Modeling weak gravitational lensing from multiple thin lenses

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(Dated: June 5, 2018)

This paper describes the simulation of the effects of weakly gravitating masses on light from a luminous source distribution in the thin lens approximation in the small angle domain. This includes magnification of the light, distortion of the distribution, and the shift in apparent location of the source. We iterate over the coplanar point mass solution to simulate the compounding effects of arbitrarily distributed thin lenses at multiple distances.

INTRODUCTION

Although the phenomenon of gravitational lensing is consistent with Newtonian gravity, as with all gravitational physics, Einstein's relativity was necessary to model it accurately, and it served as a de facto verification of the theory during the solar eclipse of May 1919. While both theories show that light bends in the presence of a gravitational potential, instead of simply applying a Newtonian gravitational acceleration, general relativistic rays follow geodesics, and so light will bend when passing near a lensing mass.

METHODS

While deriving a relativistic potential and tracing geodesics is generally quite complex, several simplifying approximations can be made when the gravitational field is weak, the lens is thin, and the angles are small (Pettini 16-18).[1] Under these assumptions, the beam is deflected towards a lensing point mass by:

$$\overline{\alpha} = \frac{4GM}{c^2\xi} \tag{1}$$

For mass distributed over a greater extent than a thin lens can accommodate, the light ray must be deflected at multiple distances. Thus, for any sufficiently separated thin lenses, their effects can be simulated by iteratively taking the last point of deflection as the observer for the next lens. This only depends on the current state of the ray, that is, its position relative to the mass in question and current angle. The lens causes the ray to deflect in the direction of the lens, at which point it continues to propagate in a straight line until it reaches another deflection, which is distant enough from the previous lens to not feel any effect.

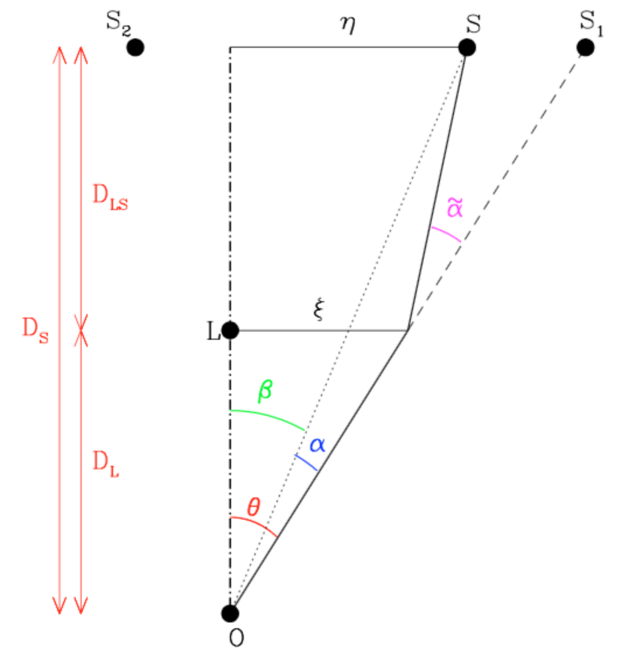

FIG. 1. **Diagram of a gravitational lens (taken from Pettini 18):** This figure depicts a strongly lensing point mass, which allows for two images from the same source (S_1 and S_2), but only the light rays from S_1 are shown. Distances on left are angular diameter distances, but for distributions of small extent (within the small angle approximation), this is basically equivalent to real distance. L represents the lens, O the observer, and S the source. ξ is the impact parameter, or distance between lens and the light ray. For small angles, this can be assumed perpendicular to the line between observer and lens.

As the we are operating in the weak field domain, each thin lens deflection can be decomposed into a linear superposition of point mass lenses at various impact distances. This allows the exact same approach that works for separate lenses to apply at the level of point masses, as deflecting by two angles at the same distance from

[1] A weak gravitational field results when the impact parameter ξ (the distance from the lens to the light ray) is much more than the Schwarzchild radius, $\frac{2GM}{c^2}$ (Pettini 17). A thin lens is distributed over a small distance from the observer, which allows for all lensing effects to result in a single adjustment to the angle of the light ray. A point mass is the simplest example of a thin lens. In the weak field domain, the angular deflection for different impact parameters within the same thin lens can be linearly summed, so any thin lens can be approximated by a sum of point masses (Schneider 159).

the observer is simply additive, so is effectively the same as applying a new deflection at the same point (that is, without allowing propagation between).[2]

RESULTS

The simulation using the above approach was carried out on a fairly simple mass distribution of 3 point masses spread over three dimensions, but still within about 1/10 of a radian angular diameter in the view of the observer. Several figures depicting the situation are shown below, along with the accompanying lensing effects. We decided to focus on predicting the true source distribution based on known lens distribution and and apparent image.[3]

Although the main body of code should also be able to handle point masses at the same distance, allowing the simulation of an arbitrary thin lens, we did not attempt this because of some quirks in the later code for calculating magnification that require distance between observing point and deflecting point, which would not be allowed in cases where the ray does not propagate.[4]

CONCLUSION

So, we are able to simulate the effect of sufficiently separated thin lenses, as well as sufficiently close lenses to be considered as part of the same thin lens. What remains is to show that between these two extremes (the exact same distance and vastly separated distances) can also be approximated using this method of allowing ray propagation to the point nearest the lensing mass.

For small separations this is easy, as adding a small separation between two closely spaced deflections will not meaningfully affect the path of the beam. If anything, it should make a slightly better prediction than assuming both masses act at the same time producing a superposed deflection.

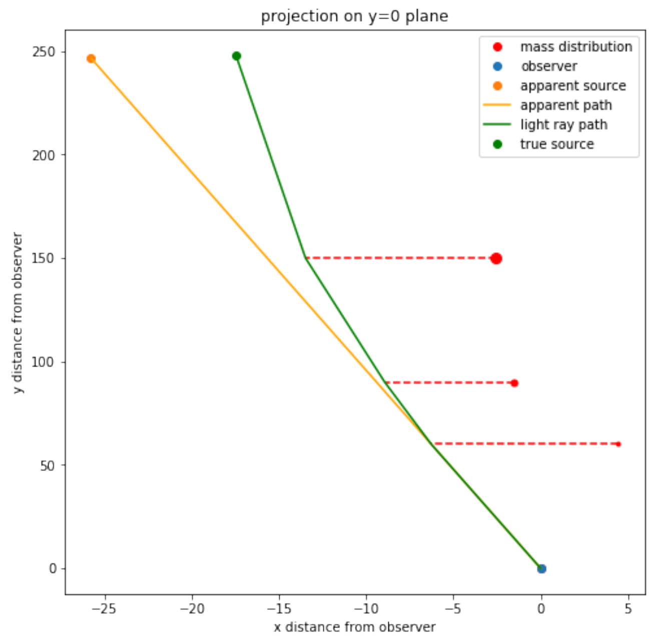

FIG. 2. **2-dimensional plot of 3 point lenses:** This rather simple system illustrates how the approach outlined above works to determine the true source of an image lensed at multiple distances. At the vertical (z) distance of each lensing point mass, the light ray is deflected by the appropriate angle according to the mass and impact parameter ξ. The ray is then allowed to propagate at its new angle, until it is lensed by the next mass. The orange line depicts the naive path taken by simply propagating the ray back along the incident angle of the image. Note that the true source appears to be almost imperceptibly above the apparent source. This is because, although the path lengths are the same, the and the true path curves, there is actually very little curvature compared to z distance traversed, so the difference in z is small.

For separations that are larger, however, I do not see a clear way to show that this approximation is valid. To reiterate, if the separation is large enough for the effects of the previous lens to be negligible, then it is clear that this approach should work, but if this assumption is false, then it is unclear if the approach is still valid.

In order to verify its validity, one could determine the effects of a mass distribution that is continuous across distance (not impact parameter) and compare it to the predictions of this simulation. For this to be informative, the mass would have to be distributed over a distance that is significantly greater than what is allowed in the thin lens approximation.

REFERENCES

Pettini, M. "Lecture 16." *Introduction to Cosmology.* Institute of Astronomy, University of Cambridge.

Schneider, P. "3.11: Galaxies as gravitational lenses." *Extragalactic Astronomy and Cosmology.* Springer,

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[2] It is worth noting that this point mass should be coplanar with the observer, source, and image, since the mass merely deflects the image ray towards itself, so if both lie within this plane, the deflected ray is also coplanar. As a plane can be defined as containing three points, or a point and a line, this plane can always be constructed from the light ray and the specific point mass.

[3] While the reverse case is also interesting physics, this situation is more analogous with experiment, as a distant body that is normally too faint to see may be made visible due to the magnifying effects of known lensing masses. Additionally, the exact same approach can be used for the reverse case, so once one has been done the other can follow fairly easily, allowing us to predict the appearance of multiple images from the same source!

[4] These difficulties are certainly surmountable and do not reflect an issue with the approach, only with the subpar implementation of a tangential part of the presentation.

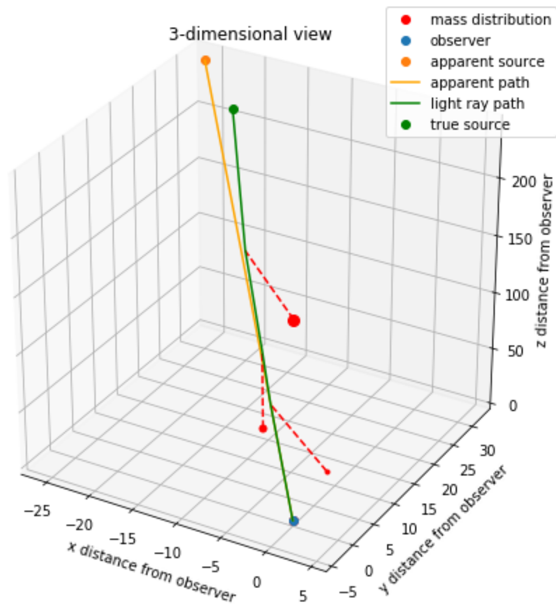

FIG. 3. **3-dimensional projection of 3 point lenses:** This plot depicts the same case as above, but in all 3 dimensions. While this plot is a more accurate representation that the projection (you can see the different mass positions), it is much harder to interpret the kinks in the path.

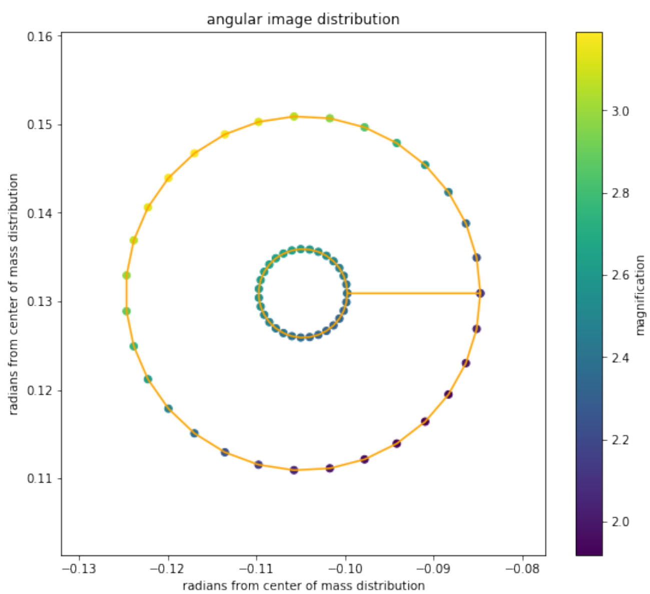

FIG. 4. **Apparent image distribution:** This is the distribution of light seen by the observer after it has been lensed by the point masses. While it is true that the image will have been distorted by the gravitational lensing, concentric circles were chosen to represent the distorted image to illustrate the determination of the true source distribution from the image. If we were instead interested in deriving the image from the source distribution, it would make more sense to choose circles for the source distribution. Magnification of the source is shown on the right, where points furthest from the lenses (not shown, bottom right) are magnified the most.

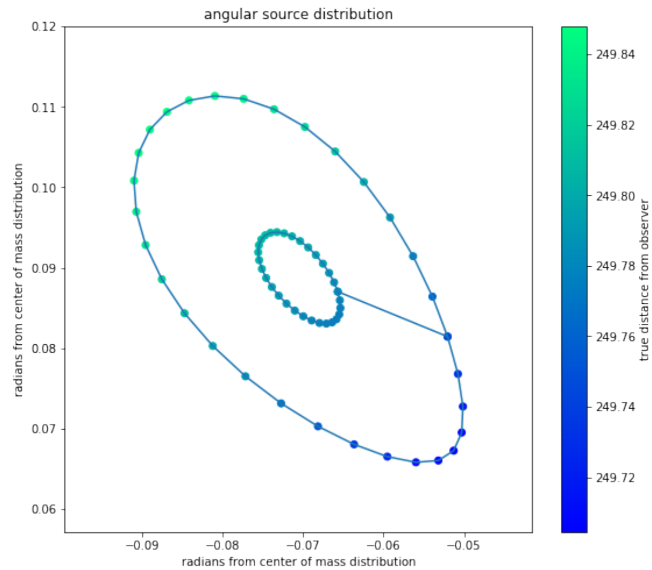

FIG. 5. **True distribution of luminous source:** Notice the difference in distortion depending on the scale of the circles: the smaller circle is distorted relatively similarly on either side, while the larger circle is more distorted closer to the lensing masses (not shown, bottom right). On the right is the straight line distance to each point, with points further from the lenses actually being more distant, though all points have identical redshift, so the path length of each ray is the same.

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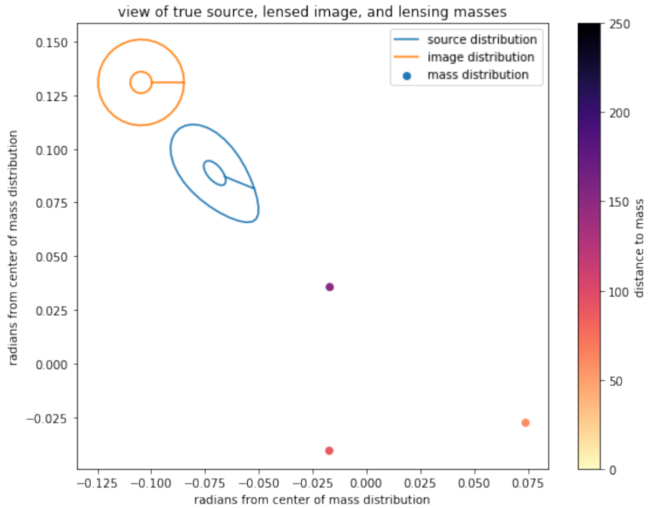

FIG. 6. **Observer view, plus true source** This depicts the angular distributions of the apparent image, the known lensing masses, and the derived source (if it were visible). On the right is the distance to each point mass lens. Note that the horizontal line in the image is tilted towards the lenses in the source distribution.