$$D(\mu,\sigma)(x) = \frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{+\infty}(x-\mu)^2 e^{-\frac{(x-\mu)^2}{2\sigma^2}}dx$$

let $x = \mu + \sigma t$

$$D(\mu,\sigma)(t) = \frac{\sigma^2}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} t^2 e^{-\frac{t^2}{2}}dt$$

Because of $(\mathrm{uv})' = u'v + uv'$

$$(\mathrm{e}^x)' = e^x$$

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$

So $\left(-\mathrm{t}e^{-\frac{t^2}{2}}\right)' = -e^{-\frac{t^2}{2}} + t^2 e^{-\frac{t^2}{2}}$

$$D(\mu,\sigma)(t) = \frac{\sigma^2}{\sqrt{2\pi}}\left(-te^{-\frac{t^2}{2}}\Big|_{-\infty}^{+\infty} + \int_{-\infty}^{+\infty} e^{-\frac{t^2}{2}}dt\right)$$

According to L 'Hopital rule

$$\lim_{\mathrm{x}\to+\infty}\frac{f(x)}{g(x)} = \lim_{\mathrm{x}\to+\infty}\frac{f'(x)}{g'(x)}$$

$$\mathrm{x}^{yz} = (x^y)^z$$

$$\lim_{t\to+\infty}(-te^{-\frac{t^2}{2}}) = \lim_{t\to+\infty}\left(\frac{-t}{e^{\frac{t^2}{2}}}\right) = \lim_{t\to+\infty}\left(\frac{-1}{te^{\frac{t^2}{2}}}\right) = 0$$

So $\lim_{t\to-\infty}(-te^{-\frac{t^2}{2}}) = -\lim_{t\to+\infty}(-te^{-\frac{t^2}{2}}) = 0$

$$-te^{-\frac{t^2}{2}}\Big|_{-\infty}^{+\infty} = 0$$

$$\int_{-\infty}^{+\infty} e^{-\frac{t^2}{2}} dt \overset{t=\sqrt{2}\mu}{=} \sqrt{2}\int_{-\infty}^{+\infty} e^{-\mu^2} d\mu$$

Let $A = \int_{-\infty}^{+\infty} e^{-x^2} dx = \int_{-\infty}^{+\infty} e^{-y^2} dy$

$$A^2 = \iint_R e^{-\left(x^2+y^2\right)} dxdy$$

转换极坐标 $x^2 + y^2 = r^2$

$$dxdy = rdrd\theta + dr^2 d\theta / 2$$

$$A^2 = \int_0^{2\pi} d\theta \int_0^{+\infty} e^{-r^2} rdr = 2\pi * -\frac{e^{-r^2}}{2} \Big|_0^{+\infty} = \pi$$

$$A = \sqrt{\pi}$$

$$\int_{-\infty}^{+\infty} e^{-\frac{t^2}{2}} dt = \sqrt{2\pi}$$

$$D(\mu,\sigma)(t) = \frac{\sigma^2}{\sqrt{2\pi}}\left(- te^{-\frac{t^2}{2}} \Big|_{-\infty}^{+\infty} + \int_{-\infty}^{+\infty} e^{-\frac{t^2}{2}} dt \right) = \frac{\sigma^2}{\sqrt{2\pi}}(0 + \sqrt{2\pi}) = \sigma^2$$

Got it!