Aufg. 2

$h_0 = \frac{\pi}{2^0}$, $h_1 = \frac{\pi}{2^1}$, $h_2 = \frac{\pi}{2^2}$, $h_3 = \frac{\pi}{2^3}$, $h_4 = \frac{\pi}{2^4}$

0. Extrapolation

$$T_{00} = h_0 \cdot \left(\frac{f(a)+f(b)}{2} + \sum_{i=1}^{0} f(x_i) \right) = \pi \cdot \left(\frac{\cos(0^2)+\cos(\pi^2)}{2} \right) = 0.153$$

$$T_{10} = h_1 \cdot \left(\frac{f(a)+f(b)}{2} + \sum_{i=1}^{1} f(x_i) \right) = \frac{\pi}{2^1} \cdot \left(\frac{\cos(0^2)+\cos(\pi^2)}{2} + \cos\left(\left(0+1\cdot\frac{\pi}{2^1}\right)^2\right) \right) = -1.151$$

$$T_{20} = h_2 \cdot \left(\frac{f(a)+f(b)}{2} + \sum_{i=1}^{3} f(x_i) \right) = \frac{\pi}{2^2} \cdot \frac{\cos(0^2)+\cos(\pi^2)}{2} + \cos\left(\left(0+1\cdot\frac{\pi}{2^2}\right)^2\right) + \ldots + \cos\left(\left(0+3\cdot\frac{\pi}{2^2}\right)^2\right)$$

$$= 0.650$$

$$T_{30} = h_3 \cdot \left(\frac{f(a)+f(b)}{2} + \sum_{i=1}^{7} f(x_i) \right) = 0.603$$

$$T_{40} = h_4 \cdot \left(\frac{f(a)+f(b)}{2} + \sum_{i=1}^{15} f(x_i) \right) = 0.575$$

1. Extrapolation:

$$T_{01} = \frac{4T_{10} - T_{00}}{3} = \frac{4\cdot(-1.151) - 0.153}{3} = -1.586$$

$$T_{11} = \frac{4T_{20} - T_{10}}{3} = 1.250, \quad T_{21} = 0.587, \quad T_{31} = 0.565$$

2. Extrapolation

$$T_{02} = \frac{16T_{11} - T_{01}}{15} = 1.439, \quad T_{12} = \frac{16T_{21} - T_{11}}{15} = 0.543, \quad T_{22} = 0.564$$

3. Extrapolation:

$$T_{03} = 0.528, \quad T_{13} = 0.564$$

4. Extrapolation:

$$T_{04} = 0.564$$