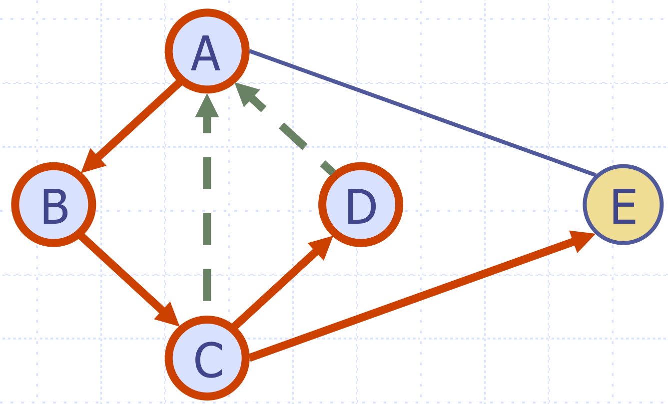
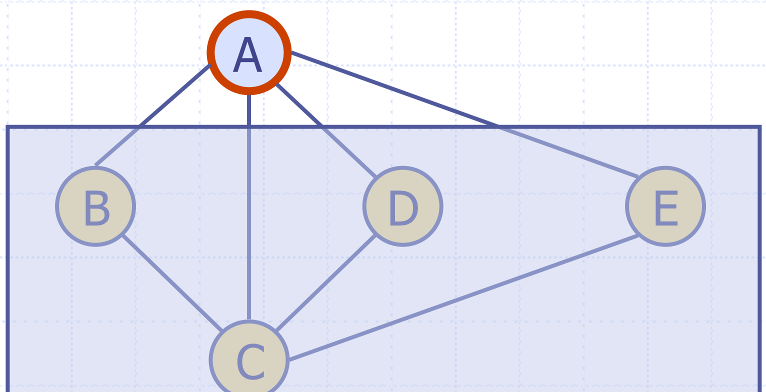
Graph Traversal
Depth-First Search
Breadth-First Search



Graph traversal - Idea

- Problem:
  - you visit each node in a graph, but all you have to start with is:
    - One vertex A
    - A method getNeighbors(vertex v) that returns the set of vertices adjacent to v



Graph traversal - Motivations

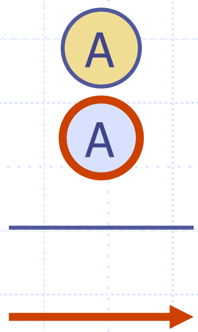
- Applications
  - Exploration of graph not known in advance, or too big to be stored:
    - Web crawling
    - Exploration of a maze
  - Graph may be computed as you go. Example: game strategy:
    - Vertices = set of all configurations of a Rubik's cube
    - Edges connect pairs of configuration that are one rotation away.

Depth-First Search

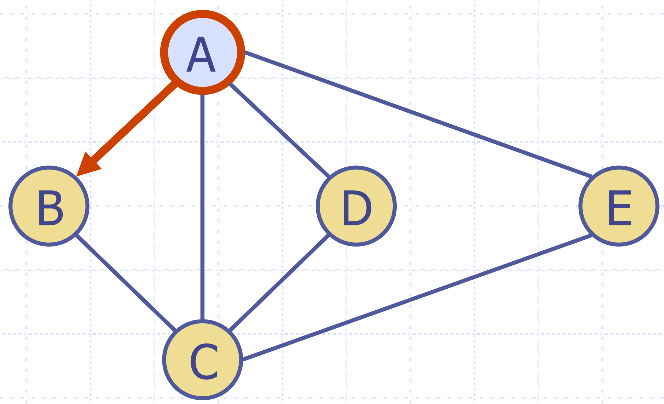
- Idea: Go Deep!
  - Intuition: Adventurous web browsing: always click the first unvisited link available. Click "back" when you hit a deadend.

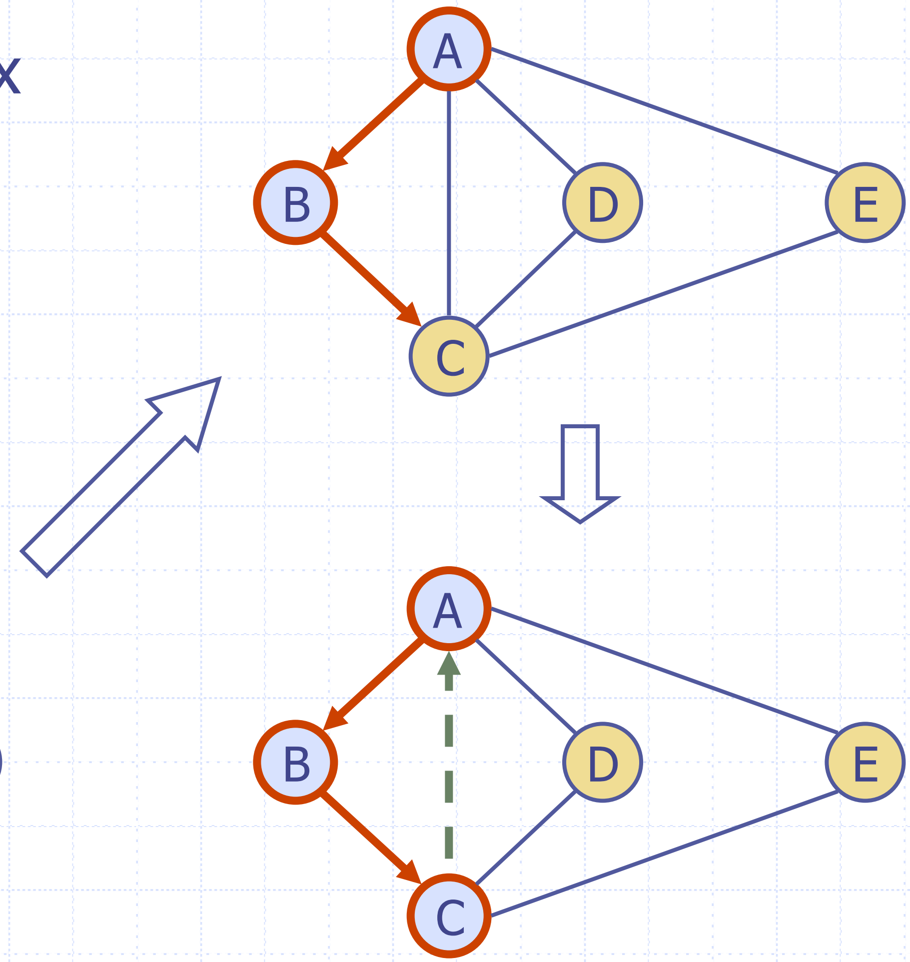
  - Start at some vertex v
  - Let w be the first neighbor of v that is not yet visited. Move to w.
  - If no such **unvisited** neighbor exists, move back to the vertex that lead to v

Example

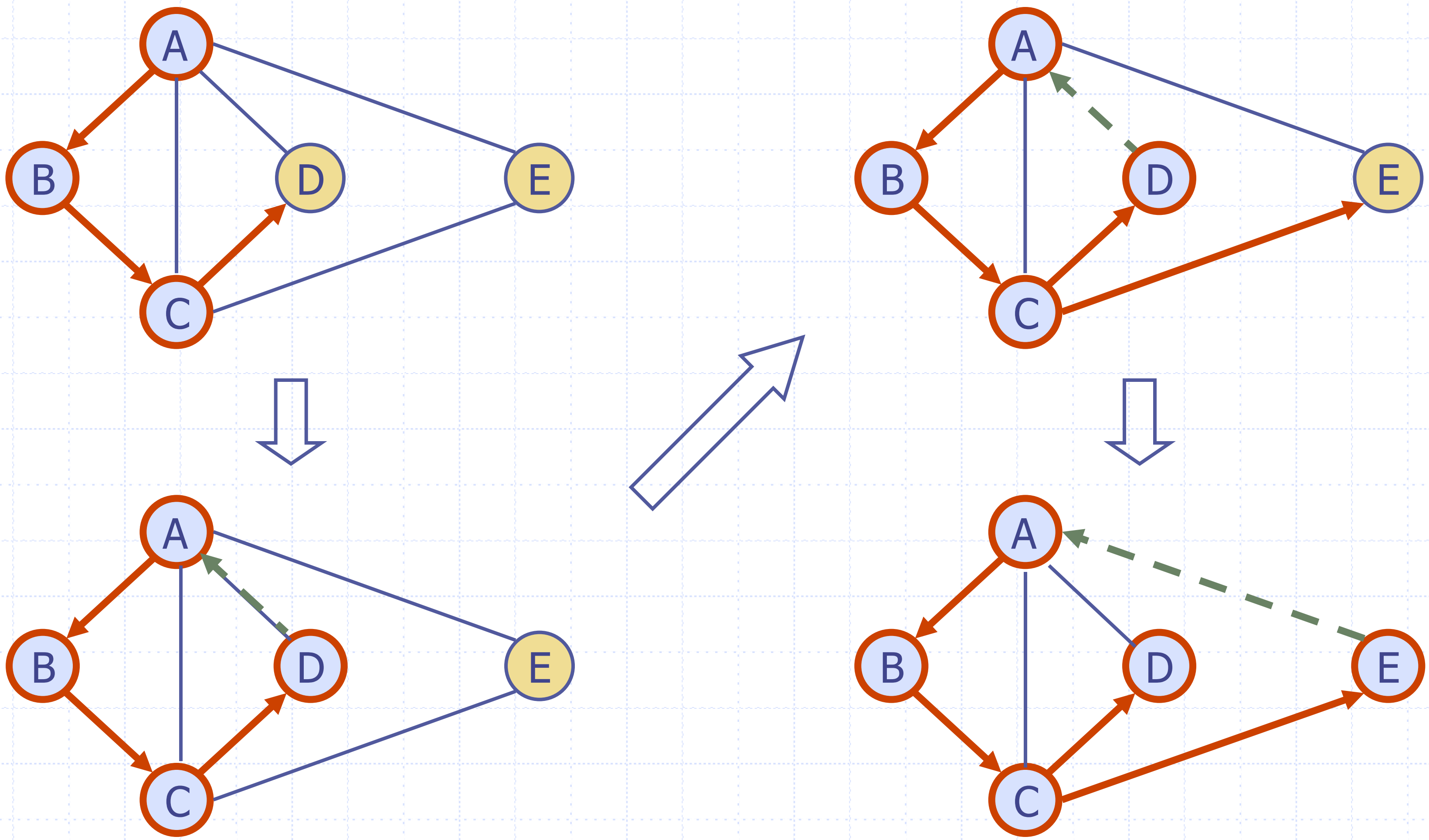


- unexplored vertex
- visited vertex
- unexplored edge
- discovery edge





Example (cont.)



DFS Algorithm

Algorithm ***DFS*(*G*, *v*)**

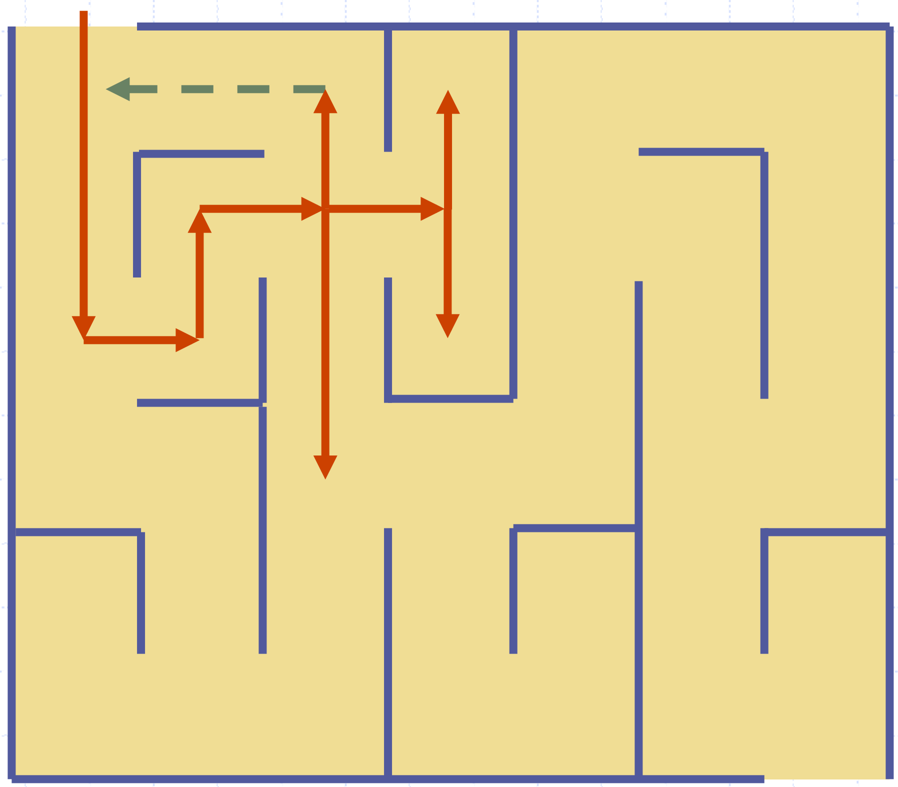
Input: graph ***G*** with no parallel edges and a start vertex ***v*** of ***G***

Output: Visits each vertex once (as long as G is connected)

```
 print v                // or do some kind of processing on v
v.setLabel(VISITED)
  for all  u ∈ v.getNeighbors()
    if (  u.getLabel() != VISITED ) then  DFS(G, u)
```

DFS and Maze Traversal

- The DFS algorithm is similar to a classic strategy for exploring a maze
  - We mark each intersection, corner and dead end (vertex) visited
  - We mark each corridor (edge) traversed
  - We keep track of the path back to the entrance (start vertex) by means of a rope (recursion stack)



DFS and Rubik's cube

- Rubik's cube game can be represented as a graph:
  - Vertices: Set of all possible configurations of the cube
  - Edges: Connect configurations that are just one rotation away from each other
- Given a starting configuration S, find a path to the "perfect" configuration P
- Depth-first search could in principle be used:
  - start at S and making rotations until P is reached, avoiding configurations already visited
- Problem: The graph is huge: 43,252,003,274,489,856,000 vertices

Running time of DFS

- DFS(G, v) is called once for every vertex v (if G is connected)
- When visiting node v, the number of iterations of the for loop is deg(v).
- Conclusion: The total number of iterations of all for loops is: $\Sigma_v \deg(v) = ?$
- Thus, the total running time is O(|E|)

Applications of variants of DFS

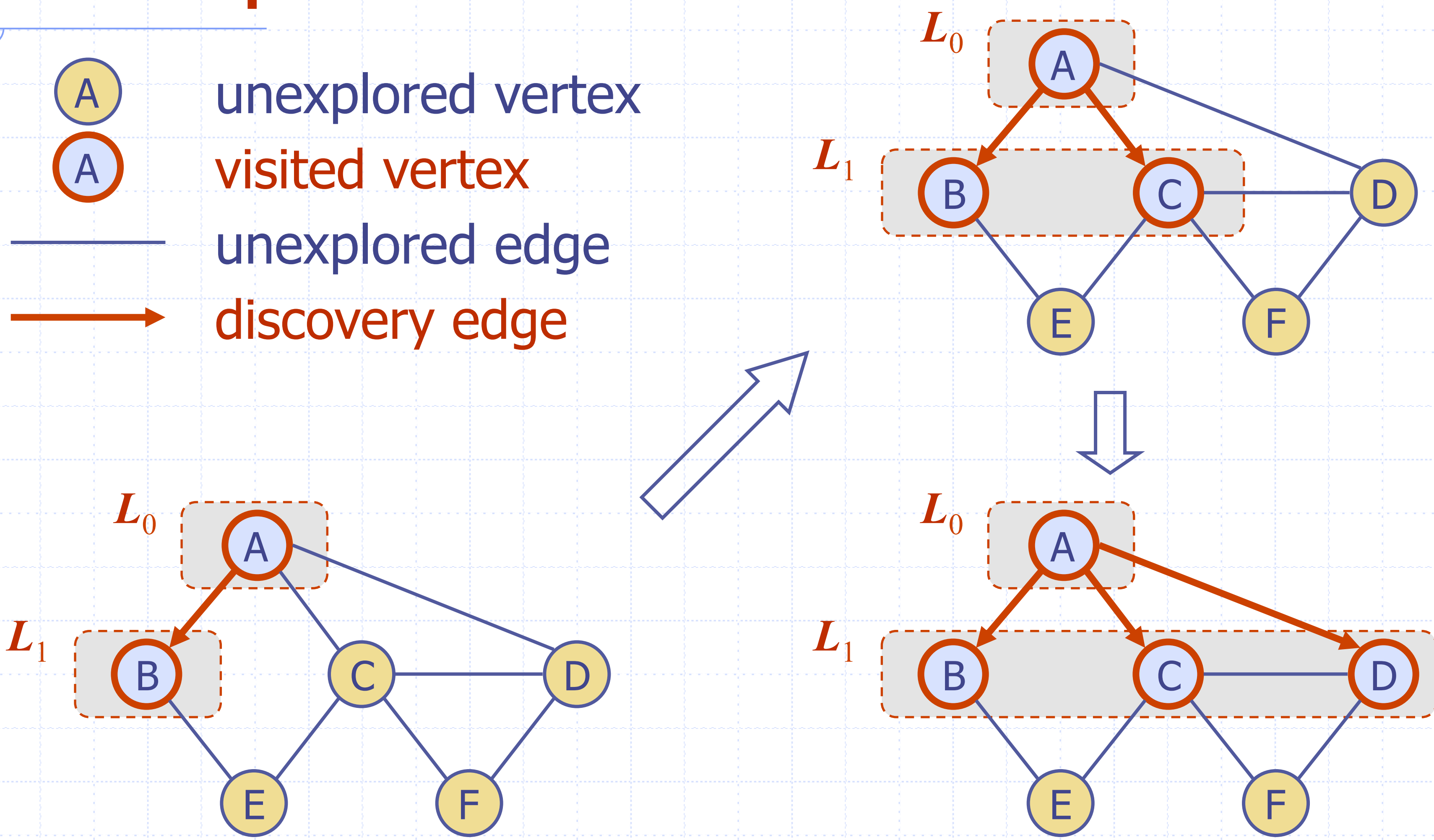
- DFS can be used to:
  - Determine if a graph is connected
  - Determine if a graph contains cycles
  - Solve games single-player games like Rubik's cube

Breadth-First Search

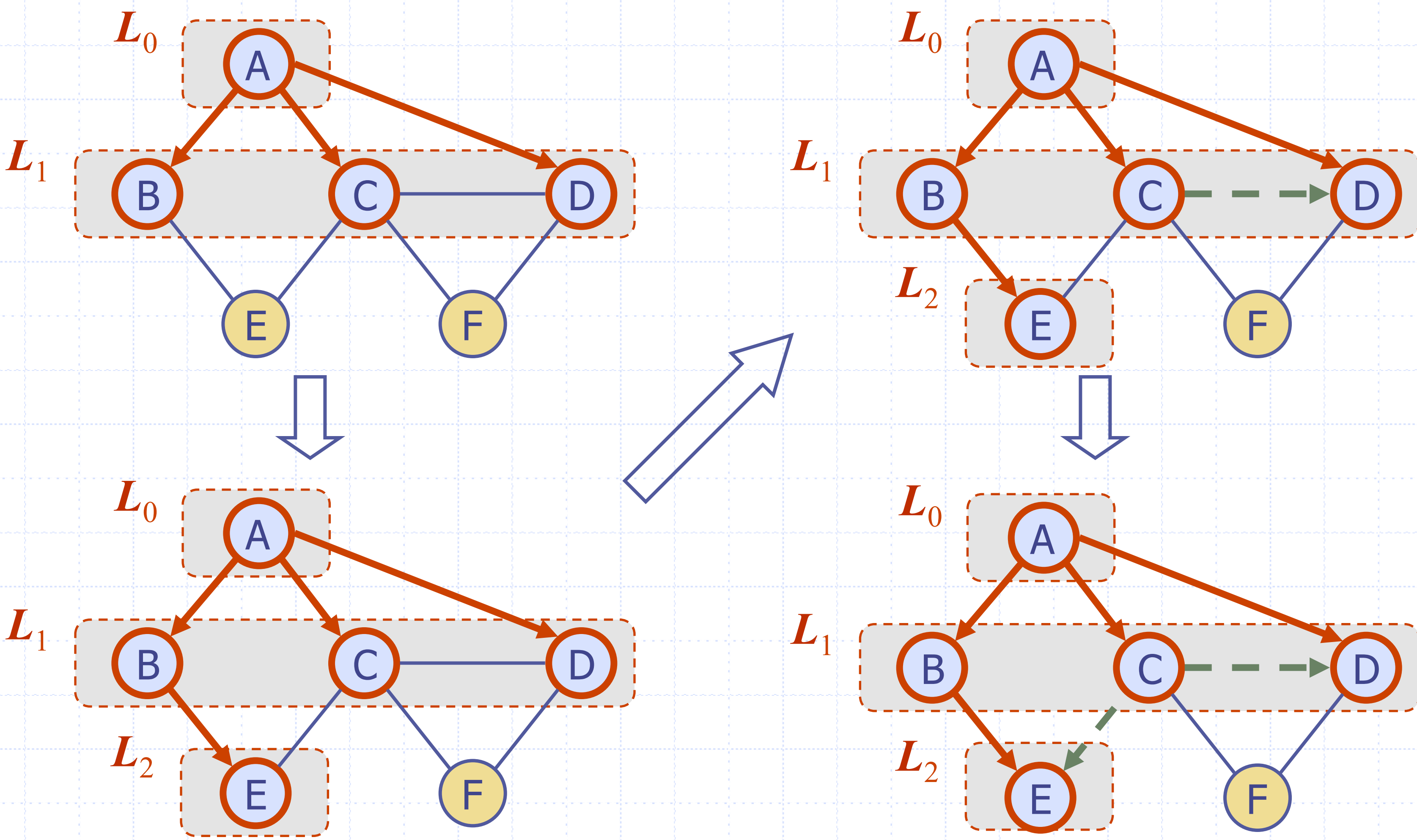
- Idea:
  - Explore graph layers by layers
  - Start at some vertex v
  - Then explore all the neighbors of v
  - Then explore all the unvisited neighbors of the neighbors of v
  - Then explore all the unvisited neighbors of the neighbors of the neighbors of v
  - until no more unvisited vertices remain

Example

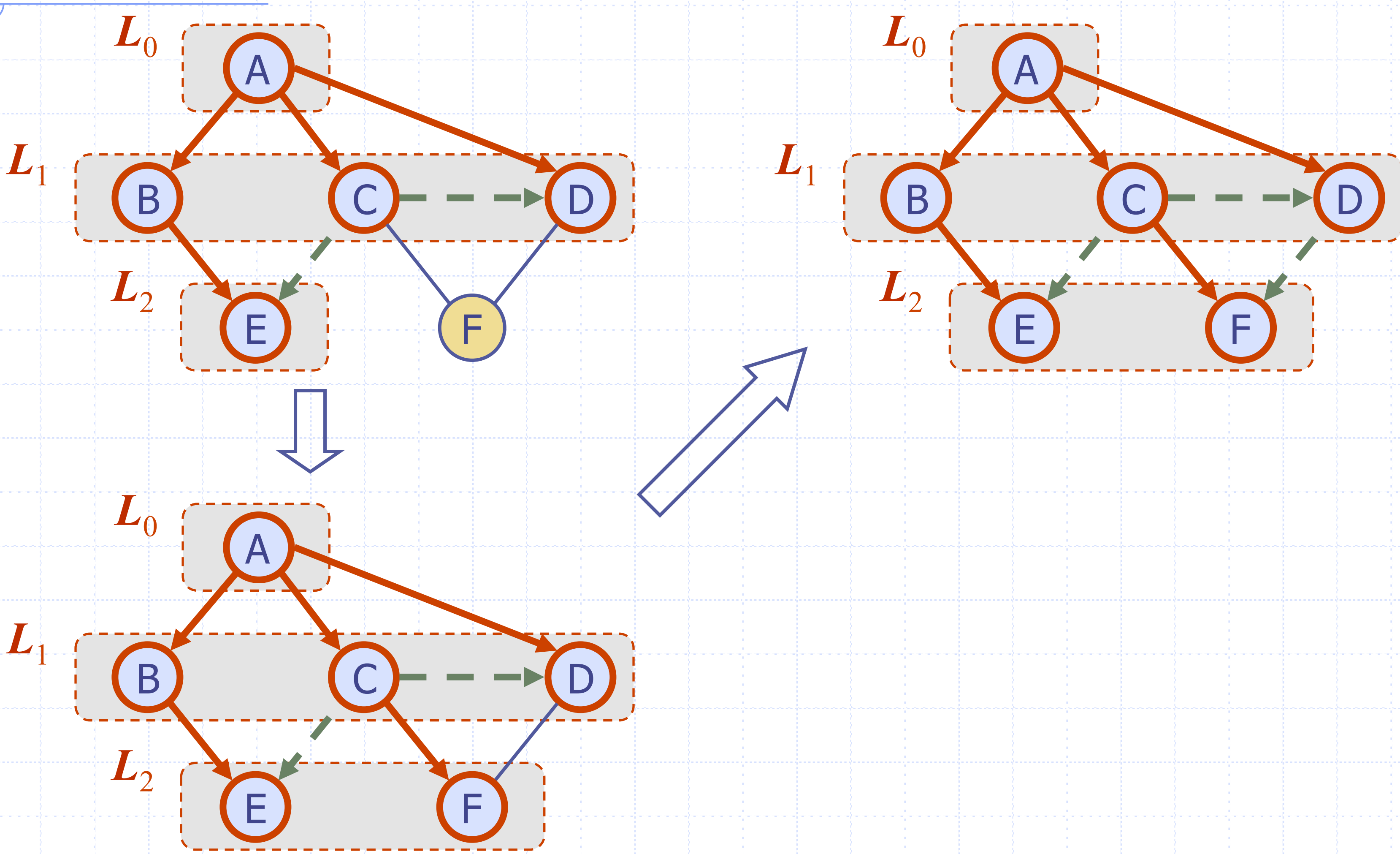
- unexplored vertex
- visited vertex
- unexplored edge
- discovery edge



Example (cont.)



Example (cont.)



Iterative BFS

- Idea: use a queue to remember the set of vertices on the frontier

```
Algorithm iterativeBFS(G, v)
  Input graph G with no parallel edges and a start vertex v of G
  Output Visits each vertex once (as long as G is connected)
  q ← new Queue()
  v.setLabel(VISITED)
  q.enqueue(v)
  while (! q.empty()) do
    w ← s.deque()
    print w          // or do some kind of processing on w
    for all  u ∈ w.getNeighbors() do
      if  ( u.getLabel() != VISITED ) then
        u.setLabel(VISITED)
        s.enqueue(u)
```

Running time and applications

- Running time of BFS: Same as DFS, O(|E|)
- BFS can be used to:
  - Find a shortest path between two vertices
    - Rubik's cube's fastest solution
  - Determine if a graph is connected
  - Determine if a graph contains cycles
  - Get out of an infinite maze...

Iterative DFS

- Use a stack to remember your path so far

Algorithm *iterativeDFS(G, v)*
Input graph *G* with no parallel edges and a start vertex *v* of *G*
Output Visits each vertex once (as long as G is connected)

```
s ← new Stack()
v.setLabel(VISITED)
s.push(v)
while (! s.empty()) do
    w ← s.pop()
    print w
    for all u ∈ w.getNeighbors() do
        if ( u.getLabel() != VISITED ) then
            u.setLabel(VISITED)
            s.push(u)
```

Notice: Code is identical to BFS, but with a stack instead of a queue