No. 1

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STA 4003 Assignments. 4.

3.7.

(a) $x_t + 1.6 x_{t-1} + 0.64 x_{t-2} = \omega_t$.

$$\phi(B) x_t = \omega_t \quad \text{where } \phi(B) = (1 + 1.6B + 0.64B^2)$$

$$\phi(z) = 0 \Rightarrow (0.8z + 1)^2 = 0$$

$$\Leftrightarrow z_1 = z_2 = -\frac{5}{4}$$

Difference Equation is given by:

$$\rho(h) + 1.6\rho(h-1) + 0.64\rho(h-2) = 0$$

~~for $\forall h \geq 1$~~

Initial condition:

$$\begin{cases} \rho(0) = 1 \\ \rho(1) + 1.6\rho(0) + 0.64\rho(1) = 0 \end{cases} \Rightarrow \begin{cases} \rho(0) = 1 \\ \rho(1) = \frac{-1.6}{1.64} \end{cases}$$

$$\rho(h) = (z_1)^{-h}(c_1 + c_2 h)$$

when $h = 0$, $c_1 = 1$

when $h = 1$, $c_1 + c_2 = -\frac{1.6}{1.64}$

$$\Rightarrow c_2 = -\frac{3.24}{1.64} \approx -\frac{81}{41}$$

$$\rho(h) = \left(-\frac{5}{4}\right)^{-h}\left(1 - \frac{81}{41}h\right)$$

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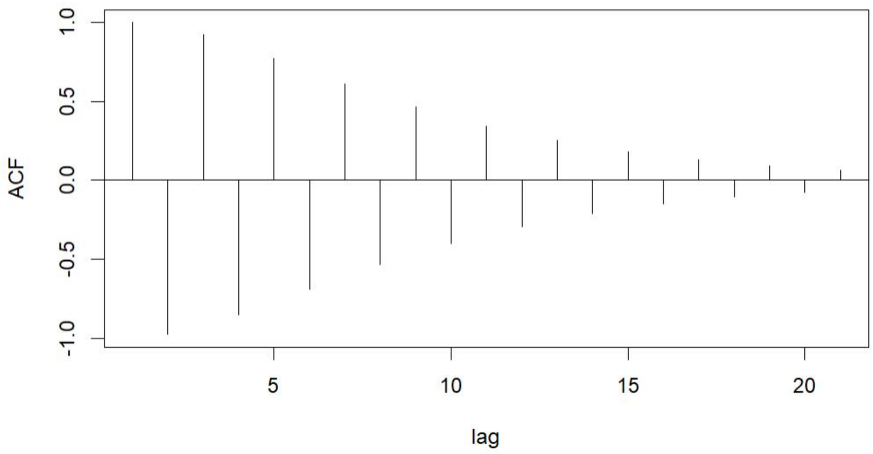
```{r}
ACF = ARMAacf(ar=c(-1.6,-0.64), ma=0, lag.max = 20)
plot(ACF, type="h", xlab="lag")
abline(h=0)
```



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(b) $x_t - 0.4x_{t-1} - 0.45x_{t-2} = w_t$

$\phi(B)x_t = w_t$ where $\phi(B) = (1-0.4B-0.45B^2)$

$\phi(z)=0 \iff (z+\frac{4}{9})^2 = \frac{196}{81}$

$z_1 = \frac{10}{9} \quad z_2 = -2.$

Difference Equation is given by

$\rho(h) - 0.4\rho(h-1) - 0.45\rho(h-2) = 0$

Initial condition:

$$\begin{cases} \rho(0) = 1 \\ \rho(1) - 0.4\rho(0) - 0.45\rho(1) = 0 \end{cases} \Rightarrow \begin{cases} \rho(0) = 1 \\ \rho(1) = \frac{8}{11} \end{cases}$$

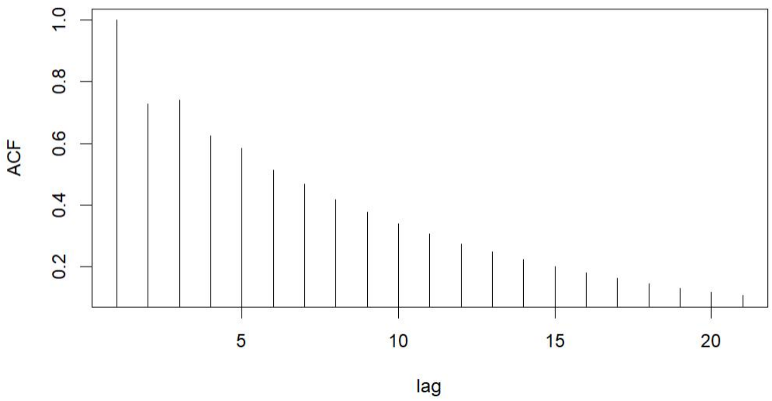
$$\rho(h) = (\tfrac{10}{9})^{-h} c_1 + (-2)^{-h} c_2$$

$$\begin{cases} \rho(1) = \frac{9}{10}c_1 - \frac{1}{2}c_2 = \frac{8}{11} \\ \rho(0) = c_1 + c_2 = 1 \end{cases}$$

$$\Rightarrow \begin{cases} c_1 = \frac{135}{154} \\ c_2 = \frac{19}{154} \end{cases}$$

$$\rho(h) = (\tfrac{10}{9})^{-h} \cdot \frac{135}{154} + (-2)^{-h} \cdot \frac{19}{154}$$

```{r}
ACF = ARMAacf(ar=c(0.4,0.45), ma=0, lag.max = 20)
plot(ACF, type="h", xlab="lag")
abline(h=0)
```



(c) $x_t - 1.2x_{t-1} + 0.85x_{t-2} = w_t$

$\phi(B)x_t = w_t$ where $\phi(B) = 1 - 1.2B + 0.85B^2$

$$\phi(z) = 0 \iff \left(z - \frac{12}{17}\right)^2 + \frac{196}{289} = 0$$

$$z_1 = \frac{12}{17} + \frac{14}{17}i \qquad z_2 = \frac{12}{17} - \frac{14}{17}i.$$

$$|z_1| = |z_2| = \frac{2\sqrt{85}}{17}.$$

$$z_1 = |z_1|(\cos\beta + i\sin\beta) = |z_1|\cdot e^{i\beta} = \frac{2\sqrt{85}}{17}e^{i\beta}$$

where $\cos\beta = \dfrac{12/17}{2\sqrt{85}/17} = \dfrac{6\sqrt{85}}{85}$, $\sin\beta = \dfrac{7\sqrt{85}}{85}$

$$\beta = \arccos\frac{6\sqrt{85}}{85}$$

$$z_2 = \frac{2\sqrt{85}}{17}\cdot e^{-i\beta}$$

Difference Equation

$$\rho(h) = c_1\cdot\left(\frac{2\sqrt{85}}{17}\right)^{-h}\cos(\beta h + c_2)$$

Initial condition:

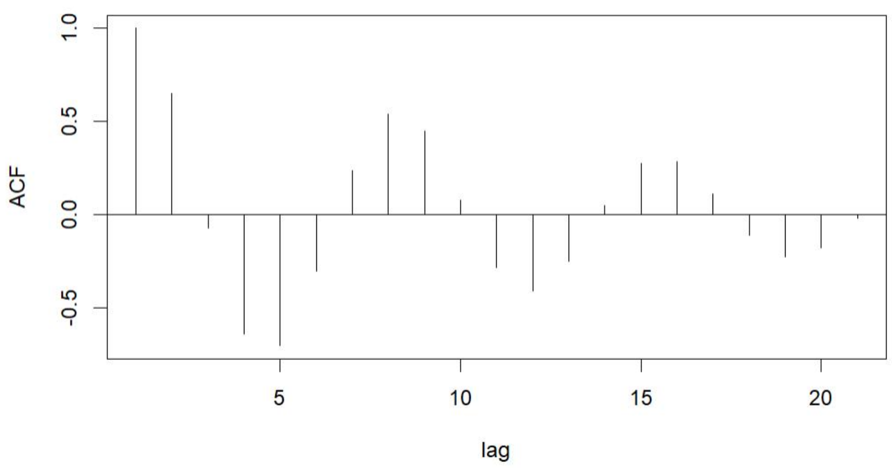
$$\begin{cases} \rho(0) = 1 \\ \rho(1) - 1.2\rho(0) + 0.85\rho(1) = 0 \end{cases} \Rightarrow \begin{cases} \rho(0) = 1 \\ \rho(1) = \frac{24}{37} \end{cases}$$

$$\begin{cases} \rho(0) = c_1\cdot\cos c_2 = 1 & \text{——— ①} \\ \rho(1) = \frac{17}{2\sqrt{85}}c_1\cos(\beta + c_2) = \frac{24}{37} & \text{— ②} \end{cases}$$

expand ②:

$$\frac{17}{2\sqrt{85}}c_1\left(\frac{6\sqrt{85}}{85}\cos c_2 - \frac{7\sqrt{85}}{85}\sin c_2\right) = \frac{24}{37}$$

```{r}
ACF = ARMAacf(ar=c(1.2,-0.85), ma=0, lag.max = 20)
plot(ACF, type="h", xlab="lag")
abline(h=0)
```



$$\Rightarrow \frac{3}{5} - \frac{7}{10}\sin C_2 = \frac{26}{37} \Rightarrow \sin C_2 = -\frac{18}{259}$$

$$\cos C_2 = \pm \frac{\sqrt{66757}}{259}$$

$$\Rightarrow C_1 = \pm \frac{259}{\sqrt{66757}}$$

$$C_2 = \arcsin\left(-\frac{18}{259}\right)$$

$$\rho(h) = \cancel{\frac{17}{2\sqrt{85}}} \pm \frac{259}{\sqrt{66757}} \cdot \left(\frac{2\sqrt{85}}{17}\right)^{-h} \cos\left[\beta + \arcsin\left(-\frac{18}{259}\right)\right]$$

where

$$\cos\beta = \frac{6\sqrt{85}}{85}, \quad \sin\beta = \frac{7\sqrt{85}}{85}$$

3.13.

Proof:

By Equation (3.63):

$$\Gamma_h \alpha_h = \gamma_h$$

Divided by $\gamma(0)$

$$\Rightarrow R_h \alpha_h = \rho_h$$

$$\Leftrightarrow \begin{bmatrix} R_{h-1} & \check{\rho}_{h-1} \\ \check{\rho}_{h-1}^T & \rho(0) \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_{hh} \end{bmatrix} = \begin{bmatrix} \rho_{h-1} \\ \rho(h) \end{bmatrix}$$

$$\Rightarrow \begin{cases} R_{h-1}\alpha_1 + \check{\rho}_{h-1}\alpha_{hh} = \rho_{h-1} & \text{—— ①} \\ \check{\rho}_{h-1}^T \alpha_1 + \rho(0)\alpha_{hh} = \rho(h) & \text{—— ②} \end{cases}$$

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① written as:

$$\alpha_1 = R_{h-1}^{-1}(\rho_{h-1} - \tilde{\rho}_{h-1}\alpha_{hh})$$

⇒ ② written as

$$\tilde{\rho}_{h-1}^T R_{h-1}^{-1}(\rho_{h-1} - \tilde{\rho}_{h-1}\alpha_{hh}) + \rho(0)\alpha_{hh} = \rho(h)$$

$$(\rho(0) - \tilde{\rho}_{h-1}^T R_{h-1}^{-1}\tilde{\rho}_{h-1})\alpha_{hh} = \rho(h) - \tilde{\rho}_{h-1}^T R_{h-1}^{-1}\rho_{h-1}$$

Since $\rho(0) = 1$,

$$\alpha_{hh} = \frac{\rho(h) - \tilde{\rho}_{h-1}^T R_{h-1}^{-1}\rho_{h-1}}{1 - \tilde{\rho}_{h-1}^T R_{h-1}^{-1}\tilde{\rho}_{h-1}}$$

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