Theorem. *For all trees* `t`*,* `size t + acc` $=$ `size_acc t acc`*.*

Proof. By structural induction on the depth of `t`.

- **Base case**: `t = Leaf`

$$
\begin{aligned}
\texttt{size t + acc} &= \texttt{size Leaf + acc} \\
&\Rightarrow \texttt{0 + acc} && \text{By definition of } \texttt{size} \\
&= \texttt{acc} \\
&\Leftarrow \texttt{size\_acc Leaf} && \text{By definition of } \texttt{size\_acc} \\
&= \texttt{size\_acc t}
\end{aligned}
$$

- **Step**: `t` $=$ `Node(l,x,r)` for some trees `l`,`r` and value `x`.
  Inductive Hypothesis (1): `size l + acc` $=$ `size_acc l acc`, for some tree `l`
  Inductive Hypothesis (2): `size r + acc` $=$ `size_acc r acc`, for some tree `r`

$$
\begin{aligned}
&\texttt{size\_acc t acc} \\
&= \texttt{size\_acc Node(l,x,r) acc} \\
&\Downarrow \texttt{size\_acc l (x + size\_acc r acc)} && \text{By definition of } \texttt{size\_acc} \\
&= \texttt{size l + (x + size\_acc r acc)} && \text{By IH(1)} \\
&= \texttt{size l + (x + size r + acc)} && \text{By IH(2)} \\
&= \texttt{size l + x + size r + acc} && \text{By associativity of } \texttt{+} \\
&= \texttt{x + size l + size r + acc} && \text{By commutativity of } \texttt{+} \\
&\Leftarrow \texttt{size Node(l,x,r) + acc} && \text{By definition of } \texttt{size} \\
&= \texttt{size t + acc}
\end{aligned}
$$

□