STATS 451 Homework 8

Yuzhou Peng

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```
library(rstan)
```

```
## 载入需要的程辑包：StanHeaders
```

```
##
## rstan version 2.32.6 (Stan version 2.32.2)
```

```
## For execution on a local, multicore CPU with excess RAM we recommend calling
## options(mc.cores = parallel::detectCores()).
## To avoid recompilation of unchanged Stan programs, we recommend calling
## rstan_options(auto_write = TRUE)
## For within-chain threading using `reduce_sum()` or `map_rect()` Stan functions,
## change `threads_per_chain` option:
## rstan_options(threads_per_chain = 1)
```

```
## Do not specify '-march=native' in 'LOCAL_CPPFLAGS' or a Makevars file
```

Problem 1

```
rat_tumour_data <- list(J = 68,
                        y = c(0,0,0,0,0,0,0,0,0,0,0,0,1,1,1,1,1,1,1,2,2,2,2,2,2,2,2,
       2,1,5,2,5,3,2,7,7,3,3,2,9,10,4,4,4,4,4,4,4,10,4,4,4,5,
       11,12, 5,5,6,5,6,6,6,6,16,15,15,9,4),
                        n = c(20,20,20,20,20,19,19,19,19,18,17,20,20,20,20,19,19,18,18,25,24,
       23,20,20,20,20,20,20,10,49,19,46,27,17,49,47,20,20,13,48,50,20,
       20,20,20,20,20,20,48,19,19,19,22,46,49,20,20,23,19,22,20,20,20,
       52,46,47,24,14)
)

sum(rat_tumour_data$y)
```

```
## [1] 266
```

```
sum(rat_tumour_data$n)
```

```
## [1] 1681
```

```
rstan_options(auto_write = TRUE)
options(mc.cores = parallel::detectCores())
```

```
rat_tumour_fit <- stan(file = 'rat_tumour.stan', data = rat_tumour_data,
            iter = 1000, chains = 4, core = 4)
```

```
## recompiling to avoid crashing R session
```

```
## Warning: Bulk Effective Samples Size (ESS) is too low, indicating posterior means and median
s may be unreliable.
## Running the chains for more iterations may help. See
## https://mc-stan.org/misc/warnings.html#bulk-ess
```

Problem 2

```
print(rat_tumour_fit)
```

```
## Inference for Stan model: anon_model.
## 4 chains, each with iter=1000; warmup=500; thin=1;
## post-warmup draws per chain=500, total post-warmup draws=2000.
##
##             mean se_mean   sd   2.5%    25%    50%    75%  97.5% n_eff
## theta[1]    0.07    0.00 0.04   0.01   0.04   0.07   0.10   0.18  2281
## theta[2]    0.07    0.00 0.04   0.01   0.04   0.06   0.09   0.17  2006
## theta[3]    0.07    0.00 0.04   0.01   0.04   0.06   0.10   0.17  2313
## theta[4]    0.07    0.00 0.04   0.01   0.04   0.07   0.10   0.17  1829
## theta[5]    0.07    0.00 0.04   0.01   0.04   0.07   0.10   0.17  1896
## theta[6]    0.07    0.00 0.05   0.01   0.04   0.07   0.10   0.19  2603
## theta[7]    0.07    0.00 0.04   0.01   0.04   0.07   0.10   0.17  1918
## theta[8]    0.07    0.00 0.04   0.01   0.04   0.07   0.10   0.17  1576
## theta[9]    0.07    0.00 0.04   0.01   0.04   0.07   0.10   0.16  1684
## theta[10]   0.08    0.00 0.05   0.01   0.04   0.07   0.10   0.19  2030
## theta[11]   0.08    0.00 0.04   0.01   0.04   0.07   0.10   0.18  1583
## theta[12]   0.07    0.00 0.04   0.01   0.04   0.06   0.10   0.17  1883
## theta[13]   0.10    0.00 0.05   0.02   0.06   0.09   0.13   0.20  2331
## theta[14]   0.10    0.00 0.05   0.02   0.06   0.09   0.13   0.20  3130
## theta[15]   0.10    0.00 0.05   0.02   0.06   0.09   0.13   0.21  3137
## theta[16]   0.10    0.00 0.05   0.02   0.06   0.09   0.13   0.22  3968
## theta[17]   0.10    0.00 0.05   0.03   0.07   0.09   0.13   0.21  2250
## theta[18]   0.10    0.00 0.05   0.03   0.07   0.10   0.13   0.22  2582
## theta[19]   0.10    0.00 0.05   0.02   0.06   0.10   0.13   0.22  2519
## theta[20]   0.11    0.00 0.04   0.04   0.08   0.10   0.14   0.21  2694
## theta[21]   0.11    0.00 0.05   0.03   0.08   0.11   0.14   0.22  3487
## theta[22]   0.11    0.00 0.05   0.04   0.08   0.11   0.14   0.23  3693
## theta[23]   0.12    0.00 0.05   0.04   0.09   0.12   0.16   0.24  3170
## theta[24]   0.12    0.00 0.05   0.04   0.08   0.12   0.16   0.24  3894
## theta[25]   0.12    0.00 0.05   0.04   0.09   0.12   0.15   0.24  3886
## theta[26]   0.12    0.00 0.05   0.04   0.09   0.12   0.15   0.24  3451
## theta[27]   0.13    0.00 0.05   0.04   0.09   0.12   0.16   0.25  3472
## theta[28]   0.12    0.00 0.05   0.04   0.09   0.12   0.15   0.23  3526
## theta[29]   0.13    0.00 0.06   0.04   0.09   0.13   0.17   0.27  3296
## theta[30]   0.12    0.00 0.04   0.05   0.09   0.11   0.14   0.20  3511
## theta[31]   0.13    0.00 0.06   0.04   0.09   0.12   0.16   0.26  4215
## theta[32]   0.12    0.00 0.04   0.05   0.09   0.12   0.15   0.21  4113
## theta[33]   0.13    0.00 0.05   0.05   0.09   0.12   0.16   0.23  3662
## theta[34]   0.13    0.00 0.06   0.04   0.09   0.13   0.17   0.26  4644
## theta[35]   0.14    0.00 0.04   0.07   0.12   0.14   0.17   0.23  3361
## theta[36]   0.15    0.00 0.04   0.07   0.12   0.15   0.18   0.24  3877
## theta[37]   0.15    0.00 0.06   0.06   0.11   0.14   0.19   0.28  3777
## theta[38]   0.15    0.00 0.06   0.06   0.11   0.14   0.19   0.27  4310
## theta[39]   0.15    0.00 0.06   0.04   0.10   0.14   0.19   0.30  3927
## theta[40]   0.18    0.00 0.05   0.09   0.14   0.17   0.21   0.27  4088
## theta[41]   0.19    0.00 0.05   0.11   0.15   0.18   0.21   0.28  4104
## theta[42]   0.18    0.00 0.06   0.08   0.13   0.17   0.21   0.31  3937
## theta[43]   0.18    0.00 0.06   0.07   0.13   0.17   0.21   0.31  4165
## theta[44]   0.18    0.00 0.06   0.08   0.13   0.17   0.21   0.32  4465
## theta[45]   0.17    0.00 0.06   0.08   0.13   0.17   0.21   0.31  3909
## theta[46]   0.18    0.00 0.06   0.07   0.13   0.17   0.21   0.31  5088
## theta[47]   0.18    0.00 0.06   0.08   0.13   0.17   0.21   0.31  4226
## theta[48]   0.18    0.00 0.06   0.08   0.13   0.17   0.21   0.32  3503
## theta[49]   0.19    0.00 0.05   0.10   0.16   0.19   0.22   0.29  4475
## theta[50]   0.18    0.00 0.06   0.08   0.14   0.18   0.22   0.31  4979
```

```
## theta[51]    0.18    0.00 0.06    0.07    0.13    0.18    0.22    0.32  3913
## theta[52]    0.18    0.00 0.07    0.07    0.13    0.17    0.22    0.33  3228
## theta[53]    0.19    0.00 0.06    0.09    0.15    0.19    0.23    0.33  4101
## theta[54]    0.21    0.00 0.05    0.12    0.18    0.21    0.25    0.32  3655
## theta[55]    0.22    0.00 0.05    0.13    0.18    0.22    0.25    0.33  3682
## theta[56]    0.20    0.00 0.07    0.09    0.15    0.20    0.24    0.35  3776
## theta[57]    0.20    0.00 0.07    0.09    0.15    0.20    0.24    0.34  3778
## theta[58]    0.21    0.00 0.06    0.10    0.16    0.21    0.25    0.35  3703
## theta[59]    0.21    0.00 0.07    0.09    0.16    0.20    0.25    0.36  3885
## theta[60]    0.21    0.00 0.07    0.10    0.17    0.21    0.26    0.36  3448
## theta[61]    0.23    0.00 0.07    0.11    0.18    0.22    0.27    0.37  3493
## theta[62]    0.23    0.00 0.07    0.11    0.18    0.22    0.27    0.37  4028
## theta[63]    0.23    0.00 0.07    0.11    0.18    0.22    0.27    0.37  3324
## theta[64]    0.26    0.00 0.05    0.17    0.23    0.26    0.30    0.37  3440
## theta[65]    0.27    0.00 0.06    0.17    0.23    0.27    0.31    0.39  2986
## theta[66]    0.27    0.00 0.06    0.17    0.23    0.27    0.31    0.38  3572
## theta[67]    0.28    0.00 0.07    0.15    0.23    0.27    0.32    0.43  2365
## theta[68]    0.21    0.00 0.07    0.09    0.16    0.20    0.25    0.37  3833
## gamma        0.15    0.00 0.01    0.13    0.14    0.15    0.16    0.18  2125
## eta          0.24    0.00 0.04    0.16    0.21    0.23    0.26    0.33   426
## alpha        2.98    0.07 1.29    1.39    2.15    2.72    3.50    6.07   321
## beta        16.96    0.38 7.12    8.04   12.39   15.67   20.02   33.82   345
## lp__      -770.18    0.58 9.66 -789.63 -776.53 -769.97 -764.02 -751.49   274
##            Rhat
## theta[1]  1.00
## theta[2]  1.00
## theta[3]  1.00
## theta[4]  1.00
## theta[5]  1.00
## theta[6]  1.00
## theta[7]  1.00
## theta[8]  1.00
## theta[9]  1.00
## theta[10] 1.00
## theta[11] 1.00
## theta[12] 1.00
## theta[13] 1.00
## theta[14] 1.00
## theta[15] 1.00
## theta[16] 1.00
## theta[17] 1.00
## theta[18] 1.00
## theta[19] 1.00
## theta[20] 1.00
## theta[21] 1.00
## theta[22] 1.00
## theta[23] 1.00
## theta[24] 1.00
## theta[25] 1.00
## theta[26] 1.00
## theta[27] 1.00
## theta[28] 1.00
## theta[29] 1.00
## theta[30] 1.00
## theta[31] 1.00
## theta[32] 1.00
```

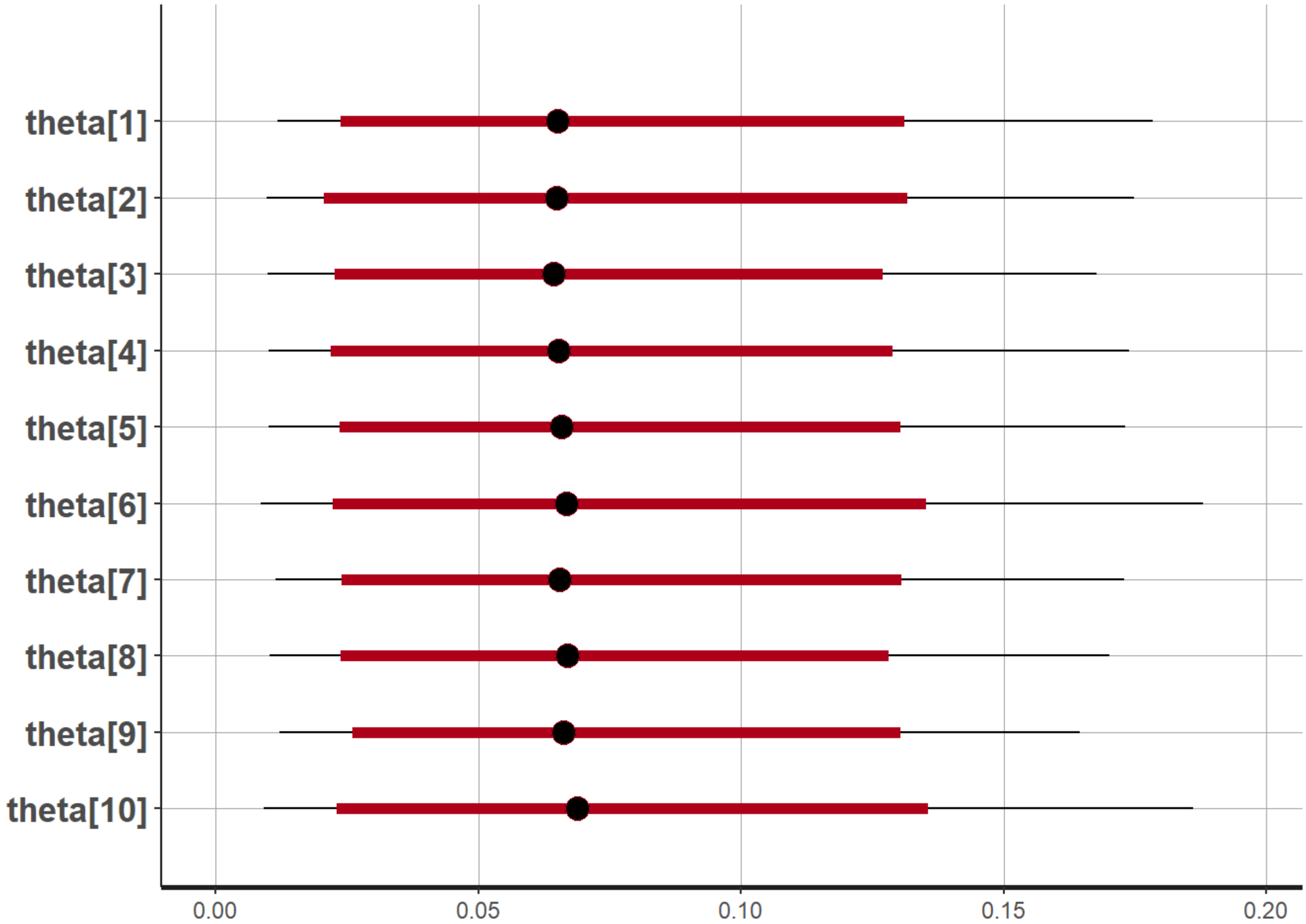
```
## theta[33] 1.00
## theta[34] 1.00
## theta[35] 1.00
## theta[36] 1.00
## theta[37] 1.00
## theta[38] 1.00
## theta[39] 1.00
## theta[40] 1.00
## theta[41] 1.00
## theta[42] 1.00
## theta[43] 1.00
## theta[44] 1.00
## theta[45] 1.00
## theta[46] 1.00
## theta[47] 1.00
## theta[48] 1.00
## theta[49] 1.00
## theta[50] 1.00
## theta[51] 1.00
## theta[52] 1.00
## theta[53] 1.00
## theta[54] 1.00
## theta[55] 1.00
## theta[56] 1.00
## theta[57] 1.00
## theta[58] 1.00
## theta[59] 1.00
## theta[60] 1.00
## theta[61] 1.00
## theta[62] 1.00
## theta[63] 1.00
## theta[64] 1.00
## theta[65] 1.00
## theta[66] 1.00
## theta[67] 1.00
## theta[68] 1.00
## gamma     1.00
## eta       1.00
## alpha     1.01
## beta      1.01
## lp__      1.01
##
## Samples were drawn using NUTS(diag_e) at Fri Apr 12 13:56:05 2024.
## For each parameter, n_eff is a crude measure of effective sample size,
## and Rhat is the potential scale reduction factor on split chains (at
## convergence, Rhat=1).
```

```
plot(rat_tumour_fit)
```

```
## 'pars' not specified. Showing first 10 parameters by default.
```

```
## ci_level: 0.8 (80% intervals)
```

```
## outer_level: 0.95 (95% intervals)
```




```
rat_tumour_sample <- extract(rat_tumour_fit)
```

```
mean(rat_tumour_sample$gamma < 0.2)
```

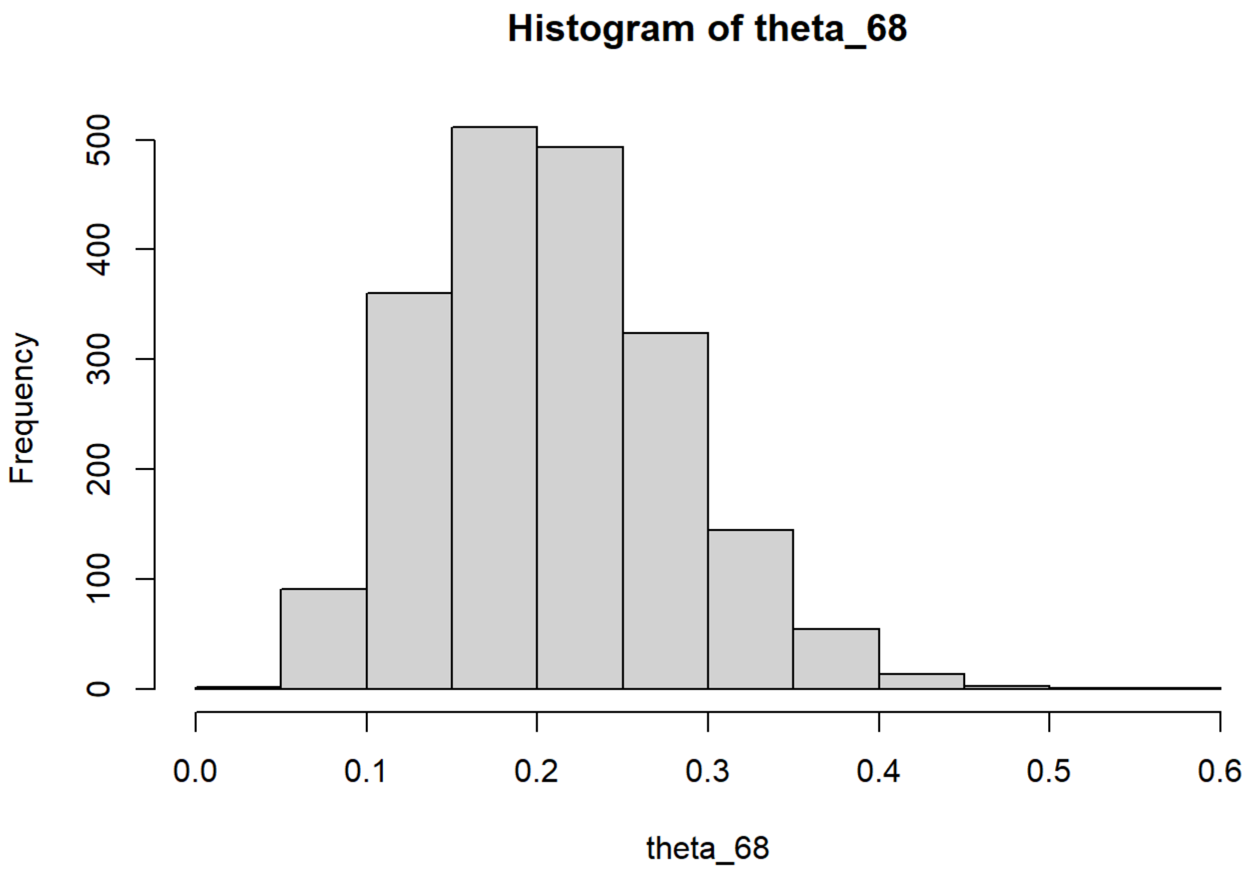
```
## [1] 1
```

We obtained samples from the target distribution.
(1) The expected value of alpha is around 2.8.
(2) Transformed parameter gamma corresponds to alpha/(alpha + beta). The probability of gamma is less than 0.2 is 0.99.

Problem 3

```
theta_68 <- rat_tumour_sample$theta[,68]

hist(theta_68)
```



The posterior distribution of theta 68 is beta distribution with mean value around 0.21,
which is smaller than single MLE estimate that yields 4/14 = 0.285 as mean value and larger than the overall MLE estimate which is 266/1681 = 0.16.
The posterior from hierarchical model is a compromise between total homogeneity model and heterogeneity model.