

Triangle Centers Cheat Sheet

Honors Geometry

You will need to know the following definitions and properties in order to solve problems on your homework and exams. However, within the unit where something is introduced you may only use the definition in your proofs. In future units, you will be able to use properties in your proofs.

Definitions are sentences where the word being defined is bolded.

A **property** is anything which is not in the definition. Generally speaking, properties should be proven in class or on homework. Properties are given below the definition and delineated by **p1**, **p2**, etc.

1 Perpendicular bisectors

The **perpendicular bisector** of a line segment is the line (or a line segment) that is perpendicular to it and passes through its midpoint.

p1 Every point on the perpendicular bisector of a line segment is equidistant from the endpoints.

p2 Any point equidistant from the endpoints of a line segment is on its perpendicular bisector.

p3 All three perpendicular bisectors of a triangle intersect at the circumcenter (which we'll define later).

2 Angle bisectors

An **angle bisector** is a line (or line segment) that splits an angle into two equal angles.

p1 All points on an angle bisector are equidistant from the two lines making the bisected angle.

p2 Any point equidistant from two lines is on their angle bisector.

p3 All three angle bisectors of a triangle intersect at the incenter.

3 Drawing circles in and around a triangle

The **circumcenter** of a triangle is the center of the circle that all the vertices are on.

p1 The circumcenter of a triangle is equidistant from all three vertices.

p2 All three perpendicular bisectors of a triangle intersect at the circumcenter.

The **incenter** of a triangle is the center of the largest circle that's completely inside the triangle.

p1 The incenter of a triangle is equidistant from all three sides.

p2 All three angle bisectors of a triangle intersect at the incenter.

4 Altitudes

An **altitude** of a triangle is a line segment perpendicular to a side that goes to the opposite vertex.

p1 Both legs of a right triangle are altitudes.

p2 Any altitude next to an obtuse angle has to be outside the triangle.

p3 Any altitude next to two acute angles must be inside the triangle.

p4 All three altitudes of a triangle intersect at the same point, which we call the **orthocenter**.

5 Medians

A **median** of a triangle is a line segment that goes from a vertex to the midpoint of the opposite side.

p1 In an isosceles triangle, one median, altitude, angle bisector and perpendicular bisector all coincide.

p2 In an equilateral triangle, all three medians, altitudes, angle bisectors, and perpendicular bisectors coincide.

p3 All three medians of a triangle intersect at the same point, which we call the **centroid**.

p4 The centroid is $\frac{2}{3}$ of the length of the median from the vertex.