Serie 11_Aufg 2

$\frac{dy}{dx} = \frac{x^2}{y}$, $y'(x) = \frac{x^2}{y(x)}$, Intervall: $0 \le x \le 1{,}4$, Startwert: $y(0) = 2$

a) $h = 0{,}7$, $x_0 = 0$, $y_0 = 2$

1. Schritt

$x_1 = x_0 + h = 0 + 0{,}7 = 0{,}7$

$y_1 = y_0 + h \cdot f(x_0, y_0) = 2 + 0{,}7 \cdot \frac{0^2}{2} = 2$

2. Schritt

$x_2 = x_1 + h = 0{,}7 + 0{,}7 = 1{,}4$

$y_2 = y_1 + h \cdot f(x_1, y_1) = 2 + 0{,}7 \cdot \frac{0{,}7^2}{2} = 2{,}1715$

b) $h = 0{,}7$, $x_0 = 0$, $y_0 = 2$

1. Schritt

$x_{h/2} = x_0 + \frac{h}{2} = 0 + \frac{0{,}7}{2} = 0.35$, $y_{h/2} = y_0 + \frac{h}{2} \cdot f(x_0, y_0) = 2 + \frac{0{,}7}{2} \cdot \frac{0^2}{2} = 2$

$x_1 = x_0 + h = 0 + 0{,}7 = 0{,}7$

$y_1 = y_0 + h \cdot f(x_{h/2}, y_{h/2}) = 2 + 0.7 \cdot \frac{0.35^2}{2} = 2.04$

2. Schritt

$x_{h/2} = x_1 + \frac{h}{2} = 0{,}7 + \frac{0{,}7}{2} = 1.05$, $y_{h/2} = y_1 + \frac{h}{2} \cdot f(x_1, y_1) = 2.04 + \frac{0.7}{2} \cdot \frac{0.7^2}{2.04} = 2.12$

$x_2 = x_1 + h = 1.4$, $y_2 = y_1 + h \cdot f(x_{h/2}, y_{h/2}) = 2.04 + 0.7 \cdot \frac{1.05^2}{2.12} = 2.4$

c) $h = 0{,}7$, $x_0 = 0$, $y_0 = 2$

1. Schritt

$x_1 = x_0 + h = 0{,}7$, $y_1^{Euler} = 2$, $k_1 = f(x_0, y_0) = \frac{0^2}{2} = 0$, $k_2 = f(x_1, y_1^{Euler}) = \frac{0{,}7^2}{2} = 0.245$

$y_1 = y_0 + h \cdot \frac{k_1 + k_2}{2} = 2 + 0{,}7 \cdot \frac{0 + 0.245}{2} = 2.08$

2. Schritt

$x_2 = 1{,}4$, $y_2^{Euler} = 2{,}08 + 0{,}7 \cdot \frac{0{,}7^2}{2{,}08} = 2{,}24$, $k_1 = f(x_1, y_1) = \frac{0{,}7^2}{2{,}08} = 0{,}24$, $k_2 = f(x_2, y_2^{Euler}) = 0.875$

$y_2 = 2.08 + 0.7 \cdot \frac{0.24 + 0.875}{2} = 2{,}47$

Fehler:

$y(0.7) \approx 2,216904749$

$y(1,4) \approx 3,364124453$

Verfahren	Absoluter Fehler $x_i = 0,7$	$x_i = 1,4$
Euler	0.216904749	1.192624453
Mittelpunkt	0.216904749	0.964124453
Mod. Euler	0.136904749	0.894124453