

Population density in Marathon (PDE)

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PDE [transport + diffusion + decay]

- **Decay:** some runners drop off from the race or give up in the middle (about 17% according to data [participation](#))
- **Transport:** runners transport (or run) at a speed of $v_k(x)$, which is a function that depends on runners' position and their cohort (basic speed)
- **Diffusion:** variation in runners speed — compact group tends to spread out.

Introduction of parameters and assumptions

x : position along the race (200m) (a small section)

t : time

S_k : basic speed of runners ($k = 1, 2, 3, 4, 5$) [2.2, 2.6, 3.0, 3.6 4.2]

$u_k(x,t)$: linear density of marathon runners from cohort k at position x and time t

$v_k(x)$: runners' actual speed at location (x) , determined by a base speed S_k multiplied by a slowdown factor $\phi(x)$ [$v_k(x) = S_k \phi(x)$]

$\phi(x)$: effect of hill on speed [$\phi(x) = 3 - \exp[-2((x-h)/w)^2]$]



PDE model

$$\frac{\partial u_k(x, t)}{\partial t} + \frac{\partial}{\partial x} (v_k(x) u_k(x, t)) = D \frac{\partial^2 u_k(x, t)}{\partial x^2} - \lambda u_k(x, t)$$

$$v_k(x) = S_k \phi(x)$$

$$\phi(x) = 3 - \exp[-2((x-h)/w)^2]$$

Boundary conditions:

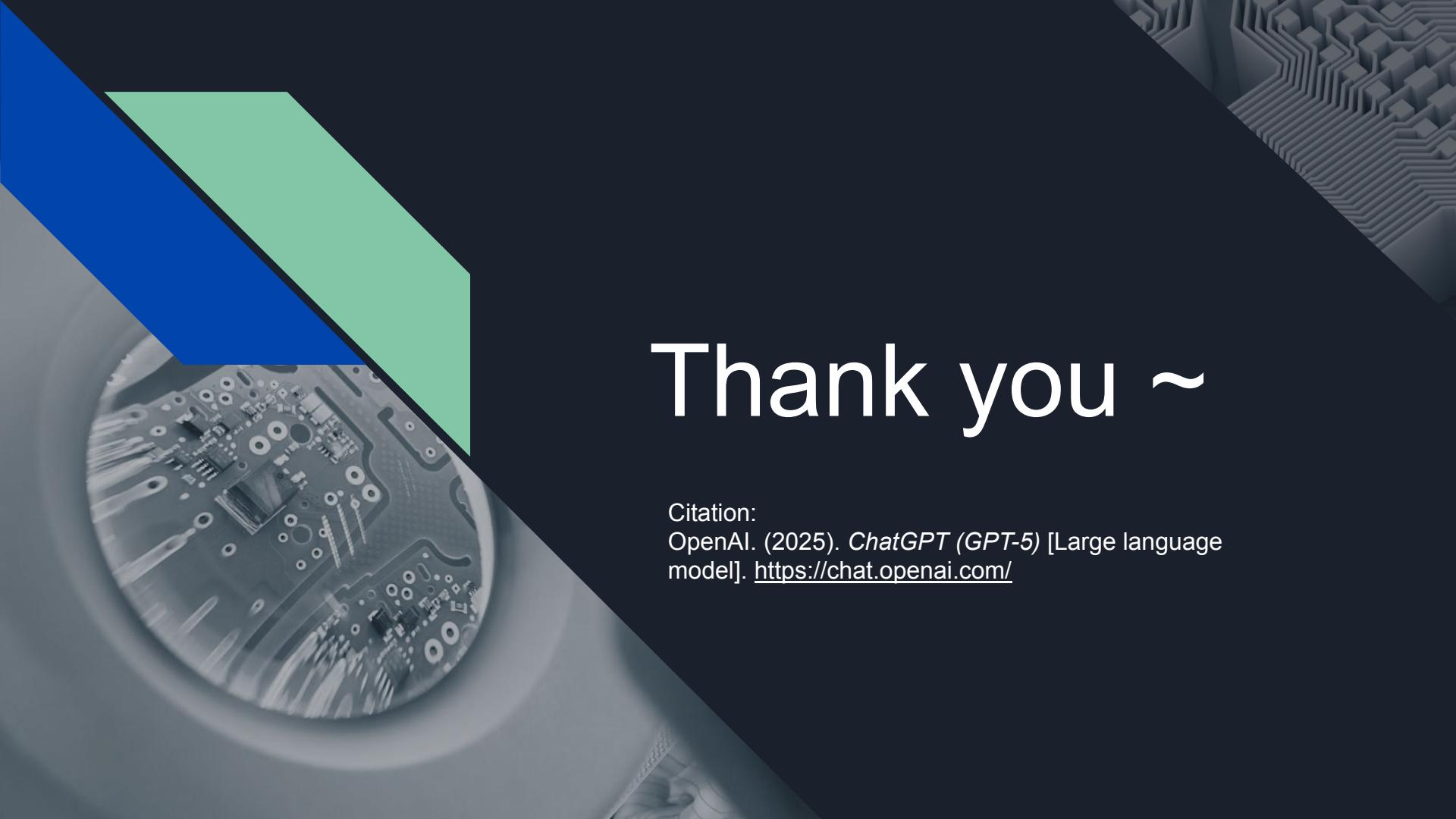
Left boundary: $v_k(0) u_k(0, t) = 0$

Right boundary: outflow flux (t) = $\sum_k v_k(L) u_k(L, t)$

Left & right:

$$\frac{\partial u_k}{\partial x}(0, t) = \frac{\partial u_k}{\partial x}(L, t) = 0$$

diagram

The background of the slide features a dark grey gradient. On the left side, there is a circular inset showing a close-up of a printed circuit board (PCB) with various electronic components like resistors and capacitors. Overlaid on this are several large, semi-transparent geometric shapes: a blue parallelogram at the top left, a green parallelogram below it, and a grey parallelogram at the bottom left. In the top right corner, there is a pattern of grey rectangular blocks of varying heights, resembling a 3D bar chart or a stylized circuit board.

Thank you ~

Citation:

OpenAI. (2025). *ChatGPT (GPT-5)* [Large language model]. <https://chat.openai.com/>