Neural Coding: Basis

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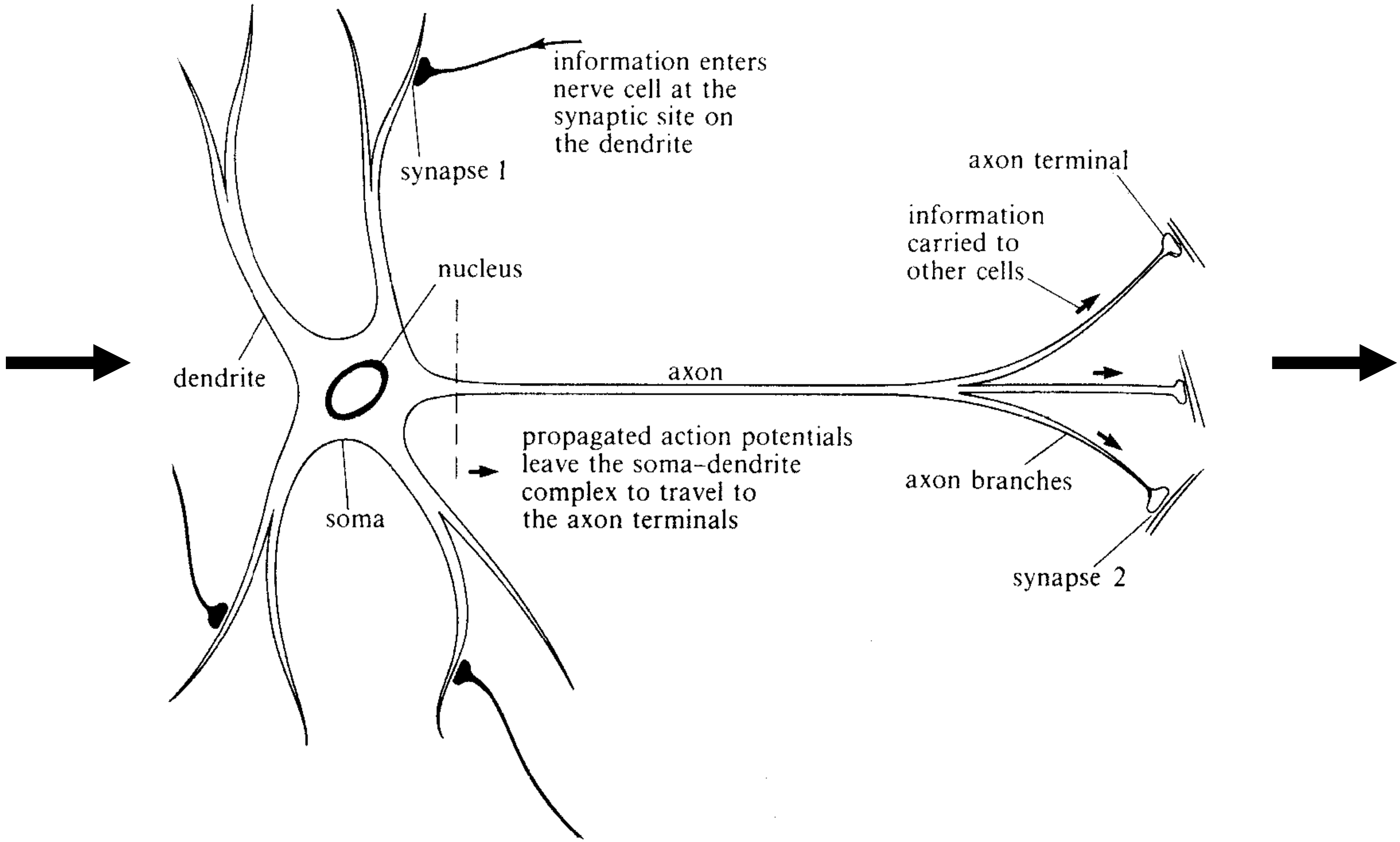
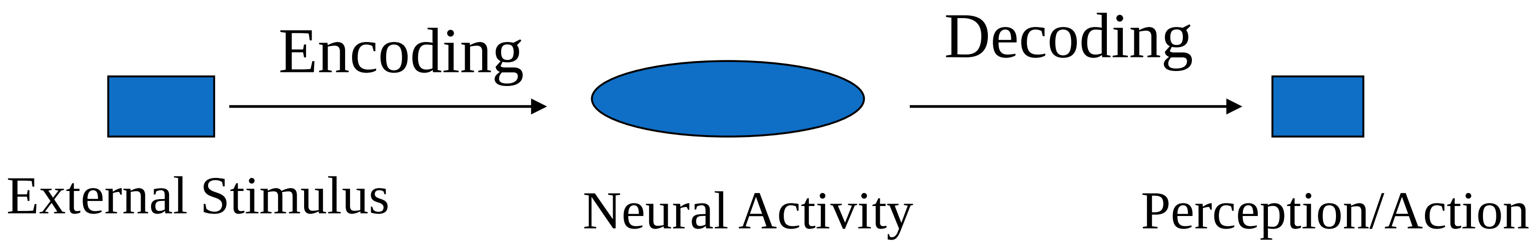
Signalling process of a single neuron



Neural Information Processing

- EVERYTHING is encoded in the stereotyped neuronal responses in the cortex, i.e., spike trains
- Elucidating brain function in the view of information processing



- Understanding neural code is prerequisite for us to fully understand high-level brain functions?

The nature of neural code?

•Rate coding: information is encoded in firing rate,

$r = N/T$ N spikes in the time duration T

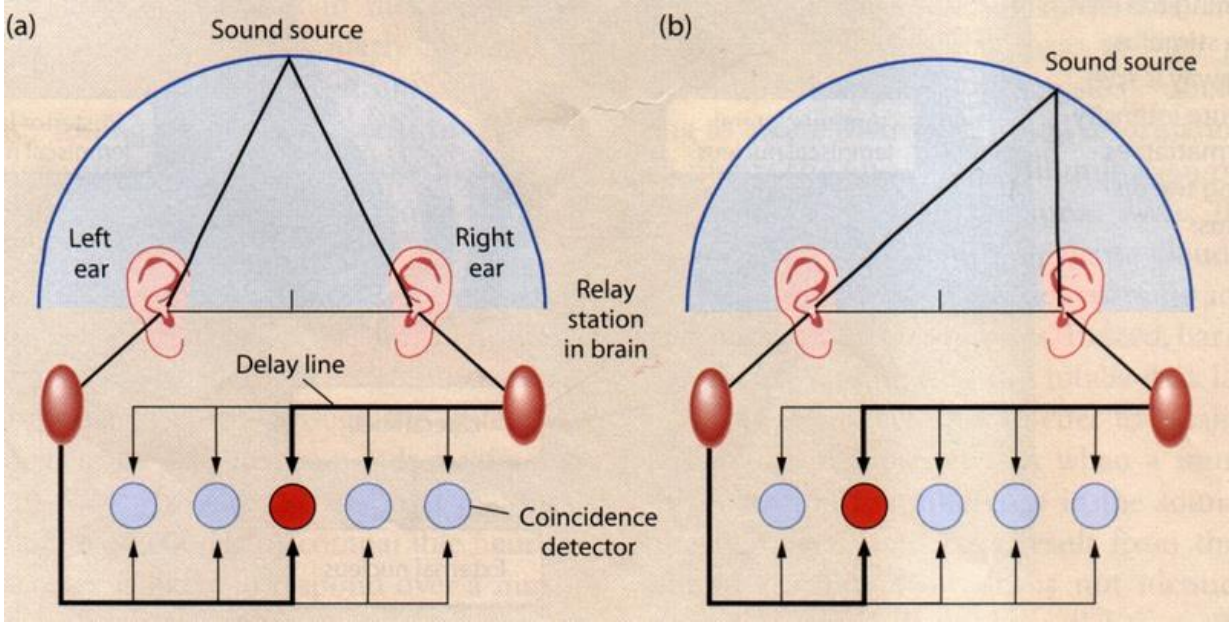
•Temporal coding: the fine structure of spike train contains information

•Coincidence detector/Synchronization: Synchronization of a group of neurons conveys information.

•Coarse structures of ISIs: Statistical features of ISIs more than the first order (firing rate) are informative.

• No such an universal neural code exists?

Temporal coding in the auditory system of owl



The cue: the time difference between a sound reaches the two ears (the order of 0.1ms).
Coincidence detector: the neuron will only be active when inputs from two ears arrive simultaneously.

Poisson Process

Poisson distribution:

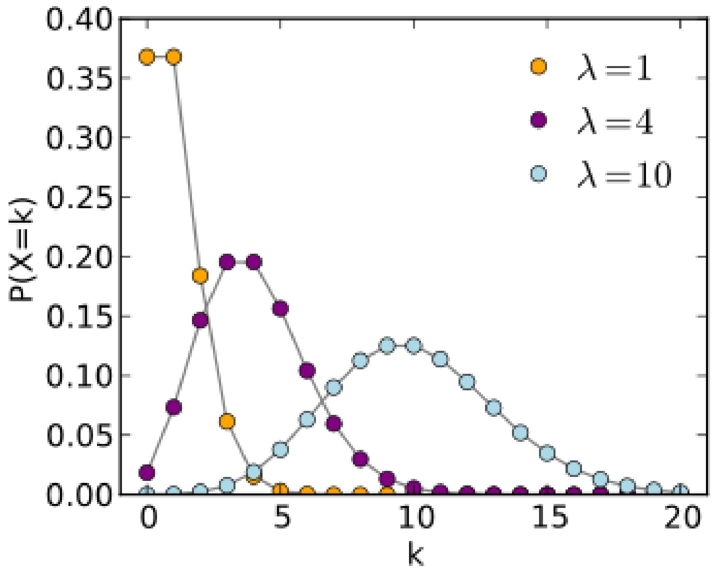
- A discrete probability distribution that expresses the probability of a given number of events occurring in a fixed interval of time or space if these events occur with a known constant rate and independently of the time since the last event.

$$P(X = k \text{ events in interval } t) = e^{-rt}\frac{(rt)^k}{k!}$$

mean: $\langle X \rangle = rt$

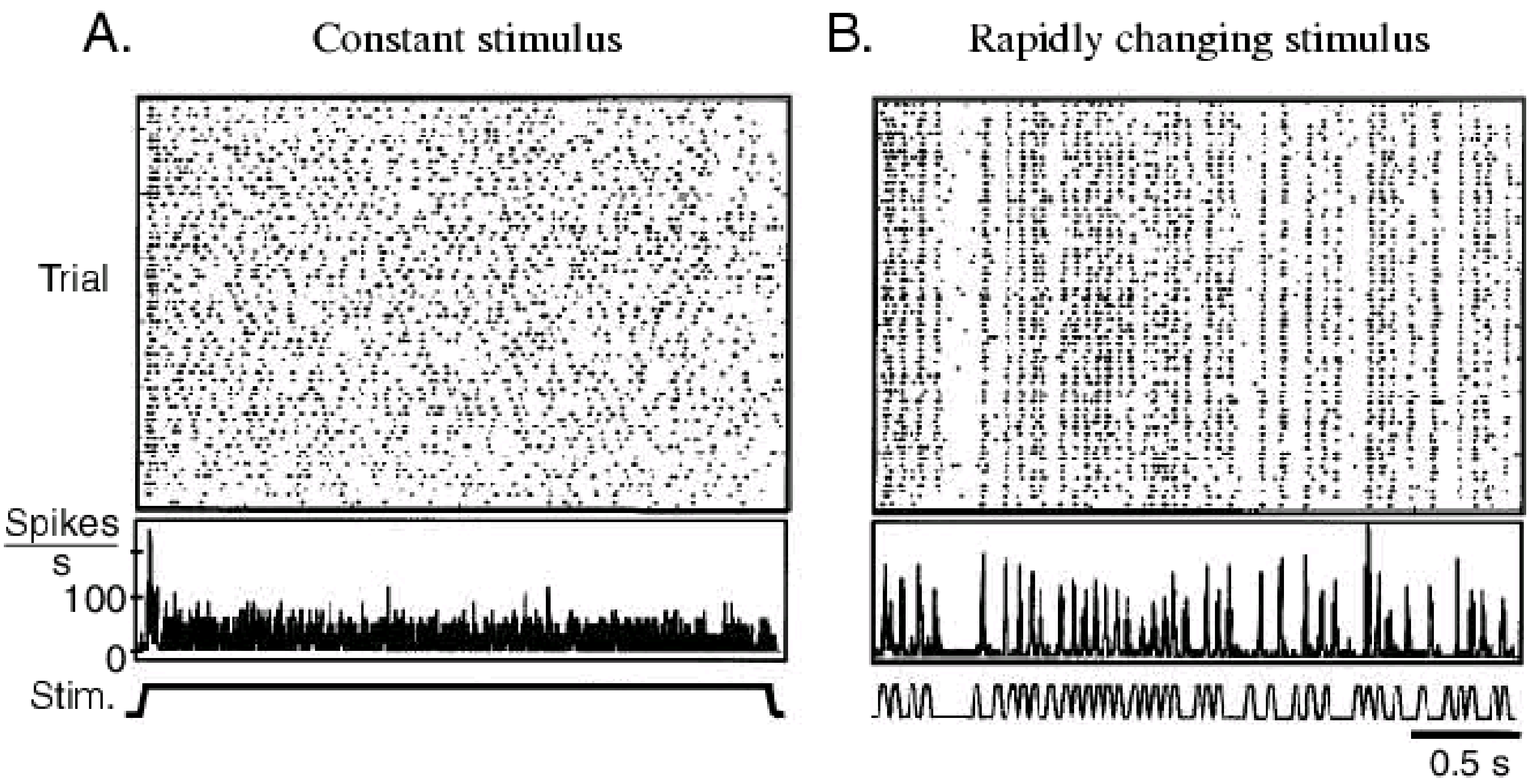
variance: $\left\langle (X - \langle X \rangle)^2 \right\rangle = rt$

$$\text{Fanofactor} = \frac{\text{variance}}{\text{mean}} = 1$$



Irregular Spiking of Neurons

Irregular spike of a neuron in vivo



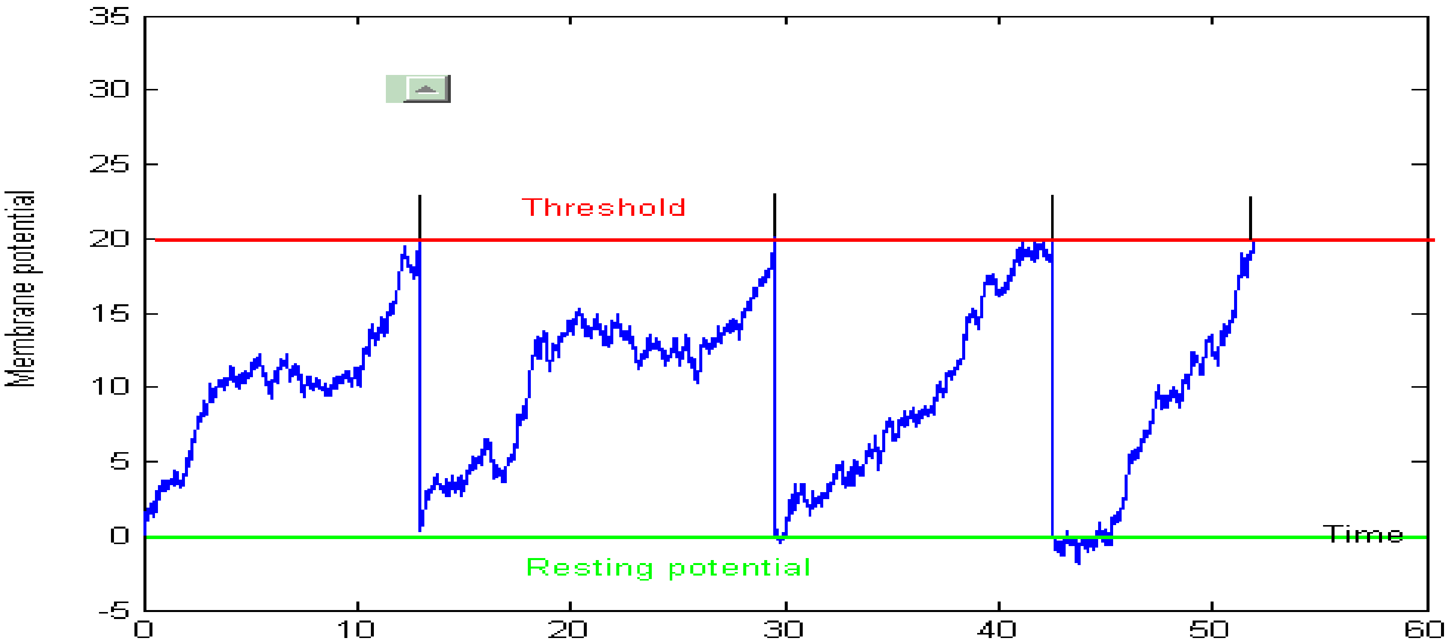
Fano factor > 1

Response of a single neuron

- The Hodgkin-Huxley model

$$C\frac{dV}{dt} = -g_L(V - E_L) - g_k(V - E_K) - g_{Na}(V - E_{Na}) + I(t)$$

- Regular spiking of a neuron in response to a constant input



A neuron fires rather regularly in response to a constant input in vitro

Central Limit Theorem

Let $\{X_1, \ldots, X_n\}$ be a random sample of size n — that is, a sequence of independent and identically distributed random variables drawn from a distribution of expected value given by μ and finite variance given by σ^2. Suppose we are interested in the sample average

$$S_n = \frac{X_1 + \cdots + X_n}{n}$$

By the law of large numbers, the sample averages converge in probability and almost surely to the expected value μ as $n \to \infty$. The classical central limit theorem describes the size and the distributional form of the stochastic fluctuations around the deterministic number μ during this convergence. For large enough n, the distribution of S_n is close to the normal distribution with mean μ and variance σ^2/n.

Neuronal spiking should be regular?

- On average, a cortical neuron receives inputs from1000~10000 connected neurons.

- Even though every neuron generates stochastic spike trains, according to the central limit theorem, after summation, the averaged input received by a neuron has very small fluctuations, implying that the neuron should fire regularly.

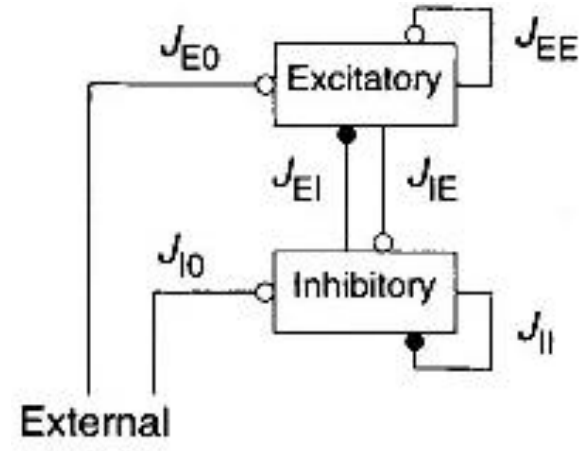
- There must be something unusual?

Good research starts from asking a good question!

A E-I Balanced Network (1)

$$\tau \frac{du_i^E}{dt} = -u_i^E + \sum_{j=1}^{K_E} J_{EE} r_j^E + \sum_{j=1}^{K_I} J_{EI} r_j^I + I_i^E$$

$$\tau \frac{du_i^I}{dt} = -u_i^I + \sum_{j=1}^{K_I} J_{II} r_j^I + \sum_{j=1}^{K_E} J_{IE} r_j^I + I_i^I$$



Sparse & random connections: $1 \ll K_E, K_I \ll N_E, N_I$
Neurons fire largely independently to each other.

Van Vreeswijk C., Sompolinsky H, Science, 1996

A E-I Balanced Network (2)

$$\tau \frac{du_i^E}{dt} = -u_i^E + \sum_{j=1}^{K_E} J_{EE} r_j^E - \sum_{j=1}^{K_I} J_{EI} r_j^I + I_i^E$$

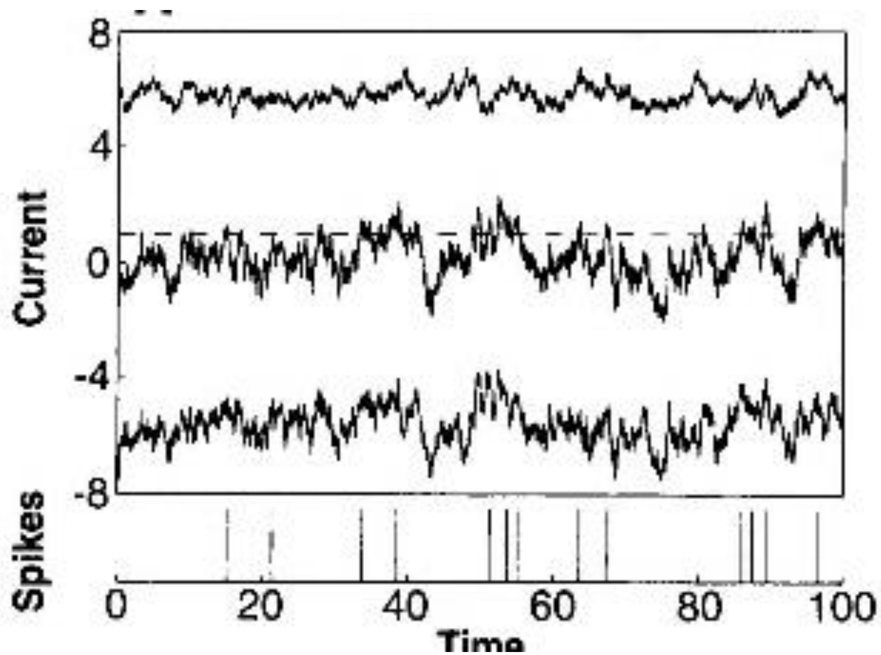
$$\tau \frac{du_i^I}{dt} = -u_i^I + \sum_{j=1}^{K_I} J_{II} r_j^I - \sum_{j=1}^{K_E} J_{IE} r_j^{\mathrm{E}} + I_i^I$$

Supposes each neuron fires irregularly with mean rate μ and variance σ^2,
The mean of recurrent input received by E neuron: $\sim K_E J_{EE}\mu - K_I J_{EI}\mu$
The variance of recurrent input received by E neuron:
$\sim K_E (J_{EE})^2\sigma^2 + K_I (J_{EI})^2\sigma^2$
The balanced condition:
$K_E J_{EE} - K_I J_{EI} \approx 0$; the mean is close to zero
$J_{EE} \sim \frac{1}{\sqrt{K_E}}, J_{EI} \sim \frac{1}{\sqrt{K_I}}$; the variance is order of one

Properties of a E-I Balanced Network (1)

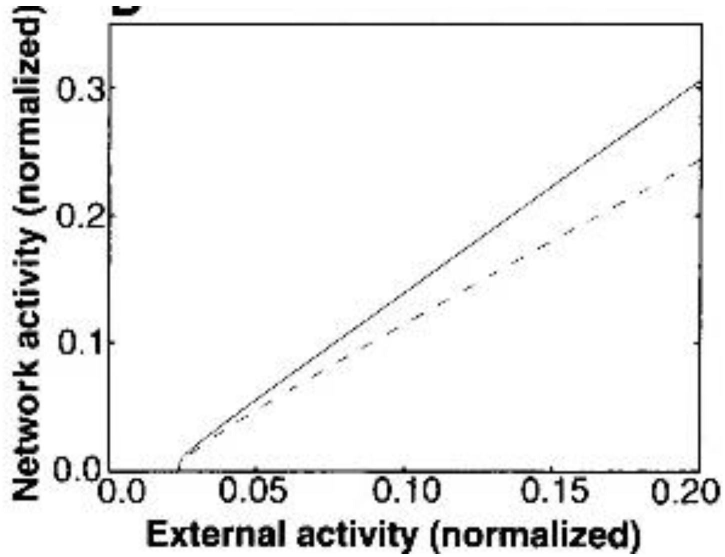
- Irregular firing (chaos) emerge from the network dynamics, without fine tuning parameters

- Neuronal firing is driven by input fluctuations

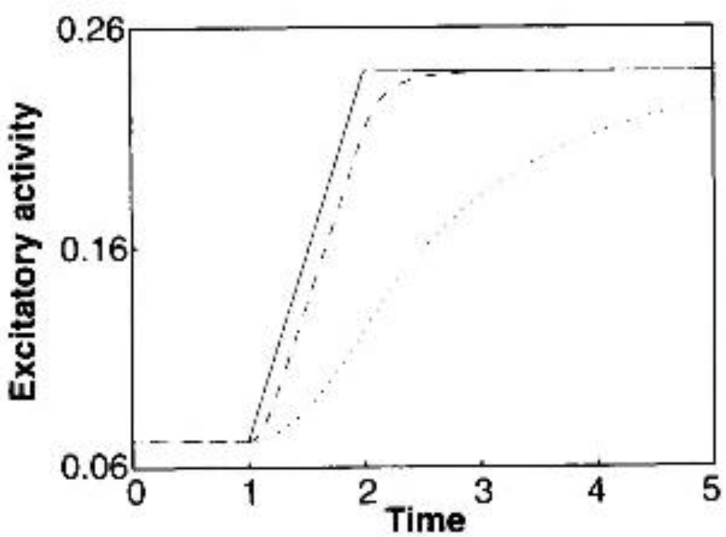


Properties of a E-I Balanced Network (2)

- External input strength is "linearly" encoded by the mean firing rate of the neural population



- The network responds rapidly to abrupt changes of the input



Ex: Building a E-I balanced network

$$\tau \frac{dV_i}{dt} = -(V_i - V_{rest}) + I_i^{ext} + I_i^{net}(t),$$

$$I_i^{net}(t) = J_{\mathrm{E}} \sum_{j=1}^{pN_e} \sum_{t_j^\alpha < t} f\left(t - t_j^\alpha\right) - J_I \sum_{j=1}^{pN_i} \sum_{t_j^\alpha < t} f\left(t - t_j^\alpha\right),$$

$$f(t) = \begin{cases} \exp\left(-\frac{t}{\tau_s}\right), & t \geq 0 \\ 0, & t < 0 \end{cases}$$

Neuron number: N = 1000，

（$Ne = 500$, $Ni = 500$）

Connection probability : p = 20%

$$\tau = 10ms, \tau_s = 2ms,$$

$$V_{rest} = -52mV, V_{reset} = -60mV, \quad V_{threshold} = -50mV$$

$$J_E = \frac{1}{\sqrt{pN_e}}, J_I = \frac{1}{\sqrt{pN_I}},$$

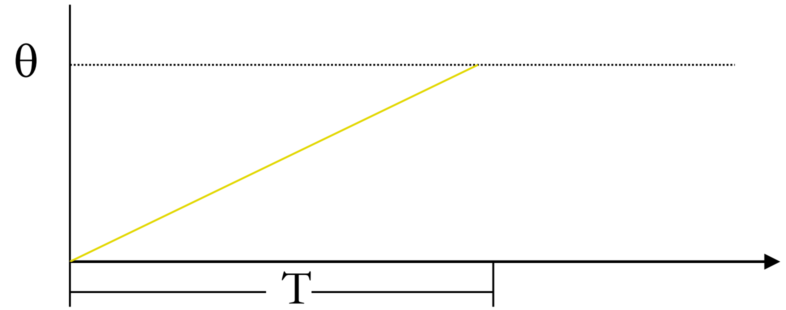
Speed limitation in a noiseless environment

The dynamics:

$$\tau \frac{dv}{dt} = I$$

The firing time:

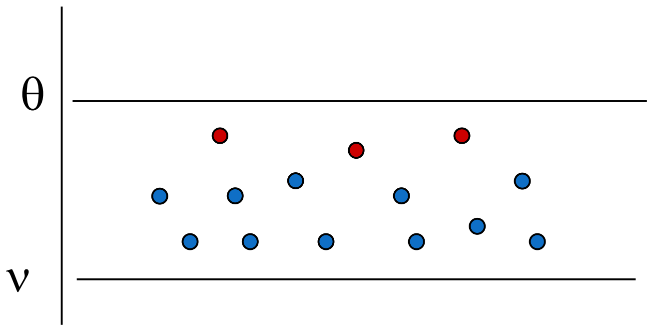
$$T = \tau \frac{\theta - v_0}{I}$$



The reaction time T is in the order of τ, the membrane time constant, which is typically 10-20 ms.

Noise speeds up computation

- A neural ensemble jointly encodes stimulus information;
- Noise randomizes the distribution of neuronal membrane potentials;
- Those neurons (red circle) whose potentials are close to the threshold will fire rapidly;
- If the noisy environment is proper, even for a small input, a certain number of neurons will fire instantly to report the presence of a stimulus.



A simple model

Integrate & firing

$$\tau \frac{dv_i}{dt} = I_i(t), \qquad \text{for } i = 1, \ldots, N$$

$I_i(t) = \mu + \sigma \xi_i(t)$

$\xi_i(t)$: Gaussian white noise

μ: the mean drift

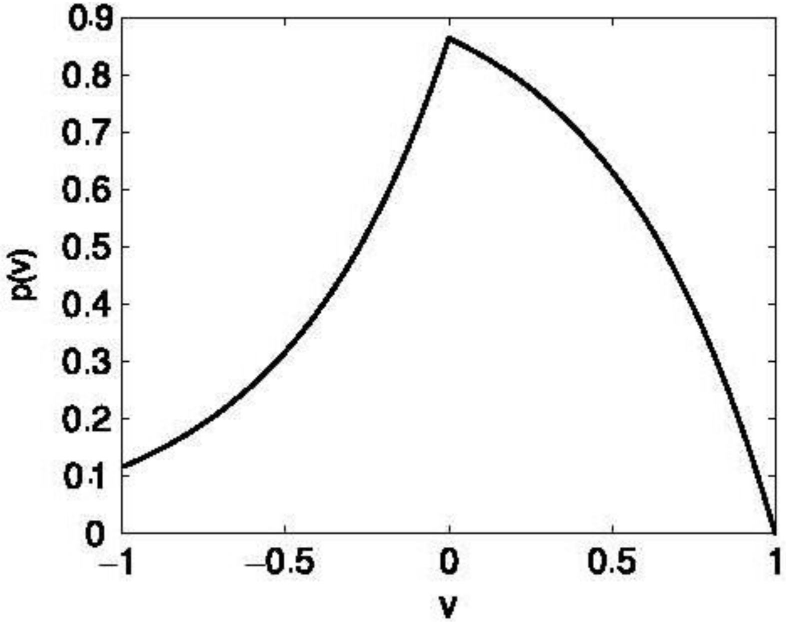
σ: the noise strength

The Poissonian noise, can be generated by a E-I balanced network?

$$\sigma^2 = \alpha\mu$$

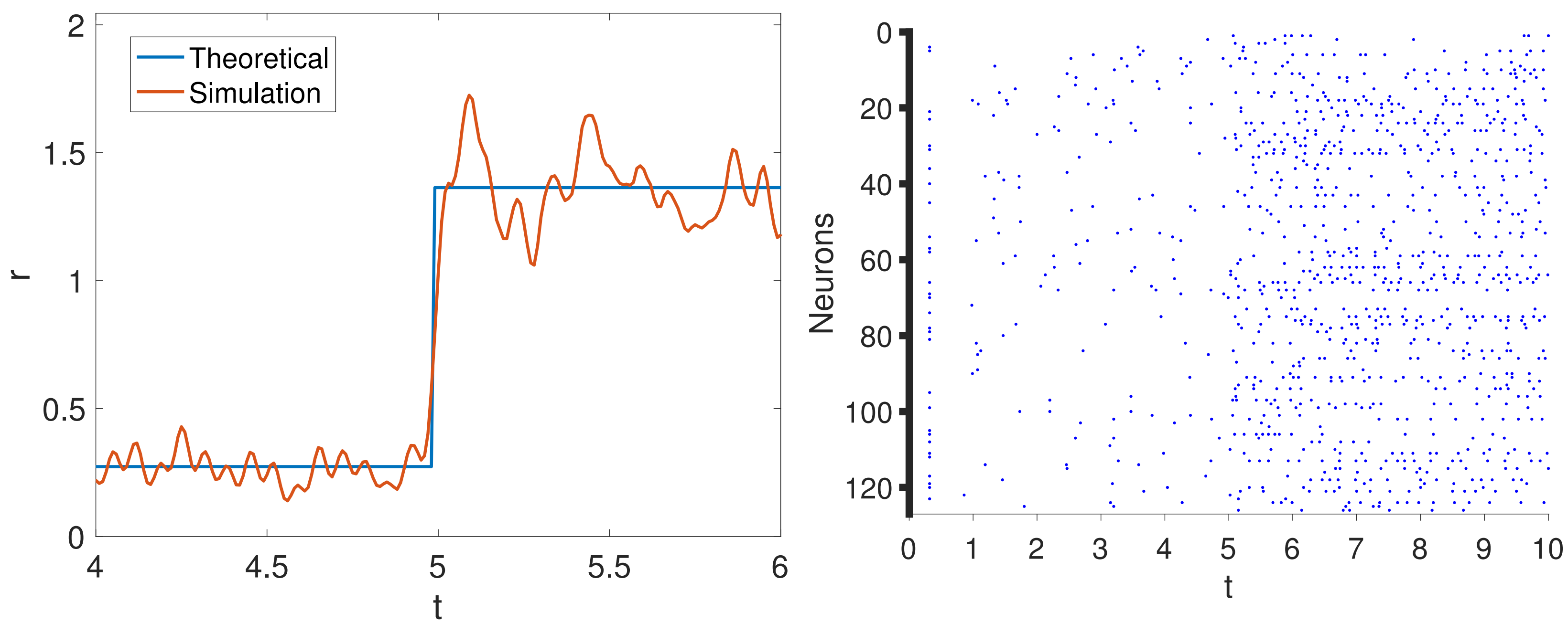
The ideal noisy environment (1)

The potential distribution at the equilibrium state is insensitive to the stimulus value. The system is always ready to detect input changes.



$$p(v) = \begin{cases} \frac{1}{\theta}(1 - \mathrm{e}^{-2\tau\theta/\beta})\mathrm{e}^{2\tau v/\beta} & v < 0 \\ \frac{1}{\theta}(1 - \mathrm{e}^{-2\tau(v-\theta)/\beta}) & 0 \leq v \leq \theta \\ 0 & v > \theta, \end{cases}$$

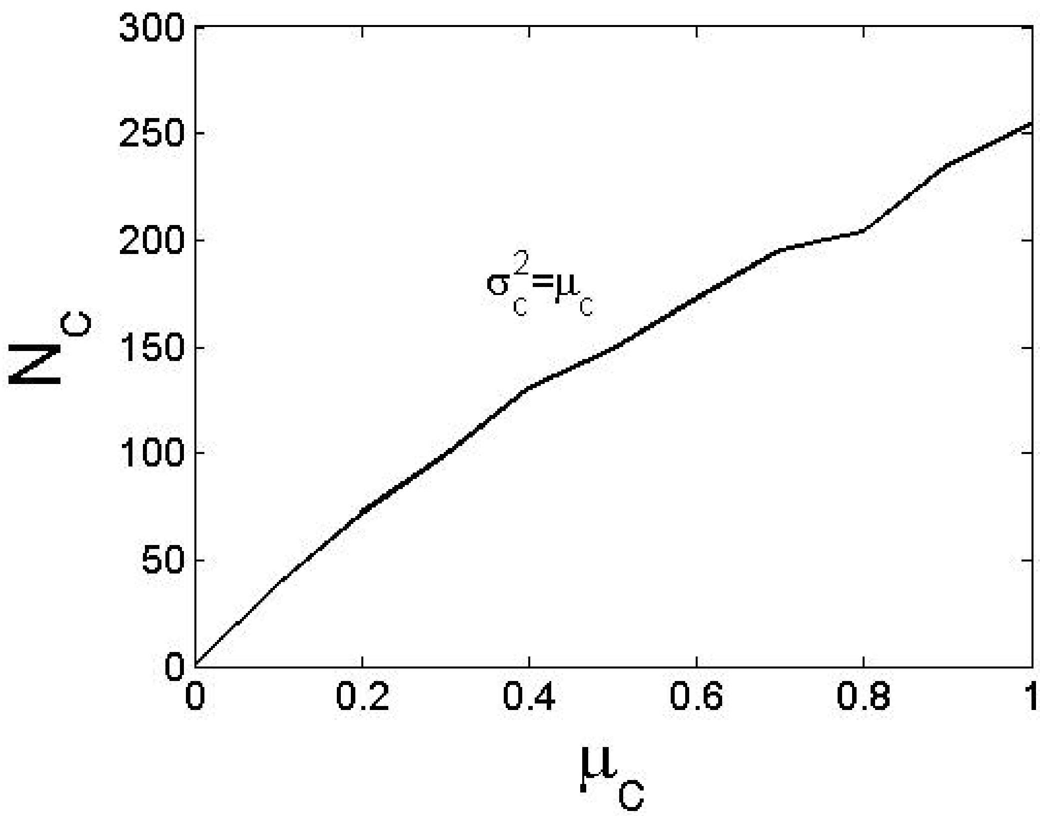
Fast Response



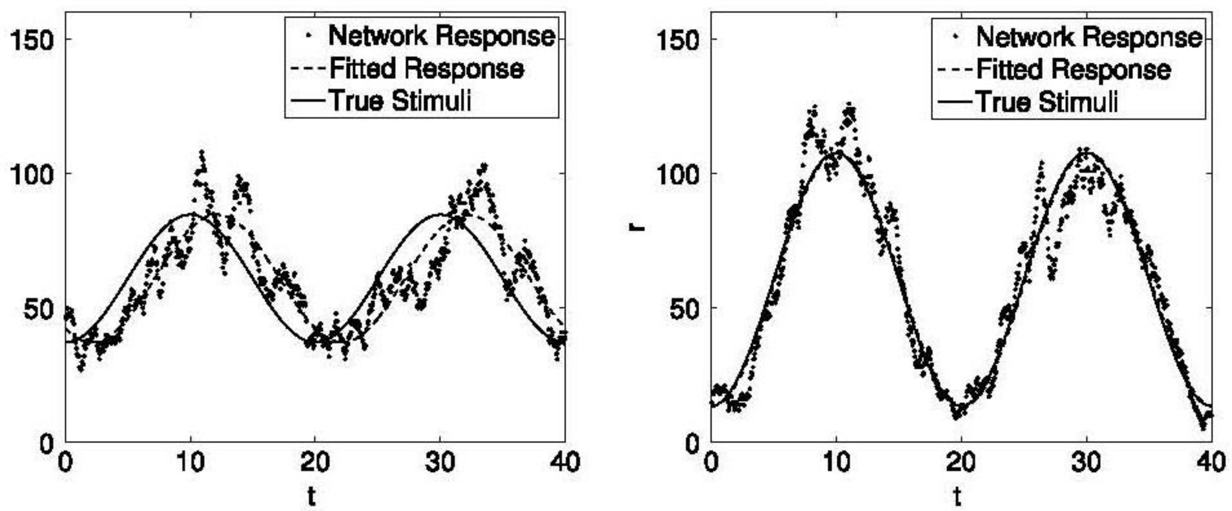
Simulation confirmed instantaneous tracking

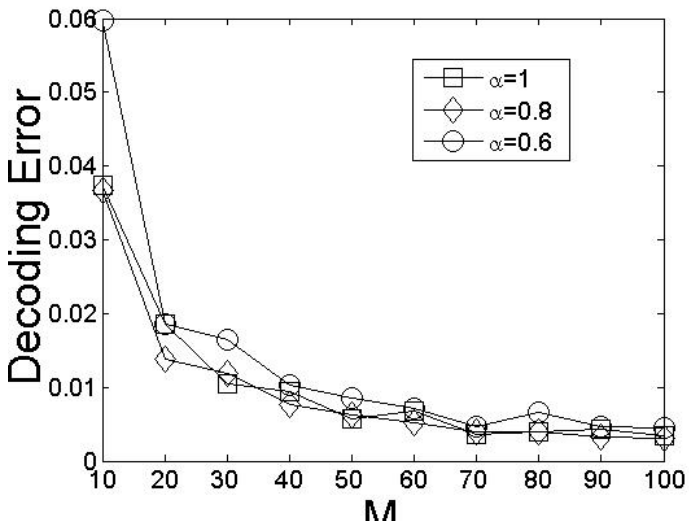
The ideal noisy environment (2)

Linear encoding at the equilibrium state



The ideal noisy environment (3)





- The decoding error at $\Delta t=0.1\tau$.
- The error linearly decrease with the number of neurons, no longer restricted by τ
- In principle, if the number of neurons is sufficiently large, the speed of neural computation can be arbitrarily fast.

ELSEVIER

Neural Networks

journal homepage: www.elsevier.com/locate/neunet

Impact of noise structure and network topology on tracking speed of neural networks

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Balanced network: why is it fast?

- In the balanced state, where neurons fire asynchronously at a relatively low rate, the presynaptic input to a neuron can be treated as the superposition of many weakly dependent, sparse renewal processes, which can be approximated by a Poisson process.

- Thus, the asynchronousness of balanced networks produces a Poissonian noise structure with constant variance-to-mean ratio for each neuronal presynaptic inputs.

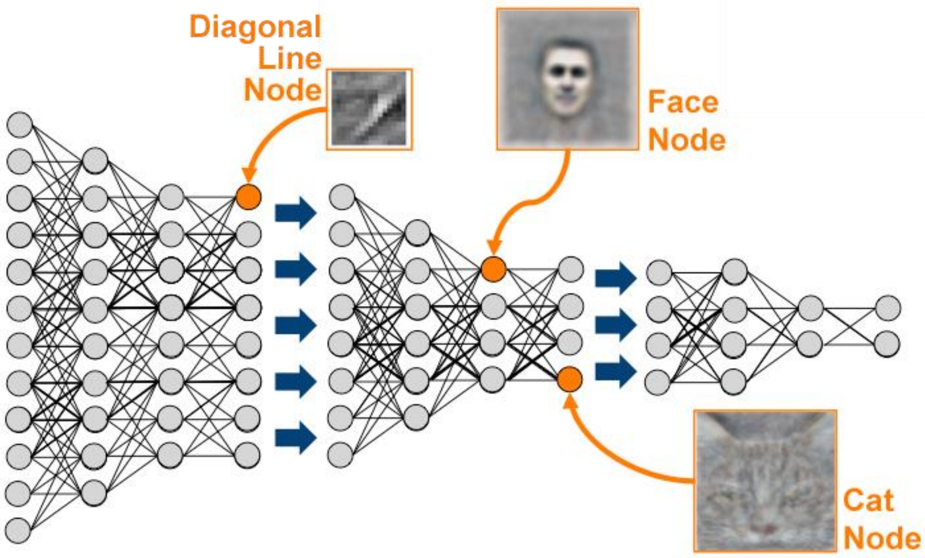
$$r = \frac{\mu_{ext}}{\theta\tau} \Big/ \left(1 - \frac{J_E K_E - J_I K_I}{\theta}\right)$$

$$\sigma_{rec}^2 = \tau^2 r (J_E^2 K_E + J_I^2 K_I)$$

$$\beta = \frac{\tau}{\theta}(J_E^2 K_E + J_I^2 K_I) + \alpha \left(1 - \frac{K_E J_E - K_I J_I}{\theta}\right)$$
$$= const.$$

Tian et al. Frontiers in Computational Neuroscience 2020.

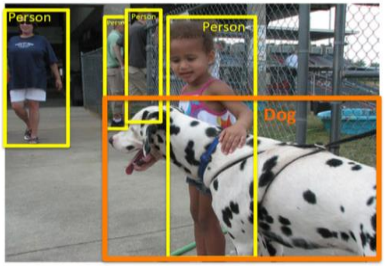
深度学习@静态图像识别



IMAGENET Large Scale Visual Recognition Challenge (ILSVRC) 2010-2014

~~20 object classes~~ ~~22,591 images~~

200 object classes	456,567 images	DET
1000 object classes	1,431,167 images	CLS-LOC



http://image-net.org/challenges/LSVRC/

大脑计算本质：处理时空动态模式

◆ The brain is targeted for processing spatio-temporal information

Optical flow to Spike train in Retina propagating to the Cortex

We can not “see” static image!

◆ Application for video analysis

➢ Play basketball

➢ Biking

➢ Diving

➢ Golfing

➢ Horse riding

➢ Soccer juggling

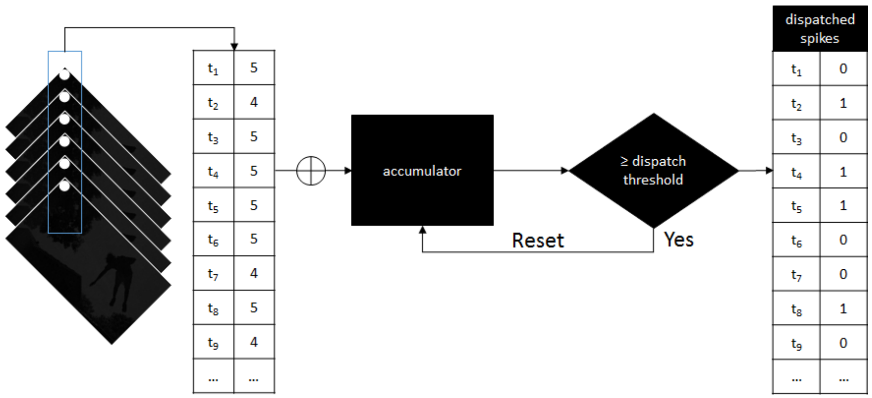
➢ Swing

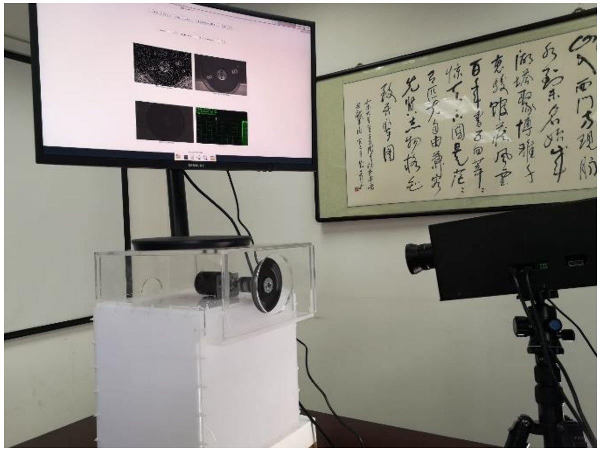
➢ Tennis

➢ Trampoline jumping

➢ Volleyball

Spike camera



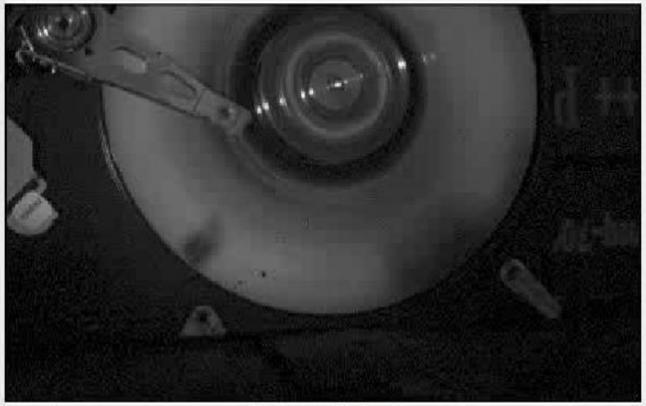


(Non-leaky integrate-and-fire model)

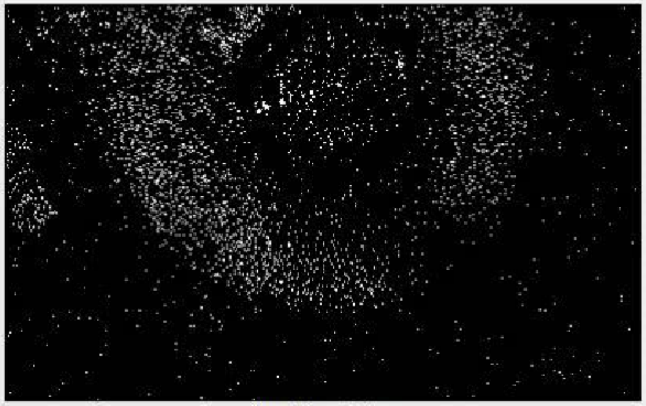
- Accumulator: integrate photocurrent $I(t)$ to a fixed threshold Φ
- Comparator: $\int I(t)dt > \Phi$
- Reset: all the charges on the integration capacitor are drained
- Readout: read the state of all the pixels every 25μs (40kHz)

Dong, S., Huang, T., & Tian, Y. (2017, April). Spike camera and its coding methods. In *2017 Data Compression Conference (DCC)* (pp. 437-437). IEEE.
黄铁军，董思维（2019）超速电眼：全时成像芯片重塑机器视觉。前沿科学

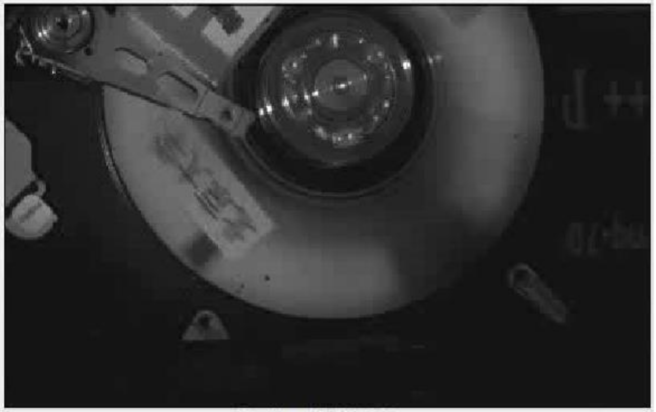
Spike camera


Texture @60fps

Rapidly rotating disk as captured by a traditional camera (60 fps)


Real-time Spike

Real-time spikes produced by spike camera (40k fps)


Texture @40000fps

Disk texture reconstructed from spike camera data (40k fps)

Chen, R., Yang, C., Jia, H., & Huang, T. (2018). Accurate full-resolution reconstruction of spike-encoded image time series using random matrix theory. *Electronics Letters*.

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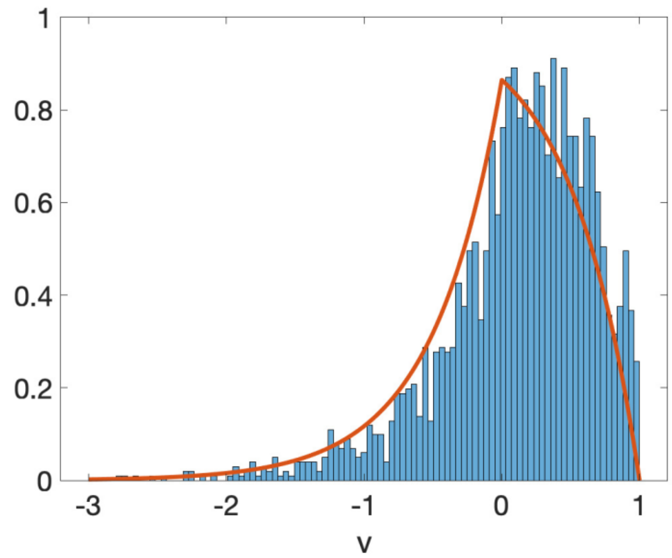
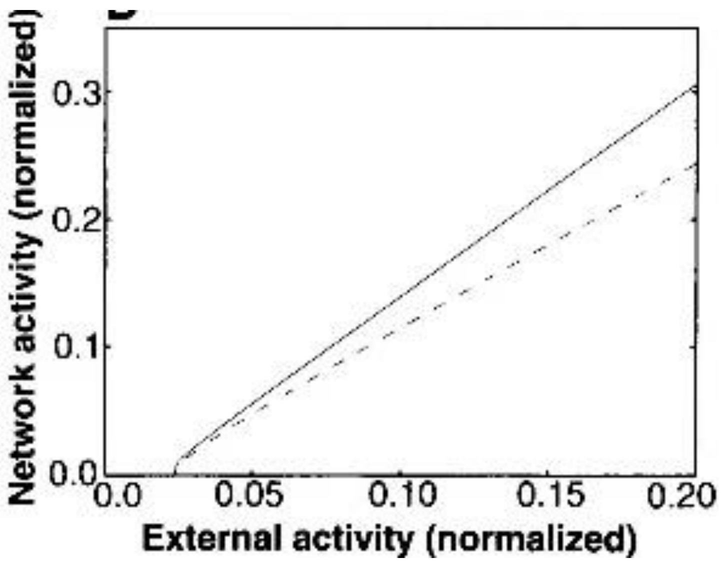
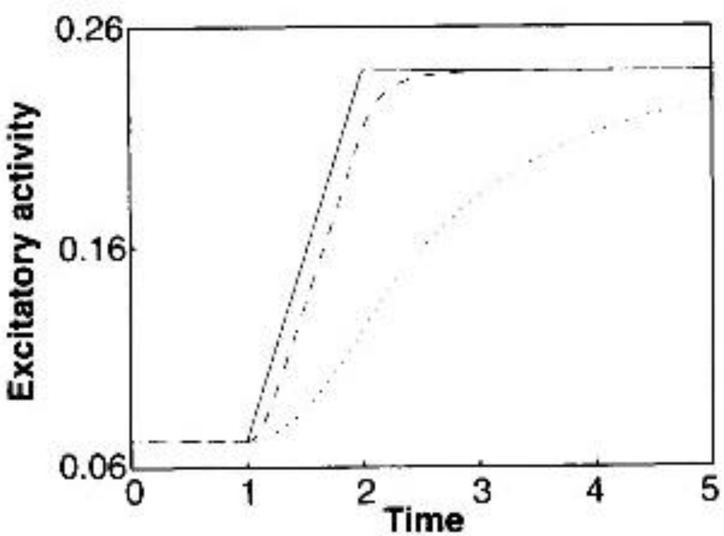
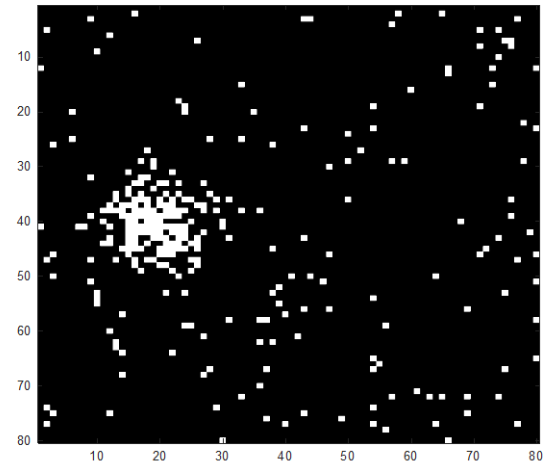




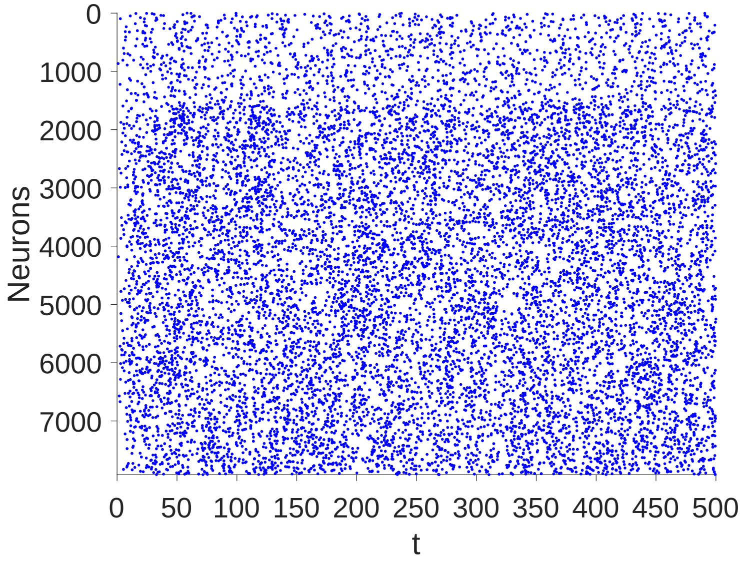


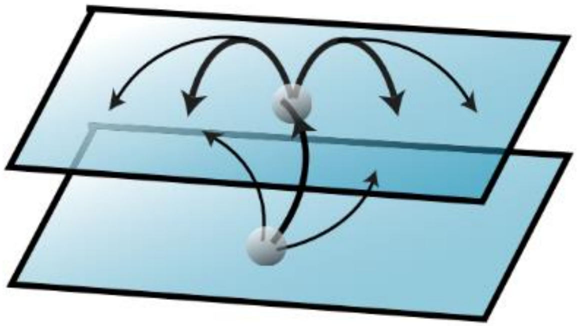
A Nearly E-I balanced network with local connectivity

Homogeneous connectivity spreads local activity across the population, and degrades spatial information.

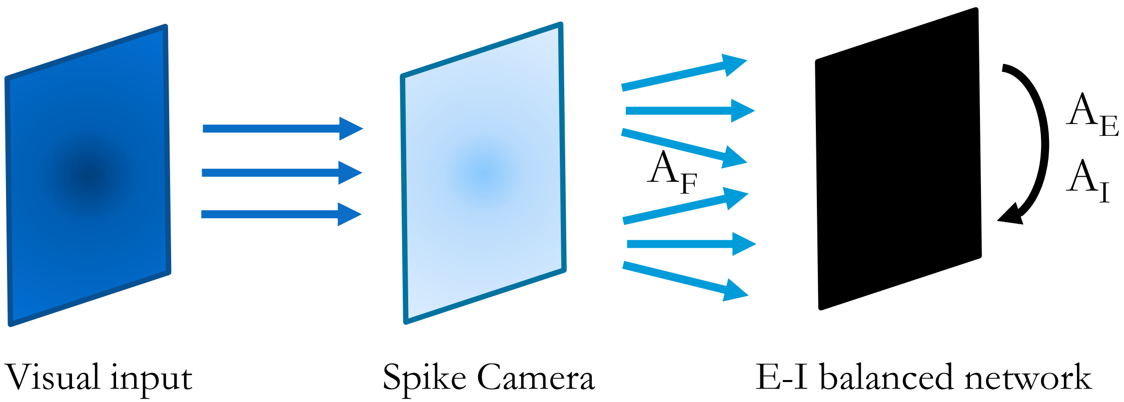


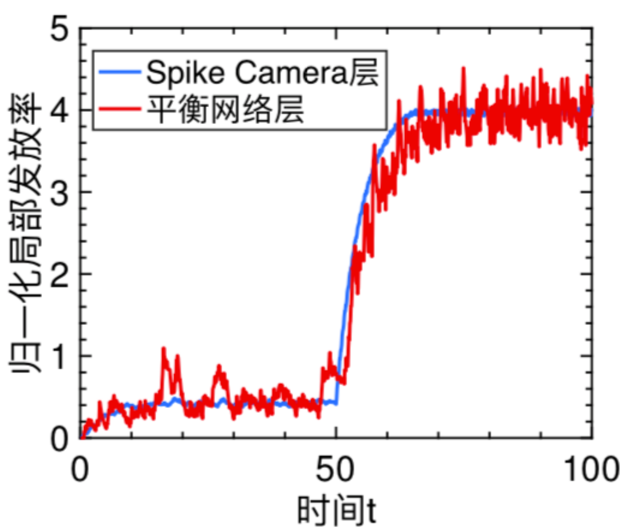
Proper localized connectivity could preserve spatial information, and meanwhile realize an approximately balanced state.

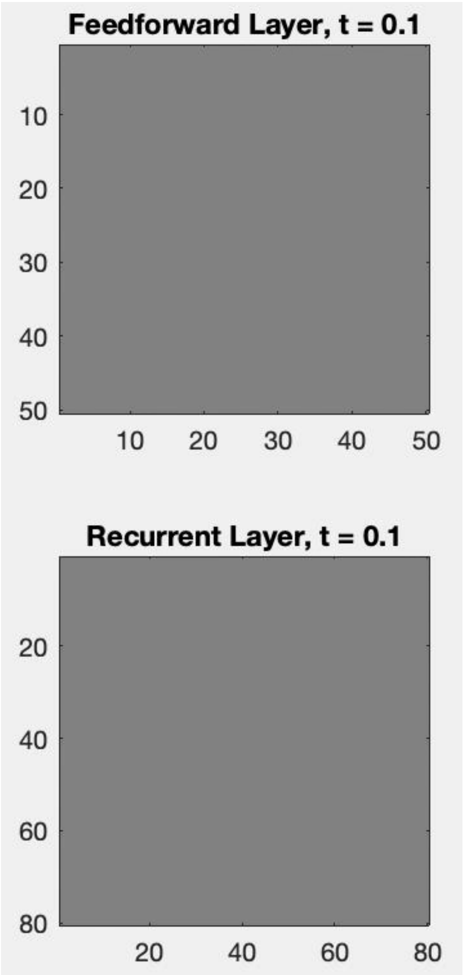




Rapid signal detection with spike camera data







Tian et al. IEEE SMC 2019