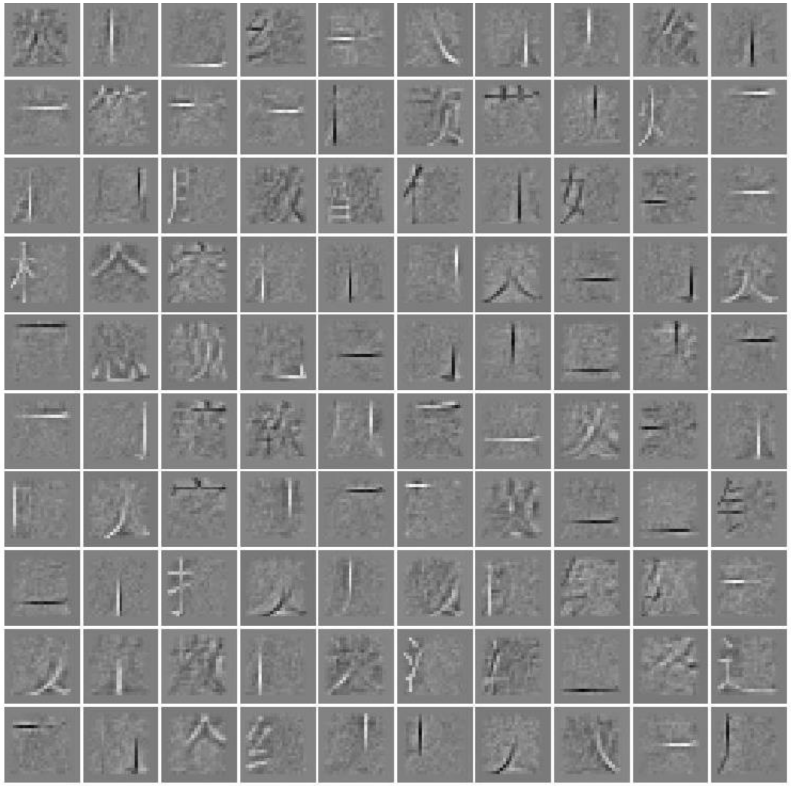


Optimal basis for human faces

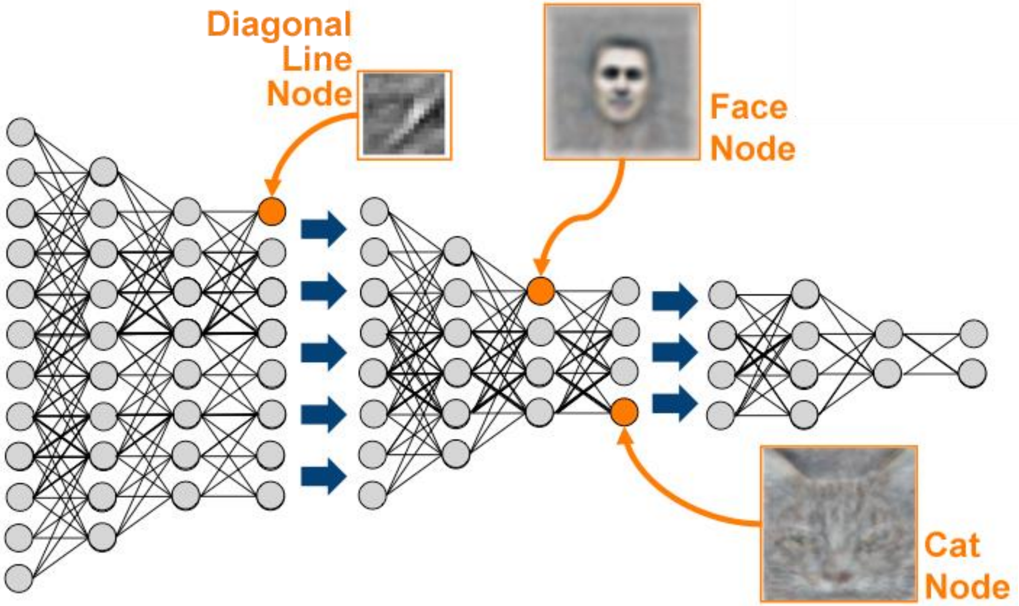


Optimal basis for Chinese





Insight from deep-learning



Representation generated by a deep neural network trained over
a large set of nature images displays similar statistics to neural coding.

Neural coding is about good representation!