Population density in Marathon (PDE)

Yvan, Qin, Sicy

PDE [transport + diffusion + decay]

- **Decay:** some runners drop off from the race or give up in the middle (about 17% according to data participation)
- **Transport:** runners transport (or run) at a speed of $v_k(x)$, which is a function that depends on runners' position and their cohort (basic speed)
- **Diffusion:** variation in runners speed — compact group tends to spread out.

Introduction of parameters and assumptions

x : position along the race (200m) (a small section)

t : time

S_k : basic speed of runners (k = 1, 2, 3, 4, 5) [2.2, 2.6, 3.0, 3.6 4.2]

$u_k(x,t)$: linear density of marathon runners from cohort k at position x and time t

$v_k(x)$: runners' actual speed at location (x), determined by a base speed S_k multiplied by a slowdown factor $\phi(x)$ [$v_k(x) = S_k\phi(x)$}

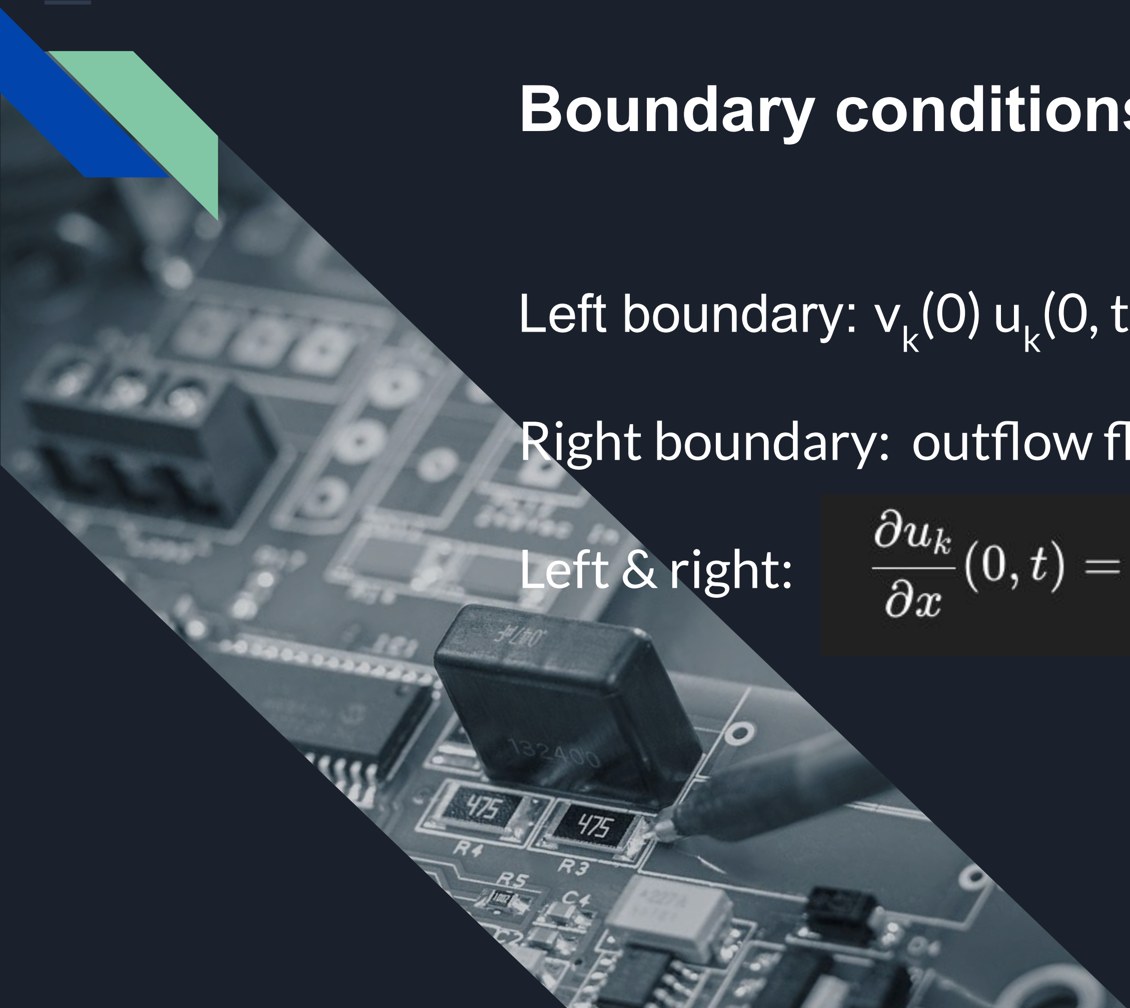
$\phi(x)$: effect of hill on speed   [$\phi(x) = 3-\exp[-2((x-h)/w)^2]$]

PDE model

$$\frac{\partial u_k(x,t)}{\partial t} + \frac{\partial}{\partial x}\left(v_k(x)u_k(x,t)\right) = D\frac{\partial^2 u_k(x,t)}{\partial x^2} - \lambda u_k(x,t)$$

$v_k(x) = S_k\phi(x)$

$\phi(x) = 3-\exp[-2((x-h)/w)^2$



Boundary conditions:

Left boundary: $v_k(0)\, u_k(0, t) = 0$

Right boundary: outflow flux $(t) = \sum_k v_k(L)\, u_k(L, t)$

Left & right: $$\frac{\partial u_k}{\partial x}(0,t) = \frac{\partial u_k}{\partial x}(L,t) = 0$$

diagram

Thank you ~

Citation:

OpenAI. (2025). *ChatGPT (GPT-5)* [Large language model]. https://chat.openai.com/