Unit 1

Descriptive Statistics

ESIGELEC
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What can I do with so many data?

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Introduction

The objective of statistics is twofold:

- Collecting data with relevant information about a given population.
- Analyzing these data in order to extract information out of them

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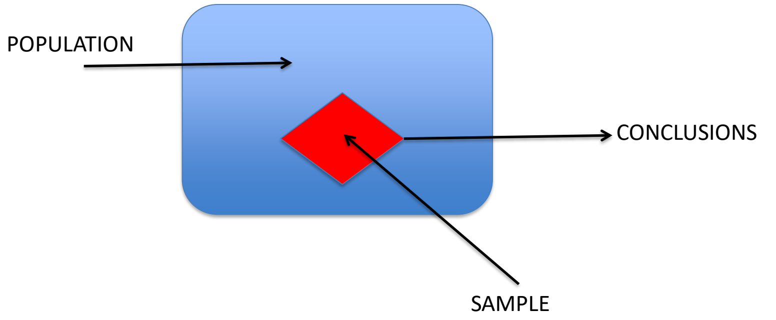
Inference (Units 4,5,6)

- We use inferential statistics when conclusions about populations are formed from sample data. Before that, we need some probability concepts!



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Descriptive statistics

- Data sets: Data obtained from observation, pools, experiments, etc.. They are called *sample,* and a sample is extracted from the *population.*
- Data are normally organized as a table or matrix (rows and columns) so that:
  - Each row represents one element of the sample.
  - Each column represents one observed characteristic.

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An example

Cardata.sf

	mpg	cylinders	displace	horsepower	accel	year	weight	origin	make	
1	43,1	4	90	48	21,5	78	1985	2	Volkswagen	R
2	36,1	4	98	66	14,4	78	1800	1	Ford	F
3	32,8	4	78	52	19,4	78	1985	3	Mazda	G
4	39,4	4	85	70	18,6	78	2070	3	Datsun	B
5	36,1	4	91	60	16,4	78	1800	3	Honda	C
6	19,9	8	260	110	15,5	78	3365	1	Oldsmobile	C
7	19,4	8	318	140	13,2	78	3735	1	Dodge	D
8	20,2	8	302	139	12,8	78	3570	1	Mercury	M
9	19,2	6	231	105	19,2	78	3535	1	Pontiac	P
10	20,5	6	200	95	18,2	78	3155	1	Chevrolet	M
11	20,2	6	200	85	15,8	78	2965	1	Ford	F
12	25,1	4	140	88	15,4	78	2720	1	Ford	F
13	20,5	6	225	100	17,2	78	3430	1	Plymouth	V
14	19,4	6	232	90	17,2	78	3210	1	AMC	C
15	20,6	6	231	105	15,8	78	3380	1	Buick	C
16	20,8	6	200	85	16,7	78	3070	1	Mercury	Z
17	18,6	6	225	110	18,7	78	3620	1	Dodge	A
18	18,1	6	258	120	15,1	78	3410	1	AMC	C
19	19,2	8	305	145	13,2	78	3425	1	Chevrolet	M
20	17,7	6	231	165	13,4	78	3445	1	Buick	R
21	18,1	8	302	139	11,2	78	3205	1	Ford	F
22	17,5	8	318	140	13,7	78	4080	1	Dodge	M
23	30	4	98	68	16,5	78	2155	1	Chevrolet	C
24	27,5	4	134	95	14,2	78	2560	3	Toyota	C
25	27,2	4	119	97	14,7	78	2300	3	Datsun	5
26	30,9	4	105	75	14,5	78	2230	1	Dodge	O
27	21,1	4	134	95	14,8	78	2515	3	Toyota	C
28	23,2	4	156	105	16,7	78	2745	1	Plymouth	S
29	23,8	4	151	85	17,6	78	2855	1	Oldsmobile	S

- First step: simple analysis of data → Descriptive statistics.
- What for?
  - To observe characteristics.
  - To summarize the information by means of :
    - Statistics (a numerical measurement describing some characteristics of a sample)
    - Graphic representations

- From now on we will use the following keywords:

Population	**Random variable**
Random sample	**Statistical data**

- Population: Is the complete collection of elements (scores, people, measurements, etc.) to be studied.

- **Example :** If you want to draw a study about the result in the following elections in France, the population will be the millions of people with right to vote in France.

- Example : If you want to study the quality of certain laptop model, the population would be all laptops of this model.

Elements : People, computers, etc.. All of them form the population. The individuals that form the population.

Randomness

- Random experiment : Is a process that, when it is repeated, generates the different elements of the population. The result is, in principle, unknown!
- Random variable(RV) : A characteristic that associates a single numerical value with each outcome of a random experiment. They can be qualitative or quantitative.

- **Example:** Random experiment: roll a dice. Random variable: number obtained

Discrete vs. Continuous

- **Discrete or continuous?**
  - Score when rolling a dice
  - Number of defective units in a production chain
  - Height of a person
  - Eye color of a person
  - Life time of a computer
  - Weight of a chair
  - Score obtained by a student in an exam
  - Number of defective screws in a box
  - Width of a screw box

Sample

- Random sample : In general, it is not possible to study all the elements in the population
  - Infinite populations (or too many elements)
  - Economical reasons
- In statistics we always work with a subset of the population. This subset is the *sample*. The sample must *represent* the population, so it is possible to extrapolate the conclusions obtained in the sample to the complete population (remember that we are interested in studying the population, not the sample).
- One way to obtain a representative sample is by using randomness, that is, by choosing a random sample.
- Statistical data : When a random sample is selected from a population, and characteristics (random variables) are observed, we have a set of statistical data.
- Usually, we denote a sample as a collection of numbers: $x_1, x_2, \ldots., x_n$ (sample size n)

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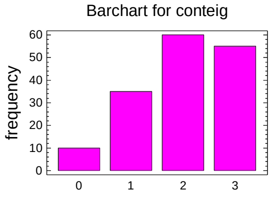
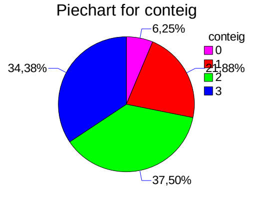
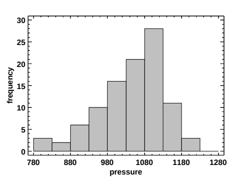
Graphical representation







Variables with few possible values: barcharts, piecharts,…,
Variables with many possible values: histograms,…
And many others. Which one to use? Depends on the sample

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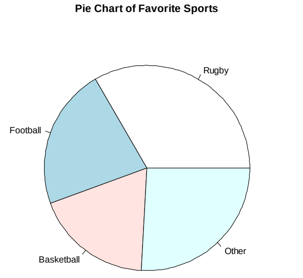
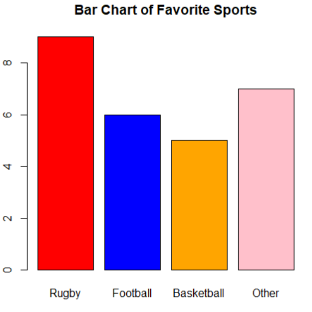
- **Exercise :** When asking a group of people about their favorite sport, 9 of them said Rugby, 6 of them Football, 5 of them Basketball, and the other 7 chose other answer. Draw a piechart and a barchart for this exercise. (*Unit1_Piechart.r*)





Measures

- Graphics are useful but limited.
- Data can be summarized numerically, so they can be more easily represented and compared.
- We will use parameters (numerical measurements describing some characteristic of a population) and statistics (numerical measurements describing some characteristics of a sample).
- Three main types: LOCATION, DISPERSION, SHAPE

The mean

- Mean: Also known as average, is the Location parameter most commonly used.
- It corresponds with the idea of "distributing in equal shares".
- Calculus: $\bar{x} = \frac{x_1 + \cdots + x_n}{n}$

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The median

- Median : In presence of asymmetry or outliers, it is more recommended than the mean.
- It follows the idea of a "central value".
- How to compute it.: Me or $\tilde{x}$ =
  - If n is odd: Central value (the one in position (n+1)/2).
  - If n is even: Average of the two central values (those in positions n/2 and n/2 +1).
- Note that data must be sorted in increasing order!

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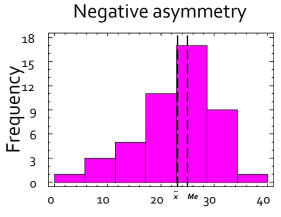
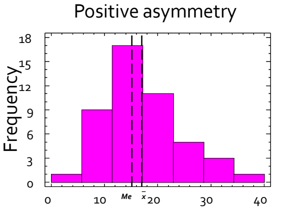
- Median: It is the value leaving 50% of data above, and 50% of data below.
- It is more "stable" (robust) than the mean, in the sense that wrong data and outliers affect it less than they affect the mean:





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- Quartiles : the three points that divide the data set into four equal groups.
  - *Q1* : Value that leaves 25% below it, and 75% above it.
  - *Q2* : Value that cuts data set in half
  - *Q3* : Value that leaves 75% below it, and 25% above it.

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- $Q2$: Coincides with the median.
- The "central" 50% of data are between Q1 and $Q3$.
- How to calculate $Q1$ and $Q3$:
  - Q1 ≈ median of the "first half" of data.
  - Q3 ≈ median of the "second half" of data.

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- Alternatively, $Q1$ and $Q3$ can be calculated as follows:
  - Sort the observations in increasing order.
  - Unless n is multiple of 4, $Q1$ is the value in position $n/4$ (rounding up to the nearest larger integer if necessary). If n is multiple of 4, Q1 is the median of the first $n/2$ data.
  - $Q3$ is calculated "symmetrically".

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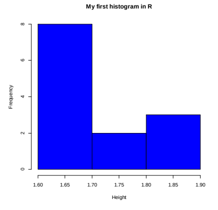
- **Exercise :** Calculate the mean, the median and the quartiles of the heights of the following randomly chosen students in ESIGELEC. Also draw a histogram with classes of length 0.1. Do the exercise both by hand and using R (*Unit1_Heights.r*)

1,64 | 1,66 | 1,74 | 1,86 | 1,87 | 1,69 | 1,65 |
1,71 | 1,68 | 1,70 | 1,62 | 1,84 | 1,61

SOL: $\bar{x} = 1.71; \mathrm{Me} = 1.69; Q_1 = 1.65; Q3 = 1.74$



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- Percentiles : They generalize the quartiles.
- Percentile *p (or pth percentile)* is the point leaving below some *p*% of data.
- It gives the proportion of data below and above a given value.

- **Example:** (Used by pediatricians) “Your baby is in the 70th height percentile, and 50th weight percentile.”

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Dispersion measures

- **Famous quote:** *"Statistics is a science that shows that if my neighbor has two cars and I none, we both have one."*

  — George Bernard Shaw (1856–1950), Irish playwright and a co-founder of the London School of Economics

- The mean represents "proportional sharing", but...

- **Example:** What is the average score in an exam if half of the students got 10 and the other half 0? How about if all of them got 5?
- **Example :** You are about to jump in a lake from a high rock. You know that the average depth of the lake is 1.40 m. Would you jump?

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- Location parameters do not give me information about how similar or different data are.
- Are my data close to each other? Or on the contrary, is there much dispersion?
- We will study the following parameters and statistics related to dispersion:

Range	**Interquartile range**
Variance	**Standard deviation**

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- Maximum : x_{max} = maximum observed value
- Minimum : x_{min} = minimum observed value
- Range.

$$\boxed{x_{max} - x_{min}}$$

- The range gives us information about the difference between the two most separated data.

- **Exercise :** Give a main drawback of this measure

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- Interquartile range . *IQR()*

$$\boxed{IR = Q3 - Q1}$$

- IR gives information about the dispersion found in "central" values.
- It is "robust" in the sense that outliers do not affect it.

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- **Exercise :** Calculate the range and the interquartile range of the data about student's heights. $SOL: Range = 0.26; I.R. = 0.09$ (*Unit1_Heights.r*)

- Ranges give some valuable information, they are easily calculated, but…
- Could we use them to calculate how far the observations from the mean on average are?
- Variance and standard deviation give this information.

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- Variance (sample) .

$$s^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}$$

$$s^2 = \frac{\sum_{i=1}^{n} x_i^2 - n(\bar{x}^2)}{n-1}$$

- In some textbooks you will find it as quasi-variance, also denoted as s^2_{n-1} or $(s')^{1.}$

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- Instead of the variance, it is more common to use its square root, because it is expressed in the same units as the data.
- Standard deviation (sample) . Calculus:

$$s = \sqrt{s^2} = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}$$

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Describing data: summary

There are many measures to describe a sample: mean, median, quartiles, percentiles, standard deviation, range, …

In this course you will use, mainly, two of them:

- The sample mean (known): $\bar{x} = \frac{x_1 + \cdots + x_n}{n}$; *mean()*
- The sample standard deviation (known): *sd()*

  $S = \sqrt{\frac{\sum_i (x_i - \bar{x})^2}{n-1}}$;

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- At times, it is useful to have a dispersion measure that does not depend on the units in which data are measured, that is, a *dimensionless* data.
- Coefficient of variation .

$$\boxed{CV = \frac{s}{\overline{x}} 100\%}$$

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- In short : Depending on the data, what parameters should you use?

	Symmetric data without outliers	Asymmetric data or presence of outliers
Location	Mean	Median
Dispersion	Standard deviation	Interquartile range

- OUTLIER: an observation that lies an *abnormal* distance from other values in the sample (e.g., more than 1.5 times the interquartile range far from the nearest quartile)

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Shape measures

- Asymmetry coefficient (also called *Skewness coefficient*) and Kurtosis coefficient are the most commonly used: *skewness(), kurtosis()*
- Both together allow us to check whether or not our data follow a "Gaussian" or "bell-shaped" curve (Normal distribution).
- Asymmetry. Calculus:

$$AC = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^3}{(n-1)s^3}$$

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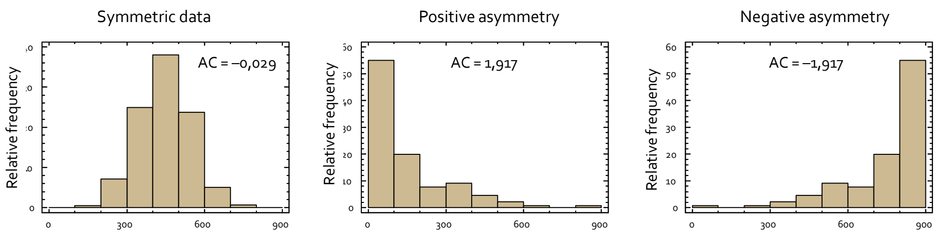
- Asymmetry: Under the hypothesis of normality, data should be symmetrical (i.e. skewness should be equal to zero)
  - AC ≈ 0 : Symmetric data
  - AC > 0 : Positive asymmetry (right tailed)
  - AC < 0 : Negative asymmetry (left tailed)



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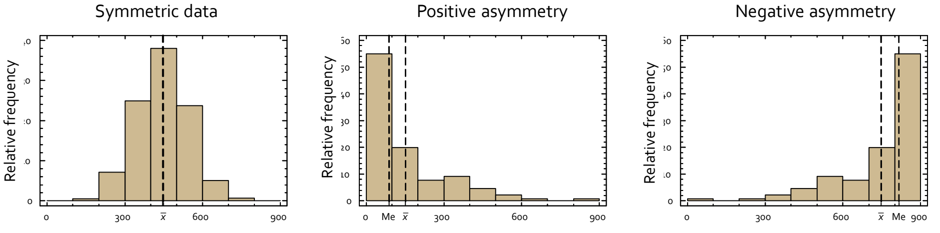
- Generally we have:

  - Symmetric data $\Leftrightarrow \bar{x} \sim Me$
  - Positive asymmetry $\Leftrightarrow \bar{x} > Me$
  - Negative asymmetry $\Leftrightarrow \bar{x} < Me$



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- Kurtosis coefficient: It measures the "tailedness" in the data
- The reference is the Gaussian curve.
- Calculus in R:

$$KU = \frac{n(n+1)}{(n-1)(n-2)(n-3)} \sum_j \frac{(x_j - \bar{x})^4}{s^4} - \frac{3(n-1)^2}{(n-2)(n-3)}$$

- Different software may compute this coefficient in different ways

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- “tailedness” degree :
- In R, under the hypothesis of normality, data should have kurtosis equal to 3

  - KU ≈ 3 : “Normal” data (bell-shaped)
  - KU > 3 : More acute peak around the mean
  - KU < 3 : Wider and lower peak around the mean

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Box-and-Whisker plot

- The Box-and-Whisker plot allows you to represent the main features about location and dispersion. *boxplot()*
- Procedure : Draw a box and two whiskers.
- Box:
  - Left side : Q1
  - Right side : *Q*3

    ⇒ Width: Interquartile range
  - Vertical line : *Q*2 (Median)
  - Point or cross: Mean (optional)

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- Whiskers:
  - The maximum length of each whisker will be 1.5 times the box width.
  - Each whisker stops in the last value that DOES NOT exceed such length.
  - Those values that are further than the whiskers, if any, are represented by dots, and called "outliers".
- Necessary computations :
  - Quartiles ($Q1$, $Q2$, $Q3$) and mean ($\bar{x}$)
  - Interquartile range (IR)
  - 1,5 · IR (to determine whether or not there are outliers)

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- The Box-and-Whisker box allows you to detect :
  - Asymmetry
  - Wrong data
  - Outliers
  - Differences between groups
- A look at the plot gives you:
  - Quartiles, median (and the mean, if it is depicted)
  - Interquartile range ⇒ 50% "central"
  - If observations are "symmetrically" distributed or not

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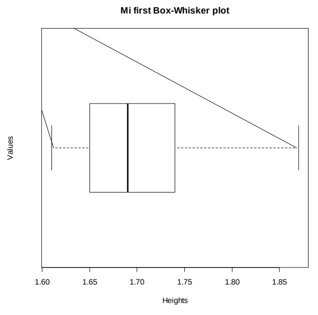
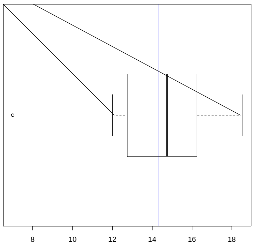
- **Exercise :** Depict the Box-and-Whisker plot for the heights exercise. (*Unit1_Heights.r*)

- **Exercise :** From the following data

  12 | 14 | 14,5 | 17 | 13,5 | 18,5 | 16 | 15,5 | 15 | 7 | 12 | 16,5

  draw the corresponding Box-and-Whisker plot. (*Unit1_BoxWhisker.r*)



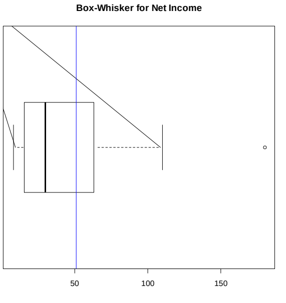


- **Exercise :** The following data represent the net incomes (in thousands of euros) of a sample constituted by 11 companies of a given sector:

  25 | 110 | 42 | 10 | 8 | 180 | 70 | 14 | 56 | 17 | 30

  a) What type of random variable do we have?
  b) Depict the Box-and-Whisker plot.
  c) What statistics would be appropriate to describe location and dispersion of data?

  SOL: $Continuous, Median\ and\ IR\ (as\ data\ are\ asymmetric\ with\ outliers).$
  (*Unit1_Netincome.r)*



Extra information: Subsets in R

- Sometimes it is useful to define one variable only for a subset of the simple.
- In R: *new.variable <-subset(variable,condition)*
- For example, define a new variable *Vble.restr* that only takes the values of another variable *Vble* if *Vble2* is equal to a certain *Value*
  Vble.restr <- subset(Vble, Vble2 == Value)

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