DECISION MAKING IN FIANCE: OPTIMIZATION. UNIT 1 PROBLEMS
SOFTWARE: EXCEL

1. A petrol company produces three types of fuel with different octane numbers, 87, 89, and 91. These three fuels are obtained by mixing three types of crude oil, named (A, B, C) bought from another company. Each type of crude oil is made out of three different components (α, β, γ) according to the following table:

		COMPONENTS (%)		
		α	β	γ
CRUDE OIL	**A**	85	10	5
	B	55	25	20
	C	35	45	20

At the end of the process, each type of fuel must be composed of certain minimum or maximum concentration of the different components. Each type of fuel can produce a certain unitary benefit in the market. These specifications are summarized in the following table

			COMPONENTS (%)		
		Unitary benefit	α	β	γ
FUEL TYPE	**91**	1600	≥ 60	≤ 25	≥ 10
	89	1500	≥ 50	≤ 30	≤ 15
	87	1600	≤ 40	≥ 35	≥ 30

- ✓ The costs of each barrel of crude A, B and C are 750, 570 and 500 €, respectively.
- ✓ Availability of each type of crude oil is 4000, 3000 and 7000 barrels.
- ✓ Government constraints impose that at least 2000 barrels of 89-fuel must be produced.
- ✓ The number of barrels need not be inte ger.

a) Specify the optimal production plan maximizing the benefit satisfying the constraints above.
b) Another company offers 10 extra barrels of crude A at 800 euros per barrel. Without solving a new problem, and using the sensitivity analysis, would you advise to pay this price for these barrels?
c) The government is willing to reduce the minimum production of 89-fuel from 2000 to 1000, as long as the company donates 50 thousand euros for a public road construction project. From the sensitivity analysis, do you think this is a good idea for the company? What is the maximum amount of money you would donate for this reduction in the 89-fuel minimum production to be profitable?

In the last two questions assume we are within the allowed ranges of variation.

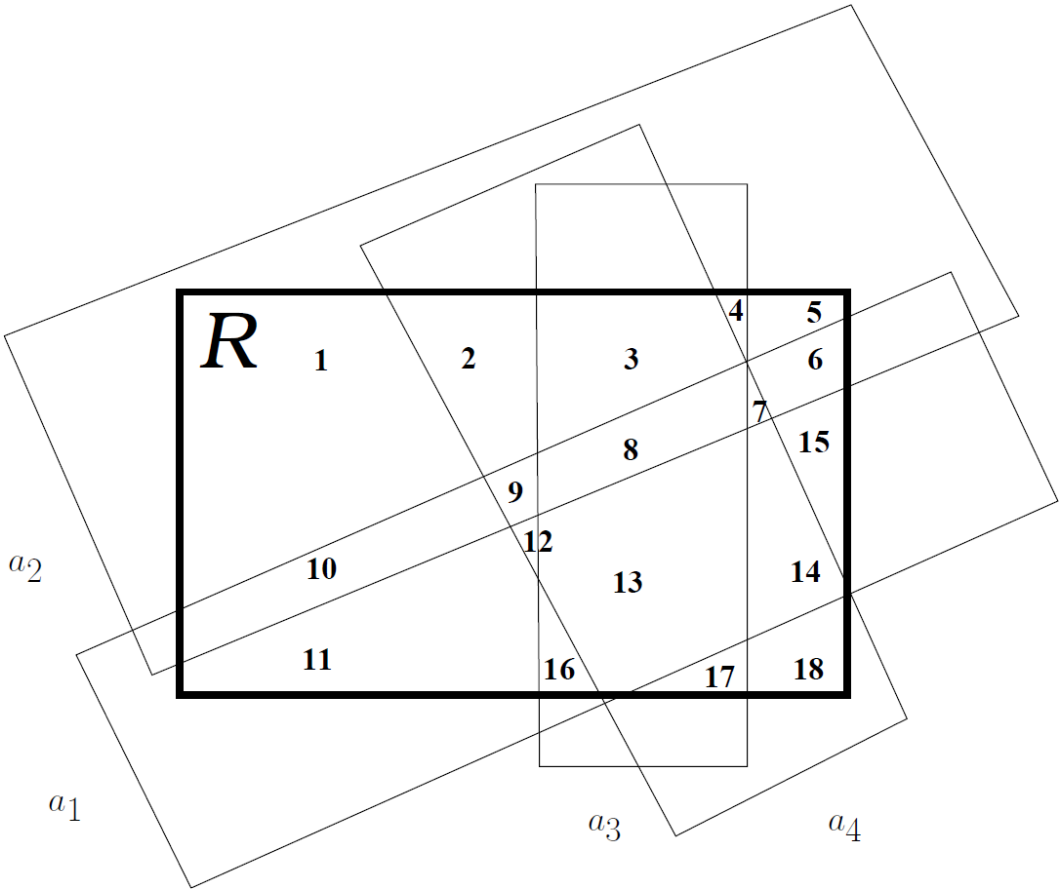
2. The manager of a company has to decide which worker has to do which task. There are four workers available and four tasks to perform. Each worker can do one task only, and all tasks must be done. The time that each task would take for each worker, in hours, is given in the following table:

	Task 1	Task 2	Task 3	Task 4
Worker 1	13	4	7	6
Worker 2	1	11	5	4

Worker 3	6	7	2	8
Worker 4	1.5	3	5	9

The company wishes to know which the assignment of workers to tasks that minimizes the total working time is, so that all tasks are done. Model a binary programming model to help you decide which workers do which tasks and solve it with EXCEL. Propose another method to solve this problem.

3. Consider the following rectangle *R*, which needs to be covered using the minimum number of satellites. There are four satellites available, and the area of *R* covered by each satellite is drawn in the following figure:



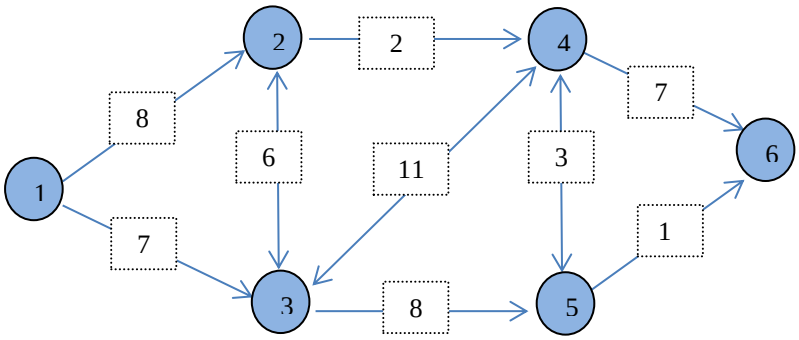
Each of the different subregions in which *R* is divided has been numbered. Model the previous problem using integer linear programming, and solve it.

4. Two providers located in Seattle and San Diego have 350 and 600 units, respectively, of a certain product in stock. Three clients, located in New York, Chicago and Topeka, need 325, 300 and 275 units of the same product, respectively.
The distance (in thousands of km) between each provider and each client is defined in the following table:

	New York	Chicago	Topeka
Seattle	2.5	1.7	1.8
San Diego	2.5	1.8	1.4

Transporting one unit of this product costs 90$ per 1000 km. Define an optimal distribution plan that satisfy the needs of the clients minimizing the transportation costs.

5. Consider the following network, in which there are 6 nodes and 9 edges. Each edge has its length in a box over it. Note that some edges can be traversed in both senses, and same others are one-direction only.



Propose an integer linear programming problem that defines the shortest path on the network from node 1 to node 6, and solve it.

Propose a general integer linear programming problem that defines the shortest path on the network from a generic source node *s* to a generic sink node *t*.

6. A company manufactures and sells household furniture. Each year the company signs contracts with raw materials suppliers to cover the needs for the year. Two raw materials are used: low and high quality wood boards. There are two suppliers that provide them. The prices that supplier A gives for the low quality wood boards depend on the quantity purchased. If up to 500 m^2 are purchased, the company pays 12 € per m^2. If more than 500 m^2 are purchased the price is 11 € per m^2. Supplier B charges 10 € per m^2 for the low quality wood boards with independence of the quantity purchased. For the high quality wood boards supplier A charges 24 €/m^2 and B 23 €/m^2. No fractions of m^2 are allowed.
The company sells three products that need these raw materials. The following table shows the raw materials needs as well as the minimum quantities that the company expects to sell during the next year (and hence those quantities must be manufactured):

Product	**Low quality wood boards (m^2/unit)**	**High quality wood boards (m^2/unit)**	**Minimum units (units)**
P1	6	1	500
P2	2	2	400
P3	4	6	280

There are two additional constraints:

1. The company wishes to force that if a given raw material is purchased to any supplier, the purchase should be no smaller than 100 m^2.
2. Supplier B is not capable of providing both raw materials to the company.

The company wants to know:

- Which suppliers provide which raw materials and in what quantity.
- Quantity to produce of each product during next year.

Write a non-linear model that would allow for the minimization of the raw materials purchase during next year. How could you linearize it?

7. We have four jobs and two machines. Each machine is capable of processing any of the four jobs. Each machine cannot process more than one job at the same time and, when a job begins its processing, it cannot be stopped until its completion. The processing time of each job j in each machine i is given by parameter p_{ij} (in hours) in the following table:

pij	Job1	Job2	Job3	Job4
M1	5	9	4	2
M2	3	7	2	5

1. Build an integer linear programming program that finds the assignment of jobs to machines, so that the maximum completion time (called *makespan*) is minimized.
2. Find the solution of a greedy algorithm to solve this problem, and explain how you would construct a GRASP algorithm.