7.1

$$20'000 x_A + 30'000 x_B + 10'000 x_C = 5.2 \cdot 10^6$$
$$10'000 x_A + 17'000 x_B + 6'000 x_C = 3 \cdot 10^6$$
$$2'000 x_A + 3'000 x_B + 2'000 x_C = 0.76 \cdot 10^6$$

a)

$$\left[\begin{array}{ccc|c} 20'000 & 30'000 & 10'000 & 5'200'000 \\ 10'000 & 17'000 & 6'000 & 3'000'000 \\ 2'000 & 3'000 & 2'000 & 760'000 \end{array}\right] \xrightarrow{:1000} \left[\begin{array}{ccc|c} 20 & 30 & 10 & 5'200 \\ 10 & 17 & 6 & 3'000 \\ 2 & 3 & 2 & 760 \end{array}\right]$$

$$\xrightarrow{k=\frac{1}{2}} \left[\begin{array}{ccc|c} 20 & 30 & 10 & 5200 \\ 0 & 2 & 1 & 400 \\ 2 & 3 & 2 & 760 \end{array}\right] \xrightarrow{k=\frac{1}{10}} \left[\begin{array}{ccc|c} 20 & 30 & 10 & 5200 \\ 0 & 2 & 1 & 400 \\ 0 & 0 & 1 & 240 \end{array}\right]$$

① $20 x_A + 30 x_B + 10 x_C = 5200$

② $2 x_B + x_C = 400$

③ $x_C = 240$

③ in ②: $2 x_B + 240 = 400$
$2 x_B = 160$
$x_B = 80$

③ und ② in ①: $20 x_A + 30 \cdot 80 + 10 \cdot 240 = 5200$
$20 x_A + 2400 + 2400 = 5200$
$20 x_A = 400$
$x_A = 20$

b)

$$\begin{array}{ccc} & A & \\ & & L \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 3/2 & 1/2 & \cdot & \cdot & \cdot \\ 10 & 17 & 6 & \cdot & \cdot & \cdot \\ 2 & 3 & 2 & \cdot & \cdot & \cdot \end{array}\right] \Rightarrow \left[\begin{array}{ccc|ccc} 1 & 3/2 & 1/2 & 1 & 0 & 0 \\ 0 & 2 & 1 & 1/2 & 1 & 0 \\ 0 & 0 & 1 & 1/10 & 0 & 1 \end{array}\right]$$

(the bracketed R under the A-part)

c) $Ax = b \Rightarrow LRx = b \Rightarrow Ly = b$ und $Rx = y$

$Ly = b$:
$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 5.72 \\ 1/2 & 1 & 0 & 3.3 \\ 1/10 & 0 & 1 & 0.836 \end{array}\right] \xrightarrow{k=\frac{1}{2}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 5.72 \\ 0 & 1 & 0 & 0.44 \\ 1/10 & 0 & 1 & 0.836 \end{array}\right] \xrightarrow{k=\frac{1}{10}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 5.72 \\ 0 & 1 & 0 & 0.44 \\ 0 & 0 & 1 & 0.264 \end{array}\right]$$

$\Rightarrow y_1 = 5.72, \quad y_2 = 0.44, \quad y_3 = 0.264$

$Rx = y$:
$20 x_A + 30 x_B + 10 x_C = 5720$ ①
$2 x_B + x_C = 440$ ②
$x_C = 264$ ③

③ in ②: $2 x_B + 264'000 = 440'000$
$2 x_B = 176'000$
$x_B = 88$

③ und ② in ①: $20 x_A + 30 \cdot 88'000 + 10 \cdot 264'000 = 5.7...$
$x_A = 22$