

STATS/DATASCI 451 Homework Assignment 2

Bayesian Inference for Discrete Observables and Discrete Unknowns

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Due: 02/09/2024 at 6 pm EST

Reference materials for lectures so far: Chapters 1-6 in “Doing Bayesian Data Analysis”, Chapter 1 in “Bayesian Data Analysis”.

Reference material for this homework assignment: Chapter 1 in “Bayesian Data Analysis”, Sections 1.1-1.5.

Approximately $\frac{1}{125}$ of all births are fraternal twins and $\frac{1}{300}$ of births are identical twins.

- Elvis Presley had a twin brother (who died at birth).
- At birth, without any human intervention, boys slightly outnumber girls, at a rate of around 105 boys per 100 girls.

What is the probability that Elvis was an identical twin? Please specify prior, likelihood, and posterior and show all your reasoning and detailed calculations.

Let $\theta = \begin{cases} 1, & \text{identical twin} \\ 0, & \text{otherwise} \end{cases}$ $\mu = \begin{cases} 1, & \text{fraternal twins} \\ 0, & \text{otherwise} \end{cases}$

$Y_i = \begin{cases} 1, & i\text{-th child is male at birth} \\ 0, & \text{otherwise} \end{cases}$

$$P\{\theta=1 \mid Y_1=1, Y_2=1\}$$

$$= \frac{P\{\theta=1\} \cdot P\{Y_1=1, Y_2=1 \mid \theta=1\}}{P\{Y_1=1, Y_2=1\}}$$

$$= \frac{P\{\theta=1\} \cdot P\{Y_1=1, Y_2=1 \mid \theta=1\}}{P\{Y_1=1, Y_2=1 \mid \theta=1\} P\{\theta=1\} + P\{Y_1=1, Y_2=1 \mid \mu=1\} P\{\mu=1\}}$$

$$= \frac{\frac{1}{300} \cdot \frac{105}{205}}{\frac{1}{300} \cdot \frac{105}{205} + \frac{1}{125} \cdot \left(\frac{105}{205}\right)^2} \quad 743.089$$

$$\approx \frac{0.0033333}{0.0033333 + 0.00409756} \approx 0.4486$$

prior: $P\{\theta\}$: probability of identical twin.

likelihood: $P\{Y_1, Y_2 | \theta\}$: probability of two brothers given they are identical twin.

posterior: $P\{\theta | Y_1, Y_2\}$: probability of identical twin given two brothers.