1. The data in Table 3.3 are a subset of the data obtained by Kaneto, Kosaka, and Nakao (1967). The experiment investigated the effect of vagal nerve stimulation on insulin secretion. The subjects were mongrel dogs with varying body weights. Table 3.3 gives the amount of immunoreactive insulin in pancreatic venous plasma just before stimulation of the left vagus nerve (X) and the amount measured 5 min after stimulation (Y) for seven dogs. Test the hypothesis of no effect against the alternative that stimulation of the vagus nerve increases the blood level of immunoreactive insulin.

Table 3.3 Blood Levels of Immunoreactive Insulin (μU/ml)

Dog i	X_i	Y_i	$\lvert Z_i \rvert$	R_i	ψ_i
1	350	480	130	5	1
2	200	130	70	4	0
3	240	250	10	1	1
4	290	310	20	2	1
5	90	280	190	6	1
6	370	1450	1080	7	1
7	240	280	40	3	1

Source: A. Kaneto, K. Kosaka, and K. Nakao (1967).

Let $Z_i = Y_i - X_i$

Assume $Z_i \sim F_i$ that are symmetric around θ.

$H_0: \theta = 0 \quad H_1: \theta > 0$

$$\hat{T}^+ = \sum_{i=1}^{7} \psi_i R_i = 24$$

Under the null, $P\{T^+ \geq \hat{T}^+\} = P\{T^+ \geq 24\} = \sum_{k=24}^{28} P\{T^+ = k\}$

$$= \frac{2+2+1+1+1}{2^7} \approx 0.0547$$

If we choose $\alpha = 5\%$, then we fail to reject the null hypothesis.

2. Change the value of X_3, in Table 3.1, from 1.62 to 16.2. What effect does this outlying observation have on the calculations performed in Example 3.1? What does this suggest about the relative insensitivity of the signed rank tests to outliers? Construct an example in which changing one observation has a marked effect on the final decision regarding rejection or acceptance of H_0.

Table 3.1 The Hamilton Depression Scale Factor IV Values

Patient i	X_i	Y_i
1	1.83	0.878
2	0.50	0.647
3	~~1.62~~ 16.2	0.598
4	2.48	2.05
5	1.68	1.06
6	1.88	1.29
7	1.55	1.06
8	3.06	3.14
9	1.30	1.29

Source: D. S. Salsburg (1970).

i	Z_i	$\lvert Z_i \rvert$	R_i	ψ_i	$R_i\psi_i$
1	−0.952	0.952	8	0	0
2	0.147	0.147	3	1	3
3	~~−1.022~~ −15.602	~~1.022~~ 15.602	9	0	0
4	−0.430	0.430	4	0	0
5	−0.620	0.620	7	0	0
6	−0.590	0.590	6	0	0
7	−0.490	0.490	5	0	0
8	0.080	0.080	2	1	2
9	−0.010	0.010	1	0	0

$T^+ = 5$

* The change results in a change of Z_3, from -1.022 to -15.602

But since -1.022 is already the largest rank, the change does not affect the calculation of T^+ and does not affect resules of test.

* This example suggests signed rank test is insensitive to the change in extreme value as long as the sign of $Z_{extreme}$ does not change.

However, if we change X_3 to -16.2. Z_3 becomes 16.798

$\psi_i = 1 \Rightarrow T^+ = 14 \qquad H_1: \theta < 0$

By large sample app imation. $T^* \approx -1.01$

$\hat{p} = P\{Z \leq T^*\} \approx 0.24 \Rightarrow$ fails to reject.

No enough evidence to support the alternative hypothesis.

17. What are the possible values of T^+ when $n = 8$? Suppose you are testing $H_0 : \theta = 0$ versus $H_2 : \theta < 0$ and you want your α level to be between .05 and .10. What are the tests available?

$1+2+3+4 \cdots + 8 = 36$

$35 = 36-1 = 2+3+\cdots 8$

$34 = 36-2 = 1+3+\cdots 8$

$\vdots$

$29 = 36-7 = 1+\cdots 6+8$

28 degenerate to case of $n=7$

Iteratively, we can see that T^+ can be any value from 0 to 36

We may use Wilcoxon sign-rank test.

43. The data in Table 3.6 are a portion of the data obtained by Cooper et al. (1967). The purpose of their investigation was to determine whether hypnotic susceptibility as measured on objective scales can be changed with practice and training. The objective measures used were the Stanford Profile Scales of Hypnotic Susceptibility, forms I and II (Hilgard, Lauer, and Morgan (1963)). The subjects were administered these Profile Scales, both forms I and II, by a hypnotist other than the experimenter. Each subject was then seen by one of the authors for an extensive period of "hypnotic training." After these sessions were concluded, each subject was retested by a different hypnotist (again not the experimenter) using equivalent forms of the Profile Scales, forms I′ and II′. Table 3.6 gives the average score obtained on forms I and II prior to hypnotic training (X) and the corresponding average score obtained on forms I′ and II′ after the training (Y) for the six subjects. Note that a high (or low) score on the Profile Scales indicates a high (or low) degree of hypnotic susceptibility.

Test the hypothesis of no change in hypnotic susceptibility versus the alternative that hypnotic susceptibility (as measured by the Profile Scales) can be increased with practice and training.

Table 3.6 Average Scores on the Stanford Profile Scales of Hypnotic Susceptibility

Subject i	X_i	Y_i	Z_i	ψ_i
1	10.5	18.5	7	1
2	19.5	24.5	5	1
3	7.5	11.0	3.5	1
4	4.0	2.5	-1.5	0
5	4.5	5.5	1	1
6	2.0	3.5	1.5	1

Source: L. M. Cooper, E. Schubot, S. A. Banford, and C. T. Tart (1967).

$H_0: \theta = 0 \quad H_1: \theta > 0, \quad B = \sum_{i=1}^{6} \psi_i$

$B_{obs} = \sum_{i=1}^{6} \psi_i = 5$

Under H_0:

$P\{B \geq B_{obs}\} = P\{B=5\} + P\{B=6\}$

$= \frac{1}{2^6}(6+1)$

$= \frac{7}{64} \approx 0.11$

Fail to reject H_0 $\Rightarrow$

No enough evidence to support the alternative hypothesis.

44. Change the value of Y_3 in Table 3.5 from 73 to 173. What effect does this outlying observation have on the calculations performed in Example 3.5? What does this suggest about the relative insensitivity of the sign tests to outliers? Construct an example in which changing one observation has an effect on the final decision regarding rejection or acceptance of H_0.

Table 3.5 Beak-Clapping Counts per Minute

Embryo i	X_i (Dark period)	Y_i (Illumination)	$Z_i = Y_i - X_i$	ψ_i
1	5.8	5	−0.8	0
2	13.5	21	7.5	1
3	26.1	~~73~~ 173	46.9	1
4	7.4	25	17.6	1
5	7.6	3	−4.6	0
6	23.0	77	54.0	1
7	10.7	59	48.3	1
8	9.1	13	3.9	1
9	19.3	36	16.7	1
10	26.3	46	19.7	1
11	17.5	9	−8.5	0
12	17.9	25	7.1	1
13	18.3	59	40.7	1
14	14.2	38	23.8	1
15	55.2	70	14.8	1
16	15.4	36	20.6	1
17	30.0	55	25.0	1
18	21.3	46	24.7	1
19	26.8	25	−1.8	0
20	8.1	30	21.9	1
21	24.3	29	4.7	1
22	21.3	46	24.7	1
23	18.2	71	52.8	1
24	22.5	31	8.5	1
25	31.1	33	1.9	1

Source: R. W. Oppenheim (1968).

If we change Y_3 to 173

$Z_3 = 146.9 \quad \psi_3 = 1$

The test statistic does not change

so the test remains the same

We see that as long as the sign of Z does not change, the test statistic and result remain the same. The signed test is relatively insensitive to extreme value.

Consider the table with $n=25$ and $\hat{B}_{obs}^{(1)} = 18$. From the Example 3.5, we may still reject H_0.

But now, we change Y_3 to -7^3 then $\psi_3 = 0$ and $\hat{B}_{obs}^{(2)} = 17$. At this point, we fail to reject H_0.