8.1

$$A=\begin{pmatrix}1 & -2 & 3\\ -5 & 4 & 1\\ 2 & -1 & 3\end{pmatrix},\quad b=\begin{pmatrix}1\\9\\5\end{pmatrix},\quad Q=I=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$$

a) $a_1=\begin{pmatrix}1\\-5\\2\end{pmatrix}$, $v_1=a_1+\operatorname{sign}(a_{11})\cdot|a_1|\cdot e_1=\begin{pmatrix}1\\-5\\2\end{pmatrix}+1\cdot\sqrt{30}\cdot\begin{pmatrix}1\\0\\0\end{pmatrix}=\begin{pmatrix}1+\sqrt{30}\\-5\\2\end{pmatrix}$

$$u_1=\frac{1}{|v_1|}\cdot v_1=\frac{1}{\sqrt{(1+\sqrt{30})^2+(-5)^2+2^2}}\cdot\begin{pmatrix}1+\sqrt{30}\\-5\\2\end{pmatrix}=\begin{pmatrix}\frac{1+\sqrt{30}}{\sqrt{2\cdot\sqrt{30}+60}}\\ \frac{-5}{\sqrt{2\cdot\sqrt{30}+60}}\\ \frac{2}{\sqrt{2\cdot\sqrt{30}+60}}\end{pmatrix}$$

$$H_1=I-2\cdot u_1\cdot u_1^T$$

$$=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}-\left(2\cdot\begin{pmatrix}0.59&-0.46&0.18\\-0.46&0.35&-0.14\\0.18&-0.14&0.06\end{pmatrix}\right)$$

$$=\begin{pmatrix}-0.18&0.91&-0.37\\0.91&0.30&0.28\\-0.37&0.28&0.89\end{pmatrix}=Q_1$$

$$A_1=Q_1\cdot A=\begin{pmatrix}-5.48&4.38&-0.73\\0&-0.93&3.88\\0&0.97&1.85\end{pmatrix}\Rightarrow\begin{pmatrix}-0.93&3.88\\0.97&1.85\end{pmatrix}$$

$$\Rightarrow a_2=\begin{pmatrix}-0.93\\0.97\end{pmatrix},\quad v_2=\begin{pmatrix}-0.93-\sqrt{1.8058}\\0.97\end{pmatrix},\quad u_2=\begin{pmatrix}\frac{-0.97-\sqrt{1.8058}}{\sqrt{(-0.93-\sqrt{1.8058})^2+0.97^2}}\\ \frac{0.97}{\sqrt{(0.93-\sqrt{1.8058})^2+0.97^2}}\end{pmatrix}$$

$$H_2=\begin{pmatrix}-0.69&0.72\\0.72&0.69\end{pmatrix}\Rightarrow Q_2=\begin{pmatrix}1&0&0\\0&-0.69&0.72\\0&0.72&0.69\end{pmatrix},\quad A_2=\underline{\underline{\begin{pmatrix}-5.48&4.38&-0.73\\0&1.34&-1.34\\0&0&4.08\end{pmatrix}}}$$

$$Q=Q_1\cdot Q_2^T=\underline{\underline{\begin{pmatrix}-0.18&-0.89&0.41\\0.91&0&0.41\\-0.37&0.45&0.82\end{pmatrix}}}$$

b) $\vec{y} = Q^T \cdot \vec{b} \Rightarrow \begin{pmatrix} -0.18 & 0.91 & -0.37 \\ -0.83 & 0 & 0.45 \\ 0.41 & 0.41 & 0.82 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 9 \\ 5 \end{pmatrix} = \begin{pmatrix} 6.21 \\ 1.34 \\ 8.16 \end{pmatrix}$

$R \cdot \vec{x} = \vec{y} \Rightarrow \quad 4.08 x_3 = 8.16 \Rightarrow x_3 = \frac{8.16}{4.08} = 2$

$1.34 x_2 - 1.34 x_3 = 1.34 \Rightarrow x_2 = \frac{1.34 + 2 \cdot 1.34}{1.34} = 3$

$-5.48 x_1 + 4.38 x_2 - 0.73 x_3 = 6.21 \Rightarrow x_1 = \frac{6.21 + 0.73 \cdot 2 - 4.38 \cdot (-1)}{-5.48} = 1$

$\Rightarrow \vec{x} \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$