Louis de Broglie Nobel Lecture

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May 14, 2020

Louis de Broglie begins his Nobel lecture by discussing the trajectory of physics as he began his work, defined by a few key developments. The more concrete developments of the time were Einstein's relativity and the Bohr model of the Hydrogen atom, the first of which provided the motivation for de Broglie's famed thesis, with the latter providing strong reason to entertain his somewhat speculative results. Forming the backdrop is the original success of Newtonian optical theory being supplanted by a wave theory largely due to the discovery of interference by Thomas Young. This naturally led to the development of Maxwell's theory, which is entirely wave-based and accounted for all observed phenomena up until Planck's explanation of blackbody radiation. What this required was quantizing the possible energies of the body, but what nobody, including Planck it seems, realized is that this also implied not only a quantization of radiative frequencies, but also of radiation itself, i.e. the photon.

Einstein was the first to resuscitate this idea in order to explain the photoelectric effect, although he did not attempt to model the internal structure of the light quanta. Nevertheless, outside of a qualitative observation that light is quantized, it was clear that any attempt to apply standard dynamics to this construction would contradict the wavelike phenomena which are readily established experimentally, so nobody really knew what to make of this. This impasse is the world into which de Broglie entered.

de Broglie placed two consideration as paramount to his motivation in resolving this seeming contradiction: firstly the existence of frequency as well-defined even in a single photon, and secondly the discrete integer-valued electron orbits of the Bohr atom. The first of these implies the need to consider a wave aspect to the "corpuscle" of light, while the second only has physical counterparts in interference and vibrational modes, both of which only arise out of wave phenomena. This naturally led de Broglie to believe that, in particular both electrons and light, but more generally all matter, must have both a wave and a particle aspect that must be accounted for.

To quote de Broglie:

> However, since corpuscles and waves cannot be independent because, according to Bohr's expression, they constitute two complementary forces of

> reality, it must be possible to establish a certain parallelism between the motion of a corpuscle and the propagation of the associated wave. The first objective to achieve had, therefore, to be to establish this correspondence.
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> With that in view I started by considering the simplest case: that of an isolated corpuscle, i.e. a corpuscle free from all outside influence.

In the rest frame of the particle we associate it with a stationary wave which has the same phase at every point and is given by the expression $A(x, y, z) \sin\big(2\pi\nu_0(t_0 - \tau_0)\big)$. In this construction τ_0 is a phase (shift) constant, ν_0 is the intrinsic frequency (which would be attributed to a photon in the case of light), and t_0 is the proper time of the particle, so the time in this frame. We will now leave out A, since we cannot as of yet say anything about amplitudes.

If we now consider this same system in an inertial frame which is moving with velocity $-v$ in the x direction with respect to the rest frame, we now see the particle moving with velocity $v = \beta c$. Applying a Lorentz transformation to relate the time t of the observer,

$$t_0 = \gamma\left(t - \beta\frac{x}{c}\right), \gamma = \frac{1}{\sqrt{1-\beta^2}} \tag{0.1}$$

We are now able to plug this in to the expression for the wave to the express the wave as a function of the time in the observer's frame:

$$\sin\left(2\pi\gamma\nu_0\big(t - \frac{\beta}{c}x - \tau_0\big)\right) \tag{0.2}$$

From this it is clear that the observer will see a wave of frequency $\nu = \gamma\nu_0$ moving in the x direction. Moreover, we can determine the phase velocity of this wave by plugging in $x = Vt$ and requiring the phase to remain stationary:

$$2\pi\gamma\nu_0\big(t - \frac{\beta}{c}Vt - \tau_0\big) = 2\pi\gamma\nu_0(-\tau_0) \Rightarrow 1 - \frac{\beta}{c}V = 0 \Rightarrow V = \frac{c}{\beta} = \frac{c^2}{v} \tag{0.3}$$

Thus, the phase velocity V of the wave differs from the velocity of the particle v. We can now determine the refractive index of the vacuum

$$n = \frac{c}{V} = \beta = \sqrt{1 - \frac{1}{\gamma^2}} = \sqrt{1 - \frac{\nu_0^2}{\nu^2}} \tag{0.4}$$

Recall our initial picture of a particle being carried along by a wave. It is natural to assume that the trajectory of the particle follows a node of the envelope. The group velocity U is defined as the velocity of the envelope, so the nodes are transported along at the group velocity. Thus, we expect U to be equal to the particle velocity v.

$$\partial_\nu(n\nu) = n + \nu\partial_\nu n = \beta + \frac{\nu_0^2}{\nu^2 n} = \beta + \frac{1}{\gamma^2\beta} = \beta + \frac{1-\beta^2}{\beta} = \beta^{-1} = \frac{c}{v} \tag{0.5}$$

By the relation $U = c\big(\partial_\nu(n\nu)\big)^{-1} = c\beta = v$ obtained by Lord Rayleigh, we confirm that the particle velocity does in fact match the group velocity.

With respect to this development "to establish the parallelism of which we have spoken, we must seek to link these parameters to the mechanical parameters, energy and quantity of motion [momentum]." Prior to de Broglie, the relation $E = h\nu_0$ was only considered as a rest energy in relation to the energies of photons. It is here in his Nobel lecture where he introduces his use of this relation in a more general context, relating the relativistic rest energy of a particle to its internal frequency by

$$h\nu_0 = E = m_0c^2 \tag{0.6}$$

We know that a body with rest mass m_0 has mass $m = \gamma m_0$ in the moving frame, so

$$p = mv = \gamma m_0 v = \gamma \frac{E}{c^2} v = \gamma \nu_0 h \frac{1}{V} = h\frac{\nu}{V} \tag{0.7}$$

By definition, the wavelength λ is the distance traversed by the wave during one period:

$$\lambda = VT = \frac{V}{\nu} = \frac{h}{p} \tag{0.8}$$

This concludes the development of the free particle with corresponding wave in an inertial frame, and brings us to the more general case of a particle in a constant potential field $F(x, y, z)$. In his lecture, de Broglie states without proof the resulting refractive index:

$$n(x,y,z) = \sqrt{\left(1 - \frac{F}{h\nu}\right)^2 - \frac{\nu_0^2}{\nu^2}} = \sqrt{\frac{(\gamma E - F)^2}{h^2\nu^2} - \frac{h^2\nu_0^2}{h^2\nu^2}} = \sqrt{\frac{(\gamma^2-1)E^2 - 2\gamma EF - F^2}{\gamma^2 E^2}} \tag{0.9}$$

Assuming a low relativistic correction $\gamma \sim 1$ (and $F^2 << E^2$) this yields

$$n(x,y,z) = \sqrt{\frac{(\gamma+1)(\gamma-1)E^2 - 2\gamma EF}{\gamma E^2}} = \sqrt{2\frac{(\gamma-1)E - F}{E}} \tag{0.10}$$

Since the potential F is symmetric with respect to time, the total energy of the particle in this inertial frame is conserved. This implies that when we fix the frame, the particle energy W will remain equal to γE. Therefore, we find that $\nu = W/h = \gamma E/h = \gamma\nu_0$ remains constant. However, a nonzero potential will likely change p, which means

$$p(x,y,z) = h\frac{V(x,y,z)}{\nu} = \frac{h}{\lambda(x,y,z)} \tag{0.11}$$

Since W is constant this leads to the relation $n(x, y, z) \propto p(x, y, z)$

$$n = \frac{c}{V} = \frac{c}{\nu\lambda} = \frac{c}{h\nu}\frac{h}{\lambda} = \frac{c}{W}p \tag{0.12}$$

The two of initial formulations of the principle of least action are that of Fermat (pertaining to light) and Maupertuis, the first of which minimizes time while the second minimizes the action directly. Both "actions" are respectively outlined below:

$$L = \int v dt \Rightarrow v = \frac{dL}{dt} = \frac{dl}{dt} \Rightarrow \Delta t = \int \frac{1}{v} dl = \int \frac{n}{c} dl \tag{0.13}$$

$$S = \int \partial_{\dot{q}} L(q, \dot{q}) dq = \int p dq = \int p dl \tag{0.14}$$

It is worth noting that the original formulation by Maupertuis did not include the mass, but the modern form has been used here for convenience. Using the relation we established between the index of refraction and the momentum, we know that these two "actions" must be directly proportional:

$$p = W \frac{n}{c} \Rightarrow S = \int p dl = \int W \frac{n}{c} dl = W \int \frac{n}{c} dl = W \Delta t \tag{0.15}$$

Therefore minimizing the traditional action also minimizes the time, so the two formulations are equivalent in the scenario of constant field.

For a closed path, de Broglie says it makes sense "to assume that the phase should be a uniform function along this trajectory." To my understanding, this is a requirement that λ be fixed on the path:

$$\oint \frac{p}{h} dl = \oint \frac{1}{\lambda} dl = \frac{L}{\lambda} = k \in \mathbb{Z} \tag{0.16}$$

This gives $\oint pdl = hk$, which "is exactly the condition of the stability of atomic periodic motion, according to Planck." It is possible that de Broglie misspoke and meant to cite Bohr's stability condition. Nonetheless, in the words of de Broglie, "the conditions of quantum stability thus emerge as analogous to resonance phenomena and the appearance of integers becomes as natural here as in the theory of vibrating cords and plates."

Concluding this development, de Broglie takes the limit which should apply to photons, that of very small rest mass m_0. From this he finds Einstein's postulates for light:

$$\begin{aligned} v &\to c \\ V &= \frac{c^2}{v} \to c \\ p &= \frac{h\nu}{V} \to \frac{h\nu}{c} \end{aligned} \tag{0.17}$$

It is unclear how this first limit is obtained, although it probably comes from holding ν, and thus the energy, fixed. He also claims to derive the relation $W = h\nu$. This was fixed in his theory from the assumption of constant potential, and it is unclear how the quantity can be obtained from first principles in the case of negligible rest mass. It is possible that de Broglie meant to highlight that $\nu \to \nu_0$ and so $W \to E$. The rest of de Broglie's lecture largely consists in discussion of the experimental confirmations of his work. Although interesting, there is not much else of substantive mathematical curiosity.