The ICT University

Summer 2023 Continuous Assessment №1

School of *ICT*

Course Code:	CS 4110
Course Title:	Compiler Construction
Instructor:	Engr. Daniel Moune

Instructions:

1. Your assignment should represent your own effort. However, you are not expected to work alone. It is fine to discuss the exercises and try to find solutions together, but **each student shall write down and submit his/her solutions separately**.

2. Please, include name, matricule, ICTU email and phone number in your submission.

3. For a each question, provide a solid argumentation of your solution. Answers with no proof or no solid argumentation will be graded **F-SCORE**.

4. This assessment is made of **03** pages

5. Convert your Answer sheet into PDF before uploading to Moodle

6. Documents in WORD format will not be accepted !!!

7. Make your answer sheet conformant to the template attached with this assessment

July 19, 2023

EXERCISES

1. With $S_1 = \{2, 3, 5, 7\}$, $S_2 = \{2, 4, 5, 8, 9\}$, and $U = \{1 : 10\}$, compute $\overline{S}_1 \cup \overline{S}_2$.
2. With $S_1 = \{2, 3, 5, 7\}$ and $S_2 = \{2, 4, 5, 8, 9\}$, compute $S_1 \times S_2$ and $S_2 \times S_1$.
3. For $S = \{2, 5, 6, 8\}$ and $T = \{2, 4, 6, 8\}$, compute $|S \cap T| + |S \cup T|$.
4. What relation between two sets S and T must hold so that $|S \cup T| = |S| + |T|$?
5. Show that for all sets S and T, $\overline{S} - \overline{T} = \overline{S} \cap \overline{T}$.
6. Prove DeMorgan's laws, Equations (1.2) and (1.3), by showing that if an element x is in the set on one side of the equality, then it must also be in the set on the other side of the equality.
7. Show that if $S_1 \subseteq S_2$, then $\overline{S}_2 \subseteq \overline{S}_1$.
8. Show that $S_1 = S_2$ if and only if $S_1 \cup S_2 = S_1 \cap S_2$.
9. Use induction on the size of S to show that if S is a finite set, then $|2^S| = 2^{|S|}$.
10. Show that if S_1 and S_2 are finite sets with $|S_1| = n$ and $|S_2| = m$, then

$$|\overline{S}_1 \cup \overline{S}_2| \leq n + m.$$

11. If S_1 and S_2 are finite sets, show that $|S_1 \times S_2| = |S_1||S_2|$.

12. Consider the relation between two sets defined by $S_1 \equiv S_2$ if and only if $|S_1| = |S_2|$. Show that this is an equivalence relation.

13. Occasionally, we need to use the union and intersection symbols in a manner analogous to the summation sign Σ. We define

$$\bigcup_{p \in \{i,j,k,\dots\}} S_p = S_i \cup S_j \cup S_k \cdots$$

with an analogous notation for the intersection of several sets. With this notation, the general DeMorgan's laws are written as

$$\overline{\bigcup_{p \in P} S_p} = \bigcap_{p \in P} \overline{S_p}$$

and

$$\overline{\bigcap_{p \in P} S_p} = \bigcup_{p \in P} \overline{S_p}.$$

Prove these identities when P is a finite set.

14. Show that

$$\overline{\overline{S}_1 \cup \overline{S}_2} = S_2 \cap S_1.$$

15. Show that $S_1 = S_2$ if and only if

$$\left(S_1 \cap \overline{S}_2\right) \cup \left(\overline{S}_1 \cap S_2\right) = \varnothing.$$

16. Show that

$$\bar{S}_1 \cup \bar{S}_2 - \left(\bar{S}_1 \cap \bar{S}_2\right) = \bar{S}_2.$$

17. Show that the distributive law

$$S_1 \cap (S_2 \cup S_3) = (S_1 \cap S_2) \cup (S_1 \cap S_3)$$

holds for sets.

18. Show that

$$S_1 \times (S_2 \cup S_3) = (S_1 \times S_2) \cup (S_1 \times S_3).$$

19. Give conditions on S_1 and S_2 necessary and sufficient to ensure that

$$S_1 = (S_1 \cup S_2) - S_2.$$

20. Use the equivalence defined in Example 1.4 to partition the set {2, 4, 5, 6, 9, 22, 24, 25, 31, 37} into equivalence classes.

21. Show that if $f(n) = O(g(n))$ and $g(n) = O(f(n))$, then $f(n) = \Theta(g(n))$.

22. Show that $2^n = O(3^n)$, but $2^n \neq \Theta(3^n)$.

23. Show that the following order-of-magnitude results hold.

(a) $n^2 + 5 \log n = O(n^2)$.

(b) $3^n = O(n!)$.

(c) $n! = O(n^n)$.

24. Show that $(n^3 - 2n)/(n+1) = \Theta(n^2)$.

25. Show that $\dfrac{n^3}{\log(n+1)} = O\left(n^3\right)$ but not $O(n^2)$.

26. What is wrong with the following argument? $x = O(n^4)$, $y = O(n^2)$, therefore $x/y = O(n^2)$.

27. What is wrong with the following argument? $x = \Theta(n^4)$, $y = \Theta(n^2)$, therefore $\frac{x}{y} = \Theta\left(n^2\right)$.

28. Prove that if $f(n) = O(g(n))$ and $g(n) = O(h(n))$, then $f(n) = O(h(n))$.

29. Show that if $f(n) = O(n^2)$ and $g(n) = O(n^3)$, then

$$f(n) + g(n) = O(n^3)$$

and

$$f(n)\, g(n) = O(n^5).$$

In this case, is it true that $g(n) / f(n) = O(n)$?

30. Assume that $f(n) = 2n^2 + n$ and $g(n) = O(n^2)$. What is wrong with the following argument?

$$f(n) = O(n^2) + O(n),$$

so that

$$f(n) - g(n) = O(n^2) + O(n) - O(n^2).$$

Therefore,

$$f(n) - g(n) = O(n).$$

31. Show that if $f(n) = \Theta(\log_2 n)$, then $f(n) = \Theta(\log_{10} n)$.

32. Draw a picture of the graph with vertices $\{v_1, v_2, v_3\}$ and edges $\{(v_1, v_1), (v_1, v_2), (v_2, v_3), (v_2, v_1), (v_3, v_1)\}$. Enumerate all cycles with base v_1.

33. Construct a graph with five vertices, ten edges, and no cycles.

34. Let $G = (V, E)$ be any graph. Prove the following claim: If there is any walk between $v_i \in V$ and $v_j \in V$, then there must be a simple path of length no larger than $|V| - 1$ between these two vertices.

35. Consider graphs in which there is at most one edge between any two vertices. Show that under this condition a graph with n vertices has at most n^2 edges.

36. Show that

$$\sum_{i=0}^{n} 2^i = 2^{n+1} - 1.$$

37. Show that

$$\sum_{i=1}^{n} \frac{1}{i^2} \leq 2 - \frac{1}{n}.$$

38. Prove that for all $n \geq 4$ the inequality $2^n < n!$ holds.

39. The *Fibonacci sequence* is defined recursively by

$$f(n+2) = f(n+1) + f(n),\ n = 1, 2, \ldots,$$

with $f(1) = 1$, $f(2) = 1$. Show that

(a) $f(n) = O(2^n)$,

(b) $f(n) = \Omega(1.5^n)$.

40. Show that $\sqrt{8}$ is not a rational number.

41. Show that $2 - \sqrt{2}$ is irrational.

42. Show that $\sqrt{3}$ is irrational.

43. Prove or disprove the following statements.

(a) The sum of a rational and an irrational number must be irrational.

(b) The sum of two positive irrational numbers must be irrational.

(c) The product of a nonzero rational and an irrational number must be irrational.

44. Show that every positive integer can be expressed as the product of prime numbers.

45. Prove that the set of all prime numbers is infinite.

46. A prime pair consists of two primes that differ by two. There are many prime pairs, for example, 11 and 13, 17 and 19. Prime triplets are three numbers $n \geq 2$, $n + 2$, $n + 4$ that are all prime. Show that the only prime triplet is (3, 5, 7).

EXERCISES

1. How many substrings *aab* are in ww^Rw, where $w = aabbab$?
2. Use induction on n to show that $|u^n| = n\,|u|$ for all strings u and all n.
3. The reverse of a string, introduced informally above, can be defined more precisely by the recursive rules

$$a^R = a,$$

$$(wa)^R = aw^R,$$

for all $a \in \Sigma$, $w \in \Sigma^*$. Use this to prove that

$$(uv)^R = v^R u^R,$$

for all $u, v \in \Sigma^+$.

4. Prove that $\left(w^R\right)^R = w$ for all $w \in \Sigma^*$.
5. Let $L = \{ab, aa, baa\}$. Which of the following strings are in L^*: *abaabaaabaa*, *aaaabaaaa*, *baaaaabaaaab*, *baaaaabaa*? Which strings are in L^4?
6. Let $\Sigma = \{a, b\}$ and $L = \{aa, bb\}$. Use set notation to describe L.
7. Let L be any language on a nonempty alphabet. Show that L and L cannot both be finite.
8. Are there languages for which $L^* = \left(L\right)^*$?

9. Prove that

$$(L_1L_2)^R = L_2^R L_1^R$$

for all languages L_1 and L_2.

10. Show that $(L^*)^* = L^*$ for all languages.

11. Prove or disprove the following claims.

(a) $(L_1 \cup L_2)^R = L_1^R \cup L_2^R$ for all languages L_1 and L_2.

(b) $(L^R)^* = (L^*)^R$ for all languages L.

12. Find a grammar for the language $L = \{a^n$, where n is even$\}$.

13. Find a grammar for the language $L = \{a^n$, where n is even and $n > 3\}$.

14. Find grammars for $\Sigma = \{a, b\}$ that generate the sets of

(a) all strings with exactly two a's.

(b) all strings with at least two a's.

(c) all strings with no more than three a's.

(d) all strings with at least three a's.

(e) all strings that start with a and end with b.

(f) all strings with an even number of b's.

In each case, give convincing arguments that the grammar you give does indeed generate the indicated language.

15. Give a simple description of the language generated by the grammar with productions

$$S \to aaA,$$

$$A \to bS,$$

$$S \to \lambda.$$

16. What language does the grammar with these productions generate?

$$S \to Aa,$$

$$A \to B,$$

$$B \to Aa.$$

17. Let $\Sigma = \{a, b\}$. For each of the following languages, find a grammar that generates it.

(a) $L_1 = \{a^n b^m : n \geq 0, m < n\}$.

(b) $L_2 = \{a^{3n} b^{2n} : n \geq 2\}$.

(c) $L_3 = \{a^{n+3} b^n : n \geq 2\}$.

(d) $L_4 = \{a^n b^{n-2} : n \geq 3\}$.

(e) $L_1 L_2$.

(f) $L_1 \cup L_2$.

(g) L_1^3.

(h) L_1^*.

(i) $L_1 - L_4$.

18. Find grammars for the following languages on $\Sigma = \{a\}$.

(a) $L = \{w : |w| \bmod 3 > 0\}$.

(b) $L = \{w : |w| \bmod 3 = 2\}$.

(c) $w = \{|w| \bmod 5 = 0\}$.

19. Find a grammar that generates the language

$$L \left\{ ww^R : w \in \{a, b\}^+ \right\}.$$

Give a complete justification for your answer.

20. Find three strings in the language generated by

$$S \to aSb | bSa | a.$$

21. Complete the arguments in Example 1.14, showing that $L(G_1)$ does in fact generate the given language.

22. Show that the grammar $G = (\{S\}, \{a, b\}, S, P)$, with productions

$$S \to SS \mid SSS \mid aSb \mid bSa \mid \lambda,$$

is equivalent to the grammar in Example 1.13.

23. Show that the grammars

$$S \to aSb|ab|\lambda$$

and

$$S \to aaSbb|aSb|ab|\lambda$$

are equivalent.

24. Show that the grammars

$$S \to aSb|bSa|SS|a$$

and

$$S \to aSb|bSa|a$$

are not equivalent.