

1.

$$f(x_1, x_2) = \begin{pmatrix} 20 - 18x_1 - 2x_2^2 \\ -4x_2 \cdot (x_1 - x_2^2) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Jacobi-Matrix: $Df(x^{(n)}) = \begin{pmatrix} -18 & -4x_2 \\ -4x_2 & -4x_1 + 12x_2^2 \end{pmatrix}$ (Mit Python berechnet)

Startpunkt: $x^{(0)} = \begin{pmatrix} 1.1 \\ 0.9 \end{pmatrix}$

Schritt 1: $Df(x^{(0)}) \delta^{(0)} = -f(x^{(0)})$

$$\Rightarrow \begin{pmatrix} -18 & -18/5 \\ -18/5 & 133/25 \end{pmatrix} \cdot \delta^{(0)} = \begin{pmatrix} 71/50 \\ 261/250 \end{pmatrix}$$

$$\Rightarrow \delta^{(0)} = \begin{pmatrix} -13/125 \\ 63/500 \end{pmatrix}$$

$$x^{(1)} = x^{(0)} + \delta^{(0)} = \begin{pmatrix} 11/10 \\ 9/10 \end{pmatrix} + \begin{pmatrix} -13/125 \\ 63/500 \end{pmatrix} = \begin{pmatrix} 249/250 \\ 513/500 \end{pmatrix}$$

$$\|f(x^{(0)})\|_2 = 881/500, \quad \|x^{(1)} - x^{(0)}\|_2 = 1267/1000$$

Schritt 2: $Df(x^{(1)}) \cdot \delta^{(1)} = -f(x^{(1)})$

$$\Rightarrow \begin{pmatrix} -18 & -41/10 \\ -41/10 & 216/25 \end{pmatrix} \cdot \delta^{(1)} = \begin{pmatrix} 3/100 \\ -23/100 \end{pmatrix}$$

$$\Rightarrow \delta^{(1)} = \begin{pmatrix} 1/250 \\ -1/50 \end{pmatrix}$$

$$x^{(2)} = x^{(1)} + \delta^{(1)} = \begin{pmatrix} 249/250 \\ 513/500 \end{pmatrix} + \begin{pmatrix} 1/250 \\ -1/50 \end{pmatrix} = \begin{pmatrix} 1 \\ 503/500 \end{pmatrix}$$

$$\|f(x^{(1)})\|_2 = 233/1000, \quad \|x^{(2)} - x^{(1)}\|_2 = 0.0252 = 252/10'000$$