1. The data in Table 4.3 are a subset of the data obtained by Thomas and Simmons (1969), who investigated the relation of sputum histamine levels to inhaled irritants or allergens. The histamine content was reported in micrograms per gram of dry weight of sputum. The subjects for this portion of the study consisted of 22 smokers; 9 of them were allergics and the remaining 13 were asymptomatic (nonallergic) individuals. Care was taken to avoid people who carried out part of their daily work in an atmosphere of noxious gases or other respiratory toxicants. Table 4.3 gives the ordered sputum histamine levels for the 22 individuals in the study.

Test the hypothesis of equal levels versus the alternative that allergic smokers have higher sputum histamine levels than nonallergic smokers. Use the large-sample approximation.

Table 4.3 Sputum Histamine Levels ($\mu g/g$ Dry Weight Sputum)

Allergics	Nonallergics
1651.0	48.1
1112.0	48.0
102.4	45.5
100.0	41.7
67.6	35.4
65.9	34.3
64.7	32.4
39.6	29.1
31.0	27.3
	18.9
	6.6
	5.2
	4.7

Source: H. V. Thomas and E. Simmons (1969).

Let Y_i be Sputum Histamine levels for Allergics

X_i be ~~for~~ Nonallergics.

Assume $X_i \overset{i.i.d.}{\sim} F$ $\quad Y_i \overset{i.i.d.}{\sim} G$.

H_0: $F(x-\Delta) = G(t)$, $\Delta = 0$ $\quad H_1$: $\Delta > 0$

Let S_j be the rank of Y_j

$\hat{W} = \sum_{j=1}^{9} S_j = 151$ $\quad$ Under H_0: $E(\hat{W}) = 103.5$ $\quad Var(\hat{W}) = 224.25$

$W = \hat{W} - E(\hat{W}) / \sqrt{Var(\hat{W})} \approx 3.17$ $\quad \hat{p} = P\{Z \geq W\} < 0.05$

We may reject H_0.

14. Suppose $m = n = 7$. Compare the exact $\alpha = .049$ test of $H_0 : \Delta = 0$ versus $H_1 : \Delta > 0$ based on W with its corresponding test based on large-sample approximation. What is the exact α value of the test based on the large-sample approximation whose nominal α value is .049?

For exact $\alpha = 0.049$. $\quad \hat{W} = 66$.

Under H_0: $E(\hat{W}) = 7(14+1)/2 = 52.5$

$Var(\hat{W}) = 61.25$

$W = \hat{W} - E(\hat{W}) / \sqrt{Var(\hat{W})} \sim N(0,1)$, $\quad W = 1.725$

$\hat{p} = P\{Z \geq W\} \approx 0.042$

1. Consider the chorioamnion permeability data in Table 4.1. In Section 4.1 we saw that a test procedure based on the Wilcoxon rank sum statistic did not reject the null hypothesis that the human chorioamnion is as permeable to water transfer at 12–26 weeks gestational age as it is at term. With this in mind and using the same data, test the hypothesis of equal dispersions versus the alternative that the variation in tritiated water diffusion across human chorioamnion is different at term than at 12–26 weeks gestational age.

Table 4.1 Tritiated Water Diffusion Across Human Chorioamnion

$Pd(10^{-4}$ cm/s)	
At term	12–26 Weeks gestational age
0.80	1.15
0.83	0.88
1.89	0.90
1.04	0.74
1.45	1.21
1.38	
1.91	
1.64	
0.73	
1.46	

Source: S.J. Lloyd, K.D. Garlid, R.C. Reba and A.E. Seeds (1969).

$N=15$ $n=5$, Assign scores $1,2,3,\cdots 7,8,7,\cdots\cdots 3,2,1$.

Assume X_i is "At term" Y_i is "12-26 weeks"

H_0: Two samples have the same variation.

$\hat{C} = 8+5+6+2+1 = 28$.

Under H_0, $E(\hat{C}) = n(N+1)^2/4N \approx 21.33$

$Var(\hat{C}) = nm(N+1)(N^2+3)/48N^2$

≈ 202.67.

$C = (\hat{C} - E(\hat{C}))/\sqrt{Var(\hat{C})} = 0.469$.

$\hat{p} = P\{Z \geq C\} \approx 0.319 \geq 0.05$.

fail to reject H_0.

4. Verify the expressions for $E_0(C)$ and $\text{var}_0(C)$ in (5.12) and (5.13), respectively, when $N = (m+n)$ is an odd integer. (See Comment 9 for guidance.)

N is odd

$$\mu_{pop} = \frac{1}{N}\left([1+(N-1)/2][(N-1)/2] + \frac{N+1}{2}\right) = (N+1)^2/4N$$

$$\sigma^2_{pop} = \left\{\frac{2}{N}\left[\sum_{i=1}^{(N-1)/2} i^2 + \frac{1}{N}\left(\frac{N+1}{2}\right)^2 - \left((N+1)^2/4N\right)^2\right\}\right.$$

$$= (N+1)(N^2+3)(N-1)/48N^2$$

Under H_0.

$$E(\hat{C}) = n\mu_{pop} = \frac{n(N+1)^2}{4N}$$

$$Var(\hat{C}) = n^2 Var(\hat{C}/n) = mn(N+1)(N^2+3)/48N^2$$